Hamilton-style paper (problems only)

1. The five-digit integer $6a4b8$ is divisible by 99. Find the digits a and b.

2. Stamps cost 4p, 6p and 9p. A collector buys at least one stamp of each type and spends exactly 60p. In how many different ways can this be done? (Two ways are different if the numbers of any stamp type differ.)

3. $ABCD$ is a trapezium with $AB \parallel CD$ and $AB = 2CD$. The diagonals AC and BD intersect at P. Prove that $AP : PC = BP : PD = 2 : 1$, and find the ratio of the areas of triangles APB and CPD.

4. Four distinct integers are chosen. The six pairwise sums (in no particular order) are $3, 4, 7, 8, 11, 12$. Find the possible value(s) of the largest integer.

5. Find all integers n with $0 \le n \le 2026$ and $n \ne 2$ for which $\dfrac{1}{n^2 - 4}$ is a terminating decimal.

6. Find all real numbers x such that

$$\lfloor x \rfloor + \lfloor 2x \rfloor + \lfloor 3x \rfloor = \lfloor 6x \rfloor.$$