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A two-phase method of forming a granular representation of signals

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ABSTRACT

This study focuses on a granular representation of signals. The development process dwells upon the use of the principle of justifiable granularity encountered in Granular Computing and the least square error method. This process consists of two phases where the construction of granular representatives of a family of signals (temporal data) is realized by invoking the design at the local and global level. At the local design level involving individual elements of the universe of discourse (time moments), the principle of justifiable granularity is applied to construct (a vertical part) information granules. At the global level, the least square error method is invoked to develop the bounds (envelopes) of the information granules already formed at the local level. Experimental studies are reported for the granular representation of synthetic data and publicly available ECG signals. Furthermore we demonstrate that the proposed approach can be used to construct fuzzy sets of type-2.

1. Introduction

For the signal processing, we are faced with several fundamental types of issues such as classification [48,53,54,67], prediction [50–52], compression and representation [45–47,66]. Owing to the uncertainties and the nonlinearities of the signals as well as the diversity of signals belonging to the same class, it becomes difficult or impractical to establish the exact mathematical models of signals. In order to cope with these issues, Computational Intelligence has been investigated as a viable conceptual and algorithmic framework. For example, some research is carried out with a focus on the technology of fuzzy sets with applications reported in forecasting time series, identification of non-linear dynamic systems and so on [35–39,62,64], as it could capture uncertainty and diversity of signals and ensuing systems, for instance, uncertainty of the rules and the noisy data. Signal processing is also carried out with the aid of neural networks, evolutionary algorithms (such as particle swarm optimization, differential evolution and others) and Granular Computing (owing to the development and usage of granular models) for signal representation, compression, classification and prediction [40–44,49,63,65]. The importance and usefulness of Granular Computing stem from the fact that information granules involved in the modeling temporal data help capture and quantify a diversity of signals and build their abstract representation.

As an emerging paradigm of information processing, Granular Computing plays a pivotal role in the construction of human-centric models [5,7,14,15,59–61]. Let us recall that Granular Computing is about acquiring, processing, interpreting and communicating information granules [3,4]. Information granules are formalized as sets, fuzzy sets, rough sets and the like [2]. Note that the central thought behind information granules is a notion of abstraction: instead of considering individual elements exhibiting some closeness or resemblance, we arrange them together by forming a single information granule. Information granules are abstract entities which arise through the process of formation of an abstract view at a certain real-world phenomenon or system. While designing granular models, the construction of information granules is of paramount relevance and constitutes a challenging problem. This problem has been intensively studied [1,9,29–34,55–57], however a number of issues are still open. A way of forming granular representatives of a collection of data or signals is discussed in [9] and [29]. In [9], a granular representative of a collection of signals is obtained by using fuzzy clustering, more specifically fuzzy C-means (FCM) [16]. While in [29], the method just concentrate on the construction of interval granules.

The key objective of this study is to establish a two-phase development process of a granular representation of signals. In the design, we engage the principle of justifiable granularity (when constructing individual information granules for the corresponding time moments; such granules are locally formed information granules) and the Least Square Error (LSE) approximation (leading

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to the refinement and construction of an information granule of a global nature). There are three evident advantages for the proposed method: at every time moment, the local information granule is reflective of the locally available data and in the design we capture the essential properties of the data expressed in terms of the measures of coverage and specificity. The formation of the global information granule is completed with the least square error method, in which way the properties of locally formed information granules are retained and reconciled. In comparison to the methods introduced in [9] and [29], the proposed method could be applied to construct both interval and fuzzy granules and is not impacted by a choice of numeric values of some parameters of the clustering method.

In what follows, we assume that we are concerned with space $\boldsymbol{X}=\{x_1,x_2, \ldots, x_N\}$, $\boldsymbol{X}\subseteq\boldsymbol{R}$ and we are also concerned with n signals located in some real-valued space $\boldsymbol{Y}\subseteq\boldsymbol{R}$ for each x_i ($x_i \in \boldsymbol{X}$) being provided in the following format:

$x_1 \quad y_{11}, y_{12}, \ldots, y_{1n}$
$x_2 \quad y_{21}, y_{22}, \ldots, y_{2n}$

$x_N \quad y_{N1}, y_{N2}, \ldots, y_{Nn}$

In numerous real-world problems, we encounter signals belonging to a certain class. The intent is to find a representative of such signals so that this representative captures the essence of the data to the highest extent. We advocate that when a collection of numeric signals is characterized, the description (prototype) has to account for the inherent diversity existing within the family of signals and be made at the higher level of abstraction in the form of an information granule rather than a numeric signal (for example, an average of all signals belonging to the same class).

The rest of this paper is structured as follows: In Section 2, the underlying concepts are briefly reviewed; the formulation of the problem along with its detailed two-phase solution is presented in Section 3; experimental studies are reported in Section 4; in Section 5, the conclusions are drawn.

2. Preliminaries

This section serves as a brief introduction to the concepts used throughout the paper. In particular, we bring forward the principle of justifiable granularity [8,17].

The principle of justifiable granularity, originally introduced in [8], serves as a fundamental design vehicle to form information granules [13] realized in the presence of experimental evidence (data). It constitutes a generic method for the designing of information granules in Granular Computing [23–27]. The principle exhibits a significant deal of generality. First, it applies to a variety of formal settings of Granular Computing. Second, it applies to various formats of experimental evidence that could be composed of information granules coming as intervals, fuzzy sets, probability density functions and others not being exclusively confined to numeric data.

In its generic version, the construction of information granule is carried out in the presence of numeric one-dimensional data. It translates into a certain optimization problem, in which we maximize the following intuitively appealing criteria: (i) high data coverage and (ii) high specificity of the constructed information granule. Concisely speaking, coverage, cov(.), expresses an extent to which the constructed information granule "covers" (includes) the available experimental data. The higher the coverage, the more legitimate (justifiable) the formed information granule tends to become. The specificity, sp(.), as the name stipulates, quantifies how detailed the information granule is. Intuitively, the highest specificity is achieved for an information granule consisting of a single element, $\mathrm{sp}(\{z\})=1$. The more elements contribute to the information granule, the lower its specificity tends to become. It is easy to note that the increase in the coverage yields lower specificity values and vice versa. As both of the requirements have to be satisfied, one can consider a product of coverage and specificity and regard it as a suitable performance index whose value have to be maximized with respect to the parameters of the information granule. In case of interval information granule, these parameters are the bounds of the interval. In situation when we focus on fuzzy sets, those are the parameters of fuzzy sets (for instance, in case of a triangular fuzzy set, the parameters are modal value and lower and upper bounds).

3. The two-phase design of granular representation of signals

The development of the granular representation of signals is completed in two phases. The local phase is concerned with the formation of an information granule for individual elements of the universe (viz. discrete time moments). The global phase focuses on the development of envelopes (bounds) completed over all information granules already produced at the local basis.

3.1. Formulation for the two-phase optimization method

As noted, the development process consists of the two essential construction phases:

(i) the determination of the parameters of each vertical information granule. For each x_i in $\boldsymbol{X}$, we construct the corresponding information granule A_i by invoking the principle of justifiable granularity. Because of the focus of this construct, we refer to A_i as the vertical information granule. It is referred to as the local granule also, as the vertical information granule constructed for each x_i is part of the final result. The analytical form of A_i is predetermined and its parameters are estimated. Obviously, the parameters depend on the type of the considered information granule. For instance, in case of an interval, we have a triple of numbers (a_i, m_i, b_i) (the detailed algorithm about how to obtain these parameters is given in Section 3.2) describing the lower bound, the central point and the upper bound of the interval. If a fuzzy set, which is described by a triangular, parabolic or square root membership function is considered, the individual coordinates concern the lower bound, modal value, and the upper bound of the fuzzy set.

(ii) the determination of the envelope functions (envelopes) of the vertical granules formed over $\boldsymbol{X}$. Having (a_i, m_i, b_i) constructed at the local phase, the parameters of the envelopes are determined through curve fitting. This optimization problem falls under the rubric of the least square error approximation. More specifically, given the parametric form of the fitting functions, their parameters are estimated. With this regard, we form three functions pertaining to the bounds and modal values of the vertical information granules. For the data (x_i, a_i), ($i=1,2, \ldots, N$), one constructs f^-. The formation of f is carried out on a basis of (x_i, m_i) whereas $f^+$is constructed on a basis of (x_i, b_i). Furthermore the obvious inequality $f^- \le f \le f^+$ implies a constraint-based optimization, which has to be taken into consideration in the overall development process. It is noticeable that now the parameters of the individual fitting functions are considered together in the approximation of the envelopes of the information granules. This step realizes the global phase and the final result produces an information granule of a global character. The essence of this two-phase design is depicted in Fig. 1.

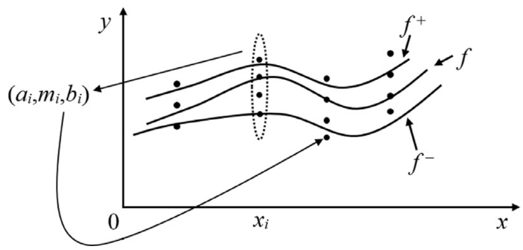

Fig. 1. A two-phase design process of the granular representative of signals.

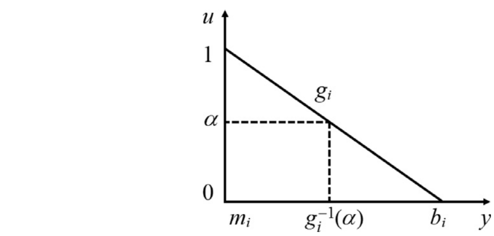

Fig. 2. Plot of a triangular membership function $g_i(y)$ used in computing specificity.

