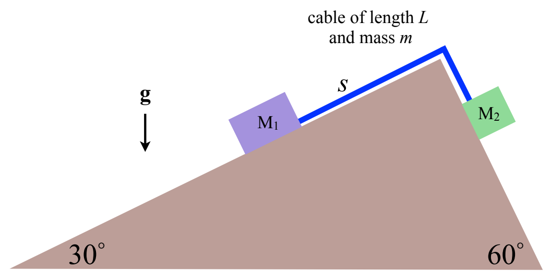
2. Slippery Slopes [30 points]

Workers hope to haul a crate (the "load") of initial mass $M_1 = M$ up a frictionless hill (of 30° slope) with the help of a steel cable connecting this load to a counterweight of mass $M_2 = \frac{2}{3}M$ on the other, steeper side of the hill (60° slope), also frictionless. The cable is thin, flexible but inextensible, and uniform, of total length L, but of non-negligible mass $m = \frac{1}{3}M$. The hill is near sea level, but air resistance may be neglected.



Two masses connected by a steel cable on a frictionless hill.

Let s represent the length of cable on the *left* side of the hill. Assume that the cable always remains taught and parallel to the hillsides, except where it bends very sharply around the (frictionless) apex of the hill, which effectively acts as an ideal pulley. The hill is tall enough that neither mass can reach the bottom while the masses remain connected by the cable but located on opposite sides of the hill.

After getting everything connected, suppose that the workers pause a lunch while the masses remain at rest:

(a) [5 points] How much of the cable is on the left hand side of the hill? Express this length s as a function of L and any other needed quantities.

(b) [5 points] What is the magnitude of the tension in the cable, at the very top of the mountain? Express your answer in terms of M, g, and any other needed quantities.

(c) [5 points] What is the magnitude of the tension in the cable where it connects to the load? Express your answer in terms of M, g, and any other needed quantities.

Alas, while the workers are still enjoying their lunch break, the crate breaks open, *suddenly* dumping *part* of its contents, and leaving the crate with mass $M_1' = \frac{1}{3}M$.

After this sudden loss of part of the mass, but before the crate reaches the top of the hill:

(d) [5 points] Find the acceleration $\boldsymbol{a}$ of the crate as a function of its position along the hillside. Express your answer as a function of s, M, L, g, and any other needed quantities, in terms of either a magnitude and direction, or horizontal and vertical components.

(e) [5 points] For a given value of s, where along the cable is the tension the greatest (in magnitude)? What is the magnitude of the tension there? Express your answer in terms of s, M, L, g, and any other needed quantities.

(f) [5 points] Find the speed of the crate just as it reaches the top of the hill. HINT: Use a chain rule to relate infinitesimal changes in speed, time, and position along the hillside. Rearrange and integrate in both velocity and position.