

- a. From $t = 0$ to $t = 2$ hr
 - b. From $t = 0$ to $t = 4$ hr
 - c. From $t = 0$ to $t = 6$ hr
 - d. From $t = 0$ to $t = 8$ hr
 - e. If the temperature of the puddle was 30°C at $t = 0$, what was the temperature of the water at its coldest point?
 - f. Sketch a graph of the temperature versus the time from $t = 0$ to $t = 8$ hr. Label the vertical axis with numerical temperature values based on your answers from previous questions.
8. A quantity Q changes over time t according to the formula $Q(t) = 10 + 4t - t^2$, where t is in seconds. See Figure 28.

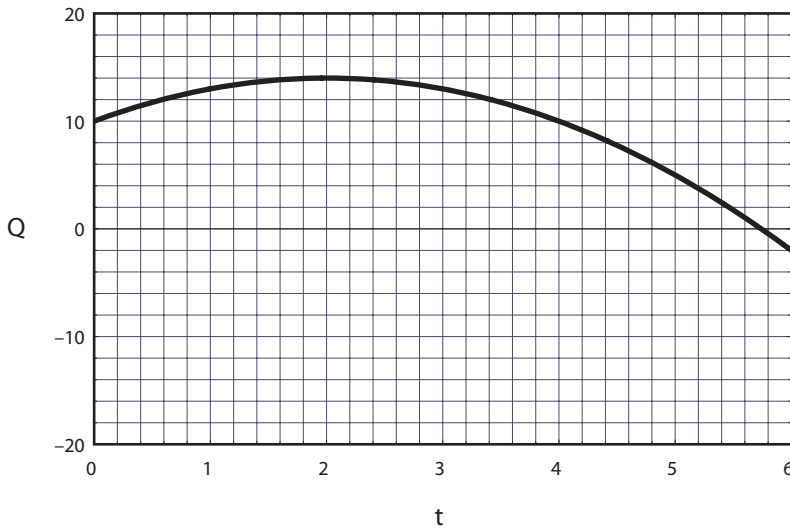


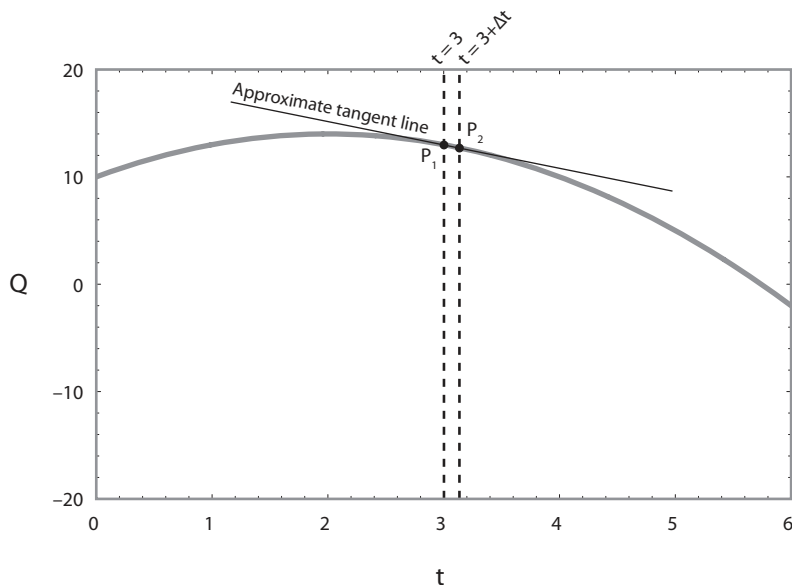
FIGURE 28.
Focused Problem 8.
The quantity
 $Q(t) = 10 + 4t - t^2$
from $t = 0$ to $t = 6$.

- a. Use the graph to estimate $\frac{d}{dt}Q$ at time $t = 3$.

The *exact* value of $\frac{d}{dt}Q$ at time $t = 3$ can be found in the following way. First, identify two particular points on the curve, a point P_1 at the time $t = 3$ we are focusing on and a point P_2 nearby. See Figure 29. The coordinates of these points are $P_1 = (3, Q(3))$ and $P_2 = (3 + \Delta t, Q(3 + \Delta t))$. Here Δt represents a small increment of time.

- b. Use the given formula, $Q = 10 + 4t - t^2$, together with the coordinates of P_1 and P_2 , to show that the slope of the line connecting P_1 and P_2 is given by $\frac{\Delta Q}{\Delta t} = -2 - \Delta t$.

FIGURE 29.
Beginning to solve
Focused Problem 8.
The thin solid line
through points P_1
and P_2 is a good
estimate of the
tangent line at time
 $t = 3$, when the
increment Δt is
small.



- c. The smaller Δt is, the better the tangent line in Figure 29 “kisses” the curve. So if we put $\Delta t = 0$, we’ll have the exact tangent line! Setting Δt to zero in the answer from part (b), we find the exact rate of change at time $t = 3$ to be $\frac{d}{dt}Q = -2$. Is this value close to the rate of change you estimated in part (a)?

Next we’ll consider the rate of change at any chosen time t instead of just at the specific time $t = 3$.

- d. Show that at any chosen time t , the rate of change of Q at time t is given by $4 - 2t$. (Hint: Adapt your method from parts (b) and (c).)
- e. Graph the rate-of-change function $\frac{d}{dt}Q = 4 - 2t$ from $t = 0$ to $t = 6$. Is the graph a plausible description of the slope of the curve in Figure 28?