RoboArm
Project Report #1

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Robotix Axis Definition

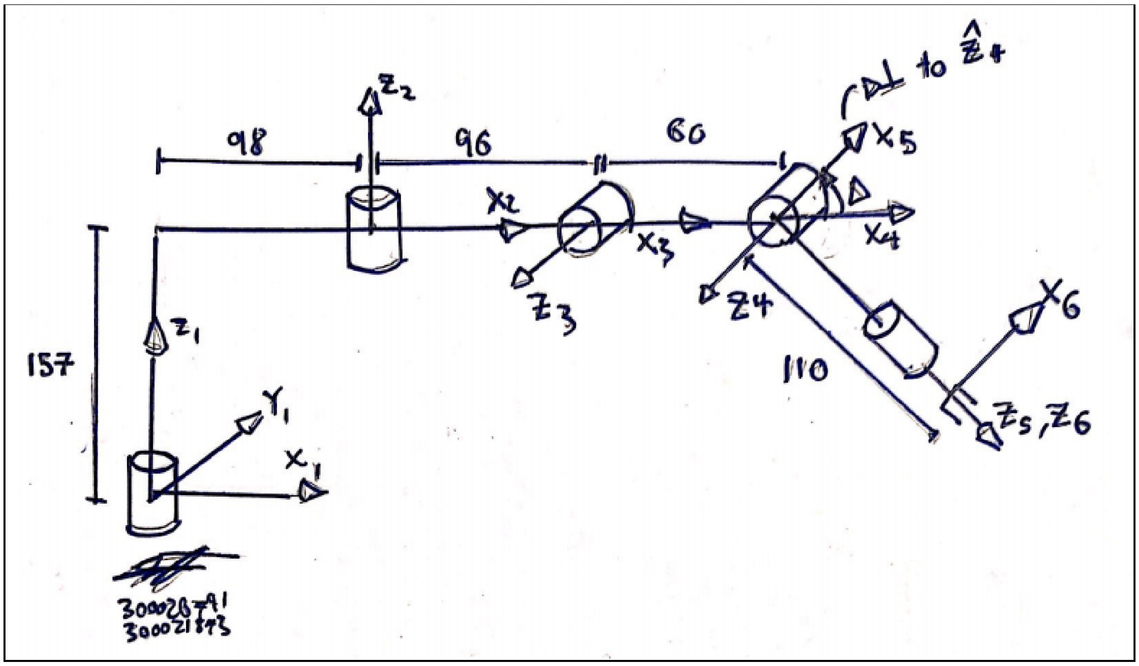


Figure 1: Axis Definition

DH Parameters Table

Table 1: DH parameters

Joint	$l(mm)$	$d(mm)$	$\alpha(Deg.)$	$\theta(Deg.)$
1	98	157	0	θ_1
2	96	0	90	θ_2
3	60	0	0	θ_3
4	0	0	90	$\theta_4 + \Delta$
5	0	110	0	θ_5

A Matrices

The A Matrices are each a forward transformation matrix for an evolving reference frame. For 6 joints, we have 5 transformations which are described fully by the DH parameters given in Table 1 above. Using the definition for an A matrix, A_i, as given below in equation 1, we proceed to calculate an A matrix for each ithjoint, A_i, while keeping each θ_i as a variable.

$$A_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & l_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & l_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Equation 1: A matrix definition, obtained from Computer Control in Robotics, Chapter 4

$$A_1 = \begin{bmatrix} cos\theta_1 & -sin\theta_1 & 0 & 98cos\theta_1 \\ sin\theta_1 & cos\theta_1 & 0 & 98sin\theta_1 \\ 0 & 0 & 1 & 157 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} cos\theta_2 & 0 & sin\theta_2 & 96cos\theta_2 \\ sin\theta_2 & 0 & -cos\theta_2 & 96sin\theta_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} cos\theta_3 & -sin\theta_3 & 0 & 60cos\theta_3 \\ sin\theta_3 & cos\theta_3 & 0 & 60sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_4 = \begin{bmatrix} cos(\theta_4 + \Delta) & 0 & sin(\theta_4 + \Delta) & 0 \\ sin(\theta_4 + \Delta) & 0 & -cos(\theta_4 + \Delta) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} cos\theta_5 & -sin\theta_5 & 0 & 0 \\ sin\theta_5 & cos\theta_5 & 0 & 0 \\ 0 & 0 & 1 & 110 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Forward Kinematics Model

We can then multiply all of the A matrices subsequently to obtain the full forward kinematics model, represented by the $Q_{effector/base}$ matrix. The $Q_{effector/base}$ matrix, for the DH parameters shown in Table 1, can be calculated by multiplying through each matrix and as before, keeping each θ_i as a variable, as shown in equation 2. The results of the subsequent matrix products are given below, eventually leading to the final $Q_{effector/base}$ matrix.

