Abstract— Index Terms—

#### NOMENCLATURE

#### **Indices**

a time period (time interval from the beginning through the end of a period)

TS network generator index g

DS network bus number index

#### **Parameters**

Tlength of the planning horizon

Gtotal number of generators in the TS

 $\overline{R}_a$ TS generator upper ramp rate limit

 $\frac{\underline{R}_g}{\overline{P}_g}$   $\underline{P}_g$  LineTS generator lower ramp rate limit TS generator generation upper bound

TS generator generation lower bound

TS line limit

TS generater g cost

TS generater g reserve cost

TS generator g commitment cost

GSFTS generation shift factor matrix

 $W^f$ forecasted wind power

 $W^{up}$ positive wind deviation from the forecast

 $W^{dn}$ negative wind deviation from the forecast

 $N_{b}$ number of buses in the TS

 $N_b^d$ number of buses in the DS

TS load power vector in hour t

DS dispatchable load lower bound at bus i

DS dispatchable load upper bound at bus i

DS inelastic load in time t at bus i

DS power flow to bus i upper bound

 $\begin{array}{c} L_t \\ \underline{L}_i^d \\ \overline{L}_i^d \\ \underline{P}_i \\ \underline{P}_i^{d1} \\ C_i^{d2} \\ C^d \end{array}$ DS power flow to bus i lower bound

DS DG generation linear cost at bus i

DS DG generation quadratic cost at bus i

DS utility for consuming dispatchable load

 $C^{dr1}$ DS linear demand response cost

 $C^{dr2}$ DS quadratic demand response cost

 $\overline{P}_{i}^{d} \\
\underline{P}_{i}^{d}$ DS DG generation upper bound at bus i

DS DG generation lower bound at bus i

 $\overline{S}^{i}(dr)$ DR scale factor

## Variables

TS generator g upward reserve in hour t

TS generator g downward reserve in hour t

demand response price in hour t

 $p_{g,t}$ TS generator g generation in hour t

DS imported power in hour t

DS dispatchable load in hour t at bus i

DS downward DR in hour t at bus i

DS DG power in hour t at bus i

DS power flow to bus i in hour t

 $P_{i,t} \\ dr_t^{up}$ DS upward DR in hour t at bus i DS energy import price in hour t

 $p_t^{inj}$ bus power injection vector in hour t

TS generator commitment variable in hour t for gen $w_{g,t}$ erator g

## Acronyms

TS transmission system

DL dispatchable load

WP wind penetration

DS distribution system

DR demand response

DG distributed generation

#### I. PROBLEM DESCRIPTION

## A. Transmission System Problem

The TS has a network of transmission lines and buses. Traditional and renewable generation units, loads and DS are connected to different buses in the network. The TS solves a day ahead market unit commitment problem, which is a co-optimization of the energy market and ancillary service market. For the energy market optimization, the TS tries to minimize the cost of meeting the system demand with its own generation or energy from DS. For the ancillary service market optimization, the TS minimizes the cost of providing enough reserve to account for the renewable forecast uncertainty. The reserve service could either come from the TS generator's reserve or the DS's DR. The TS objective function minimizes the energy and ancillary service cost at the same time.

## B. Distribution System Problem

The distribution system has dispatchable load and nondispatchable load, and distributed generation in a radial network. In the day-ahead market, the DS solves an optimal dispatch problem with power flow. The dispatchable load is optimized at some point in between its upper and lower bound. The difference between the upper/lower bound and its set point could be used to provide upward/downward DR. The objective of the DS is to minimize the cost of meeting its demand either by its distributed generation or importing power from the TS and maximize the revenue of providing DR

# C. Co-operation Mode

Under the bilevel co-optimization framework, the TS decides the price of DS energy import as well as the price for purchasing DS DR, the DS responds to those prices by exchanging a certain amount of energy with the TS and selling a certain amount of DR to the TS.

