Transmission System and Microgrid Co-optimization Model

Nomenclature

Indices

t	a time period (time interval from the beginning through the end of a period)
g	transmission network generator index

Parameters

T	length of the planning horizon
G	total number of generators in the transmission system
$\overline{R}_g$	generator upper ramp rate limit
$\underline{R}_g$	generator lower ramp rate limit
N_t	number of periods
$\overline{P}_g$	generator generation upper bound
$\underline{P}_g$	generator generation lower bound
$Line$	line limit
W^f	forecasted wind power
$\overline{B}$	microgrid storage energy state max level
$\underline{B}$	microgrid storage energy state min level
C^b	microgrid storage energy maintenance cost
C_t^m	microgrid generation cost
C_g	generater cost for generator g
C_t^d	utility for consuming dispatchable load in the microgrid in hour t
C_g^r	generater reserve cost for generator g

L_t load power vector in hour t

L^i_t inelastic load in time t

C^{ex}_t microgrid exported power cost in hour t

C^{im}_t microgrid imported power cost in hour t

$P^m_{g,t}$ microgrid generation in hour t

$\overline{P}^m$ microgrid generation upper bound

$\underline{P}^m$ microgrid generation lower bound

$\underline{P}^b$ microgrid storage discharging limit

$\overline{P}^b$ microgrid storage charging limit

$\underline{R}_g$ microgrid generator downward ramping limit

$\overline{R}_g$ microgrid generator upward ramping limit

$\overline{L}^d_t$ MG aggregated dispatchable load upper bound in hour t

$\underline{L}^d_t$ MG aggregated dispatchable load lower bound in hour t

GSF generator shift factor matrix

N_b number of buses in the transmission system

W^{up} positive wind deviation from the forecast

W^{dn} negative wind deviation from the forecast

ϵ constraint violation probability

C^c_g transmission generator commitment cost for generator g

C^s_g transmission generator start-up cost for generator g

Variables

$R^{up}_{g,t}$ transmission generator upward reserve in hour t for generator g

$R^{dn}_{g,t}$ transmission generator downward reserve in hour t for generator g

P^{dr}_t demand response price in hour t

$P_{g,t}$ transmission generator generation in hour t for generator g

B_t microgrid storage energy state in hour t

P_t^{ex} microgrid exported power in hour t

P_t^{im} microgrid imported power in hour t

P_t^{b} microgrid storage power output in hour t

L_t^{d} MG aggregated dispatchable load in hour t

DR_t^{up} microgrid storage power output in hour t

DR_t^{dn} MG aggregated dispatchable load in hour t

P_t^{inj} bus power injection vector in hour t

$w_{g,t}$ transmission generator commitment variable in hour t for generator g

$z_{g,t}$ transmission generator start-up variable in hour t for generator g

1. Optimization Model

In this section, the structure of the bi-level optimization problem is described. The general formulation of a bilevel optimization is given below:

$$\min_{x \in X, y \in Y} F(x, y) \tag{1}$$

$$\begin{aligned} \text{st: } & G_i(x, y) \leq 0, \text{for } i \in \{1, 2, ..., I\} \\ & H_k(x, y) = 0, \text{for } k \in \{1, 2, ..., K\} \\ & y \in \underset{y \in Y}{\operatorname{argmin}} \{f(x, y) : g_j(x, y) \leq 0, \text{for } j \in \{1, 2, ..., J\}, \\ & H_m(x, y) = 0, \text{for } m \in \{1, 2, ..., M\}\} \end{aligned}$$

In the above formulation, $x, F(x, y), (G_i, H_i)$ are the optimization variables, objective function and constraints of the upper level problem. Whereas, $y, f(x, y), (g_i, h_i)$ are the optimization variables, objective function and constraints of the lower level problem. According to this formulation, the bilevel formulation of the transmission system and MG co-optimization is given below.

