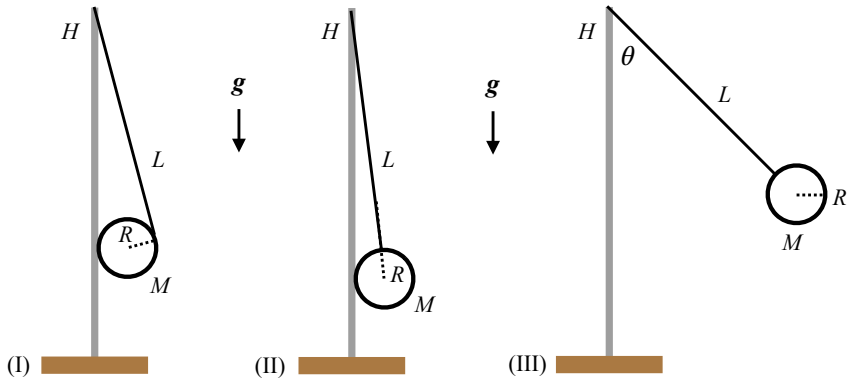
3. At the End of Our Tether [40 points]

At a playground near sea level, a tetherball apparatus consists of a fixed, rigid, vertical support post of height H and small diameter D, with a massless, inextensible rope of length L attached at one end to the top of the post and at the other end to a ball, which can be taken to be a thin spherical shell of radius R and total mass M. (The very top of the pole may be slightly tapered to allow rotation of the rope).



A tetherball (I) at rest, relying on sufficient friction, (II) at rest at a lower point, due to insufficient friction, and (III) revolving at a constant elevation around the post

Suppose that the various length-scales satisfy $0 \leq D \ll R < L < L + R < H$. The gravitational field $\boldsymbol{g}$ of the Earth will be important, but any other gravitational influences, as well as any air resistance, may be neglected. The coefficient of static friction between the side of the post and the surface of the ball is μ.

I. First, suppose the ball is at rest, supported by the side of the post and the rope, with the rope *tangent* to the ball at their point of connection:

(a) [5 points] What is the minimum possible value of the coefficient of static friction μ between the ball and support post, needed to keep the ball from slipping?

(b) [5 points] What is the force exerted by the post on the ball?

(c) [5 points] What is the force exerted by the rope on the post?

II. If the friction between the ball and post is insufficient, the ball will instead come to rest at the lowest possible point, with the rope in tension and the ball in contact with the side of the post:

(d) [5 points] What is the force exerted by the post on the ball?

(e) [5 points] What is the force exerted by the post on the rope?

III. Finally, the ball is picked up and then launched in an ideal circular trajectory with the rope remaining taught while making a *fixed angle* θ with the support pole, and with center of the ball and the points of attachment of the rope to ball and support all remaining in one line. Assume that the angle θ is sufficiently large that the rope can rotate about the top of the post, without friction, and without the rope wrapping around the post:

(f) [5 points] What is the period of revolution of the ball, around the vertical support?

(g) [5 points] What is the tension in the rope?