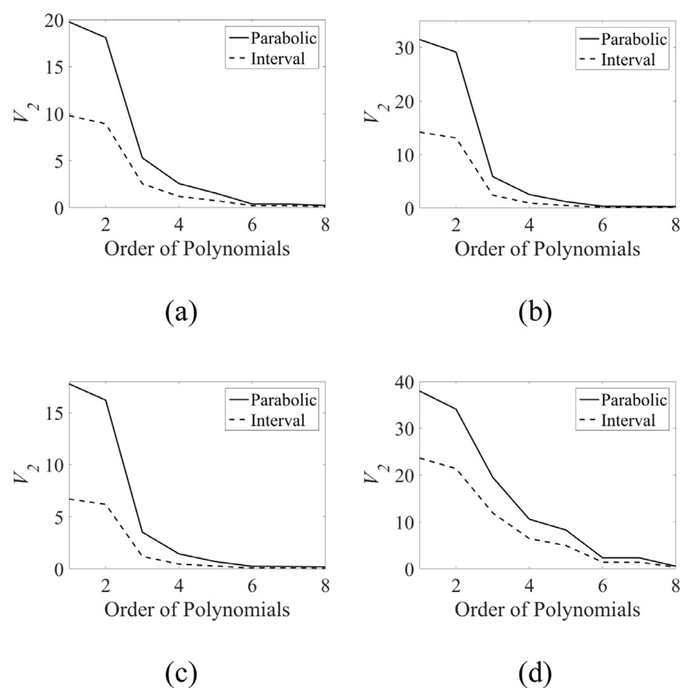

Fig. 3. Plots of V_2 for four datasets along with the change of the order of the basis (a) triangular; (b) parabolic; (c) square root; (d) Gaussian; solid line—granular representation of signals formed via fuzzy set, dashed line—interval information granules.

3.2. Determination of the information granules with the aid of the principle of justifiable granularity

Now we proceed with the detailed construction of the information granules. As noted, its parameters are determined for each element of the universe of discourse (time domain). For the given x_i, the optimization of this information granule is completed in the presence of experimental data $\boldsymbol{D_i} = \{y_{i1}, y_{i2}, \ldots, y_{in}\}$ by engaging the principle of justifiable granularity. The product of the coverage and specificity is maximized by changing the values of the bounds of the membership function. The optimized performance index is expressed as $\tilde{Q}_i = cov(A_i) * sp(A_i)$, where A_i is the information granule formed for some x_i. Prior to running the detailed optimization algorithm, one has to choose the functional form of the membership function A_i (for example, a triangular membership function). We start by forming the numeric representative of the data $\boldsymbol{D_i}$. Several alternatives (for example, the median, weighted average or a mode value) can be chosen here. In what follows, we confine ourselves to a weighted average that is a sound and commonly used alternative

$$m_i = \frac{\sum_{j=1}^{n} y_{ij} u_{ij}}{\sum_{j=1}^{n} u_{ij}}, y_{ij} \in \boldsymbol{D_i}. \quad (1)$$

where u_{ij} is the corresponding weight for each y_{ij}, i=1,2,…,N and j=1,2,…,n; if for each y_{ij} the corresponding weight u_{ij} is unknown, then just regard the average of all points y_{ij} as m_i.

Next, we carry out an optimization of the bounds a_i and $b_i$of the support of the membership function. In light of the principle of justifiable granularity, these parameters are optimized by maximizing the performance index $\tilde{Q}_i$. Here we describe only the optimization of the upper bound b_i (as the optimization of a_i is completed in the same manner). Compute the $range_i$, $range_i = y_i^{\max} - m_i$, where $y_i^{\max} = \max(\{y_{i1}, y_{i2}, \ldots, y_{in}\})$. According to [17], the functions describing coverage and specificity are described by (2) and (3). As the concept "coverage" is considered, the minimum between $g_i(y_{ik})$ and u_{ik} is taken. Then the $cov(A_i)$ is just the sum of these minimal values for all of the data points y_{ik} ($m_i \le y_{ik} \le y_i^{\max}$). If u_{ik} is unknown, just take the sum of the membership grades$g_i(y_{ik})$of all the points y_{ik} ($y_{ik} \in [m_i, y_i^{\max}]$) to be $cov(A_i)$

$$cov(A_i) = \sum\nolimits_{k: m_i \le y_{ik} \le y_i^{\max}} \min(g_i(y_{ik}), u_{ik}). \quad (2)$$

The specificity of granule is used to evaluate how specific the granule is and the highest specificity (equal to 1) is achieved for a single point set, which means the more data the granule contains (covers), the less specific the granule tends to become. To express this phenomenon and involve the character of the granule (the character of its membership function for the fuzzy granule), the specificity of A_i (for example, if $g_i(y)$ is the membership function, see Fig. 2 in which the triangular membership function is taken as an example) is described by (3). Following Fig. 2, it could be concluded that as m_i is fixed, for a given α, with the increase of b_i, the value of $1 - \frac{g_i^{-1}(\alpha) - m_i}{range_i}$does not increase, then $sp(A_i)$ does not increase. On the other hand, for a given y_{ik} ($y_{ik} \in [m_i, b_i]$) with the increase of b_i, the value of $\min(g_i(y_{ik}), u_{ik})$ does not decrease, hence $cov(A_i)$ does not decrease. There is an apparent conflict between the measures of $cov(A_i)$ and $sp(A_i)$.

$$sp(A_i) = \int_0^1 \left(1 - \frac{g_i^{-1}(\alpha) - m_i}{y_i^{max} - m_i}\right) d\alpha = \int_0^1 \left(1 - \frac{g_i^{-1}(\alpha) - m_i}{range_i}\right) d\alpha. \quad (3)$$

As the functions $cov(A_i)$ and $sp(A_i)$ are in conflict, the performance function $\tilde{Q}_i(\tilde{Q}_i = cov(A_i) * sp(A_i))$ is a product of these two criteria. Thus we obtain

$$b_i^{opt} = \operatorname{argmax}(\tilde{Q}_i(b_i)). \quad (4)$$

Once a_i has been optimized based on the data points positioned to the left of m_i, we compute the sum of the following components

$$Q_i = \tilde{Q}_i(a_i^{opt}) + \tilde{Q}_i(b_i^{opt}). \quad (5)$$

which could be regarded as a performance of the constructed information granule. The higher the value of Q_i, the better the relevance of the constructed information granule. For each x_i, the relevance of the associated granule can be weighted in terms of its performance. To accomplish this, we determine the maximal value among Q_is, say

$$Q_{\max} = \max_{i=1,2,\ldots,N} Q_i.$$

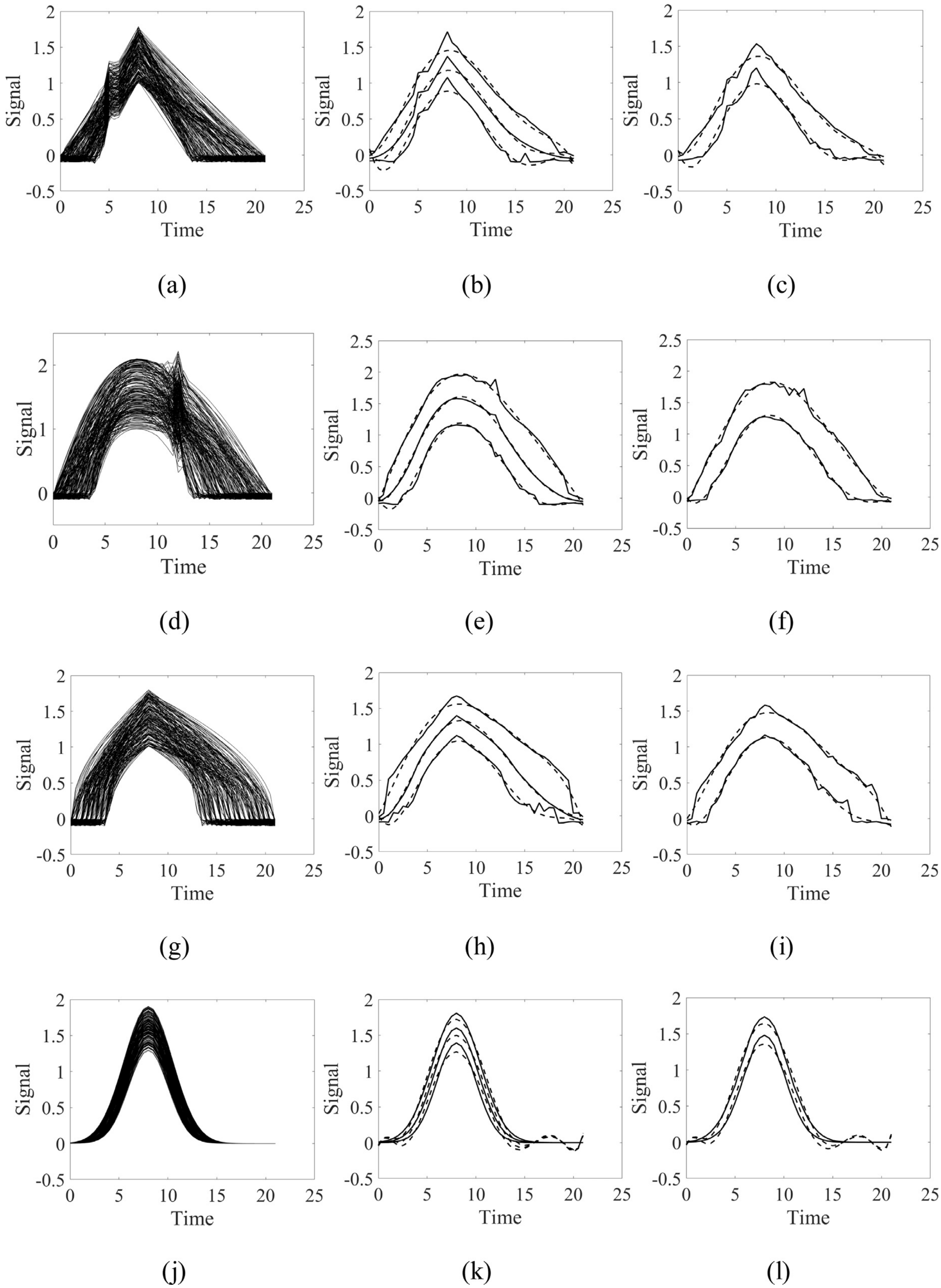

Fig. 4. Plots of four types of signals and their corresponding granular representation considering fuzzy sets and interval granules: (a), (b), (c) triangular; (d), (e), (f) parabolic; (g), (h), (i) square root; (j), (k), (l) Gaussian; (b), (e), (h), (k) granular representation under fuzzy sets; (c), (f), (i), (l) granular representation under interval granules; solid line—the characteristic functions (formed in the first phase), dashed line—the fitting lines ($f^-(x)$, $f(x)$ and $f^+(x)$).