$$Q_{effector/base} = A_1 A_2 A_3 A_4 A_5 \qquad equation\ 2$$

The calculations using the A matrices defined early, with equation 2 are shown in Figures 2-4

$$A_1A_2=\begin{bmatrix} \cos\theta_1\cos\theta_2-\sin\theta_1\sin\theta_2 & 0 & \cos\theta_1\sin\theta_2+\sin\theta_1\cos\theta_2 & 96\cos\theta_1\cos\theta_2-96\sin\theta_1\sin\theta_2+98\cos\theta_1 \\ \sin\theta_1\cos\theta_2+\cos\theta_1\sin\theta_2 & 0 & \sin\theta_1\sin\theta_2-\cos\theta_1\cos\theta_2 & 96\sin\theta_1\cos\theta_2+96\cos\theta_1\sin\theta_2+98\sin\theta_1 \\ 0 & 1 & 0 & 157 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1A_2A_3=\begin{bmatrix} \cos\theta_3(\cos\theta_1\cos\theta_2-\sin\theta_1\sin\theta_2) & -\sin\theta_3(\cos\theta_1\cos\theta_2-\sin\theta_1\sin\theta_2) & \cos\theta_1\sin\theta_2+\sin\theta_1\cos\theta_2 & \begin{array}{l}60\cos\theta_3(\cos\theta_1\cos\theta_2-\sin\theta_1\sin\theta_2)\\+96\cos\theta_1\cos\theta_2-96\sin\theta_1\sin\theta_2\\+98\cos\theta_1\end{array} \\ \cos\theta_3(\sin\theta_1\cos\theta_2+\cos\theta_1\sin\theta_2) & -\sin\theta_3(\sin\theta_1\cos\theta_2+\cos\theta_1\sin\theta_2) & \sin\theta_1\sin\theta_2-\cos\theta_1\cos\theta_2 & \begin{array}{l}60\cos\theta_3(\sin\theta_1\cos\theta_2+\cos\theta_1\\\sin\theta_2)+96\sin\theta_1\cos\theta_2+96\\\cos\theta_1\sin\theta_2+98\sin\theta_1\end{array} \\ \sin\theta_3 & \cos\theta_3 & 0 & 60\sin\theta_3+157 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4A_5=\begin{bmatrix} \cos(\theta_4+\Delta)\cos\theta_5 & -\cos(\theta_4+\Delta)\sin\theta_5 & \sin(\theta_4+\Delta) & 110\sin(\theta_4+\Delta) \\ \sin(\theta_4+\Delta)\cos\theta_5 & -\sin(\theta_4+\Delta)\sin\theta_5 & -\cos(\theta_4+\Delta) & -110\cos(\theta_4+\Delta) \\ \sin\theta_5 & \cos\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Figure 2: 3 intermediate products of A matrices

$$A_1A_2A_3A_4A_5=\begin{bmatrix} \begin{array}{l}\cos(\theta_4+\Delta)\cos\theta_5\cos\theta_3(\cos\theta_1\cos\theta_2-\sin\theta_1\sin\theta_2)\\-\sin(\theta_4+\Delta)\cos\theta_5\sin\theta_3(\cos\theta_1\cos\theta_2-\sin\theta_1\sin\theta_2)\\+\sin\theta_5(\cos\theta_1\sin\theta_2+\sin\theta_1\cos\theta_2)\end{array} & \begin{array}{l}-\cos(\theta_4+\Delta)\sin\theta_5\cos\theta_3(\cos\theta_1\cos\theta_2-\sin\theta_1\sin\theta_2)\\+\sin(\theta_4+\Delta)\sin\theta_5\sin\theta_3(\cos\theta_1\cos\theta_2-\sin\theta_1\sin\theta_2)\\+\cos\theta_5(\cos\theta_1\sin\theta_2+\sin\theta_1\cos\theta_2)\end{array} & \\ \begin{array}{l}\cos(\theta_4+\Delta)\cos\theta_5\cos\theta_3(\sin\theta_1\cos\theta_2+\cos\theta_1\sin\theta_2)\\-\sin(\theta_4+\Delta)\cos\theta_5\sin\theta_3(\sin\theta_1\cos\theta_2+\cos\theta_1\sin\theta_2)\\+\sin\theta_5(\sin\theta_1\sin\theta_2-\cos\theta_1\cos\theta_2)\end{array} & \begin{array}{l}-\cos(\theta_4+\Delta)\sin\theta_5\cos\theta_3(\sin\theta_1\cos\theta_2+\cos\theta_1\sin\theta_2)\\+\sin(\theta_4+\Delta)\sin\theta_5\sin\theta_3(\sin\theta_1\cos\theta_2+\cos\theta_1\sin\theta_2)\\+\cos\theta_5(\sin\theta_1\sin\theta_2-\cos\theta_1\cos\theta_2)\end{array} & \\ \cos(\theta_4+\Delta)\cos\theta_5\sin\theta_3+\sin(\theta_4+\Delta)\cos\theta_5\cos\theta_3 & \begin{array}{l}-\cos(\theta_4+\Delta)\sin\theta_5\sin\theta_3\\-\sin(\theta_4+\Delta)\sin\theta_5\cos\theta_3\end{array} & \cdots \\ 0 & 0 & \end{bmatrix}$$

