

COSC-373: HOMEWORK 1

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Question 1. (a) Suppose the 3 students are $\{A, B, C\}$ and the 3 internships are $\{a, b, c\}$. The preferences for the students are:

- A : a, b, c
- B : a, b, c
- C : c, a, c

The preferences for the internships are:

- a : C, A, B
- b : A, B, C
- c : B, C, A

The 5 rounds proceed as follows:

- (1) A applies to a , B applies to a , and C applies to c . a defers A , a rejects B , and c defers C .
- (2) B applies to b . b defers B and b rejects C .
- (3) C applies to a . a defers C and a rejects A .
- (4) A applies to b . b defers A and b rejects B .
- (5) B applies to c . c defers B .

The stable matchings found by the algorithm are thus $\{(A, b), (B, c), (C, a)\}$.

Question 2. (a) The stable matching found by the Gale-Shapley algorithm for such instances are $\{(s_1, u_1), (s_2, u_2), \dots, (s_n, u_n)\}$. n rounds are required until the algorithm terminates.

(b) TODO

Question 3. (a) The centralized greedy algorithm executes as follows.

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1  $M \leftarrow \emptyset$ 
2  $V_M \leftarrow \emptyset$  // set of vertices that are matched in  $M$ 
3 foreach  $v \in V$  do
4   foreach  $u \in \text{neighbors}(v)$  do
5     if  $v, u \notin V_M$  then
6        $M \leftarrow M \cup \{(v, u)\}$ 
7        $V_M \leftarrow V_M \cup \{v, u\}$ 
8 return  $M$ 
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(b) Since we are iterating over the neighbors for each vertex, we are thus iterating over the set of edges in G . Therefore, the runtime of our procedure is $O(m)$.

(c) Let $G = (V, E)$ where $V = \{a, b, c, d\}$ and $E = \{(a, b), (b, c), (c, d)\}$. Two maximal matchings are $M_1 = \{(a, b), (c, d)\}$ and $M_2 = \{(b, c)\}$. Observe that $|M_1| = 2|M_2|$.

Question 4. Suppose for the sake of contradiction that $\text{dist}(u, w) > \text{dist}(u, v) + \text{dist}(v, w)$. That is, the length of the shortest path from u to v plus the length of the shortest path from v to w is less than the length of the shortest path from u to w . The assumption thus implies that there exists a path from u to w that is shorter than the shortest path from u to w . Clearly, this is a contradiction. Therefore, dist satisfies the triangle inequality.