1.1. Upper Level Problem: Transmission System Economic Dispatch

In this study, renewable generation (eg: wind) in the transmission system is a key consideration. The uncertainty of the renewable forecast needs to be compensated by the transmission system generator reserves as well as the DR from the MG. The upper level transmission economic dispatch problem seeks to compute the optimal operation schedule including the generation output $P_{g,t}$, reserve requirement $R_{g,t}^{up}$,$R_{g,t}^{dn}$ and MG DR price $P_{g,t}^{dr}$ to minimize the total transmission system operation cost. The optimization variables are denoted by the vector x_t, and include:

$$x_t = [P_{g,t}, R_{g,t}^{up}, R_{g,t}^{dn}, P_t^{dr}]$$

The objective of the optimization is to minimize the transmission system operation cost including the generation cost, reserve cost, energy exchange cost with the MG and the MG DR cost. ***Objective function:***

$$\begin{aligned} F(\{x_t\}_{t=1}^{T}) &= \textstyle\sum_{t=1}^{T}\sum_{g=1}^{G}(C_{g,t}P_{g,t} + C_g^r(R_{g,t}^{up} + R_{g,t}^{dn}) \\ &- P_t^{im}C_t^{im} + P_t^{ex}C_t^{ex} \\ &+ P_t^{dr}(DR_t^{up} - DR_t^{dn})) \end{aligned}$$

The constraints for the transmission system are given below.

Power flow constraints:

$$-\,Line \le GSF * Pinj_t \le Line, t \in 1, ..., T \tag{2}$$

$$-\,Line \le GSF * Pinj_t^* \le Line, t \in 1, ..., T \tag{3}$$

$P_{inj,t}$ is the DC net power injection vector(ie: generation + wind - demand) for all the buses in each hour. $P_{inj,t}^*$ incorporates the wind forecast error, generator reserve and MG DR on top of $P_{inj,t}$. Eqn (2),(3) bounds the transmission line flows within the flow limits.

Generator constraints:

$$\underline{P}_g \le P_{g,t} \le \overline{P}_g, t \in 1, ..., T \tag{4}$$

$$(P_{g,t} + R_{g,t}^{up}) - (P_{g,t-1} - R_{g,t-1}^{dn}) \le \overline{R}_g, t \in 2, ..., T \tag{5}$$

$$\underline{R}_g \le (P_{g,t} - R_{g,t}^{dn}) - (P_{g,t-1} + R_{g,t-1}^{up}), t \in 2, ..., T \tag{6}$$

Eqn (4) bounds the generator generation within its capacities. Eqn (5),(6) ensure the generator ramping capability is not violated.

Power balance constraint:

$$\sum_{g=1}^{G} P_{g,t} + \mathbf{1}_{1\times N_b} * L_t + W^f = P_t^{im} - P_t^{ex}, , t \in 1, ..., T \tag{7}$$

Eqn (7) keeps the total system generation the same as the total system load.

Reserve constraints:

$$W_t^{up} <= DR_t^{up} + \sum_{g=1}^{G} R_{g,t}^{dn}, t \in 1, ..., T \tag{8}$$

$$W_t^{dn} <= DR_t^{dn} + \sum_{g=1}^{G} R_{g,t}^{up}, t \in 1, ..., T \tag{9}$$

W_t^{up} is deviation between the wind forecast and the scenario with the largest wind generation ,and could be compensated by less generation of the transmission generators (ie: R_t^{dn}) or more consumption of the MG dispachable load (ie: DR_t^{up}). W_t^{dn} is deviation between the wind forecast and the scenario with the smallest wind generation ,and could be compensated by more generation of the transmission generators (ie: R_t^{up}) or less consumption of the MG dispachable load (ie: DR_t^{dn}). Eqn (8), (9) requires enough generator reserve and MG DR to compensate the possible wind forecast deviation.

Demand response price constraint:

$$P_t^{dr} >= C_t^{d}, t \in 1, ..., T \tag{10}$$

Since the consumption of MG dispatchable load (to be discussed in the MG model) is valued with some utility. The MG DR provided by the dispatchable load has to get paid by a price greater than the utility which is reflected by Eqn (10).

Finally, the transmission system economic dispatch problem is formulated as:

$$\begin{aligned} &\min_{\{x_t\}_{t=1}^{T}} F\left(\{x_t\}_{t=1}^{T}\right) \\ &\quad \text{st: } (1) - (10) \\ &\qquad \mathbb{P}(eqn(3,5,6,8,9)) \geq 1 - \epsilon \end{aligned}$$

A chance-constrained approach is applied here for eqn(8,9) as [1] shows that a chance-constrained approach strikes a good balance between the system cost, reliability and wind penetration. The chance constraints in this study are required to meet the specified probability level 1 - ϵ jointly. It is also possible to write individual chance constraints, where each constraint i is required to meet a specified probability level 1 - ϵ_i individually. In the latter case, different i can be selected for each constraint allowing critical constraints or time periods to be managed in a robust way, with increased flexibility elsewhere.

