

A Multiperiod Energy Acquisition Model for a Distribution Company With Distributed Generation and Interruptible Load

Haiying Li, Yuzeng Li, and Zuyi Li, *Member, IEEE*

Abstract—This paper presents a multiperiod energy acquisition model for a distribution company (Disco) with distributed generation (DG) and interruptible load (IL) in a day-ahead electricity market. Assuming that cost information for individual generation companies (Gencos) and Discos is known, the Disco's energy acquisition strategy is modeled as a bilevel optimization problem with the upper subproblem representing individual Discos and the lower subproblem representing the independent system operator (ISO). The upper subproblem maximizes individual Discos' revenues. The lower subproblem simulates the ISO's market clearing problem that minimizes generation costs and compensation costs for interrupting load. The bilevel problem is solved by a nonlinear complementarity method. An 8-bus system is employed to illustrate the proposed model and algorithm. In particular, the roles of DGs and ILs to alleviate congestion are analyzed.

Index Terms—Distributed generation, distribution company, electricity market, interruptible load, nonlinear complementarity method.

NOMENCLATURE

Sets and Indices:

SB	Set of all branches.
$SB(l)$	Set of branches in independent loop l .
SD	Set of Disco buses.
$SD(m)$	Set of buses for Disco m .
SDG	Set of DG buses.
SG	Set of Genco buses.
SI	Set of IL buses.
SL	Set of independent loops.
SM	Set of Discos.
SN	Set of all buses.
$SN(i)$	Set of buses connected to bus i .
ST	Set of hours.
i, j	Index for buses.
m	Index for Discos.
t	Index for hours.

Unless otherwise stated, subscript it denotes "at bus i at hour t " in this paper.

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H. Li and Y. Li are with the Department of Automation, Shanghai University, Shanghai, China (e-mail: sh_lhyypf@hotmail.com; yzli@mail.shu.edu.cn).

Z. Li is with the Department of Electrical and Computer Engineering, Illinois Institute of Technology, Chicago, IL 60616 USA (e-mail: lizu@iit.edu).

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Constants:

$P_{d,it}^0$	Maximum demand of a Disco.
$P_{dg,it}^{\min}, P_{dg,it}^{\max}$	Lower and upper limits on generation of a DG.
$P_{g,it}^{\min}, P_{g,it}^{\max}$	Lower and upper limits on bid quantity submitted by a Genco.
$P_{ij,t}^{\min}, P_{ij,t}^{\max}$	Upper and lower flow limits on line flow of line $i - j$ at time period t .
$P_{IL,it}^{\min}, P_{IL,it}^{\max}$	Lower and upper limits on IL submitted by a Disco.
$R_{gu,it}, R_{gd,it}$	Ramping up and ramping down limits submitted by a Genco.
x_{ij}	Reactance of line $i - j$.
θ_{it}	Type of customer.

Curves:

$C_{IL,it}(P_{IL,it})$	IL cost curve of a Disco.
$a_{IL,it}, b_{IL,it}$	IL cost coefficients of a Disco.
$C_{IL,it}(P_{IL,it})$	$= a_{IL,it}P_{IL,it}^2 + b_{IL,it}\theta_{it}P_{IL,it}$
$C_{dg,it}(P_{dg,it})$	Generation cost curve of a DG.
$a_{dg,it}, b_{dg,it}$	Generation cost coefficients of a DG.
$C_{dg,it}(P_{dg,it})$	$= a_{dg,it}P_{dg,it}^2 + b_{dg,it}P_{dg,it}$
$P_{g,it}(P_{g,it})$	Generation bid price curve of a Genco.
$\rho_{g,it}(P_{g,it})$	$= 2a_{g,it}P_{g,it} + b_{g,it}$
$C_{g,it}(P_{g,it})$	Calculated generation cost function of a Genco.
$C_{g,it}(P_{g,it})$	$= a_{g,it}P_{g,it}^2 + b_{g,it}P_{g,it}$
$a_{g,it}, b_{g,it}$	Generation bid coefficients of a Genco.

Parameters:

$P_{g,it}$	Generation awarded to a Genco.
$P_{ij,t}$	Line flow of line $i - j$ at time period t .
$P_{IL,it}$	IL granted to a Disco.
$P_{d,it}$	Net demand of a Disco.
$P_{d,it}$	$= P_{d,it}^0 - P_{IL,it} - P_{dg,it}$
$P_{dg,it}$	Generation of a DG.
$\lambda_{d,it}$	LMP paid by a Disco.
$\lambda_{g,it}$	LMP paid to a Genco.

I. INTRODUCTION

DURING the initial development of electricity markets, more attention has been paid to the generation and transmission systems than the distribution system. With the further development of electricity markets, research related to distribution system has gradually started to flourish [1]–[8]. It is widely recognized that the markets will work better when a significant level of potential demand side response is available. Two resources, namely distributed generations (DGs) and interrupting loads (ILs), can be used by the distribution companies (Discos) to improve their market response capabilities, and accordingly change their passive positions in the markets.

The exercise of market power by generation company (Genco) during transmission congestion has been an inherent problem of electricity markets, which may be aggravated with the continuing growth of demand. On the other hand, this may actually facilitate the development of potential DGs. DG investment planning from the perspective of Discos is proposed in [1]. The model in [1] provides not only the sizes and sites but also the operating hours of DGs. The impacts of DGs on system congestion are discussed in [2]. Based on the impacts, DGs are divided into “desirable,” “less determinative,” and “undesirable” resources with a fuzzy c-means clustering approach [2]. Currently, there are several kinds of DGs [3], such as diesel turbines with the capacity under 1 MW, gas turbines with the capacity in the range of 1–20 MW, micro-turbines range from 30–200 kW, and hydro generation affected by weather. It is expected that DGs will cover 25% of the load increase in the coming 10 year in North America [4]. In the electricity market, DGs are usually not under the control of the independent system operator (ISO), but bulk customers or Discos. In this paper, only those DGs controlled by Discos are studied.

