This note was made in response to a follow-up post on the M248 forum: Covid19 tests and statistics

Statement of Bayes' Theorem

> Bayes' theorem is stated mathematically as the following equation:
>
> $$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)},$$
>
> where A and B are events and $P(B) \neq 0$.
>
> - $P(A \mid B)$ is a conditional probability: the likelihood of event A occurring given that B is true.
> - $P(B \mid A)$ is also a conditional probability: the likelihood of event B occurring given that A is true.
> - $P(A)$ and $P(B)$ are the probabilities of observing A and B respectively; they are known as the marginal probability.
>
> A and B must be different events.

Reference: *Bayes' theorem (Wikipedia)*.

Stating the question and clarifying the terms

The question was stated as follows:

> *So, if I wanted to know the probability of a person being positive for covid given that they had a negative LFT test (...)*

This is $P(\text{Has C19} \mid \text{Negative result})$. Mapping the terms to Bayes' Theorem states as stated above, let

- Has C19 $= C+ \to A$;
- Negative test $= -ve \to B$.

So therefore the probability $P(\text{Has C19} \mid \text{Neagtive result})$, is

$$P(C+ \mid -ve) = \frac{P(-ve \mid C+)\, P(C+)}{P(-ve)}$$

Sourcing the data

The probability $P(-ve \mid \text{Not } C+)$ is the **specificity** of the LFT,[1] which is approximately 99.6%,[2] then[3]

$$P(-ve \mid C+) = 100 - P(-ve \mid \text{Not } C+) \simeq 0.4\%.$$

It has been estimated[4] that the percentage of the population in England with Covid-19, $P(C+)$, in the week 12 to 18 December 2020 was approximately 1.17%. This means that the population in England without Covid-19, $P(\text{Not } C+)$, was approximately 98.83%

The probability[5] $P(-ve)$ is given by

$$\begin{aligned} P(-ve) &= P(-ve \mid C+)\, P(C+) + P(-ve \mid \text{Not } C+)\, P(\text{Not } C+) \\ &\simeq 0.004(0.0117) + 0.996(0.983) \\ &\simeq 0.979. \end{aligned}$$

Calculating the probability

Now that we have the needed figures, let us calculate the probability that a person is positive for Covid-19, given they return a negative LFT,

$$P(C+\mid -ve) = \frac{P(-ve\mid C+)\,P(C+)}{P(-ve)} \simeq \frac{0.004(0.017)}{0.979}$$
$$= 0.000069\ldots.$$

Hence, the probability of a person being positive for covid given that they had a negative LFT test is approximately 0.000069.

This is not a surprising result, given the LFT is highly specific. It also points to a reason behind why much of the focus is on the sensitivity of the LFT, as opposed to specificity.

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1. This is the true negative rate. ↩
2. "*He said that, while 0.4% (400 in 100 000) was a very low rate, with a sensitivity of 58% and specificity of 99.6%*" Covid-19: Lateral flow tests miss over half of cases, Liverpool pilot data show. BMJ ↩
3. This is because they are **mutually exlusive**: A person who returns a negative test can either have Covid-19 or not. ↩
4. "*The number of people with the coronavirus (COVID-19) in England has increased, with 645,800 people estimated to have had COVID-19 in the most recent week (12 to 18 December 2020). This equates to 1 in 85 people.*" Coronavirus (COVID-19) roundup. ONS ↩
5. "*The* $P(\text{Positive}) = \ldots$ *is a direct application of the* ***Law of total probability.***" Bayes' theorem. Wikipedia ↩