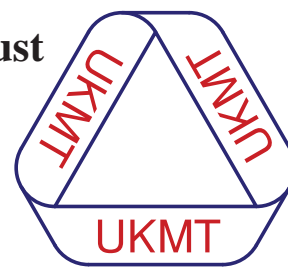# The United Kingdom Mathematics Trust

## Intermediate Mathematical Olympiad and Kangaroo (IMOK)

## Olympiad Hamilton Paper

### Thursday 16th March 2017

**All candidates must be in *School Year 10* (England and Wales), *S3* (Scotland), or *School Year 11* (Northern Ireland).**

## READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.

2. **The use of calculators, protractors and squared paper is forbidden.**
   Rulers and compasses may be used.

3. Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Cover Sheet on top.

4. Start each question on a fresh A4 sheet.
   You may wish to work in rough first, then set out your final solution with clear explanations and proofs.

   *Do not hand in rough work.*

5. Answers must be FULLY SIMPLIFIED, and EXACT. They may contain symbols such as $\pi$, fractions, or square roots, if appropriate, but NOT decimal approximations.

6. Give full written solutions, including mathematical reasons as to why your method is correct.
   Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.

7. **These problems are meant to be challenging!** The earlier questions tend to be easier; the last two questions are the most demanding.
   Do not hurry, but spend time working carefully on one question before attempting another.
   Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

## DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

- *Do not hurry, but spend time working carefully on one question before attempting another.*

- *Try to finish whole questions even if you cannot do many.*

- *You will have done well if you hand in full solutions to two or more questions.*

- *Answers must be FULLY SIMPLIFIED, and EXACT. They may contain symbols such as π, fractions, or square roots, if appropriate, but NOT decimal approximations.*

- *Give full written solutions, including mathematical reasons as to why your method is correct.*

- *Just stating an answer, even a correct one, will earn you very few marks.*

- *Incomplete or poorly presented solutions will not receive full marks.*

- ***Do not hand in rough work.***

**H1.** The diagram shows four equal arcs placed on the sides of a square. Each arc is a major arc of a circle with radius 1 cm, and each side of the square has length $\sqrt{2}$ cm.
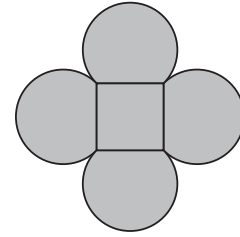
What is the area of the shaded region?

**H2.** A ladybird walks from $A$ to $B$ along the edges of the network shown. She never walks along the same edge twice. However, she may pass through the same point more than once, though she stops the first time she reaches $B$.
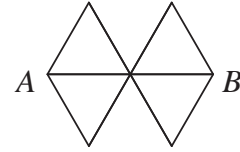
How many different routes can she take?

**H3.** The diagram shows squares $ABCD$ and $EFGD$. The length of $BF$ is 10 cm. The area of trapezium $BCGF$ is 35 cm$^2$.
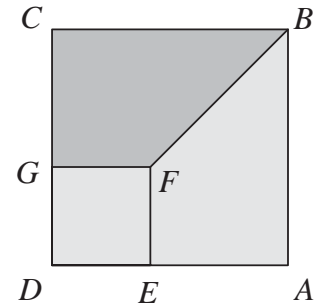
What is the length of $AB$?

**H4.** The largest of four different real numbers is $d$. When the numbers are summed in pairs, the four largest sums are 9, 10, 12 and 13.

What are the possible values of $d$?

**H5.** In the trapezium $ABCD$, the lines $AB$ and $DC$ are parallel, $BC = AD$, $DC = 2AD$ and $AB = 3AD$.
The angle bisectors of $\angle DAB$ and $\angle CBA$ intersect at the point $E$.

What fraction of the area of the trapezium $ABCD$ is the area of the triangle $ABE$?

**H6.** Solve the pair of simultaneous equations

$$x^2 + 3y = 10 \qquad \text{and}$$

$$3 + y = \frac{10}{x}.$$