Interruptible load, as a tool of hedging price volatility, is widely used in electricity markets. Modeling IL in secondary reserve markets and contingency events is discussed in [5] and [6]. In New York state markets, the IL programs include “Emergency Demand Response Program,” “Special case resources Program,” and “Economic Day-ahead Demand Response Program (DADRP)” [7]. DADRP is designed to provide an incentive to customers to reduce consumption when locational marginal prices (LMPs) are high in the day-ahead market or the real time market. Similar to [7], in this paper, we assume that Discos bid ILs in the day-ahead market on behalf of their end customers.

A Disco usually buys energy from the wholesale market to meet the requirement of its end customers. However, a Disco will have more choices to acquire energy if it possesses DGs and ILs. A Disco energy acquisition market model with DGs and ILs is presented in [8] under a market structure based on Pool and bilateral contract. However, the energy acquisition model in [8] is a static single-period model and it does not consider the impact of other Discos’ strategies.

This paper develops a multiperiod energy acquisition model for Discos with multiple options including DGs, ILs, and wholesale market purchase. The focus is to study the roles of ILs and DGs in demand side response. Discos with DGs and ILs can use this model to increase their flexibilities in energy acquisition, improve the demand side responses, and maximize their

benefits. We assume in this model that a Disco has full knowledge of its own cost information and can estimate the cost information of other Discos and Gencos based on historical data. The objective of this model is to maximize every Disco’s profits by rational schedule strategy. The interactions among individual Discos are considered in this model.

This paper formulates the energy acquisition model as a bilevel optimization problem. The upper subproblem represents individual Discos’ profit maximization problem considering the constraints of its own DGs. The lower subproblem represents the ISO’s market clearing model for minimizing generation costs and compensation costs for ILs while satisfying various system constraints. The final equilibrium point is identified where every Disco maximizes its profits.

Several techniques have been applied to solve the bilevel equilibrium problems on the generation side. Reference [9] introduces an explicit function approach, namely deriving each Genco’s profit function with their strategic variables. However this approach is only suitable to models with a few constraints. The MPEC (mathematical program with equilibrium constraints)-based procedures, such as the penalty interior point algorithm [10], primal-dual interior point algorithm [11] and noninterior point algorithm [12], are developed for calculating equilibria under the assumption that only one Genco decides its bidding strategy while other Gencos will not change their strategies. Combining all market participants’ Karush-Kuhn-Tucker (KKT) optimality conditions to obtain the equilibrium solution is employed in [13], [14]. Those KKT conditions are combined as a mixed linear complementarity problem as a result of linearization of demand functions and marginal generation costs and solved using the available PATH-GAMS LCP software. The multi-Genco equilibrium problem is formulated as an equilibrium problem with equilibrium constraints (EPEC) in [15] and a nonlinear complementary method (NCM) is proposed after applying the KKT conditions, first in the ISO problem and second in the resulting individual Genco problem, to solve this bilevel problem. All Discos’ equilibrium problem in this paper is also an EPEC, so this paper follows a similar strategy to that of [15] for solving the proposed model.

The rest of the paper is organized as follows. The structure of electricity market and relevant cost information are discussed in Section II. The energy acquisition model is presented in Section III and the solution methodology is discussed in Section IV. Section V analyzes the illustrative example and Section VI concludes this paper.

II. ELECTRICITY MARKET STRUCTURE

In this section, a general market structure and relevant cost information are described, which are the basis for the Disco energy acquisition model discussed in Section III.

A. Structure of Electricity Market

The structure of the competitive day-ahead market considered in this paper is shown in Fig. 1. The ISO operates the market through security-constrained economic dispatch. Gencos sell energy to the market by bidding into the market. The information Gencos submit to the ISO includes lower and upper generation limits ($P_{g,it}^{\min}$, $P_{g,it}^{\max}$), bidding prices ($\rho_{g,it}(P_{g,it})$), and

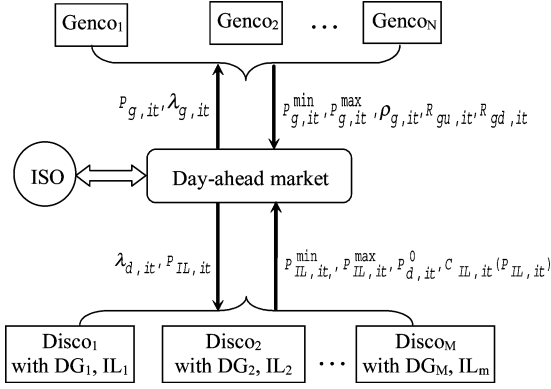


Fig. 1. Structure of electricity market.

ramping up/down limits ($R_{gu,it}$, $R_{gd,it}$). Discos purchase energy from the market to serve end customers. Discos may own DGs, and if necessary, Discos may interrupt loads for security or economic reasons. ILs are flexible contracts between Discos and their end customers. The information Discos submit to the ISO includes maximum demands ($P_{d,it}^0$), lower and upper ILs limits ($P_{IL,it}^{\min}$, $P_{IL,it}^{\max}$), and cost curves for ILs ($C_{IL,it}(P_{IL,it})$).

The day-ahead market serves as a platform for Gencos and Discos to sell and buy energy. The ISO clears the day-ahead market and publish LMPs. The amount Gencos get paid is LMPs ($\lambda_{g,it}$) multiplied by awarded generations ($P_{g,it}$). The amount Discos pay is LMPs ($\lambda_{d,it}$) multiplied by net demands ($P_{d,it}$). Discos may utilize DGs or ILs when LMPs are high. Thus, to some extent, DGs and ILs compete with Gencos. In order to offer enough incentive to customers to curtail loads when LMPs are high, the granted ILs are compensated by corresponding LMPs. With such a compensation mechanism, more customers may be willing to participate in the demand side management program.

The objective of a Disco is to maximize its profits by determining the amount of IL, the schedule of its DG, and the quantity of the energy purchased from the day-ahead market. The following cost information is required on hourly basis for a Disco to optimize its strategy.

B. Assumptions on the Cost Information Required by Discos

Assumptions on the cost information required by Discos are given as follows.