1.2. Lower Level Problem: Microgrid Model

A comprehensive microgrid is considered in this work. The microgrid consists of a generator, a storage unit, and aggregated dispatchable and non-dispatchable loads, and is able to exchange power with the main grid. Since a microgrid typically covers a small local area with very limited power capacity, it thus has very limited power in the setting of the market price, it is assumed that the microgrid is a price taker in the problem formulation. The goal of the microgrid optimal dispatch model is to compute the generation schedule $P^m_{g,t}$, the battery energy state B_t, the battery charging decision Pb_t, microgrid energy import P^{im}_t and export P^{ex}_t schedule, dispatchable load profile L^d_t and the upward/downward DR^{up}_t, DR^{dn}_t it provides. The optimization variables are denoted by the vector y_t, and include:

$$y_t = [P^m_{g,t}, B_t, P^{im}_t, P^{ex}_t, DR^{up}_t, DR^{dn}_t, L^d_t]$$

The objective of the microgrid optimization is to minimize the microgrid operation cost including its generation cost, battery maintenance cost, energy exchange cost with the transmission system and maximize its dispatchable load utility and demand response revenue.

Objective function:

$$\begin{aligned} f(\{y_t\}^T_{t=1}) = &\sum_{t=1}^{T}(C^m_g P^m_{g,t} + C^b * B_t + P^{im}_t C^{im}_t - P^{ex}_t C^{ex}_t \\ &-C^d_t L^d_t - P^{dr}_t (DR^{up}_t + DR^{dn}_t)) \end{aligned}$$

The constraints for the MG are given below. The λ and μ variables next to the constraints are the dual variables for the inequality and equality constraints.

Generator constraints:

$$\underline{P}^m \leq P^m_{g,t} \leq \overline{P}^m, \lambda1_t, \lambda2_t, t \in 1, ..., T \tag{12}$$

Eqn (12) limits the generator's output to an upper and lower bound.

Dispatchable load constraint:

$$\underline{L}^d_t \leq L^d_t \leq \overline{L}^d_t, \lambda3_t, \lambda4_t, t \in 1, ..., T \tag{13}$$

Eqn (13) shows that the dispatchable loads need to stay within predefined bounds. The room between the dispatchable set point and the bounds could be used as DR to compensate wind forecast errors.

Demand response constraints:

$$L_t^d + DR_t^{up} \leq \overline{L}_t^d, \lambda 5_t, t \in 1, ..., T, , \tag{14}$$

$$Ld_t - DR_t^{dn} \geq \underline{L}_t^d, \lambda 6_t, t \in 1, ..., T, , \tag{15}$$

$$0 \leq DR_t^{up} \leq W^{up}, \lambda 7_t, \lambda 8_t, t \in 1, ..., T, , \tag{16}$$

$$0 \leq DR_t^{dn} \leq W^{dn}, \lambda 9_t, \lambda 10_t, t \in 1, ..., T, , \tag{17}$$

$$\tag{18}$$

Eqn (17), (15) restrains the demand response of dispatchable load within the diapatachable load bounds.

Storage constraints:

$$\underline{P}^b \leq P_t^b \leq \overline{P}^b, \lambda 11_t, \lambda 12_t, t \in 1, ..., T \tag{19}$$

$$\underline{B} \leq B_t \leq \overline{B}, \lambda 13_t, \lambda 14_t, t \in 1, ..., T \tag{20}$$

$$B_t = B_{t-1} + P_{t-1}^b, \mu 1_t, t \in 1, ..., T \tag{21}$$

Equations (19) and (20) restrict the storage's output power and the energy state to their upper and lower bounds. Equation (21) shows the transition of the storage energy state from one hour to the next. A positive/negative P_t^b value corresponds to charging/discharging of the battery.

Import and Export constraints:

$$0 \leq P_t^{im}, \lambda 15_t, t \in 1, ..., T \tag{22}$$

$$0 \leq P_t^{ex}, \lambda 16_t, t \in 1, ..., T \tag{23}$$

The MG import and export power is defined to be non-negative.