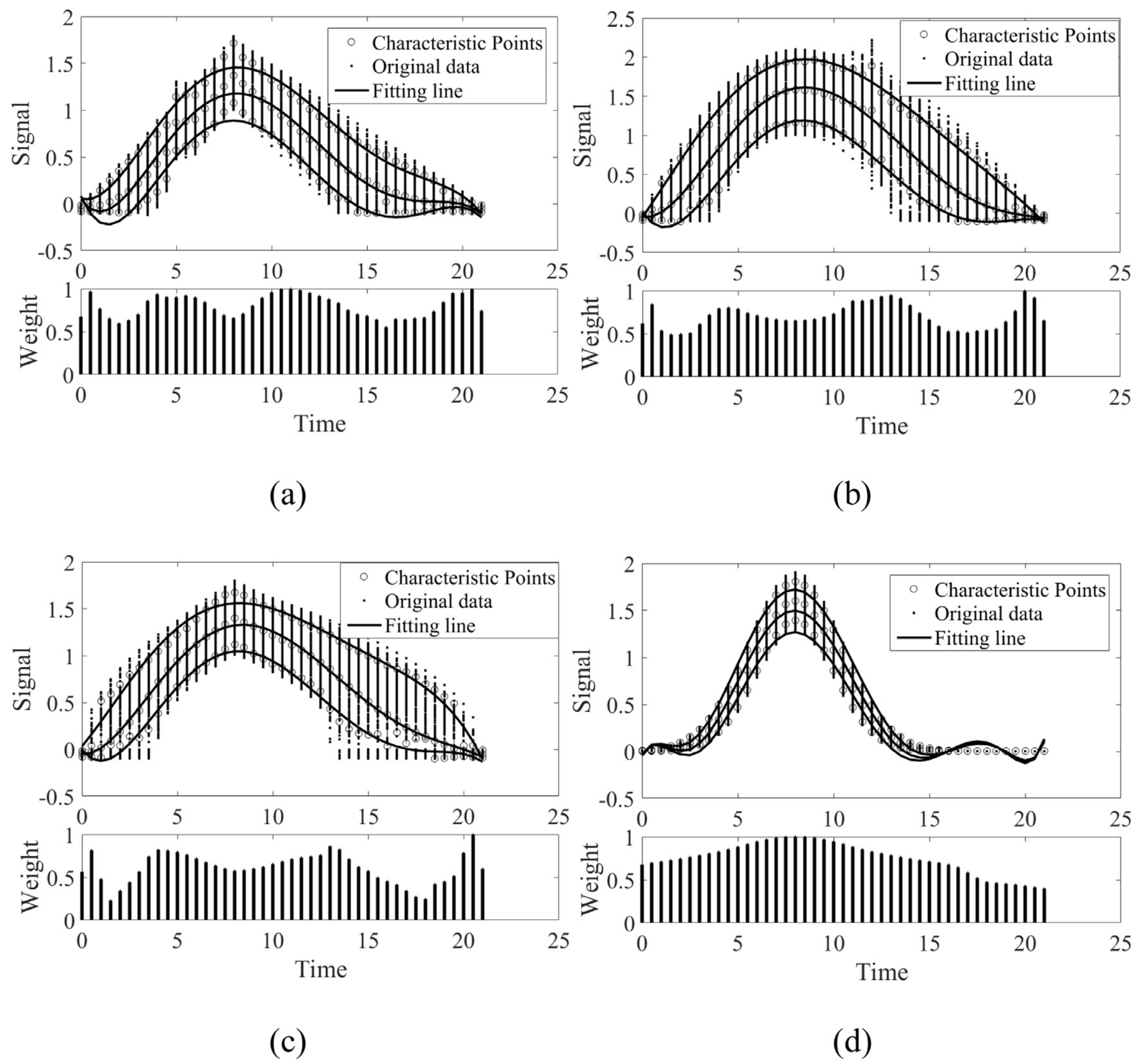

Fig. 5. Original data versus the global granular representations along with the weights of the corresponding formed information granules under fuzzy sets for four types of synthetic signals (a) triangular; (b) parabolic; (c) square root; (d) Gaussian.

Then we introduce the ratio $\xi_i = \frac{Q_i}{Q_{\max}}$, which serves as the performance indicator of information granule A_i. This indicator will be used at the second phase of the development of the global fitting. For convenience, use a_i, m_i and b_i to denote the optimized parameters of the information granules at x_i.

The overall quality of the developed information granules expressed over all the elements of the universe of discourse (space) is captured by the following sum

$$V_1 = \sum_{i=1}^{N} Q_i. \tag{6}$$

The higher the value of V_1, the better the representation of membership grades realized in terms of the information granules.

3.3. Construction of the functions of the bounds and modal values

The use of the principle of justifiable granularity has resulted in the collection of optimized lower and upper bounds, namely $\{a_1,a_2,\ldots,a_N\}$ and $\{b_1,b_2,\ldots,b_N\}$, and the modal values $\{m_1,m_2,\ldots,m_N\}$ associated with the corresponding values in the space $\boldsymbol{X} = \{x_1,x_2,\ldots,x_N\}$. For them, we complete the construction of fitting the bounds of the envelope at the global level. Recall that the available data sets come in the form of the weighted triples (a_i, m_i, b_i), namely

$$\boldsymbol{D_a} = \{(x_i, a_i, \xi_i) | x_i \in \boldsymbol{X}, a_i \in \boldsymbol{Y}, i = 1, 2, \ldots, N\},$$

$$\boldsymbol{D}_m = \{(x_i, m_i, \xi_i) | x_i \in \boldsymbol{X}, m_i \in \boldsymbol{Y}, i = 1, 2, \ldots, N\},$$

$$\boldsymbol{D_b} = \{(x_i, b_i, \xi_i) | x_i \in \boldsymbol{X}, b_i \in \boldsymbol{Y}, i = 1, 2, \ldots, N\}.$$

The functions $f^-(x)$, $f(x)$ and $f^+(x)$ are optimized by minimizing the weighted least square error criterion. The performance index (to be minimized) and the associated constraints are formulated in the following way

$$V_2 = \sum_{i=1}^{N} \xi_i [(f^-(x_i) - a_i)^2 + (f(x_i) - m_i)^2 + (f^+(x_i) - b_i)^2] \tag{7}$$

subject to constraints

$$f^-(x_i) - f(x_i) \le 0, f(x_i) - f^+(x_i) \le 0, i = 1, 2, \ldots, N.$$

The constraints present in the problem are handled by introducing a collection of Lagrange multipliers thus converting the problem into an unconstraint one. According to [6], the optimization procedure is realized as follows