- 1) We assume in this paper that a Genco's bid function is given as

$$\rho_{g,it}(P_{g,it}) = 2a_{g,it}P_{g,it} + b_{g,it}, \quad i \in SG, \quad t \in ST. \quad (1a)$$

Accordingly, the generation cost curve can be calculated as

$$C_{g,it}(P_{g,it}) = a_{g,it}P_{g,it}^2 + b_{g,it}P_{g,it}, \quad i \in SG, \quad t \in ST \quad (1b)$$

- 2) We assume, based on [16], that the cost curve of a customer to curtail its load is given as

$$C_{IL,it}(P_{IL,it}) = a_{IL,it}P_{IL,it}^2 + b_{IL,it}\theta_{it}P_{IL,it}, \quad i \in SI, \quad t \in ST \quad (2)$$

where θ_{it} is called customer type in this paper and takes value from 0 to 1. θ_{it} represents a customer's willingness

to curtail its load. As θ_{it} increases, the cost of ILs increases and the customer is less willingness to curtail its load.

- 3) We assume that the cost function of a DG is given as

$$C_{dg,it}(P_{dg,it}) = a_{dg,it}P_{dg,it}^2 + b_{dg,it}P_{dg,it}, \quad i \in SDG, \quad t \in ST. \quad (3)$$

The Gencos' bidding curves are researched using game theory [11] and artificial neural network method [17], which are based on the predicted load or LMP published on the ISO website. [16] presents a calibration process to estimate the cost coefficients and types of ILs using existing historical utility data. The historical information of the New York State market in the U.S., as an example, can be obtained from its web at [18]. [3] is a good resource for the information on distributed generation.

III. ACQUISITION MODEL OF DISCO

A. Market Clearing Model

IL bids are evaluated along with generation supply bids in the day-ahead markets. We assume that the ISO clears the market using a security-constrained economic dispatch model (4a)–(4g), which is to minimize the generation costs and the costs for compensating ILs, subject to the bids and line flow constraints

$$\begin{aligned} \min \quad & \sum_{i \in SG, t \in ST} (a_{g,it}P_{g,it}^2 + b_{g,it}P_{g,it}) \\ & + \sum_{i \in SI, t \in ST} (a_{IL,it}P_{IL,it}^2 + b_{IL,it}\theta_{it}P_{IL,it}) \end{aligned} \quad (4a)$$

$$\text{s.t.} \quad P_{g,it}^{\min} \leq P_{g,it} \leq P_{g,it}^{\max}, \quad i \in SG, \quad t \in ST \quad (4b)$$

$$-P_{ij,t}^{\max} \leq P_{ij,t} \leq P_{ij,t}^{\max}, \quad ij \in SB, \quad t \in ST \quad (4c)$$

$$P_{IL,it}^{\min} \leq P_{IL,it} \leq P_{IL,it}^{\max}, \quad i \in SI, \quad t \in ST \quad (4d)$$

$$P_{g,it} - P_{g,i(t-1)} \leq R_{gu,it}, \quad i \in SG, \quad t \in ST \quad (4e)$$

$$P_{g,it} - P_{g,it} \leq_{gd,it} \quad i \in SG, \quad t \in ST \quad (4f)$$

$$P_{g,it} - (P_{d,it}^0 - P_{IL,it} - P_{dg,it}) + \sum_{j \in SN(i)} P_{ij,t} = 0, \quad i \in SN, \quad t \in ST \quad (4g)$$

$$\sum_{ij \in SB(l)} P_{ij,t}x_{ij} = 0, \quad l \in SL, \quad t \in ST. \quad (4h)$$

The decision variables in this model include $P_{g,it}$, $P_{ij,t}$, $P_{IL,it}$. The first term in the objective function (4a) represents the generation costs, and the second term represents the compensation costs for ILs.

Constraints (4b)–(4f) represent generation capacity constraints, transmission line flow constraints, IL capacity constraint, and ramping up/down constraints, respectively. Constraint (4g) is the load balance constraints on individual buses. LMP at a bus is the Lagrangian multiplier of the corresponding constraint in (4g). Constraint (4h), which is based on dc power flow equations, means that the summation of branch voltages for any independent loop should be zero [10].

The ramping constraint (4e) and (4f) makes the ISO market clearing model a linked multiperiod optimization problem. Without (4e) and (4f), the ISO market clearing model could be treated as multiple single-period models. The impact of ramping constraint, and accordingly the difference between a multiperiod model and multiple single-period models, will be analyzed in Section V.

It should be noted that there are other constraints that link successive hours, such as unit commitment and energy constraints for hydrogenation. The reason we exclude the unit commitment link is that the existence of equilibrium cannot be proven if there are integer variables, which is why nearly all power market oligopolistic models omit integer variables [14]. Unlike the discrete linking unit commitment constraints, continuous constraints such as energy constraints for hydrogenation could be modeled without much difficulty, which could be a subject for future work.

B. Disco Profit Model

Assumptions on the Disco profit model in this paper are given as follows.

- 1) We assume that a Disco does not bid its DGs into the day-ahead market but schedules its DGs according to estimated LMPs. Relaxation of this assumption would make a DG similar to a generating unit owned by a Genco.
- 2) We assume that if the load of a Disco is interrupted, the Disco will be paid according to LMP and load reduction.
- 3) We assume that a Disco returns all the compensation collected from the interruption of load to the interrupted end customer and does not benefit from the IL compensation. A Disco's profit only comes from the difference between the revenue it collects from customers and the cost it pays for the same amount of energy.
- 4) We assume that a Disco's retail energy rate is given as a fixed price $\bar{\lambda}$.

Once the energy market is cleared, every Disco pays the ISO according to LMP and the energy purchased from the day-ahead market. The cost of energy purchasing is

$$C_{d,it}(P_{d,it}) = \lambda_{d,it} P_{d,it} \quad i \in SD(m), \quad m \in SM, \quad t \in ST \quad (5)$$

where $P_{d,it}$ is the energy purchased from the market by a Disco at bus i at hour t , $P_{d,it} = P_{d,it}^0 - P_{IL,it} - P_{dg,it}$.