The co-optimization between the TS and DS is illustrated in Fig. 1

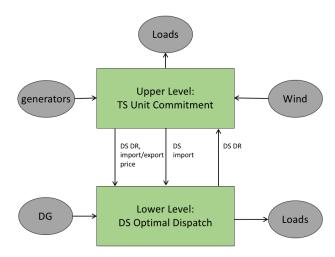


Fig. 1: Co-optimization between the TS and MG

#### D. Renewable Forecast Uncertainty Management

In this work, we use a robust approach to manage the uncertainty in renewable forecast. Specifically, for a renewable generation forecast and a set of possible generation scenarios, we calculate the upward/downward forecast deviation by taking the difference of the hourly maximum/minimum generation scenario and the hourly forecast. The downward/upward TS generation reserve and upward/downward MG DR are used to account for the upward/downward renewable forecast deviation.

# II. MODEL FORMULATION

In this section, the structure of the bi-level optimization problem is described. The general formulation of a bilevel optimization is given below:

$$\begin{aligned} & \min_{x \in X, y \in Y} F\left(x, y\right) \\ & \text{st: } G_i\left(x, y\right) \leq 0, \text{for } i \in \{1, 2, ..., I\} \\ & H_k\left(x, y\right) = 0, \text{for } k \in \{1, 2, ..., K\} \\ & y \in \underset{y \in Y}{\operatorname{argmin}} \{f(x, y) : g_j(x, y) \leq 0, \text{for } j \in \{1, 2, ..., J\}, \\ & h_m(x, y) = 0, \text{for } m \in \{1, 2, ..., M\} \} \end{aligned}$$

In the above formulation,  $x, F(x,y), (G_i, H_k)$  are the optimization variables, objective function and constraints of the upper level problem. Whereas,  $y, f(x,y), (g_i, h_m)$  are the optimization variables, objective function and constraints of the lower level problem. According to this formulation, the bilevel formulation of the TS and DS co-optimization is given in the following sections.

#### A. Upper Level Problem: TS Unit Commitment Problem

The upper level transmission day-ahead unit commitment problem seeks to compute the optimal operation schedule including the generator commitment status  $w_{g,t}$ , generation output  $p_{g,t}$ , upward and downward generator's reserve  $R_{g,t}^{up}$ ,

 $R_{g,t}^{dn}$ , DS DR price  $P_{g,t}^{dr}$ , and the DS energy import  $c_t^{im}$  to minimize the total TS operation cost. The optimization variables are denoted by the vector  $x_t$ , and include:

$$x_t = [w_{g,t}, p_{g,t}, r_{g,t}^{up}, r_{g,t}^{dn}, p_t^{dr}, c_t^{im}]$$

The objective of the upper level optimization problem is to minimize the TS operation cost including the generator commitment cost, generation cost, reserve cost, the DISCO DR cost, and negative DISCO energy import cost.

#### Objective function:

$$F(\lbrace x_t \rbrace_{t=1}^T) = \sum_{t=1}^T \sum_{g=1}^G (C_{g,t}^c w_{g,t} + C_{g,t} p_{g,t} + C_g(r_{g,t}^{up} + r_{g,t}^{dn}) + p_t^{dr} (dr_t^{up} + dr_t^{dn}) - p_t^{im} c_t^{im})$$

The constraints of the TS are listed below.

#### Power flow constraints:

$$-Line \le GSF * p_t^{inj} \le Line, t \in 1, ..., T$$
 (2)

$$-Line \le GSF * p_t^{inj*} \le Line, t \in 1, ..., T$$
 (3)

 $p_t^{inj}$  is the DC net power injection vector (ie: generation + wind - demand) for all the buses in each hour.  $p_t^{inj*}$  incorporates the wind forecast error, generator reserve and MG DR on top of  $p_t^{inj}$ . Eqn (2),(3) bound the transmission line flows within the flow limits.

## Generator constraints:

$$\underline{P}_q \le p_{g,t} \le \overline{P}_g, t \in 1, ..., T \tag{4}$$

$$(p_{g,t} + r_{g,t}^{up}) - (p_{g,t-1} - r_{g,t-1}^{dn}) \le \overline{R}_g, t \in 2, ..., T$$
 (5)

$$\underline{R}_{g} \le (p_{g,t} - r_{g,t}^{dn}) - (p_{g,t-1} + r_{g,t-1}^{up}), t \in 2, ..., T$$
 (6)

Eqn (4) bounds the generator generation within its capacities. Eqn (5),(6) satisfy the generator ramping capability.