1) Solution with the use of the method of Lagrange multipliers

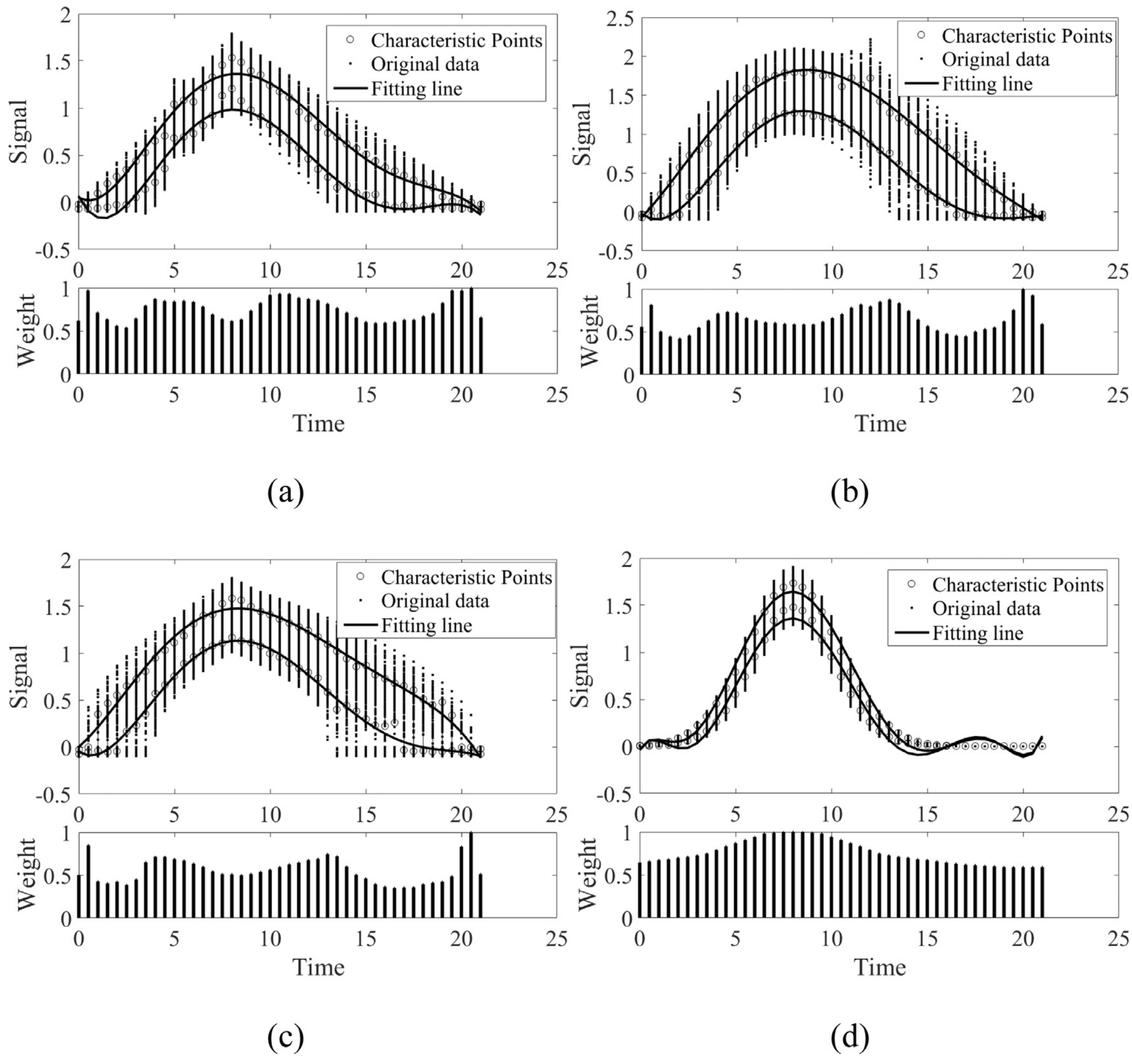

Fig. 6. Original data versus the global granular representations along with the weights of the corresponding formed information granules under interval information granules for four types of synthetic signals (a) triangular; (b) parabolic; (c) square root; (d) Gaussian.

Let us assume that the functions present in the inequality constrains are convex and twice continuously differentiable [6,11]. According to (7) and the method discussed in [6], the inequality constraints could be converted to equality constraints by introducing a vector of auxiliary variables $\mathbf{z} = [z_1, z_2, ..., z_{2N}]$. This leads to the following format of the optimization problem

$$V_2 = \sum_{i=1}^{N} \xi_i[(f^-(x_i) - a_i)^2 + (f(x_i) - m_i)^2 + (f^+(x_i) - b_i)^2] \tag{8}$$

subject to constraints

$$f^-(x_i) - f(x_i) + z_i^2 = 0,\, f(x_i) - f^+(x_i) + z_{i+N}^2 = 0,\, i = 1, 2, ..., N.$$

In the sequel, we introduce the penalty factor $c > 0$ and a vector of Lagrange multipliers $\lambda = [\lambda_1, \lambda_2, ..., \lambda_{2N}]$, which transform the problem of constrained optimization to an unconstrained one. Denote the vector of parameters to be optimized as $\mathbf{w} = [w_1, w_2, ..., w_d]$, where d stands for the dimensionality of the solution space encountered in the optimization problem. Let us also use the notation $\psi_i(\mathbf{w}) = f^-(x_i) - f(x_i), \psi_{i+N}(\mathbf{w}) = f(x_i) - f^+(x_i)$, $i = 1, 2, ..., N$. Then the augmented objective function involving the Lagrange multipliers reads as

$$\bar{L}(\mathbf{w}, \lambda, \mathbf{z}, c) = \sum_{i=1}^{N} \xi_i\left[(f^-(x_i) - a_i)^2 + (f(x_i) - m_i)^2 + (f^+(x_i) - b_i)^2\right]$$
$$+ \sum_{i=1}^{2N}[\lambda_i(\psi_i(\mathbf{w}) + z_i^2) + \frac{1}{2}c(\psi_i(\mathbf{w}) + z_i^2)^2]. \tag{9}$$

The overall performance index to be optimized comes in the form [6,11]

$$L(\mathbf{w}, \lambda, c) = \sum_{i=1}^{N} \xi_i[(f^-(x_i) - a_i)^2 + (f(x_i) - m_i)^2 + (f^+(x_i) - b_i)^2]$$
$$+ \frac{1}{2c}\sum_{i=1}^{2N}(s_i^2 - \lambda_i^2) \tag{10}$$

where $s_i = \max\{0, \lambda_i + c\psi_i(\mathbf{w})\}, i = 1, 2, ..., 2N$.

The resulting problem can be solved by any technique of unconstrained optimization, say the Powell-Hestenes-Rockfellar (PHR) method [18].

The detailed optimization processes of the PHR method aimed at the optimization of (10) comes as a sequence of the following steps:

Step 0: Initialization: fix some initial values $\mathbf{w_0}$, $\boldsymbol{\lambda_1}$, $c_1 = 2$, $0 < \varepsilon = 10^{-5} << 1$, $\varsigma = 0.8 \in (0, 1)$, $\eta = 2 > 1$ and set $k = 1$.

Step 1: Solve the following unconstrained sub-problem with the initial point $\mathbf{w_{k-1}}$:

$$\min L(\mathbf{w}, \lambda_\mathbf{k}, c_k)$$

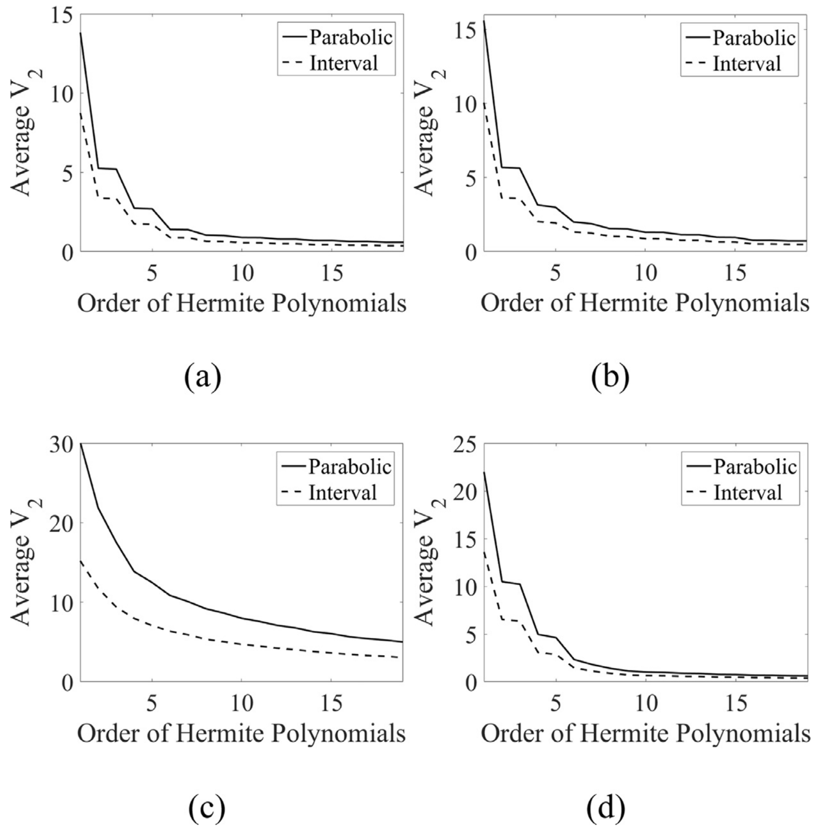

Fig. 7. Plot of the Average V_2 for all the values of σ in [0.5,20] with step width 0.5 for four classes of heart beats: (a) normal beat; (b) left bundle branch block beat; (c) premature ventricular contraction; (d) atrial premature beat; solid line—granular representation of signals formed via fuzzy set, dashed line—interval information granules.

using the Broyden–Fletcher–Goldfarb–Shanno algorithm [19]; the optimal solution is denoted as $\mathbf{w_k}$;

Step 2: Check the terminate condition: if $\beta_k = \left(\sum_{i=1}^{2N}\left[\max\left(\psi_i(\mathbf{w_k}), -\frac{\lambda_{ki}}{c_k}\right]^2\right)^{1/2} \le \varepsilon$, stop and return the feasible solution $\mathbf{w_k}$; else go to step 3;

Step 3: Update the penalty factor: if $\beta_k \ge \varsigma\beta_{k-1}$, set $c_{k+1} = \eta c_k$ else set $c_{k+1} = c_k$;

Step 4: Update the multipliers: $\lambda_{(k+1)i} = \max\{0, \lambda_{ki} + \psi_i(\mathbf{w_k})\}, i = 1, 2, \ldots, 2N$;

Step 5: Increment $k = k + 1$ and go to step 1.

2) Solution with the differential evolution algorithm

If the required assumptions (differentiability) are not satisfied, one can confine to some methods coming from the area of population-based optimization. Differential evolution (DE) [10] [12] arises here as a viable option. There are several reasons behind this choice. DE has been found efficient and being competitive to other techniques such as particle swarm optimization (PSO), genetic algorithm (GA) and others. DE is easy to use, robust and requires few variables to tune. Here the symbols are the same as being defined above.