Based on (3) and (5), the profit of Disco m can be formulated as

$$R_m = \sum_{i \in SD(m), t \in ST} \left[\bar{\lambda} (P_{d,it}^0 - P_{IL,it}) - C_{dg,it}(P_{dg,it}) - \lambda_{d,it} (P_{d,it}^0 - P_{IL,it} - P_{dg,it}) \right] \quad m \in SM. \quad (6)$$

C. Disco Energy Acquisition Model

A Disco intends to maximize its own profit after considering other Discos' purchasing strategies subject to its DG constraints and the ISO's market clearing model. Thus, for every Disco

($M \in SM$), its multiperiod energy acquisition model is

$$\max R_m \quad (7a)$$

$$\text{s.t. } P_{dg,it}^{\min} \leq P_{dg,it} \leq P_{dg,it}^{\max}, \quad i \in SDG(m), \quad t \in ST \quad (7b)$$

$$(4a)-(4g). \quad (7c)$$

The decision variables in this model are $P_{dg,it}$ and those in (4a)–(4h).

IV. SOLUTION TO THE ACQUISITION MODEL OF DISCO

The Disco energy acquisition model (7a)–(7c) is a bilevel optimization problem, where the upper level represents the decision maker Disco, while the lower level is for the ISO's market clearing. Similar to [15], NCM is used to solve this model. For the convenience of description, we first introduce the basic idea of NCM, and then derive the solution approach by applying KKT optimality conditions and the NCM.

A. Nonlinear Complementarity Method

For a complementarity condition $a \geq 0, b \geq 0, ab = 0$, we can define a nonlinear complementarity function (NCF) $_{\varphi(a,b)}$ so that

$$\varphi(a, b) = 0 \Leftrightarrow a \geq 0, \quad b \geq 0, \quad ab = 0. \quad (8)$$

Equation (8) means if a and b satisfy the nonlinear complementarity equation (NCE) $\varphi(a, b) = 0$, then the complementarity condition $a \geq 0, b \geq 0, ab = 0$ is satisfied automatically.

The method of replacing a complementarity condition with a single nonlinear complementarity equation is called NCM. The importance of NCM is to transform a complementarity problem (in most cases due to the application of KKT conditions) into a set of nonlinear equations, which can be solved by a Newton-type method. The merit of NCM compared to the interior-point method (IPM) is that it relaxes the strict positive conditions mandatory for IPM [19].

The key to NCM is to find an appropriate NCF. In this paper, we have used the NCF defined in [20] as follows:

$$\varphi(a, b) = a + b - \sqrt{a^2 + b^2}. \quad (9a)$$

A vector version of $\varphi(a, b)$ is defined as

$$\varphi(\mathbf{a}, \mathbf{b}) = \begin{pmatrix} \varphi(a_1, b_1) \\ \varphi(a_2, b_2) \\ \vdots \\ \varphi(a_n, b_n) \end{pmatrix}. \quad (9b)$$

B. Generalized Market Clearing Model

For the convenience of further derivation, the ISO's market clearing model (4a)–(4h) can be generalized as follows:

$$\min \text{Cost}(\mathbf{x}) \quad (10a)$$

$$\text{s.t. } \mathbf{Ax} = \mathbf{b} \quad (10b)$$

$$\mathbf{x}_l \leq \mathbf{x} \leq \mathbf{x}_u \quad (10c)$$

where

- \mathbf{x} represents all variables in (4a)–(4h), including $P_{g,it}, P_{ij,t}, P_{IL,it}, R_{g,it}$;
 \mathbf{x}_l and \mathbf{x}_u represent the lower and upper bounds of \mathbf{x} , including (4a)–(4f);
 $\mathbf{Ax} = \mathbf{b}$ represents all the equality constraints, including (4g)–(4h);
 $\text{Cost}(\mathbf{x})$ represents the objective function (4a);

$$\text{Cost}(\mathbf{x}) = \sum_{i \in SG, t=ST} (a_{g,it} P_{g,it}^2 + b_{g,it} P_{g,it}) + \sum_{i \in SI, t=ST} (a_{IL,it} P_{IL,it}^2 + b_{IL,it} \theta_{it} P_{IL,it}).$$

C. Generalized Market Clearing Model as a Set of Nonlinear Equations

The KKT conditions of the generalized market clearing model (10a)–(10c) are

$$\nabla L_{\mathbf{x}} = \mathbf{0} \quad (11a)$$

$$\mathbf{Ax} = \mathbf{b} \quad (11b)$$

$$\mu_{\mathbf{x}_l} \geq \mathbf{0}, \quad \mathbf{x} - \mathbf{x}_l \geq \mathbf{0}, \quad \mu_{\mathbf{x}_l} \circ (\mathbf{x} - \mathbf{x}_l) = \mathbf{0} \quad (11c)$$

$$\mu_{\mathbf{x}_u} \geq \mathbf{0}, \quad \mathbf{x}_u - \mathbf{x} \geq \mathbf{0}, \quad \mu_{\mathbf{x}_u} \circ (\mathbf{x}_u - \mathbf{x}) = \mathbf{0} \quad (11d)$$

where

L Lagrange function of (10a)–(10c);

$$L = \text{Cost}(\mathbf{x}) + \mathbf{t}^T (\mathbf{Ax} - \mathbf{b}) + \mathbf{t}_{\mathbf{x}_l}^T (\mathbf{x} - \mathbf{x}_l) + \mathbf{t}_{\mathbf{x}_u}^T (\mathbf{x}_u - \mathbf{x});$$

$\nabla L_{\mathbf{x}}$ partial derivative vector of L with respect to \mathbf{x} ;

$\mu_{\mathbf{x}_l}, \mu_{\mathbf{x}_u}$ dual variable vectors related to \mathbf{x}_l and \mathbf{x}_u ;
 \circ represents the dot product of two vectors (element by element).