Figure 3:Columns 1 and 2 of the final product of all 5 A matrices

$$
\cdots
\begin{bmatrix}
\sin(\theta_4+\Delta)\cos\theta_3(\cos\theta_1\cos\theta_2-\sin\theta_1\sin\theta_2) + \cos(\theta_4+\Delta)\sin\theta_3(\cos\theta_1\cos\theta_2-\sin\theta_1\sin\theta_2) & 110\sin(\theta_4+\Delta)\cos\theta_3(\cos\theta_1\cos\theta_2-\sin\theta_1\sin\theta_2) + 110\cos(\theta_4+\Delta)\sin\theta_3(\cos\theta_1\cos\theta_2-\sin\theta_1\sin\theta_2) + 60\cos\theta_3(\cos\theta_1\cos\theta_2-\sin\theta_1\sin\theta_2) + 96\cos\theta_1\cos\theta_2 - 96\sin\theta_1\sin\theta_2 + 48\cos\theta_1 \\
\sin(\theta_4+\Delta)\cos\theta_3(\sin\theta_1\cos\theta_2+\cos\theta_1\sin\theta_2) + \cos(\theta_4+\Delta)\sin\theta_3(\sin\theta_1\cos\theta_2+\cos\theta_1\sin\theta_2) & 110\sin(\theta_4+\Delta)\cos\theta_3(\sin\theta_1\cos\theta_2+\cos\theta_1\sin\theta_2) + 110\cos(\theta_4+\Delta)\sin\theta_3(\sin\theta_1\cos\theta_2+\cos\theta_1\sin\theta_2) + 60\cos\theta_3(\sin\theta_1\cos\theta_2+\cos\theta_1\sin\theta_2) + 96\sin\theta_1\cos\theta_2 + 96\cos\theta_1\sin\theta_2 + 48\sin\theta_1 \\
\sin(\theta_4+\Delta)\sin\theta_3 - \cos(\theta_4+\Delta)\cos\theta_3 & 110\sin(\theta_4+\Delta)\sin\theta_3 - 110\cos(\theta_4+\Delta)\cos\theta_3 + 60\sin\theta_3 + 157 \\
0 & 1
\end{bmatrix}
$$

Figure 4:Columns 3 and 4 of the final product of all 5 A matrices

$$
Q_{eff/base} =
\begin{bmatrix}
\cos\theta_3\cos(\theta_1+\theta_2)\cos(\theta_3+\theta_4+\Delta) + \sin\theta_3\sin(\theta_1+\theta_2) & -\sin\theta_3\cos(\theta_1+\theta_2)\cos(\theta_3+\theta_4+\Delta) + \cos\theta_3\sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2)\sin(\theta_3+\theta_4+\Delta) & \cdots \\
\cos\theta_3\sin(\theta_1+\theta_2)\cos(\theta_3+\theta_4+\Delta) - \sin\theta_3\cos(\theta_1+\theta_2) & -\sin\theta_3\sin(\theta_1+\theta_2)\cos(\theta_3+\theta_4+\Delta) - \cos\theta_3\cos(\theta_1+\theta_2) & \sin(\theta_1+\theta_2)\sin(\theta_3+\theta_4+\Delta) & \cdots \\
\cos\theta_3\sin(\theta_3+\theta_4+\Delta) & -\sin\theta_3\sin(\theta_3+\theta_4+\Delta) & -\cos(\theta_3+\theta_4+\Delta) & \cdots \\
0 & 0 & 0 & \cdots
\end{bmatrix}
$$

$$
\cdots
\begin{bmatrix}
110\cos(\theta_1+\theta_2)\sin(\theta_3+\theta_4+\Delta) + 60\cos\theta_3\cos(\theta_1+\theta_2) + 96\cos(\theta_1+\theta_2) + 48\cos\theta_1 \\
\sin(\theta_1+\theta_2)[110\sin(\theta_3+\theta_4+\Delta) + 60\cos\theta_3 + 96] + 98\sin\theta_1 \\
-110\cos(\theta_3+\theta_4+\Delta) + 60\sin(\theta_3) + 157 \\
1
\end{bmatrix}
$$

→ final $Q_{eff/base}$ to validate model w/ Matlab (SEE Appendix C)

Figure 5: Simplified Q_eff/base Matrix

To validate the $Q_{effector/base}$ matrix we found above, we test it against the matlab code used to derive the $Q_{effector/base}$ numerically. The results are shown in Figures 6-8 below, testing 3 different sets of theta values, all for **delta=0**(see Figure 1), and all measurements in mm. Note that for the first two sets of theta values, the validation can easily be seen by adding up the dimensions with respect to the reference frame of joint 1, as in Figure 1. For example, for set 1 the x coordinate of the end effector is given by (96+98+60)=254, the y coordinate = 0, and the z coordinate is given by (157-110)=47 as shown by both models in Figure 6. The same can be said for the 3rd set, by also doing a little bit of basic trigonometry.

```
>> Lab1_Analytical_Q

Q =

     1     0     0   254
     0    -1     0     0
     0     0    -1    47
     0     0     0     1

>> Lab1_Numerical_Q
     1     0     0   254
     0    -1     0     0
     0     0    -1    47
     0     0     0     1
```