The DE optimization applied to this particular problem can be outlined as follows [12]

(i) Initialization: p individuals are selected randomly (usually p equals to be 10 times higher than the dimensionality of the parameter vector to be optimized) in the searching space (for each parameter which should be optimized, choose a suitable space for the corresponding problem, for instance, it could be given as a symmetric interval and the length of the interval could be tuned until the performance index doesn't decrease with several runs of the algorithm or formulated as $[x^-, x^+]$ where $x^- = x_{\min} - \rho * \text{sign}(x_{\min}) * x_{\min}$, $x^+ = x_{\max} + \rho * \text{sign}(x_{\max}) * x_{\max}$ and $x_{\min} = \min_{i=1,2,\ldots,N} x_i$, $x_{\max} = \max_{i=1,2,\ldots,N} x_i$, $\rho \ge 0$). Set up the initial generation $\boldsymbol{P^0} = \{\mathbf{w}_1^0, \mathbf{w}_2^0, \ldots, \mathbf{w}_p^0\}$ and evaluate the value of the fitness function V_2;

(ii) Variation: for the ith intermediate product $\tilde{\mathbf{w}}_i^{G+1}$, select 3 different trial vectors from the Gth generation $\boldsymbol{P^G}$: $\mathbf{w}_j^G, \mathbf{w}_k^G, \mathbf{w}_l^G$, $1 \le j \ne k \ne l \le p$ and make $\tilde{\mathbf{w}}_i^{G+1} = \mathbf{w}_j^G + F * (\mathbf{w}_k^G - \mathbf{w}_l^G)$, $i \ne j \ne k \ne l$ where F is the mutation factor which can be set as 0.5 ($i = 1, 2, \ldots, p$);

(iii) Crossover: the crossover happens between the Gth generation and its mutation intermediate products. It is realized as follows:

$$\hat{w}_{ij}^{G+1} = \begin{cases} \tilde{w}_{ij}^{G+1}, & \text{if } rand \le CR \\ w_{ij}^G, & \text{otherwise} \end{cases} \tag{11}$$

where CR is the crossover factor whose value is set to 0.8, $i = 1, 2, \ldots, p$ and $j = 1, 2, \ldots, d$;

(iv) Selection: we select the individuals of the next generation with the greedy selection scheme and check the terminate condition; if it is satisfied (reached within the maximum number of iterations), stop and return the optimum or go to step (ii). Thus the ith individual of the $(G+1)$th generation is constructed as follows

$$\mathbf{w}_i^{G+1} = \begin{cases} \hat{\mathbf{w}}_i^{G+1}, & \text{if } V_2(\mathbf{w}_i^{G+1}) \le V_2(\mathbf{w}_i^G) \\ \mathbf{w}_i^G, & \text{otherwise} \end{cases}. \tag{12}$$

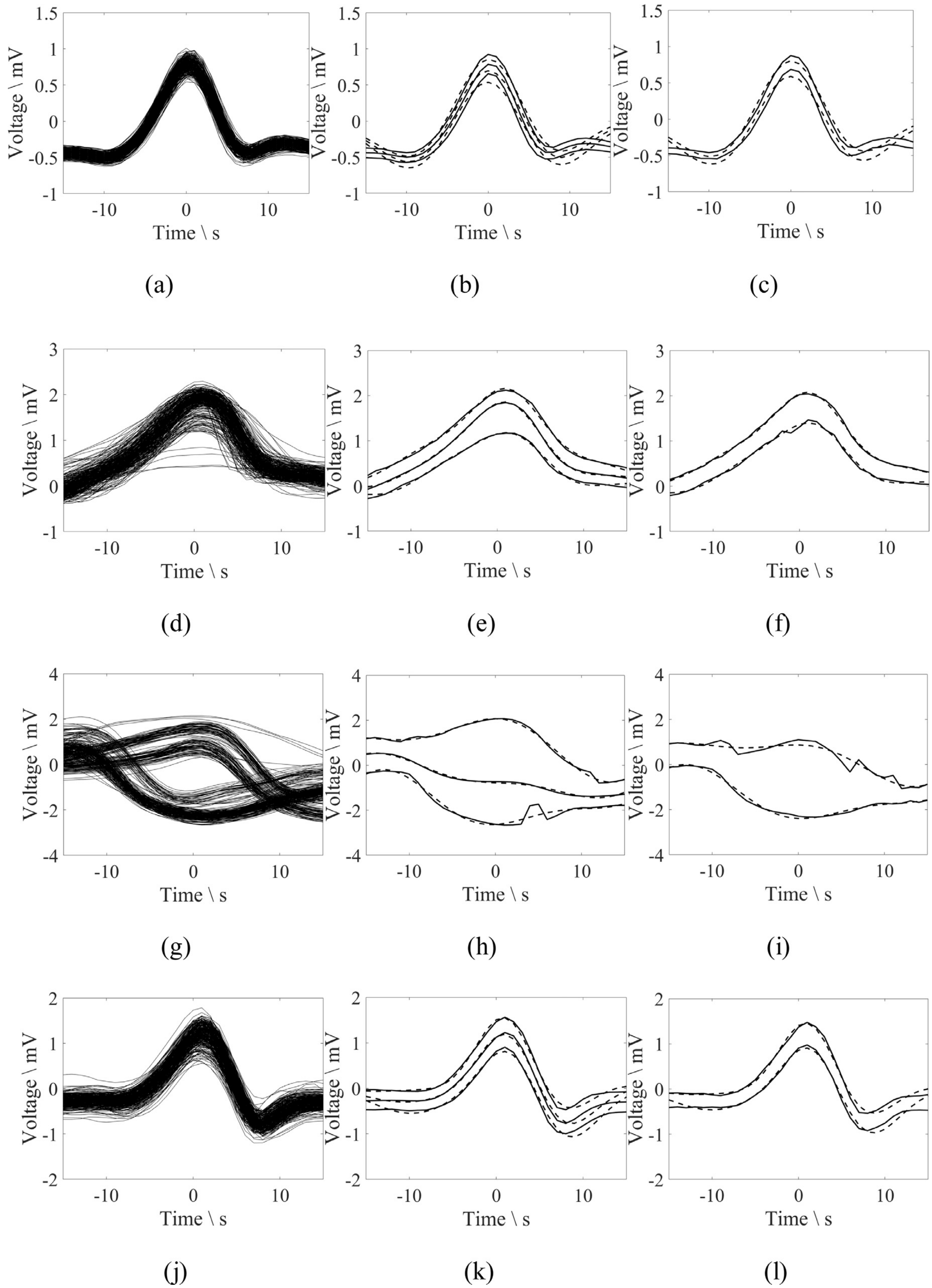

Fig. 8. Plots of the heart beats and the corresponding granular representation involving fuzzy set and interval granules: (a), (b), (c) normal beat; (d), (e), (f) left bundle branch block beat; (g), (h), (i) premature ventricular contraction; (j), (k), (l) atrial premature beat; (b), (e), (h), (k) granular representation under fuzzy sets; (c), (f), (i), (l) granular representation under interval granules; solid line—the characteristic functions (formed in the first phase), dashed line—the fitting lines ($f^{-}(x)$, $f(x)$ and $f^{+}(x)$).

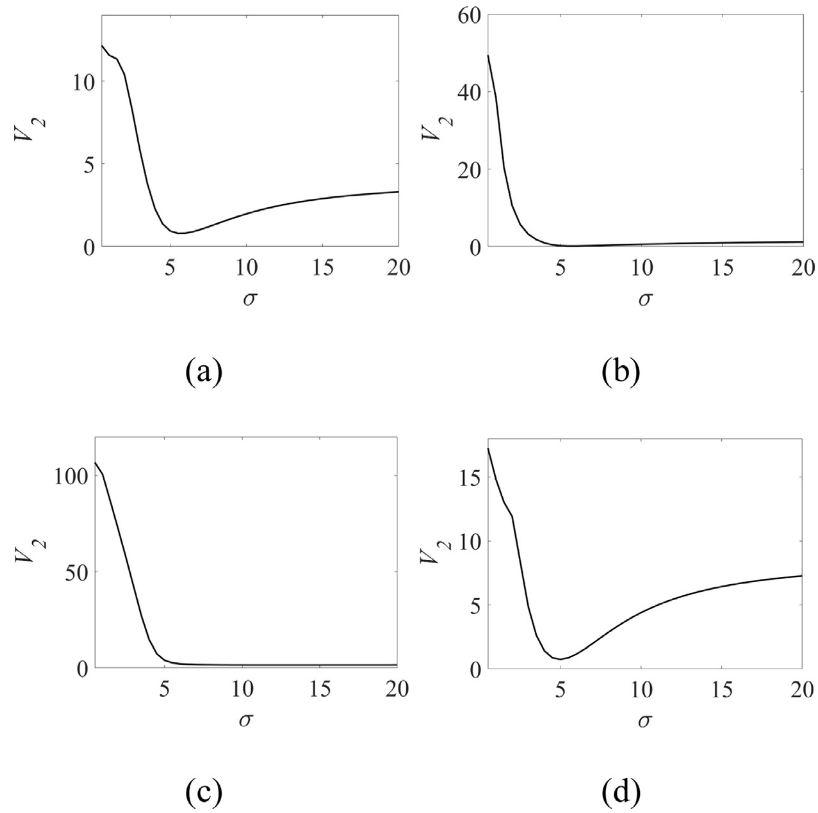

Fig. 9. Plots of V_2 along with the values of σ in [0.5,20] with step width 0.5 for four datasets: (a) 205 (normal beat); (b) 214 (left bundle branch block beat); (c) 233 (premature ventricular contraction); (d) 209 (atrial premature beat).

The performance index used in the DE optimization is expressed in the form

$$V_2 = \begin{cases} \sum_{i=1}^{N} \xi_i\left[(f^-(x_i) - a_i)^2 + (f(x_i) - m_i)^2 \right. \\ \left. + (f^+(x_i) - b_i)^2 \right] & f^-(x_i) \le f(x_i) \le f^+(x_i), i = 1, 2, ..., N \\ +\infty & \text{otherwise} \end{cases} \tag{13}$$

Note that the infinity standing in the second row of the formula eliminates the generated individuals which do not satisfy the constraints. In any implementation, instead of the infinity, one can take a large real number.

