Statement of Bayes' Theorem

> Bayes' theorem is stated mathematically as the following equation:
>
> $$P(A \mid B) = \frac{P(B \mid A) P(A)}{P(B)},$$
>
> where A and B are events and $P(B) \neq 0$.
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> - $P(A \mid B)$ is a conditional probability: the likelihood of event A occurring given that B is true.
> - $P(B \mid A)$ is also a conditional probability: the likelihood of event B occurring given that A is true.
> - $P(A)$ and $P(B)$ are the probabilities of observing A and B respectively; they are known as the marginal probability.
>
> A and B must be different events.

Reference: *Bayes' theorem (Wikipedia)*.

Stating the question and clarifying the terms

What is the probability that a person is positive for Covid-19, given they return a positive LFT?

This is $P(\text{Has C19} \mid \text{Positive result})$. Mapping the terms to Bayes' Theorem states as stated above, let

- Has C19 $= C+ \to A$;
- Positive test $= +ve \to B$.

So therefore the probability $P(\text{Has C19} \mid \text{Positive result})$, otherwise known as the **Positive Predictive Power**,[1] is

$$P(C+ \mid +ve) = \frac{P(+ve \mid C+)\, P(C+)}{P(+ve)}$$

Sourcing the data

The $P(+ve \mid C+)$ is the **sensitivity** of the LFT,[2] which is approximately 76.8%.[3]

It has been estimated[4] that the percentage of the population in England with Covid-19, $P(C+)$, in the week 12 to 18 December 2020 was approximately 1.17%. This means that the population in England without Covid-19, $P(\text{Not } C+)$, was approximately 98.83%

The probability $P(+ve) = P(+ve \mid C+)\, P(C+) + P(+ve \mid \text{Not } C+)\, P(\text{Not } C+)$.[5] The probability $P(+ve \mid \text{Not } C+)$ is the **false positive rate** of the LFT, which was found to be approximately 0.38%.[6]

Calculating the probability

Now that we have the needed figures, let us calculate the probability that a person is positive for Covid-19, and returns a positive LFT,

$$\begin{aligned} P(C+ \mid +ve) = \frac{P(+ve \mid C+)\, P(C+)}{P(+ve)} &= \frac{P(+ve \mid C+)\, P(C+)}{P(+ve \mid C+)\, P(C+) + P(+ve \mid \text{Not } C+)\, P(\text{Not } C+)} \\ &= \frac{0.768(0.0117)}{0.768(0.0117) + 0.0038(0.9883)} \\ &= 0.70524\ldots \simeq 0.705. \end{aligned}$$

Hence, the positive predictive power of the LFT in the week 12-18 December 2020 was approximately 70.5%.

Caveat on the sensitivity

The sensitivity of the test was stated as 76.8%. This is the *overall* sensitivity of the test. However, the sensitivity dropped to 58% when used by "*self-trained staff at a Boots track-and-trace centre*".[6:1]

Using this other figure for the sensitivity would mean the PPV of the LFT would fall to approximately 64.4% in the week quoted!

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1. *The Positive predictive value (PPV) of a test is the proportion of persons who are actually positive out of all those testing positive.* Bayes' theorem. Wikipedia. ↩
2. This is the true positive rate ↩
3. "*The findings contrast with an earlier assessment of the Innova test by Public Health England's Porton Down laboratory and the University of Oxford, which found an overall sensitivity of 76.8%.*" Covid-19: Lateral flow tests miss over half of cases, Liverpool pilot data show. BMJ ↩
4. "*The number of people with the coronavirus (COVID-19) in England has increased, with 645,800 people estimated to have had COVID-19 in the most recent week (12 to 18 December 2020). This equates to 1 in 85 people.*" Coronavirus (COVID-19) roundup. ONS ↩
5. "*The* $P(\text{Positive}) = \dots$ *is a direct application of the* ***Law of total probability.***" Bayes' theorem. Wikipedia ↩
6. "*The report said that the test's sensitivity was 58% when used by the public and that the false positive rate was 0.38%*" Covid-19: Innova lateral flow test is not fit for "test and release" strategy, say experts. BMJ ↩ ↩