Conditions (11c) and (11d) represent complementarity conditions, which can be converted into nonlinear complementarity equations using NCM

$$\varphi(\mu_{\mathbf{x}_l}, \mathbf{x} - \mathbf{x}_l) = \mathbf{0} \quad (12a)$$

$$\varphi(\mu_{\mathbf{x}_u}, \mathbf{x}_u - \mathbf{x}) = \mathbf{0}. \quad (12b)$$

Then, the KKT conditions of the generalized market clearing model (11a)–(11d), and correspondingly the original market clearing model (4a)–(4h), are equivalent to a set of nonlinear (13a)–(13d). The equivalency is true since the Hessian of the Lagrangian is positive definite ($a_{g,it}$ and $a_{IL,it}$ are assumed to be positive)

$$\nabla L_{\mathbf{x}} = \mathbf{0} \quad (13a)$$

$$\mathbf{Ax} = \mathbf{b} \quad (13b)$$

$$\varphi(\mu_{\mathbf{x}_l}, \mathbf{x} - \mathbf{x}_l) = \mathbf{0} \quad (13c)$$

$$\varphi(\mu_{\mathbf{x}_u}, \mathbf{x}_u - \mathbf{x}) = \mathbf{0}. \quad (13d)$$

D. Individual Disco Energy Acquisition Model as a Set of Nonlinear Equations

We use (14c) to replace (4a)–(4h) in the Disco energy acquisition model (7a)–(7c), which is reformulated as follows:

$$\max R_m \quad (14a)$$

$$\text{s.t. } P_{dg,it}^{\min} \leq P_{dg,it} \leq P_{dg,it}^{\max}, \quad i \in SDG(m), \quad t \in ST \quad (14b)$$

$$(13a) \text{--}(13d). \quad (14c)$$

The KKT conditions of the equivalent Disco energy acquisition model (14a)–(14c) are

$$\nabla L_{m,y} = \mathbf{0} \quad (15a)$$

$$\mu_{1,it} \geq 0, \quad P_{dg,it} - P_{dg,it}^{\min} \geq 0,$$

$$\mu_{1,it} (P_{dg,it} - P_{dg,it}^{\min}) \geq 0 \quad i \in SDG(m), \quad t \in ST \quad (15b)$$

$$\mu_{2,it} \geq 0, \quad P_{dg,it}^{\max} - P_{dg,it} \geq 0$$

$$\mu_{2,it} (P_{dg,it}^{\max} - P_{dg,it}) \geq 0 \quad i \in SDG(m), \quad t \in ST \quad (15c)$$

$$(13a) \text{--}(13d) \quad (15d)$$

where

\mathbf{y} vector of all variables in (15a)–(15d), including \mathbf{x} (i.e., $P_{g,it}, P_{ij,t}, P_{IL,it}, R_{g,it}$), $\mu_{\mathbf{x}_l}, \mu_{\mathbf{x}_u}$, and $P_{dg,it}$;

L_m Lagrange function of (15a)–(15d).

$$L_m = \sum_{i \in SD(m), t \in ST} [\bar{\lambda}(P_{d,it}^0 - P_{IL,it}) - C_{dg,it}(P_{dg,it}) - \lambda_{d,it}(P_{d,it}^0 - P_{IL,it} - P_{dg,it})] + \sum_{i \in SDG(m), t \in ST} [\mu_{1,it}(P_{dg,it} - P_{dg,it}^{\min}) + \mu_{2,it}(P_{dg,it}^{\max} - P_{dg,it})] + \mu_3^T \nabla L_{\mathbf{x}} + \mu_4^T (\mathbf{Ax} - \mathbf{b}) + \mu_5^T \varphi(\mu_{\mathbf{x}_l}, \mathbf{x} - \mathbf{x}_l) + \mu_6^T \varphi(\mu_{\mathbf{x}_u}, \mathbf{x}_u - \mathbf{x})$$

$\nabla L_{m,y}$ partial derivative vector of L_m with respect to \mathbf{y} ;

$\mu_{1,it}, \mu_{2,it}$ dual variables related to the lower and upper bounds of $P_{dg,it}$ in (14b);

$\mu_3, \mu_4, \mu_5, \mu_6$ dual variable vectors related to other equality constraints in (15d).

Conditions (15b) and (15c) represent a set of complementarity conditions, which can be converted into nonlinear complementarity equations using NCM

$$\varphi(\mu_{1,it}, P_{dg,it} - P_{dg,it}^{\min}) = 0, \quad i \in SDG(m), \quad t \in ST \quad (16a)$$

$$\varphi(\mu_{2,it}, P_{dg,it}^{\max} - P_{dg,it}) = 0, \quad i \in SDG(m), \quad t \in ST. \quad (16b)$$

Then, the KKT conditions of the equivalent Disco energy acquisition model (15a)–(15d), and correspondingly the original Disco energy acquisition model (7a)–(7c), are equivalent to a set of nonlinear equations (17a)–(17g). The equivalency is true

since the Hessian of the Lagrangian is negative definite ($a_{dg,it}$ is assumed to be positive)

$$\nabla L_{m,y} = \mathbf{0} \quad (17a)$$

$$\varphi(\mu_{1,it}, P_{dg,it} - P_{dg,it}^{\min}) = 0, \quad i \in SDG(m), \quad t \in ST \quad (17b)$$

$$\varphi(\mu_{2,it}, P_{dg,it}^{\max} - P_{dg,it}) = 0, \quad i \in SDG(m), \quad t \in ST \quad (17c)$$

$$\nabla L_{\mathbf{x}} = \mathbf{0} \quad (17d)$$

$$\mathbf{Ax} = \mathbf{b} \quad (17e)$$

$$\varphi(\mu_{\mathbf{x}l}, \mathbf{x} - \mathbf{x}l) = 0 \quad (17f)$$

$$\varphi(\mu_{\mathbf{x}u}, \mathbf{x}u - \mathbf{x}) = 0. \quad (17g)$$

E. Complete Disco Energy Acquisition Model as a Set of Nonlinear Equations

Based on (17a)–(17g), the complete energy acquisition model including all Discos is as follows:

$$\nabla L_{m,y} = \mathbf{0}, \quad m \in SM \quad (18a)$$

$$\varphi(\mu_{1,it}, P_{dg,it} - P_{dg,it}^{\min}) = 0, \quad i \in SDG(m), \quad t \in ST, \quad m \in SM \quad (18b)$$

$$\varphi(\mu_{2,it}, P_{dg,it}^{\max} - P_{dg,it}) = 0, \quad i \in SDG(m), \quad t \in ST, \quad m \in SM \quad (18c)$$

$$\nabla L_{\mathbf{x}} = \mathbf{0} \quad (18d)$$

$$\mathbf{Ax} = \mathbf{b} \quad (18e)$$

$$\varphi(\mu_{\mathbf{x}l}, \mathbf{x} - \mathbf{x}l) = 0 \quad (18f)$$

$$\varphi(\mu_{\mathbf{x}u}, \mathbf{x}u - \mathbf{x}) = 0. \quad (18g)$$

The above model is formulated by integrating individual Discos' equivalent models. Note that we have only one set of equations representing the ISO clearing model since it is the same for all Discos.