4. Experimental studies

In this section, we present a suite of experiments carried out for the granular representation of synthetic signals and real-world data—the electrocardiogram (ECG) signals and the application of the proposed method to the construction of general type-2 fuzzy set [20,21].

4.1. Granular representation of signals

Granular representation of synthetic data

Here we generate four types of signals: triangular, parabolic, square root, and Gaussian as shown in Fig. 4. Time x is taken in the [0,21] range with a step length of 0.5. The detailed expressions for the triangular, parabolic, square root and Gaussian signals are given in the form triangular:

$$y = \begin{cases} (1 + 0.8h_2)\frac{x - x_l}{8 - x_l} & \\ \quad + \max\left(\min((-0.1 + 0.2h_1\frac{x-3}{2}), \right. & \\ \quad \left. 0.2(6 - x)), 0\right) & x \in [x_l, 8] \\ (1 + 0.8h_2)\frac{x - x_r}{8 - x_r} & x \in [8, x_r] \\ -0.1 + 0.1h_3 & \\ \quad + \max\left(\min((-0.1 + 0.2h_1\frac{x-3}{2}), \right. & \\ \quad \left. 0.2(6 - x)), 0\right) & x \in [0, x_l) \text{or} (x_r, 21] \end{cases} . \tag{14}$$

parabolic:

$$y = \begin{cases} (1 + 1.1h_1)(1 - (\frac{x-8}{x_l - 8})^2) & x \in [x_l, 8] \\ (1 + 1.1h_1)\left(1 - (\frac{x-8}{x_r - 8})^2\right) & \\ \quad + \max(\min(\frac{x-10}{2}), (13 - x)), 0) * 0.2 & x \in [8, x_r] \\ -0.1 + 0.1h_2 & \\ \quad + \max(\min(\frac{x-10}{2}), (13 - x)), 0) * 0.2 & x \in [0, x_l) \text{or} (x_r, 21] \end{cases} . \tag{15}$$

square root:

$$y = \begin{cases} (1 + 0.8h_1)(\frac{x - x_l}{8 - x_l})^{0.5} & x \in [x_l, 8] \\ (1 + 0.8h_1)(\frac{x - x_r}{8 - x_r})^{0.5} & x \in [8, x_r] \\ -0.1 + 0.1h_2 & x \in [0, x_l) \text{or} (x_r, 21] \end{cases} . \tag{16}$$

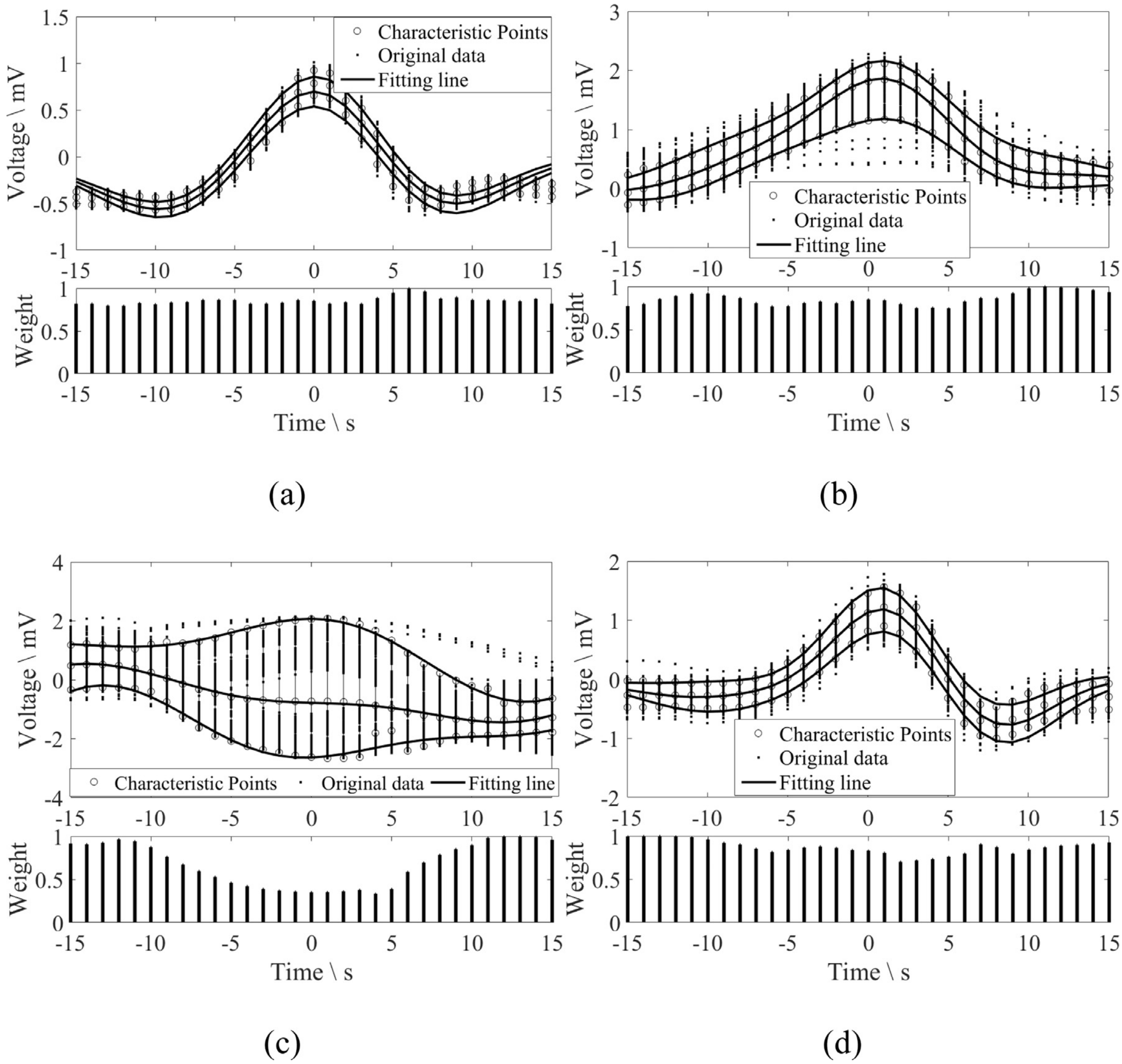

Fig. 10. Plots of the original data against the fitting lines and the weights of the corresponding information granules for four datasets (a) 205 (normal beat); (b) 214 (left bundle branch block beat); (c) 233 (premature ventricular contraction); (d) 209 (atrial premature beat); round circle—the characteristic points (a_i, m_i, b_i ($i=1,2, \ldots, 31$)), solid point—the original data, solid line-fitting lines ($f^-(x)$, $f(x)$ and $f^+(x)$).

Gaussian:

$$y = \frac{1.6 + 0.4h_1}{\sqrt{2\pi}(2 + 0.5h_2)} \exp(-(x-8)^2/(2(2+0.5h_2)^2)), x \in [0, 21]. \tag{17}$$

The values of parameters x_l, x_r, h_1, h_2, h_3 standing in (14)–(17), are chosen randomly (following a uniform distribution); furthermore h_1, h_2, $h_3 \in [0, 1]$, $x_l \in [0, 4]$ and $x_r \in [13, 21]$.

There are 200 samples in each dataset. To show the results, the granular representations of signals involving fuzzy sets described by the parabolic membership function and the interval information granules are taken as two examples and displayed in Figs. 5 and 6, respectively.

(i) For the fuzzy set with the parabolic membership function, the performance index becomes

$$V_2(\mathbf{w}) = \sum_{i=1}^{N} \xi_i[(f^-(x_i) - a_i)^2 + (f(x_i) - m_i)^2 + (f^+(x_i) - b_i)^2] \tag{18}$$

subject to constraints

$$f^-(x_i) - f(x_i) \le 0, f(x_i) - f^+(x_i) \le 0, i = 1, 2, \ldots, N$$

where $f^-(x) = \sum_{n=0}^{M-1} w_n\varphi_n(x)$, $f(x) = \sum_{n=0}^{M-1} w_{M+n}\varphi_n(x)$, $f^+(x) = \sum_{n=0}^{M-1} w_{2M+n}\varphi_n(x)$, w_j ($j=0, 1, 2, \ldots, 3M-1$) are the coefficients and $\mathbf{w}=[w_0, w_1, \ldots, w_{3M-1}]$, while $\varphi_n(x) = x^n$ ($n=0, 1, 2, \ldots, M-1$) are the polynomial basis.

(ii) For the interval information granules, only the upper and lower bounds of the granules are formed [22], so the performance index becomes:

$$V_2(\mathbf{w}) = \sum_{i=1}^{N} \xi_i[(f^-(x_i) - a_i)^2 + (f^+(x_i) - b_i)^2] \tag{19}$$

subject to constraints

$$f^-(x_i) - f^+(x_i) \le 0, i = 1, 2, \ldots, N$$

where $f^-(x) = \sum_{n=0}^{M-1} w_n\varphi_n(x)$, $f^+(x) = \sum_{n=0}^{M-1} w_{M+n}\varphi_n(x)$, w_j ($j=0, 1, 2, \ldots, 2M-1$) are the coefficients and $\boldsymbol{w}=[w_0, w_1, \ldots, w_{2M-1}]$, while $\varphi_n(x) = x^n$ ($n=0, 1, 2, \ldots, M-1$) are the polynomial basis.

