

# UK Maths Trust

## Intermediate Mathematical Olympiad

HAMILTON PAPER

**Thursday 20 March 2025**

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MARKETS

*England & Wales: Year 10 | Scotland: S3 | Northern Ireland: Year 11*

These problems are meant to be challenging.

Try to finish whole questions even if you cannot do many; you will have done well if you hand in a complete solution to two or more questions.

### Instructions

1. Time allowed: **2 hours**.
2. **Full written solutions – not just answers – are required**, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then write up your best attempt. Do not hand in rough work.
3. **One complete solution will gain more credit than several unfinished attempts.**
4. **Each question carries 10 marks.**
5. The use of rulers, set squares and compasses is allowed, but **calculators and protractors are forbidden**.
6. Start each question on an official answer sheet on which there is a **QR code**.
7. If you use additional sheets of (plain or lined) paper for a question, please write the following in the top left-hand corner of each sheet. (i) The question number. (ii) The page number for that question. (iii) The digits following the ‘:’ from the question’s answer sheet QR code. **Please do not write your name or initials on additional sheets.**
8. **Write on one side of the paper only**. Make sure your writing and diagrams are clear and not too faint. (Your work will be scanned for marking.)
9. **Arrange your answer sheets in question order before they are collected**. If you are not submitting work for a particular problem, please remove the associated answer sheet.
10. To accommodate candidates in other time zones, please do not discuss the paper online until **12pm GMT on Saturday 22 March**, when the video solutions will be released.
11. Do not turn over until told to do so.

Enquiries about the Intermediate Mathematical Olympiad should be sent to:

[challenges@ukmt.org.uk](mailto:challenges@ukmt.org.uk)      [www.ukmt.org.uk](http://www.ukmt.org.uk)

*supported by* **Overleaf**

## Advice to candidates

- ◊ *Do not hurry, but spend time working carefully on one question before attempting another.*
- ◊ *Try to finish whole questions even if you cannot do many.*
- ◊ *You will have done well if you hand in full solutions to two or more questions.*
- ◊ *Your answers should be fully simplified, and exact. They may contain symbols such as  $\pi$ , fractions, or square roots, if appropriate, but not decimal approximations.*
- ◊ *Give full written solutions, including mathematical reasons as to why your method is correct.*
- ◊ *Just stating an answer, even a correct one, will earn you very few marks.*
- ◊ *Incomplete or poorly presented solutions will not receive full marks.*
- ◊ *Do not hand in rough work.*

1. There is a water tank which is a cuboid with base area,  $x$ , and height,  $h$ . Standing in the tank are three identical metal columns, each a cylinder of height,  $h$ , and base a circle of area,  $y$ . The tank is then filled with a volume,  $v$ , of water.

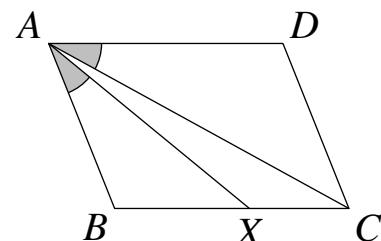
When one column is removed, the water level drops by 50.

When a second column is removed, the water level drops by a further 30.

Find how much further the water level drops when the last column is removed.

2. There are two types of tile to tile a floor. One is a square tile 5 by 5 and the other is a rectangular tile 4 by 9. The floor is 140 long by 80 wide. It is to be completely tiled with no gaps, none of them overlapping and none going outside the edges of the floor using the minimum number of tiles possible. Demonstrate how to do this and prove that it cannot be done with fewer tiles.

3. A parallelogram  $ABCD$  has a point  $X$  on  $BC$  such that  $\angle BAX = \angle CAD$  and  $AX : AC = 5 : 7$ .  
Find the ratio of  $CX : XB$ .



4. A keypad is a 3 by 3 grid with the digits 1 to 3 in order on the top row, 4 to 6 in order on the second row and 7 to 9 in order on the third row.

Fred has the keypad upside-down (i.e rotated through  $180^\circ$ ) and types his four-digit PIN as if the keypad were the right way up. The number he types in turns out to be 2025 more than  $k$  times his original PIN where  $k$  is a positive integer. Prove there is only one possible PIN and find it.

5. Find all integer values of  $n$ ,  $0 \leq n \leq 2025$ ,  $n \neq 3$ , for which  $\frac{1}{n^2 - 9}$  can be written as a terminating decimal.

6. A vegetable patch is a square grid of  $n$  by  $n$  uncleared cells with  $n \geq 3$  and  $n$  odd. Mo the mole is in the centre cell and digs it up to clear it.

Mo then digs up cells one at a time according to the following rule. Mo can dig up an uncleared cell that is both adjacent horizontally or vertically to a cleared cell and in the middle of a line of three uncleared cells in the vegetable patch.

After digging up  $M$  squares, Mo can dig no further. Find in terms of  $n$  the smallest  $M$  for which this can happen and prove there is no smaller value.