The complete Disco energy acquisition model (18a)–(18g) is solved using the inexact Levenberg-Marguardt (LM) algorithm, which is effective for solving nonlinear complementarity problems [15]. [20] presents a detail discussion on the inexact LM algorithm.

V. NUMERICAL EXAMPLE

An eight-bus system shown in Fig. 2 is used to illustrate the proposed model and solution algorithm. The system includes three competing Discos. Detailed information on Discos, IL types, Gencos, and transmission lines are shown in Tables I–IV, which are assumed to be the same for all hours. The demand profile for 24 trading periods is shown in Fig. 3.

The following information is assumed for the studies in this paper.

- Maximum quantity of IL is one tenth of its peak load.
- Cost coefficients of IL are $a_{IL} = 1\$/ (MW^2h)$, $a_{IL} = 120\$/MWh$.
- Price that Discos charge their end customers for energy is $\bar{\lambda} = 80\$/MWh$.

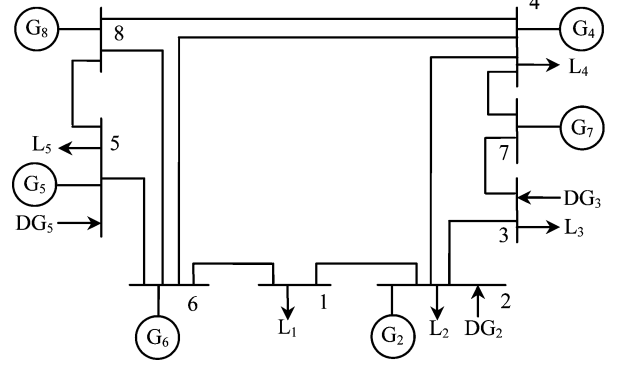


Fig. 2. Single-line diagram of the eight-bus system.

TABLE I
DISCOS DATA

Disco	Load	DG
Disco1	L ₁ , L ₂	DG ₂
Disco2	L ₃ , L ₄	DG ₃
Disco3	L ₅	DG ₅

- All DGs have the same characteristics $P_{dg}^{\min} = 0$ MW, $P_{dg}^{\max} = 1.5$ MW, $a_{dg} = 0.09\$/ (MW^2h)$, $b_{dg} = 38\$/MW$.
- All generators' ramping-up and ramping-down rates are $R_{gu} = R_{gd} = 5$ MW/h

The strategy of Disco depends on the demand and LMPs of day-ahead market. From Fig. 3, we can see that demand increases slowly for the hours before 16:00 and after 22:00, therefore the impact of ramping-up and ramping-down limits on LMPs can be ignored and the optimal solutions for those hours are the same as the independent single-period outcomes. Note that without considering ramping constraints, the multiperiod energy acquisition model is equivalent to multiple single-period models. Thus, in order to reduce the computation burden, the strategies can be determined by a simple single-period model except for trading periods 16:00–17:00–18:00 and 20:00–21:00–22:00. Three cases have been studied.

Case 1 energy acquisition for a single trading period at 19:00. In this case, no ramping limits and no transmission line flow limits are considered. Thus, the LMPs are uniform (31.02\$/MWh) for all the buses. The three Discos purchase all energy from the day-ahead market due to the low market price. Their profits are 2546.93\$/h, 2938.77\$/h, and 1469.38\$/h, respectively.

Case 2 energy acquisition for three continuous periods 16:00–17:00–18:00. In this case, ramping limits for some generators are active and flow limits for some lines also are active. As the three periods are linked with each other, the optimal solutions for each period are not the same as those obtained from single-period model. Fig. 4 and Tables V and VI list the detailed solutions, where “S” represents the single-period model, “M” represents the multiperiod model, “+” means another 1 MW of demand is increased at Bus 2 and DAM means the day-ahead market.

From Table V, we can see that the generations of G_2 , G_4 and G_7 at 16:00–17:00 and the generation of G_4 at 17:00–18:00 are

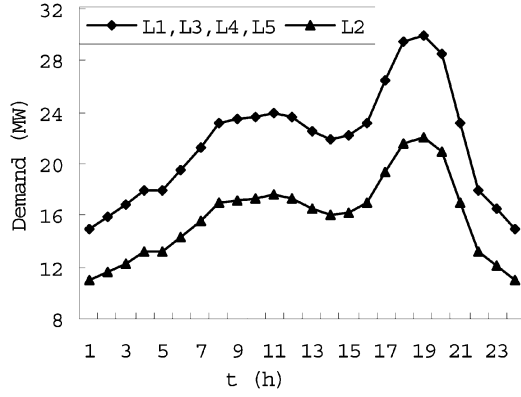


Fig. 3. Demand profile.

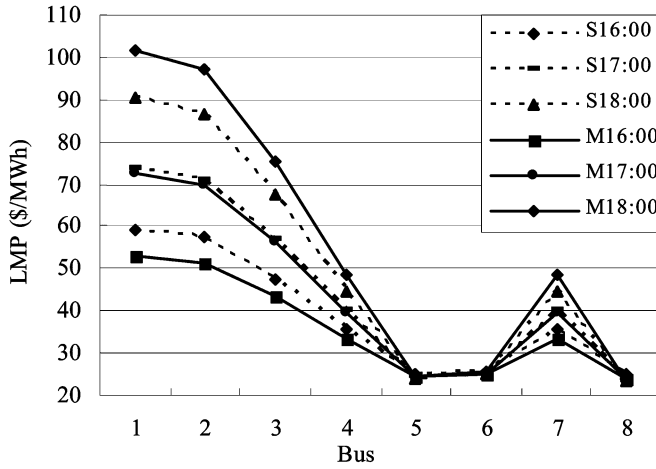


Fig. 4. Comparison of LMPs with and without ramping limits.