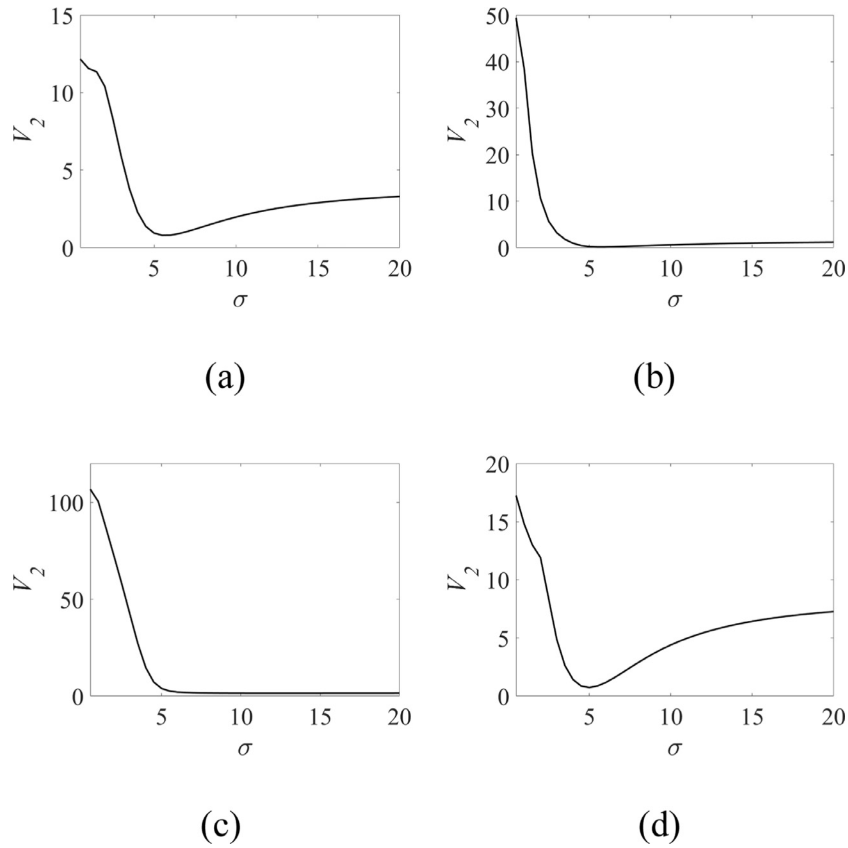

Fig. 11. Plots of V_2 along with the values of σ in [0.5, 25] with step width 0.5 for four datasets (a) 205 (normal beat); (b) 214 (left bundle branch block beat); (c) 233 (premature ventricular contraction); (d) 209 (atrial premature beat).

As shown in Fig. 3, the performance index V_2 decreases with the increase of the order of the polynomial. For the first three datasets, the performance changes slowly when the order is higher than 6, so here the order of the polynomial basis is set to be 6 for the first three datasets. For the fourth dataset, the order is set to 8.

As shown in Fig. 4, the granular representative of the signals based on the interval information granules leads to lower data coverage when compared with those formed when using fuzzy sets described by parabolic membership function. This is because of the difference of the membership functions of these two types of information granules.

Granular representation of ECG signals

The signals used in the experiments come from [28]; see also https://www.physionet.org/physiotools/wfdb/lib/ecgcodes.h. As illustrated in Fig. 8, there are four types of heart beats—the normal beats, the left bundle branch block beat, the premature ventricular contraction and the atrial premature beats—collected from files # 205, 214, 233 and 209 for 8–(a), 8–(d), 8–(g), 8–(j), respectively. There are 300 samples for each dataset.

For the formation of the granular representation of signals, the Hermite polynomials are considered as this orthogonal basis resembles the shape of the ECG signals. The Hermite basis is expressed in the following way

$$\phi_n(x,\sigma) = \frac{1}{\sqrt{\sigma 2^n n! \sqrt{\pi}}} e^{-x^2/2\sigma^2} H_n\left(\frac{x}{\sigma}\right). \tag{20}$$

$H_n(\frac{x}{\sigma})$ describes Hermite polynomials of successive orders: $H_0(\frac{x}{\sigma}) = 1$, $H_1(\frac{x}{\sigma}) = 2\frac{x}{\sigma}$ and in general

$$H_n\left(\frac{x}{\sigma}\right) = 2\frac{x}{\sigma}H_{n-1}\left(\frac{x}{\sigma}\right) - 2(n-1)H_{n-2}\left(\frac{x}{\sigma}\right), n \geq 2. \tag{21}$$

The construction of the granular representation of signals involves fuzzy sets described by the parabolic membership function and interval information granules, respectively.

For fuzzy sets with parabolic membership function, the optimization problem reads as follows:

$$V_2(\mathbf{w}) = \sum_{i=1}^{N} \xi_i[(f^-(x_i) - a_i)^2 + (f(x_i) - m_i)^2 + (f^+(x_i) - b_i)^2] \tag{22}$$

subject to constraints

$f^-(x_i) - f(x_i) \leq 0, f(x_i) - f^+(x_i) \leq 0, i = 1, 2, ..., N$

where $f^-(x) = \sum_{n=0}^{M-1} w_n\phi_n(x,\sigma)$, $f(x) = \sum_{n=0}^{M-1} w_{M+n}\phi_n(x,\sigma)$, $f^+(x) = \sum_{n=0}^{M-1} w_{2M+n}\phi_n(x,\sigma)$, w_j ($j=0, 1, 2, \ldots, 3M-1$) are the coefficients and $\boldsymbol{w} = [w_0, w_1, \ldots, w_{3M-1}]$, while $\phi_n(x, \sigma)$ ($n=0, 1, 2, \ldots, M-1$) are the Hermite basis.

For interval information granules, only the upper and lower bound of these granules come as a solution to the optimization problem:

$$V_2(\mathbf{w}) = \sum_{i=1}^{N} \xi_i[(f^-(x_i) - a_i)^2 + (f^+(x_i) - b_i)^2] \tag{23}$$

subject to constraints

$f^-(x_i) - f^+(x_i) \leq 0, i = 1, 2, ..., N$

where $f^-(x) = \sum_{n=0}^{M-1} w_n\phi_n(x,\sigma)$, $f^+(x) = \sum_{n=0}^{M-1} w_{M+n}\phi_n(x,\sigma)$, w_j ($j=0, 1, 2, \ldots, 2M-1$) are the coefficients and $\boldsymbol{w} = [w_0, w_1, \ldots, w_{2M-1}]$, while $\phi_n(x, \sigma)$ ($n=0, 1, 2, \ldots, M-1$) are the Hermite basis.

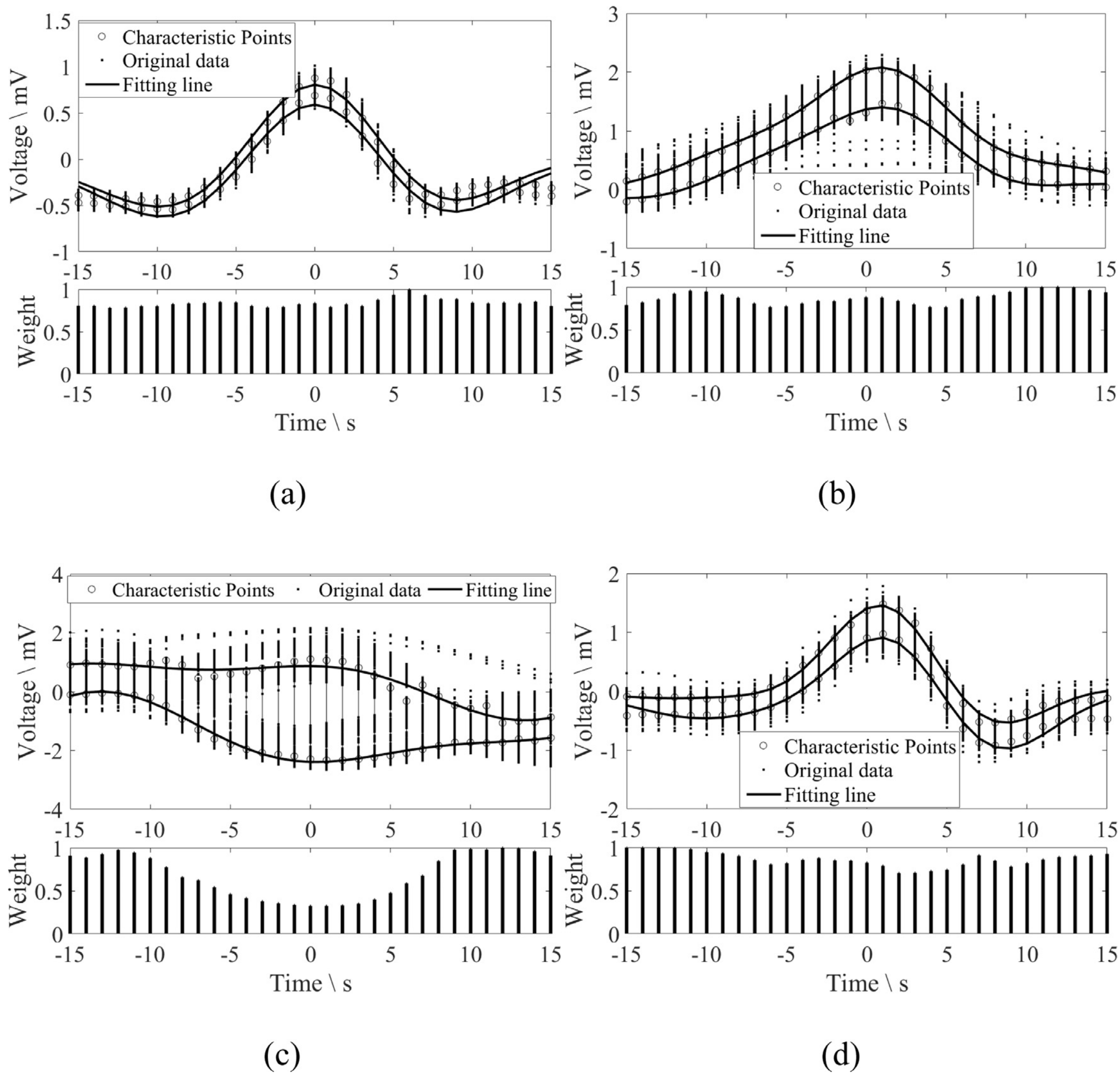

Fig. 12. Plots of the original data against the fitting lines and the weights of the corresponding information granules for four datasets (a) 205 (normal beat); (b) 214 (left bundle branch block beat); (c) 233 (premature ventricular contraction); (d) 209 (atrial premature beat); round circle-the characteristic points (a_i, m_i, b_i ($i=1,2,\ldots,31$)), solid point—the original data, solid line—fitting lines ($f^-(x)$ and $f^+(x)$).