## Power balance constraint:

$$\sum_{g=1}^{G} p_{g,t} + \mathbf{1}_{1 \times N_b} * L_t + W^f = p_t^{im} - p_t^{ex}, t \in 1, ..., T \quad (7)$$

where  $\mathbf{1}_{1\times N_b}$  is a vector of length  $N_b$  filled with 1's. The dot product  $\mathbf{1}_{1\times N_b}*L_t$  gives the total load in the system. Eqn (7) balances the system power supply and demand.

# Reserve constraints:

$$W_t^{up} \le dr_t^{up} + \sum_{g=1}^G r_{g,t}^{dn}, t \in 1, ..., T$$
 (8)

$$W_t^{dn} \le dr_t^{dn} + \sum_{g=1}^G r_{g,t}^{up}, t \in 1, ..., T$$
 (9)

Eqn (8), (9) ensure enough generator reserve and MG DR to compensate the possible wind forecast deviation.

Finally, the TS unit commitment problem is formulated as:

$$\min_{\{x_t\}_{t=1}^T} F\left(\{x_t\}_{t=1}^T\right)$$

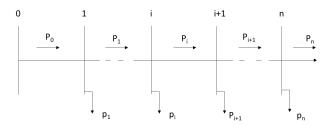


Fig. 2: Diagram of a radial distribution network

### B. Lower Level Problem: DS Operation Optimization

The DS considered in this work has a radial network as shown in Fig. 2. There are n buses in the network indexed by i=0,1,...,n. A simplified version of power flow that only considers active power according to [1] is considered in this work. The power flow equation at each node i could be expressed as

$$P_{i+1} = P_i - p_{i+1}$$
$$p_i = L^i + l^d - p_i^d$$

 $p_i$  is the net load at bus i.

The DS could only import energy from the transmission system. The load in the DS is served by the DS DG as well as the TS energy import.

The goal of the DS optimization problem is to compute the DG schedule  $p_{g,t}^d$ , and energy import  $p_t^{im}$  schedule, dispatchable load profile  $l_t^d$ , the upward/downward DR  $dr_t^{up}$ ,  $dr_t^{dn}$  provided by the dispachable load. The lower level optimization variables are denoted by the vector  $y_t$ , and include:

$$y_t = [p_{i,t}^d, p_t^{im}, dr_{i,t}^{up}, dr_{i,t}^{dn}, l_{i,t}^d]$$

The objective of the DS optimization is to minimize the DS operation cost including its DG cost, energy import cost from the TS, and DR cost and maximize its DL utility and DR revenue.

## Objective function:

$$\begin{split} f(\{y_t\}_{t=1}^T) &= \sum_{t=1}^T \sum_{i=1}^{N_b^d} (C_i^{d1} p_{i,t}^d + C_g^{d2} p_{i,t}^d p_{i,t}^d + p_t^{im} c_t^{im} \\ &+ C^{dr1} (dr_{i,t}^{up} + dr_{i,t}^{dn}) \\ &+ C^{dr2} (dr_{i,t}^{up} dr_{i,t}^{up} + dr_{i,t}^{dn} dr_{i,t}^{dn}) \\ &+ C^d (\overline{P}_i^d - p_{i,t}^d) (\overline{P}_i^d - p_{i,t}^d) \\ &- p_t^{dr} (dr_{i,t}^{up} + dr_{i,t}^{dn})) \end{split}$$

The constraints for the DS are given below. The  $\lambda$  and  $\mu$  variables next to the constraints are the dual variables associated with the corresponding inequality and equality constraints.

# DG constraints:

$$\underline{P}_{i}^{d} \leq p_{i}^{d} \leq \overline{P}_{i}^{d}, \lambda_{1,i,t}, \lambda_{2,i,t}, t \in 1, ..., T, i \in 1, ..., N_{b}^{d} \quad (10)$$

Eq (10) limits the generator's output within the upper and lower bound.

## Power flow constraints:

$$P_{i+1,t} = P_{i,t} - (L_{i,t}^{ie} + l_{i,t}^{d} - p_{i,t}^{d})$$

$$t \in 1, ..., T, \mu_{1,i,t}, i \in 1, ..., N_{b}^{d}$$

$$\underline{P}_{i} \leq P_{i,t} \leq \overline{P}_{i}, t \in 1, ..., T, \lambda_{3,i,t}, \lambda_{4,i,t}, i \in 1, ..., N_{b}^{d}$$
(11)

Eq (11) regulates the power flow between nodes in the distribution system. Eq (12) bounds the power flow within the line limits in the distribution system. Note that  $P_{1,t}$  is the DS energy import  $p_t^{im}$ .