TABLE II
TYPES OF INTERRUPTIBLE LOADS

bus	1	2	3	4	5
θ_i	0.75	0.78	0.81	0.83	0.95

TABLE III
GENCOS DATA

	P_{\min} (MW)	P_{\max} (MW)	a_g (\$/(MW) ² h)	b_g (\$/MWh)
G_2	0	40	0.08	45.62
G_4	0	58	0.11	35.35
G_5	0	40	0.09	22.47
G_6	0	50	0.095	23.37
G_7	0	24	0.085	33.47
G_8	0	60	0.078	21.39

TABLE IV
TRANSMISSION LINES DATA

	From bus	To bus	X (p.u.)	Limits (MW)
1	1	2	0.011	20
2	2	3	0.018	30
3	2	4	0.03	20
4	3	7	0.022	40
5	7	4	0.015	38
6	4	6	0.03	30
7	4	8	0.03	40
8	8	6	0.0065	40
9	8	5	0.02	20
10	5	6	0.025	38
11	6	1	0.03	14.2

TABLE V
COMPARISON OF GENERATION (MW) WITH AND WITHOUT RAMPING LIMITS

	G2	G4	G5	G6	G7	G8
S16:00	40	2.59	15.18	12.34	14.41	21.82
S16:00+	40	3.89	14.54	11.84	16.09	20.98
S17:00	40	20.82	11.24	9.8	24	16.11
S18:00	40	43.13	8.11	8.21	24	11.16
M16:00	35	18.99	11.44	8.35	15.84	17.94
M16:00+	35.05	18.99	11.36	8.27	15.84	17.84
M17:00	40	23.99	11.24	9.74	20.84	16.16
M18:00	40	28.99	11.21	11.97	24	13.91

TABLE VI
COMPARISON OF ENERGY ACQUISITION (MW) WITH AND WITHOUT RAMPING LIMITS

		S16	S17	S18	M16	M17	M18
Disco1	DAM	38.54	44.26	48.81	38.88	44.26	44.28
	DG	1.5	1.5	1.5	1.16	1.5	1.5
	IL	0	0	0.13	0	0	4.66
	Total	40.04	45.76	50.44	40.04	45.76	50.44
	Profit	573.69			369.49		
Disco2	DAM	44.7	51.3	56.7	45.57	51.3	56.7
	DG	1.5	1.5	1.5	0.63	1.5	1.5
	IL	0	0	0	0	0	0
	Total	46.2	52.8	58.2	46.2	52.8	58.2
	Profit	4456.12			4456.9		
Disco3	DAM	23.1	26.4	29.1	23.1	26.4	29.1
	DG	0	0	0	0	0	0
	IL	0	0	0	0	0	0
	Total	23.1	26.4	29.1	23.1	26.4	29.1
	Profit	4362.83			4195.6		

limited by ramping-up rates. The ramping-up limits cause those units' generations to be different from those obtained from independent single-period model. The LMPs are also different accordingly. In particular, LMPs at 16:00 with ramping-up limits are lower than those obtained from independent single-period model. This counterintuitive result can be explained as follows.

- 1) G_4 is an expensive unit compared with G_5 , G_6 , G_7 , and G_8 . Its generation will be as small as possible if ramp limits are not considered. This can be found from the data of single period results. When there is another 1 MW of load increment at Bus 2 in the system, G_4 's generation increases

by more than 1 MW while the generations of other cheap units decrease due to transmission congestion, which leads to high LMPs.

- 2) However, generation of G_4 increases at 16:00 once ramp limits are enforced. When the same 1 MW load increment occurs at Bus 2, G_4 's generation does not change at all. Thus, LMPs are lower although ramping limits are active. This also indicates that the decrease of cheaper power or the increase of more expensive power can cause less congestion to a certain extent.

Because the LMPs at 16:00 calculated from the multiple-period model are lower, Disco1 and Disco2 would like to purchase more energy from the day-ahead market, thus their DGs generations are less than those when no ramping limits are enforced.

Ramping limits have smaller impact on the LMPs at 17:00. So, the strategies are almost the same with and without ramping

TABLE VII
IMPACT OF DG AND IL

Case		3A	3B	3C	3D
Disco1	IL (MW)	0	0	2.72	1.41
	DG (MW)	0	1.5	0	1.5
	DAM (MW)	52	50.5	49.28	49.09
	Total (MW)	52	52	52	52
	Profit (\$)	-3232.19	-795.13	-670.42	-605.22
Disco2	IL (MW)	0	0	0	0
	DG (MW)	0	1.5	0	1.5
	DAM (MW)	60	58.5	60	58.5
	Total (MW)	60	60	60	60
	Profit (\$)	279.25	1183.08	1275.77	1276.36
Disco3	IL (MW)	0	0	0	0
	DG (MW)	0	0	0	0
	DAM (MW)	30	30	30	30
	Total (MW)	30	30	30	30
	Profit (\$)	1725.84	1688.39	1686.16	1683.09
LMP (\$/MWh)	Bus 1	138.41	96.31	95.44	92.82
	Bus 2	147.27	92.13	91.32	88.85
	Bus 3	102.63	71.64	71.1	69.4
	Bus 4	48.06	46.6	46.38	45.62
	Bus 5	22.47	23.72	23.79	23.9
	Bus 6	27.21	24.81	24.87	24.94
	Bus 7	48.06	46.6	46.38	45.62
	Bus 8	18.68	22.85	22.93	23.07
Operation costs (\$/h)		5726.41	5564.07	5685.73	5559.63
Cost for Gencos (\$/h)		10371.85	7752.41	7689.06	7421.21
CPE (\$/h)		12587.10	8929.26	8850.72	8538.63
MP (\$/h)		2215.25	1176.85	902.06	986.54

limits. With the increase of demand and due to ramping-up limits, the LMPs at 18:00 are much higher, so more ILs are utilized in order to maximize Discos' profits.