Figure 6: Model Comparison for $\theta_1 = \theta_2 = \theta_3 = \theta_4 = \theta_5 = 0$

```
>> Lab1_Analytical_Q

Q =

     0     1     0     0
     0     0     1   364
     1     0     0   157
     0     0     0     1

>> Lab1_Numerical_Q
     0     1     0     0
     0     0     1   364
     1     0     0   157
     0     0     0     1

>>
```

Figure 7: Model Comparison for $\theta_1 = \theta_4 = 90,\ \theta_3 = \theta_2 = \theta_5 = 0$

```
>> Lab1_Numerical_Q
   -0.3685    0.9068    0.2048  124.0640
   -0.0885   -0.2535    0.9633  326.9575
    0.9254    0.3368    0.1736  186.5202
         0         0         0    1.0000

>> Lab1_Analytical_Q

Q =

   -0.3685    0.9068    0.2048  124.0640
   -0.0885   -0.2535    0.9633  326.9575
    0.9254    0.3368    0.1736  186.5202
         0         0         0    1.0000

>>
```

Figure 7: Model Comparison for $\theta_1 = 45, \theta_2 = 33, \theta_3 = 10, \theta_4 = 90, \theta_5 = -20$

Demonstration

To demonstrate the forward kinematic model derived we apply our model to the physical robot itself. A set of theta values are given as input, and the output of the model vs the measured physical outputs of the robotic arm itself are compared. To compare outputs, we measure the x, y, and z coordinates of the tip of the robotic arm and compare these values to x_T, y_T, and z_T given in the $Q_{effector/base}$ matrix, in inches. To calculate the $Q_{effector/base}$ matrix for a given set of theta values we use Matlab a script(see Appendix A for the full code). When calculating $Q_{effector/base}$ matrix we choose $\Delta = 90\ Deg$(see Figure 1) to keep the model consistent with the calibrations(this results in the arm being completely extended for $\theta = 0$).

During the lab session we tested 4 sets of values for each theta, as shown in Table 2(for the pictures of physical robotic arm positions for each set of theta values, see Appendix B). To measure the set of physical coordinates, x, y, and z, we measure the tip of the end effector of the robotic arm, as shown from the 2 camera angles in Figure 9. Also note that the coordinate system was defined initially in Figure 1 (X1, Y1, and Z1) and so all measurements are taken with respect to this definition.

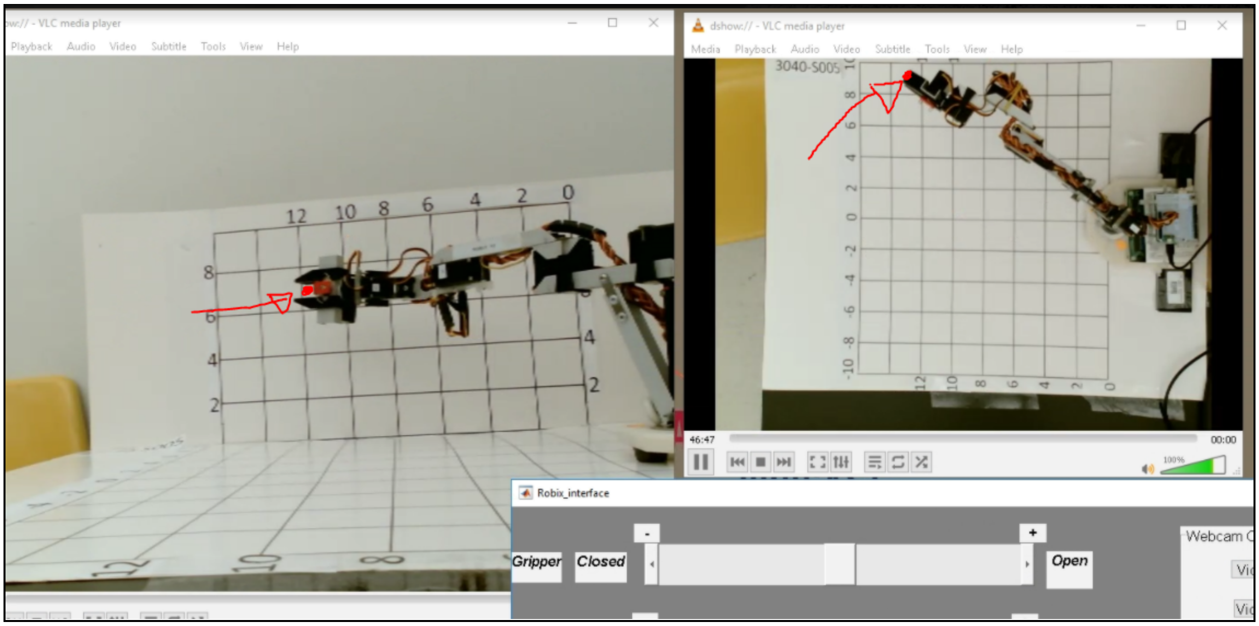


Figure 9: Defining the physical coordinates of the end effector

Table 2: Physical Results vs Predicted Results for theta_i(degrees) and x,y, and z(inches)