#### Dispatchable load constraint:

$$\underline{L}_{i}^{d} \leq l_{i}^{d} \leq \overline{L}_{i,t}^{d}, \lambda_{5,i,t}, \lambda_{6,i,t}, t \in 1, ..., T, i \in 1, ..., N_{b}^{d}$$
 (13)

Eq (13) constrains the dispatchable loads within predefined bounds.

#### DR constraints:

$$0 \le dr_{i,t}^{up} \le S^{(dr)} * l_t^{i,d}, \lambda_{7,i,t}, \lambda_{8,i,t}, t \in 1, ..., T, i \in 1, ..., N_b^d$$
(14)

$$0 \le dr_{i,t}^{dn} \le S^{(dr)} * l_t^{i,d}, \lambda_{9,i,t}, \lambda_{10,i,t}, t \in 1, ..., T, i \in 1, ..., N_b^d$$
(15)

The amount of DR is set to be within a certain percentage defined by the DR scale factor  $S^{(dr)}$  of the DL, which is reflected in Eq (14), (15).

Finally, the MG optimal dispatch can be formulated as:

$$\min_{\{y_t\}_{t=1}^T} f\left(\{y_t\}_{t=1}^T\right)$$
s.t. (10) – (15)

# C. Reformulation to a Single Level Problem

If the lower level problem is convex and satisfies certain regularity conditions [2], it can be replaced by its Karush-Kuhn-Tucker(KKT) conditions [3], the formulation in (1) becomes a single -level problem reformulation as follows:

$$\begin{split} \min_{x \in X, y \in Y} F\left(x, y\right) \\ \text{st: } G_i\left(x, y\right) &\leq 0, \text{for } i \in \{1, 2, ..., I\} \\ H_k\left(x, y\right) &= 0, \text{for } k \in \{1, 2, ..., K\} \\ g_i\left(x, y\right) &\leq 0, \text{for } j \in \{1, 2, ..., M\} \\ h_m\left(x, y\right) &\leq 0, \text{for } m \in \{1, 2, ..., M\} \\ \text{dual feasibility: } \lambda_i &\geq 0, \text{for } i \in \{1, 2, ..., I\} \\ \text{complementary slackness: } \lambda_i * g_i\left(x, y\right) &= 0, \text{for } i \in \{1, 2, ..., I\} \\ \text{stationarity: } \nabla L(x, y, \lambda, \mu) &= 0 \\ where: \\ L(x, y, \lambda) &= f(x, y) + \sum_{i=1}^J \lambda_i g_i(x, y) + \sum_{m=1}^M \mu_m h_m(x, y) \end{split}$$

The lower level problem in this study is a convex optimization problem and satisfies Slater's condition, which is one of the sufficient conditions for strong duality theorem [2]. Therefore, the lower level problem could be replaced by its KKT conditions. The KKT conditions are given below:

#### Stationarity:

For stationarity, the derivative of the lagrangian function  $L(x, y, \lambda)$  is taken with respect to each optimization variable. For example, eq(23) is the derivative of the lagrangian function with respect to  $p_{i,t}^d$ .

$$2 * C_i^{d2} * p_{i,t}^d + C_i^{d1} + \lambda_{2,i,t} - \lambda_{1,i,t} + \mu_{1,i,t} = 0$$

$$t \in 1, ..., T, i \in \{1, ..., N_b^d\}$$

$$2 * C_i^{dr2} * dr_{i,t}^{up} + C_i^{dr1} - p_t^{dr} + \lambda_{8,i,t} - \lambda_{7,i,t} = 0$$

$$(17)$$

$$t \in 1, ..., T, i \in \{1, ..., N_b^d\}$$
(18)

$$2 * C_i^{dr2} * dr_{i,t}^{up} + C_i^{dr1} - p_t^{dr} + \lambda_{8,i,t} - \lambda_{7,i,t} = 0$$