Power balance constraint:

$$P_t^m - P_t^b - L_t^i - L_t^d = P_t^{ex} - P_t^{im}, \mu 2_t, t \in 1, ..., T \tag{24}$$

Equation (24) requires the microgrid load to be met with different power sources.

Finally, the microgrid optimal dispatch can be formulated as:

$$\min_{\{y_t\}_{t=1}^T} f\left(\{y_t\}_{t=1}^T\right)$$
$$\text{st: } (12) - (25)$$

1.3. Reformulation to a Single Level Problem

When the lower level problem is convex and satisfies some regularity conditions, it can be replaced by its Karush-Kuhn-Tucker(KKT) conditions [2], yielding a single -level problem reformulation as below:

$$\begin{aligned}
&\min_{x\in X, y\in Y} F(x,y)\\
\text{st: } &G_i(x,y) \leq 0, \text{for } i \in \{1,2,...,I\}\\
&H_k(x,y) \leq 0, \text{for } k \in \{1,2,...,K\}\\
&g_i(x,y) \leq 0, \text{for } i \in \{1,2,...,j\}\\
&h_m(x,y) \leq 0, \text{for } m \in \{1,2,...,M\}\\
&\text{dual feasibility: } \lambda_i \geq 0, \text{for } i \in \{1,2,...,j\}\\
&\text{complementary slackness: } \lambda_i * g_i(x,y) = 0, \text{for } i \in \{1,2,...,j\}\\
&\text{stationarity: } \nabla L(x,y,\lambda,\mu) = 0\\
&where:\\
&L(x,y,\lambda) = f(x,y) + \sum_{i=1}^{J} \lambda_i * g_i(x,y) + \sum_{m=1}^{M} \mu_m * h_m(x,y)
\end{aligned}$$

The lower level problem in this study is a linear program and could thus be replaced by its KKT conditions. The KKT conditions are given below:

Stationarity:

$$C^m + \lambda 2_t - \lambda 1_t + \mu 2_t = 0 \tag{26}$$

$$-C^d - \lambda 4_t + \lambda 3_t - \mu 2_t = 0 \tag{27}$$

$$C^b + \lambda 13_t - \lambda 14_t + \mu 1_t = 0 \tag{28}$$

$$C_t^{im} - \lambda 14_t - \mu 2_t = 0 \tag{29}$$

$$-C_t^{ex} - \lambda 15_t + \mu 2_t = 0 \tag{30}$$

$$\lambda 8_t - \lambda 11_t - \mu 1_t - \mu 2_t = 0 \tag{31}$$

$$-P_t^{dr} - \lambda 8_t - \lambda 7_t + \lambda 5_t = 0 \tag{32}$$

$$-P_t^{dr} - \lambda 10_t - \lambda 9_t + \lambda 6_t = 0 \tag{33}$$

Dual feasibility:

$$\lambda 1 ... \lambda 16 \geq 0 \tag{34}$$

Complementary slackness:

$$\lambda 1_t * (P_t^m - \underline{P}^m) = 0 \tag{35}$$

$$\lambda 2_t * (P_t^m - \overline{P}^m) = 0 \tag{36}$$

$$\lambda 3_t * (L_t^d - \overline{L}_t^d) = 0 \tag{37}$$

$$\lambda 4_t * (L_t^d - \underline{L}_t^d) = 0 \tag{38}$$

$$\lambda 5_t * (DR_t^{up} + L_t^d - \overline{L}_t^d) = 0 \tag{39}$$

$$\lambda 6_t * (-DR_t^{dn} - L_t^d + \underline{L}_t^d) = 0 \tag{40}$$

$$\lambda 7_t * (DR_t^{up}) = 0 \tag{41}$$

$$\lambda 8_t * (DR_t^{up} - W^{up}) = 0 \tag{42}$$

$$\lambda 9_t * (DR_t^{dn}) = 0 \tag{43}$$

$$\lambda 10_t * (DR_t^{dn} - W^{dn}) = 0 \tag{44}$$

$$\lambda 11_t * (P_t^b - \overline{P}_t^b) = 0 \tag{45}$$

$$\lambda 12_t * (P_t^b - \underline{P}_t^b) = 0 \tag{46}$$

$$\lambda 13_t * (B_t - \overline{B}) = 0 \tag{47}$$

$$\lambda 14_t * (B_t - \underline{B}) = 0 \tag{48}$$

$$\lambda 15_t * (B_t) = 0 \tag{49}$$

$$\lambda 16_t * (B_t) = 0 \tag{50}$$

The reformulated single level problem has the following format:

$$\min_{\{x_t, y_t, \lambda_t, \mu_t\}_{t=1}^T} F\left(\{x_t, y_t, \lambda_t, \mu_t\}_{t=1}^T\right)$$
$$\text{st: } (1) - (50)$$