Set No.	θ_1	θ_2	θ_3	θ_4	θ_5	x	y	z	x_T	y_T	z_T
1	62	-75	-51	65	0	12.0	4.5	7.5	11.03	1.28	5.40
2	32	-43	-22	-13	0	13.5	2.5	3.5	12.61	0.23	2.81
3	-10	-20	-22	43	0	14.0	-6.0	7.5	12.47	-5.68	6.85
4	-30	-5	-7	10	0	13.0	-9.0	7.0	11.90	-7.92	6.12

Error Calculation:

Using equation 3 we calculate the percent difference for each coordinate per set, and proceed to take the average of all percent differences. We also calculate the absolute difference in the same manner.

$$Percent\ Difference\ =\ (\frac{|a - a_T|}{a})100\% \qquad equation\ 3$$

Table 3: Calculating Percent and Absolute Differences

a	a_T	$Absolute\ Difference(\%)$	$Percent\ Difference(\%)$
12	11.03	0.97	8.083333333
13.5	12.61	0.89	6.592592593
14	12.47	1.53	10.92857143
13	11.9	1.1	8.461538462
4.5	1.28	3.22	71.55555556
2.5	0.23	2.27	90.8
-6	-5.68	-0.32	5.333333333
-9	-7.92	-1.08	12
7.5	5.4	2.1	28
3.5	2.81	0.69	19.71428571
7.5	6.85	0.65	8.666666667
7	6.12	0.88	12.57142857
	Average	1.075	23.55894214

Discussion

By applying 5 elementary transformation matrices(for evolving reference frames) to the base reference frame we were able to find the coordinates of the end effector relative to the base reference frame, while keeping the rotation of each joint(theta) a variable. The result, is a model that reuters the $Q_{effector/base}$ for a specified set of theta values for each joint.

By comparing the model against our physical measurements we obtain an average percent difference of 23.56, and average absolute difference of 1.075. The absolute difference is low, and constant enough through each coordinate measurement that we suspect the kinematic model developed on Matlab works perfectly, and the error instead originates from the calibration of the robotic arm, error created by measuring using static camera angles, and any random error in reading the measurements. We also note that the physical measurements are consistently greater than the predicted values, on an average represented by the absolute difference. A reasonable explanation for this is that each joint is under calibrated, in the sense that a given theta, θ_{given}, results in a physical/measured theta, $\theta_{physical}$, such that $\theta_{physical} < \theta_{given}$. Thus, the robotic arm extends less distance in each dimension from its starting position.

Appendix A

Matlab program to numerically calculate the $Q_{effector/base}$ matrix for a given set of theta values.