$$t \in 1, ..., T, i \in \{1, ..., N_b^d\}$$
 (19)

$$p_t^{im} - \mu_{1,1,t} + \lambda_{4,1,t} - \lambda_{3,1,t} = 0, t \in 1, ..., T, i = 1$$
 (20)

$$\mu_{1,i-1,t} - \mu_{1,i,t} + \lambda_{4,i,t} - \lambda_{3,i,t} = 0,$$

$$t \in 1, ..., T, i \in \{2, ..., N_b^d\}$$
 (21)

$$2 * C^{d} * l_{1,t}^{d} + C^{d} * \overline{L}_{i,t}^{d} + \lambda_{6,1,t} - \lambda_{5,1,t} - \lambda_{8,1,t} - \lambda_{10,1,t} = 0, t \in 1, ..., T, i = 1$$

$$2 * C^{d} * l_{i,t}^{d} + C^{d} * \overline{L}_{i,t}^{d} + \lambda_{6,i,t} - \lambda_{5,i,t}$$
(22)

$$-\lambda_{8,i,t} - \lambda_{10,i,t} + \mu_{1,i,t} = 0, t \in 1, ..., T, i \in 2, ..., N_b^d$$
(23)

## Dual feasibility:

For dual feasibility, all dual variables need to be non-negative.

$$\lambda_{1,t}...\lambda_{10,t}, t \in 1, ..., T > 0$$
 (24)

# Complementary slackness:

For complementary slackness, the product of the dual variables and their corresponding inequalities need to be zero.

$$\lambda_{1,i,t} * (p_{i,t}^d - \underline{P}_i^d) = 0, t \in 1, ..., T, i \in 1, ..., N_b^d$$
 (25)

$$\lambda_{2,i,t} * (p_{i,t}^d - \overline{P}_i^d) = 0, t \in 1, ..., T, i \in 1, ..., N_b^d$$
 (26)

$$\lambda_{3,i,t} * (P_{i,t} - \overline{P}_i) = 0, t \in 1, ..., T, i \in 1, ..., N_b^d$$
 (27)

$$\lambda_{4,i,t} * (P_{i,t} - \underline{P}_i) = 0, t \in 1, ..., T, i \in 1, ..., N_b^d$$
 (28)

$$\lambda_{5,i,t} * (l_{i,t}^d - \overline{L}_t^d) = 0, t \in 1, ..., T, i \in 1, ..., N_b^d$$
 (29)

$$\lambda_{6,i,t} * (l_{i,t}^d - \underline{L}_t^d) = 0, t \in 1, ..., T, i \in 1, ..., N_b^d$$
 (30)

$$\lambda_{7,i,t} * dr_{i,t}^{up} = 0, t \in 1, ..., T, i \in 1, ..., N_b^d$$
(31)

$$\lambda_{8,i,t} * (dr_{i,t}^{up} - 0.2 * l_{i,t}^{d}) = 0, t \in 1, ..., T, i \in 1, ..., N_b^d$$
(32)

$$\lambda_{9,i,t}*dr_{i,t}^{dn}=0,t\in {1,...,T},i\in {1,...,N_b^d} \tag{33}$$

$$\lambda_{10,i,t} * (dr_{i,t}^{dn} - 0.2 * l_{i,t}^{d}) = 0, t \in 1, ..., T, i \in 1, ..., N_b^d$$
(34)

This reformulation is not easy to solve mainly due to the non-convexity in the complementary slackness and bilinear terms in the objective function. The big-M reformulation is used to transform the complementary conditions to mixed integer constraints. The technical details of the Big-M method is given in the appendix.

According to strong duality theorem, the optimal objective function value of the lower level optimization dual problem equals the optimal objective function value of the lower level optimization primal problem. As a result, the bilinear terms  $-p_t^{im}c_t^{im}+p_t^{dr}(dr_t^{up}+dr_t^{dn})$  in the lower level objective function, which also appear in the upper level objective function, could be expressed as linear terms using the objective function of the dual problem. The lower level dual problem objective function is as follow:

$$D(\{\lambda_{t}, \mu_{t}\}_{t=1}^{T}) = \sum_{t=1}^{T} \sum_{i=1}^{N_{b}^{d}} (-C^{d2} p_{i,t}^{d} p_{i,t}^{d} - C^{ld} l_{i,t}^{d} l_{i,t}^{d} l_{i,t}^{d} l_{i,t}^{d} + \lambda_{1,i,t} \overline{P}_{i,t} - \lambda_{4,i,t} \overline{P}_{i,t} + \lambda_{5,i,t} \overline{L}_{i,t}^{d} - \lambda_{6,i,t} \underline{L}_{i,t}^{d} + \lambda_{1,i,t} \overline{P}_{i}^{d} - \lambda_{2,i,t} \underline{P}_{i}^{d})$$