This reformulation is not easy to solve mainly due to the non-convexity in the complementary slackness and bilinear term in the objective function. The big-M reformulation is used to transform the complementary conditions to mixed integer constraints. The Mccormick envelope technique is applied to change the bilinear term into a single variable with additional linear constraints. The technical details of those two methods are given in the appendix.

With the big-M reformulation and Mccormick envelope methods, a bunch of binary variables and additional constants are introduced. The bilevel problem becomes a single level becomes mixed integer linear problem and could thus be solved with a wide range of commercial solvers such as Cplex and Gurobi.

2. Appendix

2.1. Big-M Method

The complementary conditions are set of constraints with the following format:

$$\lambda_i * g_i(x, y) = 0$$

Specifically, given a sufficiently large positive value M_i and binary value ϕ_i, the complementary conditions could be reformulated as below:

$$\begin{aligned} &-(1-\phi_i) * M_i \leq g_i(x, y) \\ &\lambda_i \leq \phi_i * M_i \end{aligned}$$

For a detailed treatment of the Big-M method, please refer to [8].

2.2. Mccormick Envelope

For a toy optimization problem of the following format:

$$\begin{aligned} \min\ Z = x * y \\ \underline{x} \leq x \leq \overline{x} \\ \underline{y} \leq y \leq \overline{y} \end{aligned} \tag{52}$$

The problem could be reformulated by introducing a new variable $w = x * y$ as follow:

$$\begin{aligned} \min\ Z = w \\ w \geq \overline{x} * y + x * \overline{y} - \overline{x} * \overline{y} \\ w \geq \underline{x} * y + x * \underline{y} - \underline{x} * \underline{y} \\ w \leq \overline{x} * y + x * \underline{y} - \overline{x} * \underline{y} \\ w \leq \underline{x} * y + x * \overline{y} - \underline{x} * \overline{y} \\ \underline{x} \leq x \leq \overline{x} \\ \underline{y} \leq y \leq \overline{y} \end{aligned} \tag{53}$$

For a detailed treatment of the Big-M method, please refer to [9].

[1] J. Liu, M. G. Martinez, B. Li, J. Mathieu, and C. L. Anderson, "A comparison of robust and probabilistic reliability for systems with renewables and responsive demand," in *System Sciences (HICSS), 2016 49th Hawaii International Conference on.* IEEE, 2016, pp. 2373–2380.

[2] A. Sinha, P. Malo, and K. Deb, "A review on bilevel optimization: from classical to evolutionary approaches and applications," *IEEE Transactions on Evolutionary Computation*, 2017.

[3] R. D. Zimmerman, C. E. Murillo-Sánchez, and R. J. Thomas, "Matpower's extensible optimal power flow architecture," in *Power & Energy Society General Meeting, 2009. PES'09. IEEE.* IEEE, 2009, pp. 1–7.

[4] G. L. R. W. Energy, "Eastern wind integration and transmission study," *Technical Report, National Renewable Energy Laboratory (NREL)*, 2010.

[5] C. Anderson and R. Zimmerman, "Wind output forecasts and scenario analysis for stochastic multiperiod optimal power flow," *PSERC Webinar*, pp. 1–38, 2011.

[6] E. W. E. Association *et al.*, *Wind energy-the facts: a guide to the technology, economics and future of wind power.* Routledge, 2012.

[7] J. Liu, G. Martinez, and C. L. Anderson, "Quantifying the impact of microgrid location and behavior on transmission network congestion," in *Proceedings of the 2016 Winter Simulation Conference.* IEEE Press, 2016, pp. 1745–1756.

[8] W. L. Winston, M. Venkataramanan, and J. B. Goldberg, *Introduction to mathematical programming.* Thomson/Brooks/Cole Duxbury; Pacific Grove, CA, 2003, vol. 1.

[9] P. M. Castro, "Tightening piecewise mccormick relaxations for bilinear problems," *Computers & Chemical Engineering*, vol. 72, pp. 300–311, 2015.