# Method 1

The definition of the c.d.f. of a continuous distribution[1] is

$$F(t) = \int_L^t f(x)\,dx, \quad t \in (L, U).$$

Now, an exponential distribution has range $x > 0$[2] and p.d.f.[3] $f(x) = \lambda e^{-\lambda x}$. Therefore the c.d.f. of an exponential distribiution will be

$$\begin{aligned} F(t) = \int_L^t f(x)\,dx &= \int_0^t \lambda e^{-\lambda x}\,dx \\ &= \left[\frac{\lambda}{-\lambda} e^{-\lambda x}\right]_0^t \\ &= -(e^{-\lambda t} - e^0) \\ &= -(e^{-\lambda t} - 1) \\ &= 1 - e^{-\lambda t}. \end{aligned}$$

# Method 2

We can also reach this result using the **Poisson distribution**. The probability that the time $T$ between two events $X$ is less than $t$ is equivalent to the probability that at least one event happens in time $t$. Given that $X$ is a Poisson random variable with distrubtion $X \sim \text{Poisson}(\lambda t)$[3:1] then

$$P(T > t) \equiv P(X > 0).$$

The probability $P(X > 0) = 1 - P(X \leq 0) = P(X = 0)$, given that the range of $X$ is $\{0, 1, 2, \ldots\}$,[2:1] so

$$1 - P(X \leq 0) = 1 - P(X = 0),$$

where

$$P(X = 0) = e^{-\lambda t}\left(\frac{(\lambda t)^0}{0!}\right) = e^{-\lambda t}.$$

And so finally

$$P(T > t) \equiv 1 - P(X = 0) = 1 - e^{-\lambda t}.$$

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1. HB pp 7 ↩
2. HB pp 26-27 ↩ ↩
3. HB pp 10 ↩ ↩