The bilinear terms  $-p_t^{im}c_t^{im}+p_t^{dr}(dr_t^{up}+dr_t^{dn})$  in the upper level objective function are represented as:

$$\begin{split} y &= \sum_{t=1}^{T} \sum_{i=1}^{N_b^d} (C_i^{d1} p_{i,t}^d + C_g^{d2} p_{i,t}^d p_{i,t}^d \\ &+ C^{dr1} (dr_{i,t}^{up} + dr_{i,t}^{dn}) \\ &+ C^{dr2} (dr_{i,t}^{up} dr_{i,t}^{up} + dr_{i,t}^{dn} dr_{i,t}^{dn}) \\ &+ C^d (\overline{P}_i^d - p_{i,t}^d) (\overline{P}_i^d - p_{i,t}^d)) - D(\{\lambda_{i,t}, \mu_{i,t}\}_{t=1,i=1}^{T,N_b^d}) \end{split}$$

The reformulated upper level objective function is as follow:

$$\begin{array}{lcl} F\left(\{x_t,y_t,\lambda_{t,i},\mu_{t,i}\}_{t=1,i=1}^{T,N_b^d}\right) & = & \sum_{t=1}^T \sum_{g=1}^G (C_{g,t}^c w_{g,t} + C_{g,t} p_{g,t} \\ & + & C_g^r(r_{g,t}^{up} + r_{g,t}^{dn}) - y \end{array}$$

The reformulated single level problem has the following format:

$$\begin{aligned} \min_{\{x_t, y_t, \lambda_t, \mu_t\}_{t=1}^T} & F\left(\{x_t, y_t, \lambda_t, \mu_t\}_{t=1}^T\right) \\ & \text{st: } (2) - (34) \end{aligned}$$

With the big-M reformulation, a set of binary variables and additional constants are introduced. The bilevel problem becomes a single level mixed integer linear problem and could thus be solved with a wide range of commercial solvers such as Cplex and Gurobi.

#### III. NUMERICAL RESULTS

In this section, the transmission model described in Section II is applied to the IEEE 30-bus system, shown in Fig. 3. Interested readers are referred to [4] for detailed parameters of the TS. The parameters of the 15 MW DS with 50% DL and a DR scale factor of 30% is listed in the appendix. A wind farm is positioned at bus 8 in the TS to provide renewable generation. A DS is also connected at bus 8 to provide DR to offset the renewable forecast errors.

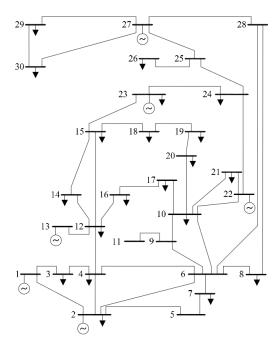


Fig. 3: IEEE 30 Bus System

The wind data for the wind farms in the TS are selected from the NREL-Eastern Wind Integration Study dataset [5]. Using three years of data, 24-hour trajectories are grouped to identify a set of 54 similar trajectories, with a common initial condition. The set of trajectories was used to represent the realizations of a similar forecast. The central trajectory of the group was selected as the wind forecast, and the remaining were used to estimate the distribution of forecast errors, as described in [6]. From the forecast error distribution, 10000 scenarios are used to generate a robust wind error scenario set.

The objective of this work is to analyze different factors that affect the WP in the TS and the operational cost of the two systems in this co-optimization framework as well as the standalone framework.

The definition of WP is based on the wind power capacity penetration defined by the European Wind Energy Association [7]. It is the ratio of the installed wind capacity divided by the peak system load.

1) Base Case: no DR: For benchmarking purpose, a base case study is carried out first. The DL level is 50%. The DR function in the DS is disabled. The results are given in Table I.