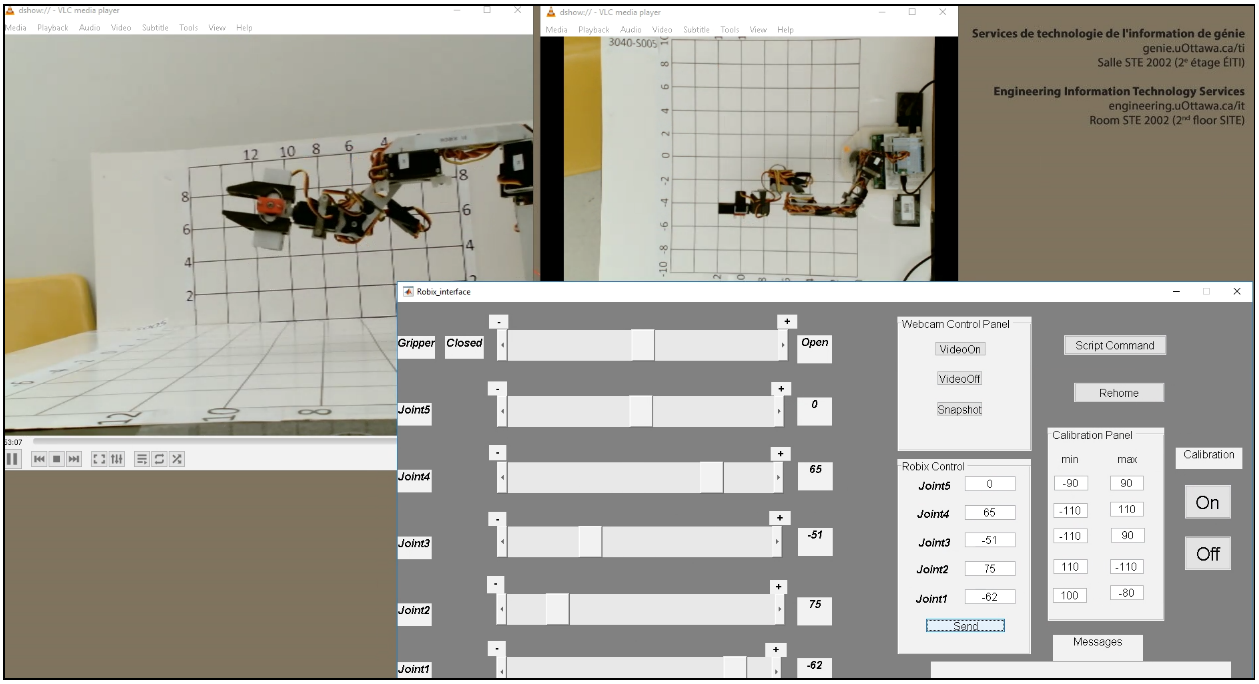
Lab1_Returns_Q_for_thetai.m

```
%Theta Values
NumberOfTransformations = 5;
theta1=0;
theta2=0;
theta3=0;
theta4=0;
theta5=0;

%DH PARAMS
%cm
l = [98 96 60 0 0];
d = [157 0 0 0 110];
%}
%inches
%{
l = [98*0.0393701 96*0.0393701 60*0.0393701 0 0];
d = [157*0.0393701 0 0 0 110*0.0393701];
%}
alpha = [0 90 0 90 0];
theta = [theta1 theta2 theta3 theta4+90 0];
%A Matrix Forward Qmatrix definitions
Amatrix = zeros(4, 4, NumberOfTransformations);
Qmatrix = zeros(4, 4);

cosa = @cosd;
sina = @sind;

%simple function for easily switching between degrees and radians
DegreeOrRadians = 0;        %0 for degrees and 1 for radians
if DegreeOrRadians == 1
    cosa = @cos;
    sina = @sin;
end

%determing the A matrix values and forward transformation matrix
for i = 1:NumberOfTransformations
    Amatrix(:,:,i) = [cosa(theta(i)) -sina(theta(i))*cosa(alpha(i))...
        sina(theta(i))*sina(alpha(i)) l(i)*cosa(theta(i));
                      sina(theta(i)) cosa(theta(i))*cosa(alpha(i))...
                      -cosa(theta(i))*sina(alpha(i)) l(i)*sina(theta(i));
                      0 sina(alpha(i)) cosa(alpha(i)) d(i);
                      0 0 0 1];
    if i == 1
        Qmatrix = Amatrix(:,:,i);
```

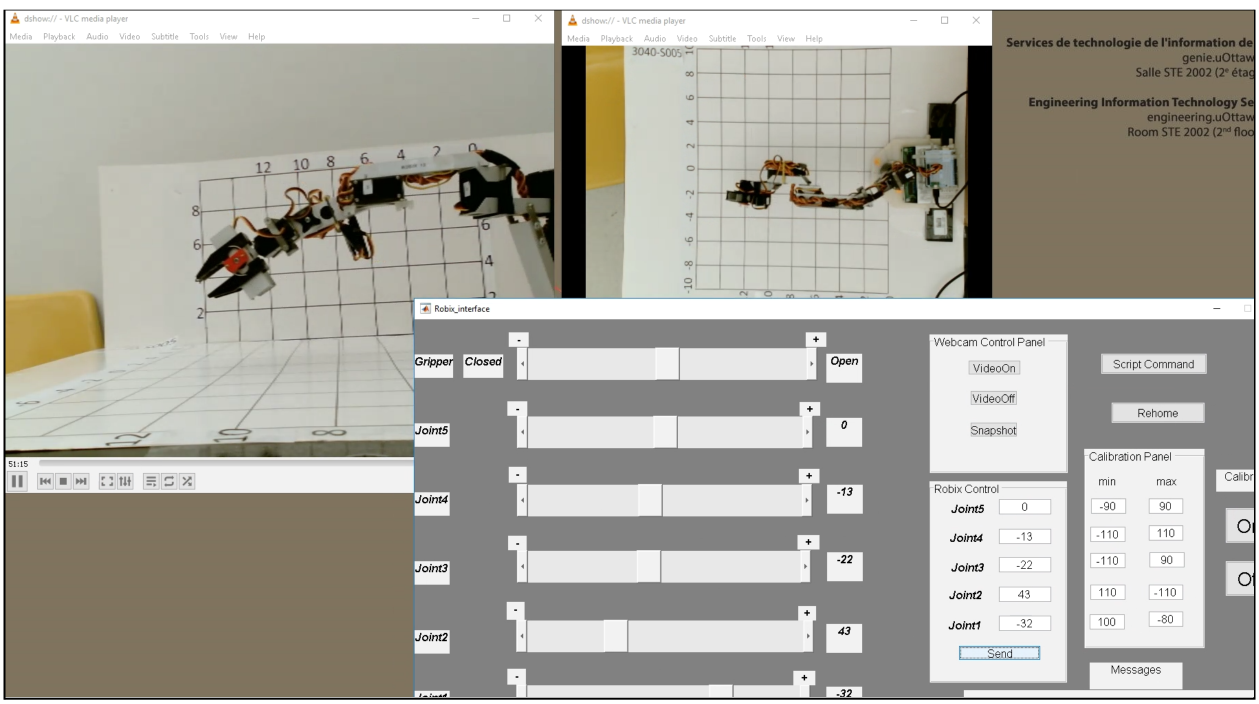
```
        else
            Qmatrix = Qmatrix*Amatrix(:,:,i);
        end
    end

    disp(Qmatrix);

    %Final end effector location and angles
    xt = Qmatrix(1,4);
    yt = Qmatrix(2, 4);
    zt = Qmatrix(3, 4);
    thetaf = atan2(Qmatrix(2,1), Qmatrix(1,1));
    phif = atan2(-Qmatrix(3,1),Qmatrix(1,1)*cosa(thetaf)+Qmatrix(2,1)...
        *sina(thetaf));
    psif = atan2(Qmatrix(2,2),Qmatrix(3,3));