From Table VI, we can see that there are no significant changes in Discos' profits if appropriate energy acquisition strategies are employed, even with the inclusion of ramping constraints. The same analysis can be made to the trading periods 20:00–21:00–22:00, where ramping-down limits are active.

Case 3 this case is to analyze the roles of DG and IL when congestion occurs in the system. Ramping limits are not considered in this case. Demands in the system are at their peak with $L_1 = L_3 = L_4 = L_5 = 30$ MW, $L_2 = 22$ MW.

The following four subcases are considered

- 3A. Discos have no DG and IL;
- 3B. Discos have DG but no IL;
- 3C. Discos have IL but no DG;
- 3D. Discos have both DG and IL.

The comparisons of Discos' optimal strategies, Discos' profits, LMPs, operation costs (summation of Generation costs, the DGs' costs and the ILs' costs), cost for Gencos, cost for purchasing energy (CPE) and merchandise surplus (MS, the difference between the revenue collected from the Discos and the cost for Gencos and ILs) are shown in Table VII.

From Table VII, the following observations can be made.

- 1) In Case 3A, congestion leads to much higher LMPs for Disco1 and Disco2 compared to those in Case1, where no transmission flow limits are considered. Their profits are negative since they have no DG and IL and can only purchase energy from the day-ahead market with high prices.
- 2) In Case 3B, Disco1 and Disco2 would prefer to schedule their own DG when market prices are high. The net demand

TABLE VIII
RUN TIME OF THE STUDIED CASES

Case	1	2	3A	3B	3C	3D
Run time (s)	12.01	702.6	47.44	15.32	27.98	28.73

decreases and LMPs decrease significantly compared to those in Case 3A. Their profits increase accordingly. This means that congestion offers potential incentive to Discos to develop investment plan in DGs.

- 3) Besides DG, IL is a useful resource to mitigate congestion, as shown in Case 3C. When Disco1's demand is interrupted, LMPs are lower than those in Case 3A. Although its profit is still negative, the loss is smaller than that in Case 3A. This means that Discos will be encouraged to sign flexible IL contracts with the end customers. It should be pointed out that the cost of IL is usually higher than the cost of DG or the spot price without congestion. So in general, IL is more suitable to be used in emergency states.
- 4) In Case 3D, where there are DGs and ILs in the system, the proposed model can help Discos make their energy purchase plan to maximize their profits.
- 5) Disco3 can always purchase cheap energy from the day-ahead market, so its DG and IL are not used in these four subcases.
- 6) When DG and IL participate in the competition of day-ahead market (Cases 3B–3D), the operation costs increase slowly but the LMPs are reduced evidently. The reason is that DG and IL have alleviated the congestion and the merchandise surplus decreases notably, which results in lower LMPs. Therefore, DG and IL can maintain the stability of market prices.
- 7) When DG and IL participate in the competition of day-ahead market (Cases 3B–3D), the ISO pays less money to Gencos, which can restrict Gencos' capabilities of earning more profits by means of capacity withholding and strategic bidding.

The case studies presented above have been performed in MATLAB on a computer with 2.8GHz Pentium CPU and 1GB RAM. There are 29 variables, 322 dual variables, and 351 equality constraints in the single-period model; and 119 variables, 1062 dual variables, and 1181 equations in the multiperiod model. The average run time to find the equilibrium of different cases with a convergence tolerance 10^{-6} is listed in Table VIII. Note that the run time is related to the starting points of LM algorithm, and the time for Case 2 is the multiple-period run time.

VI. CONCLUSION

A multiperiod energy acquisition model for a Disco in the competitive day-ahead electricity market with two new resources, DG and IL, is proposed in this paper. Due to the interactions among Discos' strategies, the energy purchasing scheme is modeled as a bilevel optimization problem in order to maximize every Disco's profits. To solve the complementarity problems formulated from the KKT conditions, a nonlinear complementarity method is employed to solve this model.

The eight-bus example demonstrates that the proposed model and solution algorithm can help Discos make optimal energy

purchasing plans in different constraint conditions. The inclusion of inter-temporal generation constraints such as ramping constraints may lead to different market outcomes from the independent hourly calculations and offer insights to appropriate energy acquisition scheme.

The roles of DG and IL are also studied in case of transmission congestion. Studies in this paper show that DG and IL can alleviate congestion, hedge the volatility of market price, and reduce Gencos' market power. Based on these results, Discos are encouraged to make full use of the two useful resources by investing on DGs and signing IL contracts with the end customers.

The examples presented in this paper are to demonstrate the roles of IL and DG by applying the proposed energy acquisition model. Extending the procedure to practical use, e.g., for larger-scale systems, would require further research. In addition, considering the uncertainties of estimated information about other Gencos and Discos and the associated risks inherent in the electricity markets could be a challenging subject for future research. One possible option is to use Monte Carlo simulation and scenario analysis [21], [22].

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Haiping Li received the B.S. and M.S. degrees from the Taiyuan University of Technology, Taiyuan, China, in 1999 and 2002, respectively. She is currently pursuing the Ph.D. degree at Shanghai University, Shanghai, China.

She is a Faculty Member of the University of Shanghai for Science and Technology, Shanghai, and currently a Visiting Scholar with the Illinois Institute of Technology, Chicago. Her research interests are demand management and transmission congestion management.

Yuzeng Li graduated from Fudan University, Shanghai, China, in 1968. He received the M.S. degree from the Mathematics and Physics Department, Academia Sinica, Shanghai, China, in 1982 and the Ph.D. degree in electrical engineering from Hong Kong Polytechnic in 1993.

He is currently a Professor at Shanghai University. His research interests include games analysis for electricity market and interruptible load management.

Zuyi Li (M'03) received the B.S. degree in electrical engineering from Shanghai Jiaotong University, Shanghai, China, in 1995, the M.S. degree in electrical engineering from Tsinghua University, Tsinghua, China, in 1998, and the Ph.D. degree in electrical engineering from the Illinois Institute of Technology (IIT), Chicago, in 2002.

He is an Assistant Professor with the Electrical and Computer Engineering Department, IIT. He is the coauthor of the book *Market Operations in Electric Power Systems* (New York: Wiley, 2002). His research interests focus on market operation and wide-area protection of power systems.