ſ	Max WP	TS Cost	DS Cost
ł	200/	9709	1607
	29%	1 8/98	1697

TABLE I: Base Case Results

2) Case 1: different DL levels with DR: In this case, the DR function of the DS is enabled and maintained at a level of 30%. The WP and costs of the two systems at different DL levels are recorded in Table II.

DL Level	Max WP	TS Cost	DS Cost
25%	30%	8568	2720
50%	31%	8771	1660
75%	33%	9110	180

TABLE II: Results at Different DL Levels

From Table II, the WP increases with an increasing DL level, however the increase in WP is not significant. That is because the DS size is relatively very small and the amount of DR it could provide is very limited. Looking at the system costs, the TS cost increases due to the increased requirement for reserves to compensate for the increased wind uncertainty. The DS cost decreases significantly with the increased DL level as it has more DR capacity with more DL to make more revenue. In addition, the flexibility with more DL could enable more load curtailing at expensive period ,which also reduces the system cost. Compared to the base case, the DR generates considerable cost savings for both systems as the base TS case cost at 29% WP is even greater than the 31% WP in this case. Moreover, the DR also reduces the DS cost for the same setting compared to the base case.

3) Case 2: different DR levels: In this case, the DR function of the DS is enabled and varied at different levels. The DL level is maintained at 50%. The WP and costs of the two systems at different DL levels are recorded in Table III.

DR Level	Max WP	TS Cost	DS Cost
15%	30%	8795	1706
30%	31%	8821	1660
45%	33%	8862	1644

TABLE III: Results at Different DL Levels

From Table III, the WP increases with an increasing DR level, once again the increase in WP is not significant. The reason is the same as before. Looking at the system costs, the TS cost increases due to the increased requirement for reserves to compensate for the increased wind uncertainty. One interesting thing to note is that the increase in TS cost in this case is smaller than the previous case. That is because in the previous case the amount of DR does not necessarily increase for all periods with an increased amount of DL as the DL setpoint could be very low for some periods and thus have very limited DR for those periods, so those periods still incur high cost increase with a higher WP. In the contrary, an increasing DR level at a constant DL guarantees more DR in this case and could more effectively reduce the cost increase in the TS with more wind uncertainty. The DS cost decreases with an increasing DR level but not as significantly as the previous level. That is due to the fact that the DS in the previous case saves money not only from DR but also from load curtailing.

# IV. APPENDIX

## A. Big-M Method

The complementary conditions are set of constraints with the following format:

$$\lambda_i * g_i(x, y) = 0$$

Specifically, given a sufficiently large positive value  $M_i$  and binary value  $\phi_i$ , the complementary conditions could be reformulated as below:

$$-(1 - \phi_i) * M_i \le g_i(x, y)$$
$$\lambda_i \le \phi_i * M_i$$

For a detailed treatment of the Big-M method, please refer to [8].

# B. Microgrid Parameters

The parameter values for a 25 MW MG are listed in the table below.

Parameters	Bus 1	Bus 2	Bus 3	Bus 4	Bus 5	Bus 6
$\overline{L}_i^d$	0MW	2MW	2.5MW	0MW	2MW	1MW
$\overline{L}_i^d$	0MW	1MW	1.25MW	0MW	1MW	0.5MW
$L_i^{ie}$	0MW	2MW	2.5MW	0MW	2MW	1MW
$\overline{\overline{P}_i}$	10MW	10MW	10MW	10MW	10MW	10MW
$\underline{P}_i$	0MW	0MW	0MW	0MW	0MW	0MW
$C_i^{d1}$	0\$/MW	0\$/MW	0\$/MW	0\$/MW	0\$/MW	5\$/MW
$C_i^{d2}$	0\$/MW	0\$/MW	0\$/MW	0\$/MW	0\$/MW	0.02\$/MW
$\overline{P}_i^d$	0MW	0MW	0MW	0MW	0MW	10MW
$\underline{P}_i^d$	0MW	0MW	0MW	0MW	0MW	0MW

Parameters	Values
$C_t^d$	3\$/MW
$C_t^{dr1}$	1\$/MW
$C_t^{dr2}$	1\$/MW

TABLE IV: DS parameter values

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