COSC-373: HOMEWORK 2

LEE JIAEN, HYERY YOO, AND ALEXANDER LEE

Question 1. Let $G_1 = (\{v_1, u_1, u_2, u_3, u_4, u_5\}, \{(v_1, u_1), (v_1, u_2), (u_1, u_3), (u_3, u_4), (u_4, u_5)\})$ and $G_2 = (\{v_2, u_1, u_2, u_3, u_4, u_5\}, \{(v_2, u_1), (v_2, u_2), (u_1, u_3), (u_2, u_3), (u_3, u_4), (u_4, u_5)\})$. Both have diameter $D = 4$. The MCM of G_1 is $\{(v_1, u_2), (u_1, u_3), (u_4, u_5)\}$, and a MCM of G_2 is $\{(v_2, u_1), (u_2, u_3), (u_4, u_5)\}$. Note that v_1 and v_2 have different partners, so G_1 and G_2 have different solutions to MCM. The distance $D/2 - 1 = 1$ neighborhoods of v_1 and v_2 are identical: $\Gamma_1(v_1) = \Gamma_1(v_2) = (\{u_1, u_2\}, \{(v_{1or2}, u_1), (v_{1or2}, u_2)\})$. G_1 and G_2 are indistinguishable in $D/2 - 1$ rounds, so at least $D/2$ rounds are required to solve MCM.

Question 2. (a) Each node does the following:

$V \leftarrow \{my\_id\}$
$E = \emptyset$
foreach $i \in \{0, 1, \ldots, d\}$ **do**
  if $i = 0$ **then**
    send my_id, V, E
  if *received messages* **then**
    foreach $m \in messages$ **do**
      $E \leftarrow E \cup m.E \cup \{my\_id, m.my\_id\}$
      $V \leftarrow V \cup m.V$
  send my_id, V, E
$\Gamma_d(v) \leftarrow (V, E)$
return $\Gamma_d(v)$

(b) Each node does the following:

$V = \{my\_id\}$
$E = \emptyset$
foreach *round* i **do**
  if $i = 0$ **then**
    send my_id, V, E
  if *received messages* **then**
    foreach $m \in messages$ **do**
      $E \leftarrow E \cup m.E \cup \{my\_id, m.my\_id\}$
      $V \leftarrow V \cup m.V$
  if V *is unchanged* **then**
    halt
  send my_id, V, E
$G \leftarrow (V, E)$
return G

(c) By locality lemma, in $D + 1$ rounds, the state of each node is determined by the node's D neighborhood. D is the diameter of the graph, so the D neighborhood of any node is the entire graph G. Every node knows about the entire graph in $D + 1$ rounds and terminates in one additional round, so any graph problem can be solved in $D + 2$ rounds total.

(d) $O(nlogn + mlogn) = O((n + m)logn)$

Question 3. (a) Let $G_1 = (V = \{v_1, u_1, u_2\}, E = \{(v_1, u_1), (u_1, u_2)\})$ and $G_2 = (V = \{v_2, u_1, u_2, u_3\}, E = \{(v_1, u_1), (u_1, u_2), (u_1, u_3)\})$.
G_1 and G_2 have different sizes (3 and 4, respectively). The diameter D of both graphs is 2. At round $D-1 = 1$, v_1 and v_2 know their 1 neighborhood, which identically consist of node u_1 and an edge to node u_1. The two graphs are indistinguishable in $D-1$ rounds. SIZE cannot be solved in fewer than D rounds.

(b) Modify the leader election algorithm as the following:

```
cur_leader ← my_id
parent ← ⊥
children ← ∅
done ← false
size = 1
updated ← true
for round  1 do
    send message 'my_id'
foreach round  r > 1 do
    if received message 'leader_id' with leader_id < cur_leader then
        cur_leader ← leader_id
        parent ← neighbor that sent leader_id
        children ← ∅
        send parent message 'parent'
        send others message 'leader_id'
        updated ← true
        done ← false
    else
        if updated then
            if received 'parent' from u then
                add u to children
            if children = ∅ then
                done ← true
                send parent message (done, size)
            updated ← false
        if not done and received (done, branch_size) from all children then
            done ← true
            size ← size+ sum of branch_sizes from all children
            if my_id = cur_leader then
                output size
            else
                send message (done, size) to parent
```