The Bootstrap method

About

This note was written in response to a forum post by Ian K

> If I could code the following, this is what I would do....
>
> - Generate a normal distribution of data and plot it
> - Take repeated samples from that data and plot the distributions of their means
> - Then if you overlayed the plots this would show a normal distribution within a normal distribution. I think that too would be a vizualisation of the CLT
>
> Reference: Python for Data Analysis

The Bootstrap method

According to Wikipedia, the bootstrap method for the sampling distribution of the mean is....

> Consider a coin-flipping experiment. We flip the coin and record whether it lands heads or tails. Let $X = x_1, x_2, \ldots, x_{10}$ be 10 observations from the experiment. $x_i = 1$ if the i^{th} flip lands heads, and 0 otherwise. From normal theory, we can use t-statistic to estimate the distribution of the sample mean,
>
> $$\overline{x} = \frac{1}{10}(x_1 + x_2 + \cdots + x_{10}).$$
>
> Instead, we use bootstrap, specifically case resampling, to derive the distribution of $\overline{x}$.
>
> We first resample the data to obtain a bootstrap resample $\ldots$ (so) the number of data points in a bootstrap resample is equal to the number of data points in our original observations.
>
> Then we compute the mean of this resample and obtain the first bootstrap mean: μ_1^*. We repeat this process to obtain the second resample X_2^* and compute the second bootstrap mean μ.
>
> If we repeat this 100 times, then we have $\mu_1^*, \mu_2^*, \ldots, \mu_{100}^*$. This represents an empirical bootstrap distribution of sample mean. From this empirical distribution, one can derive a **bootstrap confidence interval** for the purpose of hypothesis testing.
>
> Reference: Bootstrapping (statistics), Wikipedia.

My original notebook was not a bootstrap method, as the sampling was done **without replacement**.[1]

```
from util.bootstrap import bootstrap
from util.clt import clt
from scipy.stats import norm, uniform
```

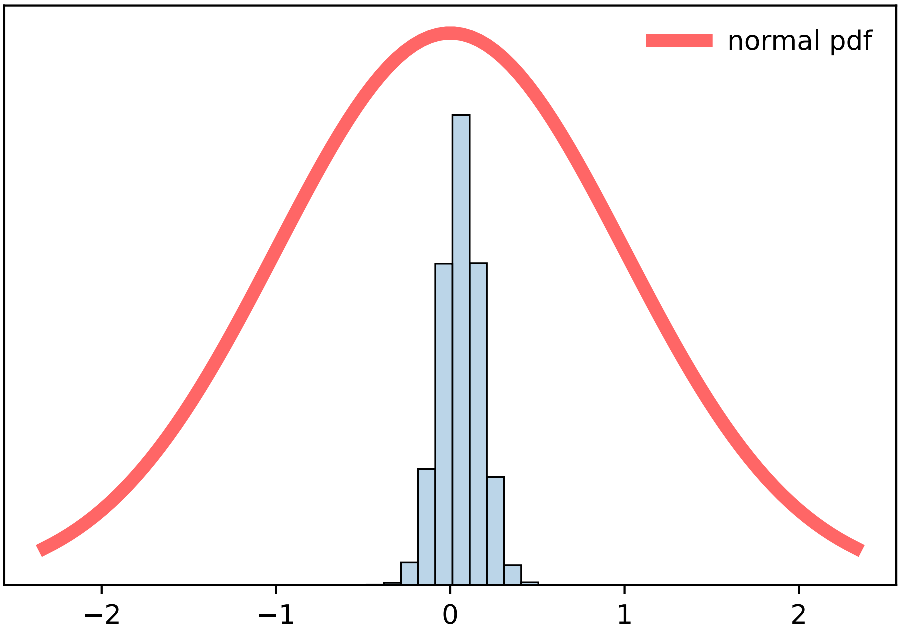
Using the bootstrap

Algorithm

I have translated the brief outline above into the following algorithm. You can see the implemented script here

```
BEGIN
  generate a_sample of size n from a_dist
  repeating n*n times
    declare a_resample as an empty list
    repeating n times
      randomly select an observation
      append observation to a_resample
    calculate a_mean of a_resample
    append a_mean to means
  plot means as a histogram
END
```

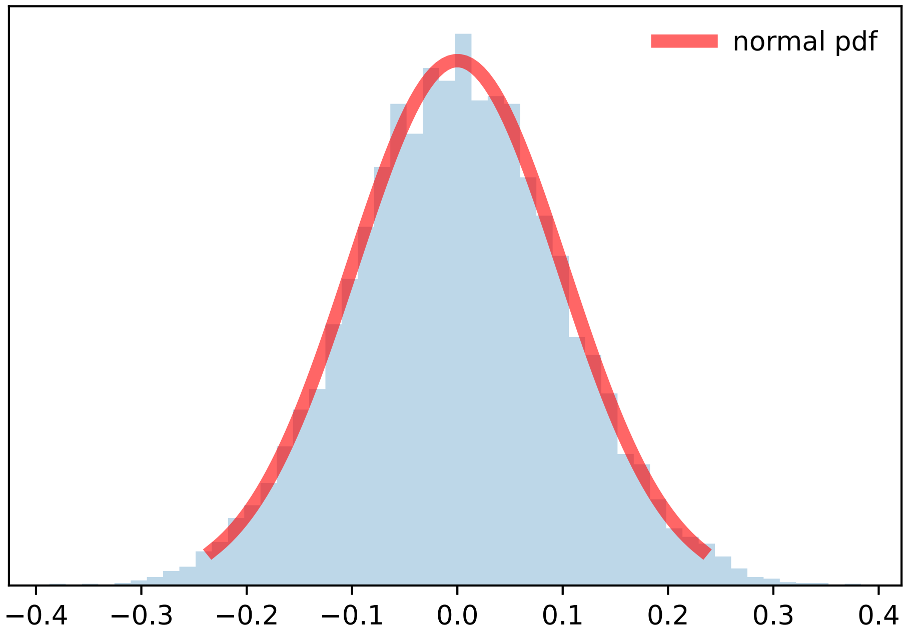
```
bootstrap(a_dist=norm(), n=100)
```



Using the CLT

Let us compare the above plot to the one generated by `clt()` script.

```
clt(a_dist=norm(), n=100, N=10000, bins=50)
```



Notes

I don't know enough about Bootstrapping to comment, but I will say it is a rather peculiar plot, and does not look much like the distribution generated from the CLT.

However, it is **very** good estimate of the mean, with a single mode at the mean of the distribution and low variance. I think I can see why it is popular, as this was done with just a single sample.

1. In fact, a new sample (from the same distribution) was generated each time. ↩