```

Appendix B

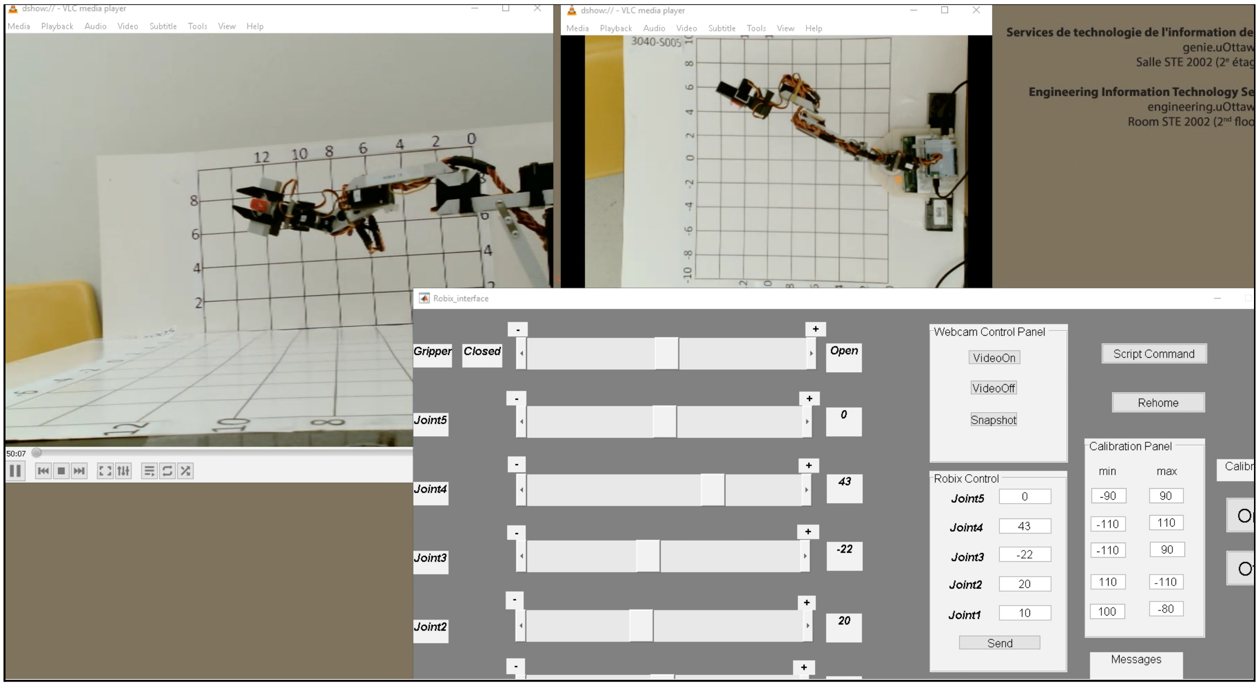
Set 1



Set 2



Set 3


dshow:// - VLC media player
Media Playback Audio Video Subtitle Tools View Help
Services de technologie de l'information de
genie.uOttaw
Salle STE 2002 (2e étag
Engineering Information Technology Se
engineering.uOttaw
Room STE 2002 (2nd floo
Robix_interface
Gripper
Closed
Open
Joint5
0
Joint4
43
Joint3
-22
Joint2
20
Webcam Control Panel
VideoOn
VideoOff
Snapshot
Robix Control
Joint5 0
Joint4 43
Joint3 -22
Joint2 20
Joint1 10
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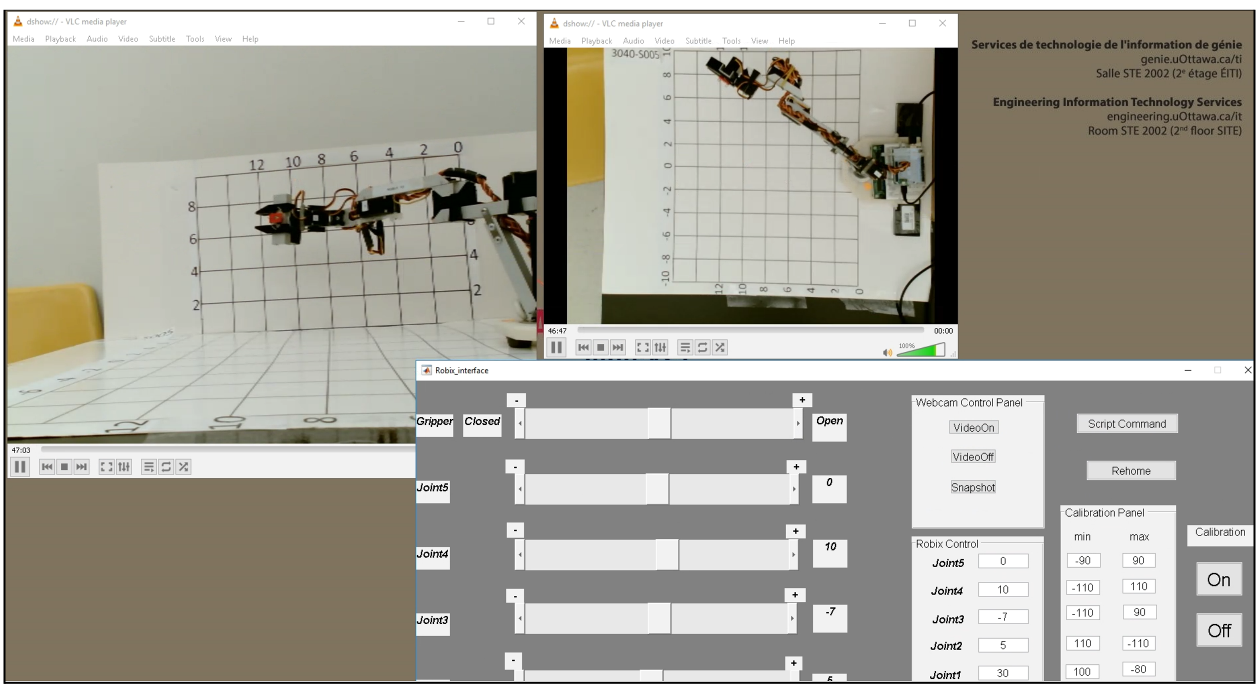
Script Command
Rehome
Calibration Panel
min max
-90 90
-110 110
-110 90
110 -110
100 -80
Messages

Set 4


dshow:// - VLC media player
Media Playback Audio Video Subtitle Tools View Help
Services de technologie de l'information de génie
genie.uOttawa.ca/ti
Salle STE 2002 (2e étage ÉITI)
Engineering Information Technology Services
engineering.uOttawa.ca/it
Room STE 2002 (2nd floor SITE)
Robix_interface
Gripper
Closed
Open
Joint5
0
Joint4
10
Joint3
-7
Webcam Control Panel
VideoOn
VideoOff
Snapshot
Robix Control
Joint5 0
Joint4 10
Joint3 -7
Joint2 5
Joint1 30
Script Command
Rehome
Calibration Panel