The order and the value of σ present in the Hermite polynomials should be predefined. The domain of M is taken as [2,20] to make sure the final approximation is acceptable. σ is taken from the range [0.5,20] and this interval is divided evenly into 40 points. For each value of M, we determine:

$$\text{Average } V_2(M) = \frac{\sum_{k=0}^{L}[V_2^{(k)}(M)]}{L} \tag{24}$$

where $V_2^{(k)}$is the performance index when $\sigma=(k+1)*0.5$, $L=\frac{20-0.5}{0.5}$.

From the results shown in Figs. 7, 9, and 11 and the approximation results included in Figs. 8, 10 and 12, the set for the domains of M ([2,20]) and σ ([0.5,20]) is suitable. If the final results are not satisfactory, the ranges of M and σ could be extended according to the problem.

As shown in Fig. 7, the performance index Average V_2 decreases along with the increase of the order of the Hermite polynomials; the performance for the approximation with the granules formed with fuzzy sets is higher than that of the interval information granules which results from the definition of the performance index function. Here we set the value of M to 5, considering the approximation results and the computation complexity. The length of the time window is 31, which is long enough to fully capture the QRS complex.

The followings are the corresponding results for the granular representation of ECG signals with the fuzzy sets and interval information granules. The results of performance index V_2 along with the change of σ in [0.5,20] with step width 0.5 are displayed in Figs. 9 and 11.

Case 1: Granular representation formed by fuzzy sets with parabolic membership functions

According to the results displayed in Fig. 9, one concludes that the best values of σ are 5.5, 5.5, 12 and 5 for the dataset 205 (normal beat), 214 (left bundle branch block beat), 233 (premature ventricular contraction) and 209 (atrial premature beat), respectively; the value of M is set to 5.

Fig. 10 displays the plot of the original granules formed in the first phase against the fitting lines. In Fig. 10, the results of the fitting lines positioned against the original data of the signals and the characteristic points of the fuzzy sets are displayed with the corresponding weights.

Case 2: Granular representation in terms of interval information granules

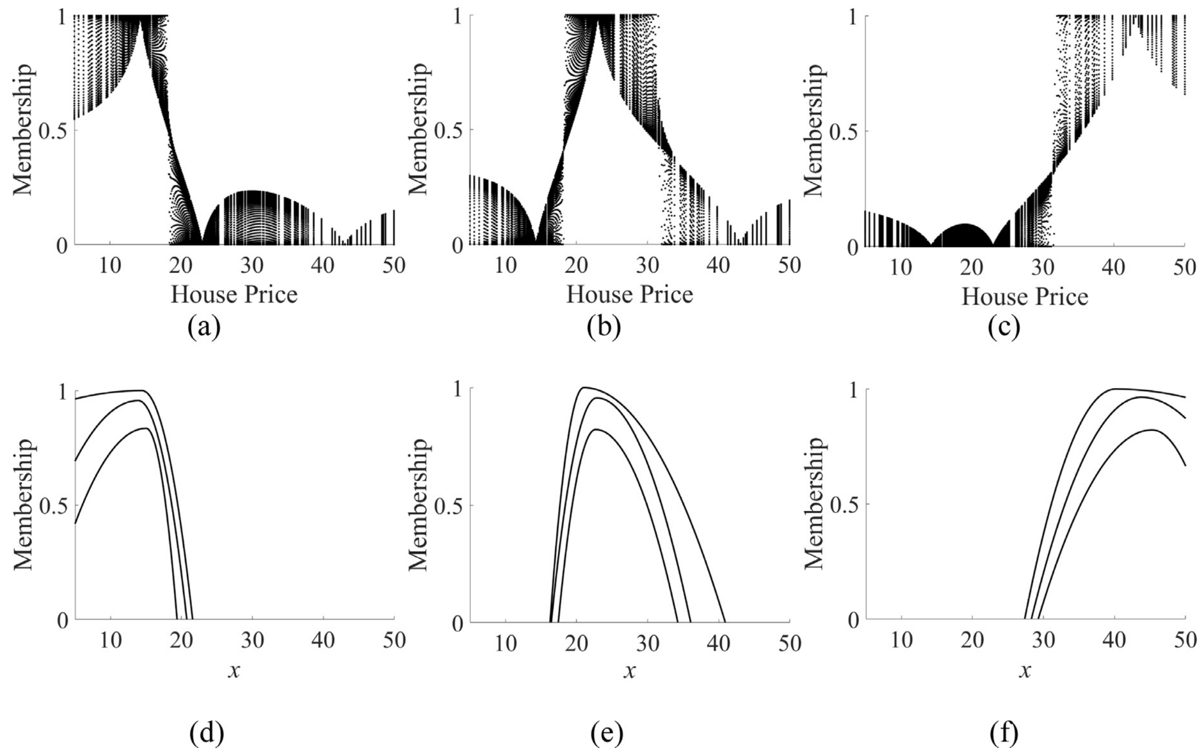

Fig. 13. Membership functions of house prices obtained for different values of m for different clusters (a) *low*; (b) *medium*; (c) *high*; the obtained functions $f^{-}(x)$, $f(x)$ and $f^{+}(x)$ with the proposed method (the optimization method is DE) for the corresponding clusters (d) *low*; (e) *medium*; (f) *high*.

Considering Fig. 11, we observe that the best values of σ are 5.5, 5.5, 12 and 5 for the dataset 205 (normal beat), 214 (left bundle branch block beat), 233 (premature ventricular contraction) and 209 (atrial premature beat), respectively. Here M is equal to 5. For the final granular representatives, they capture the essence of the signals, which are quantified through the results displayed in Figs. 4, 5, 6, 8, 10, and 12.

4.2. Application of the method to the construction of type-2 fuzzy sets

The way of constructing granular representatives for signals can be used to develop fuzzy sets of type-2 (membership functions). With the growing interest and visibility of these generalized fuzzy sets, the crucial problem of estimation of their membership functions has not been fully addressed even though it is central to the theory and applications of fuzzy systems. We show that the approach proposed here offers a constructive solution to this important estimation problem.

Fuzzy sets are often constructed on a basis of existing numeric data by engaging fuzzy clustering. The obtained clusters can be analytically described by membership functions constructed on a basis of a collection of prototypes produced by a clustering technique. The method of FCM [16] is one of the commonly considered ways of building membership functions. The fuzzy sets obtained in this way are so-called fuzzy sets of type-1 meaning that their membership functions are numeric. In contrast, type-2 fuzzy sets come with membership grades that are intervals (interval-valued fuzzy sets) or fuzzy sets defined over the unit interval. The FCM method is endowed with a free parameter referred as a fuzzification coefficient (m) assuming values greater than 1 and affecting a shape of the obtained membership functions. The choice of a certain value of this coefficient is not clear. A sound design alternative is to consider a range of values of m and aggregate the resulting fuzzy sets belonging to the same cluster. The aggregation results in the form of a type-2 fuzzy set. We follow this way to establish a membership function of type-2 fuzzy set.

As an example, the Boston housing data [58] (https://archive.ics.uci.edu/ml/datasets/Housing) are clustered into three clusters ('*low*', '*medium*' and '*high*') and divide the range of the fuzzification coefficient (m) [1.05,3.2] into 43 points with the step of 0.05. And the cluster is divided according to the each value of m ($m \in$ [1.05, 3.2]) by the FCM algorithm. For the membership grades (obtained from all values of m) of the data points to the same cluster—for instance, the cluster '*low*' of which the price of the house variable of the prototype is the minimum among the three prototypes—put them into the same set. Based on this set and following the approach proposed in the study, we form the type-2 fuzzy set associated with the cluster '*low*' for the price of house variable (typically regarded as the output variable in this problem). For the other two clusters, the corresponding type-2 fuzzy sets are formed in the same way.

The resulting plots of the membership functions are visualized in Fig. 13 with the obtained functions $f^{-}(x)$, $f(x)$ and $f^{+}(x)$.

5. Conclusions

In this study, we have developed the two-phase method of the construction of granular representation of signals. Owing to the use of the principle of justifiable granularity, the development of the granules along with the vertical axis appeals for requirements of experimental justification and semantic relevance. The least squares error method is used to derive the optimal coefficients of the fitting functions at the global level. It is important to stress that at both phases of the design, the quality of the estimated results (the vertical granules and the approximated functions) is quantified, which helps decide whether the constructed granular representative is acceptable. The established construct comes with a great deal of flexibility manifesting in several ways. First, the estimation of the vertical granules can be realized in the presence of granular experimental evidence (and this flexibility resides within the principle of justifiable granularity). Second, the choice of the fitting functions can lead to the optimization of the associated performance index.

The proposed method not only could be applied to form the granular representation of signals but also be used to construct general type-2 fuzzy sets.

Acknowledgments

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