min max
-90 90
-110 110
-110 90
110 -110
100 -80
Calibration
On
Off

Appendix C

Matlab code implementing the analytically derived $Q_{effector/base}$ matrix

```
%---------------------Theta Values ---------------------------
theta1 = 0;
theta2 = 0;
theta3 = 0;
theta4 = 0;
theta5 = 0;
delta = 0;

%--------------------Calculate Q------------------------------
%Q = zeros(4, 4);
a = theta3+theta4+delta; %arbritrary constant for simplicity
b = theta1+theta2;
c = theta5;
Q = [cosd(c)*cosd(b)*cosd(a)+sind(c)*sind(b)  -sind(c)*cosd(b)*cosd(a)+cosd(c)*sind(b)...
    cosd(b)*sind(a)  cosd(b)*(110*sind(a)+60*cosd(theta3)+96)+98*cosd(theta1);
    cosd(c)*sind(b)*cosd(a)-sind(c)*cosd(b)  -sind(c)*sind(b)*cosd(a)-cosd(c)*cosd(b)...
    sin(b)*sind(a)  sind(b)*(110*sind(a)+60*cosd(theta3)+96)+98*sind(theta1);
    cosd(theta5)*sind(a)  -sind(theta5)*sind(a)  -cosd(a)  -110*cosd(a)+60*sind(theta3)+157;
    0 0 0 1];

%----------Final end effector location and angles--------------
thetaf = (atan2(Q(2,1), Q(1,1)))*180/pi;
phif = (atan2((-Q(3,1)),(Q(1,1)*cosd(thetaf)+Q(2,1)*sind(thetaf))))*180/pi;
psif = (atan2(Q(3,2),Q(3,3)))*180/pi;

%--------------------Display Q---------------------------------
display(Q);
```