DOA Estimation Based on Sparse Signal Recovery Utilizing Weighted l_1-Norm Penalty

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***Abstract*—In this letter, a new DOA estimation method based on sparse signal recovery is proposed. We utilize the Capon spectrum to design a weighted l_1-norm penalty in order to further enforce the sparsity and approximate the original l_0-norm. A theoretical guidance for choosing a proper regularization parameter is also presented according to the dual form of the original problem. Simulation results demonstrate the effectiveness and efficiency of the proposed method.**

***Index Terms*—Convex optimization, direction of arrival (DOA), weighted l_1-norm penalty, sparse signal recovery.**

I. Introduction

DIRECTION of arrival (DOA) estimation of multiple narrowband signals is an important problem in array signal processing. Many high-resolution DOA estimation methods [1]–[3] have been developed over the years. However, the performance of these methods is generally not satisfactory in low SNR, small number of snapshots or closely spaced sources.

In recent years, a kind of novel direction finding methods based on the sparse signal recovery have been proposed [4], [6]–[10], which exploits the property that the spatial spectrum of the point source signals is sparse when the number of signals is limited. The literature [4] proposes a sparse representation model based on the l_1-norm penalty in time domain after the singular value decomposition (SVD) of the data matrix, converts the DOA estimation into a problem of sparse signal recovery, and then solves it in a second order cone (SOC) framework [5]. Based on [4], the literature [6] proposes a weighted l_1-norm penalty utilizing the property of noise subspace. However, there is a problem on how to select a proper regularization parameter especially when SNR is low. In [7] and [8], a whitened sparse covariance-based representation model is proposed by applying a global matched filter (GMF). Instead of approximating l_0-norm with the l_1-norm, the literature [9] proposes to exploit a class of Gaussian functions to deal with the $l_{2,0}$-norm minimization problem. In [10], a novel sparse iterative covariance-based estimation approach (SPICE) is presented so as to avoid parameter selection. Compared with the conventional DOA estimation methods, the methods based on the sparse signal recovery have higher resolution, require smaller number of snapshots, and need no priori knowledge about the number of signals in general.

In this letter, we propose a new method of DOA estimation based on sparse signal recovery. We design a weighted l_1-norm penalty whose weights correspond to the Capon spectrum in order to get a better approximation of l_0-norm and further enforce the sparsity. In addition, we give a theoretical guidance on how to select a proper regularization parameter according to the dual form of the original problem. Several simulations are presented to evaluate the performance of the proposed method.

II. Problem Formulation

Consider K uncorrelated narrowband far-field signals impinging on an M-element uniform linear array (ULA), where the distance between adjacent elements is equal to half the wavelength. Assume that each signal $s_k(t)$ comes from a different direction θ_k with power $\sigma_k^2, k=1,\ldots,K$. The $M\times 1$ array output vector $\mathbf{x}(t)$ is then given by

$$\mathbf{x}(t)=\sum_{k=1}^{K}\mathbf{a}(\theta_k)s_k(t)+\mathbf{n}(t)=\mathbf{A}\mathbf{s}(t)+\mathbf{n}(t) \tag{1}$$

where $\mathbf{a}(\theta)=\left[1,\ldots,e^{-j(M-1)\pi\cos\theta}\right]^T$ is the $M\times 1$ steering vector, $\mathbf{A}=[\mathbf{a}(\theta_1),\ldots,\mathbf{a}(\theta_K)]$, $\mathbf{s}(t)=[s_1(t),\ldots,s_K(t)]^T$ and $\mathbf{n}(t)$ is the $M\times 1$ noise vector with the power of each entry equal to σ_n^2. By assumption, the entries of $\mathbf{s}(t)$ and $\mathbf{n}(t)$ are zero mean wide-sense stationary random processes, and the entries of $\mathbf{n}(t)$ are uncorrelated with each other and the signals.

From (1), we can also obtain the $M\times M$ array covariance matrix which will be used in Section III.

$$\mathbf{R}_x=E\left\{\mathbf{x}(t)\mathbf{x}^H(t)\right\}=\mathbf{A}\mathbf{R}_s\mathbf{A}^H+\sigma_n^2\mathbf{I}_M \tag{2}$$

where $\mathbf{R}_s=E\left\{\mathbf{s}(t)\mathbf{s}^H(t)\right\}=\mathrm{diag}\left\{\sigma_1^2,\ldots,\sigma_K^2\right\}$, and $\mathbf{I}_m$ represents an $m\times m$ identity matrix. The symbol $\mathrm{diag}\{z_1,z_2\}$ represents a diagonal matrix with diagonal entries z_1 and z_1.In addition, the operator $(\cdot)^*,(\cdot)^T,(\cdot)^H,(\cdot)^{-1},E\{\cdot\}$ denotes conjugate, transpose, conjugate transpose, inverse and expectation, respectively.

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III. DOA Estimation Utilizing Weighted L1-Norm Penalty

A. DOA Estimation in Sparse Spatial Domain

We start to formulate the DOA estimation problem as a following sparse representation problem [4].

$$\mathbf{X}=\mathbf{BS}+\mathbf{N} \tag{3}$$

where $\mathbf{X}=[\mathbf{x}(t_1),\ldots,\mathbf{x}(t_L)]$, $\mathbf{B}=[\mathbf{b}_1,\ldots,\mathbf{b}_{\bar{K}}]$ is an overcomplete basis matrix, $\mathbf{b}_i$ corresponds to the steering vector of the angle $\bar{\theta}_i$, the set $\Theta=\left\{\bar{\theta}_1,\ldots,\bar{\theta}_{\bar{K}}\right\}$ is a sampling grid of all potential directions in spatial domain, $\mathbf{S}=[\bar{\mathbf{s}}(t_1),\ldots,\bar{\mathbf{s}}(t_L)]$, $\bar{\mathbf{s}}(t_l)$ is a sparse vector whose ith element is nonzero and equal to $s_k(t_l)$ if signal k comes from $\bar{\theta}_i$ for some k and zero otherwise, and $\mathbf{N}=[\mathbf{n}(t_1),\ldots,\mathbf{n}(t_L)]$. In general, $\bar{K}\gg M>K$. Note that $\mathbf{B}$ is known and does not depend on the actual signal directions. It means that we can estimate the signal directions as long as we find the position of nonzero values in $\bar{\mathbf{s}}(t_l)$. That is, the problem of DOA estimation is converted into one of sparse signal recovery from (3).

Furthermore, we can take the SVD of $\mathbf{X}$ to reduce the computational cost.

$$\mathbf{X}=\mathbf{ULV}^H \tag{4}$$

Define $\mathbf{X}_{\rm SV}=\mathbf{XVW}_{K,0}^L$, $\mathbf{S}_{\rm SV}=\mathbf{SVW}_{K,0}^L$, and $\mathbf{N}_{\rm SV}=\mathbf{NVW}_{K,0}^L$, where $\mathbf{W}_{K,0}^L=[\mathbf{I}_K,\mathbf{0}]$, and $\mathbf{0}$ is a $K\times(L-K)$ zero matrix. We get

$$\mathbf{X}_{\rm SV}=\mathbf{BS}_{\rm SV}+\mathbf{N}_{\rm SV}. \tag{5}$$

The above problem of noisy sparse signal recovery can then be converted into a following optimization problem using the whitened GMF form [7], [8]

$$\min\frac{\|\mathbf{X}_{SVW}-\mathbf{B}_W\mathbf{S}_{\rm SV}\|_2^2}{2}+h\|\bar{\mathbf{s}}^{(l_2)}\|_0. \tag{6}$$

where $\bar{\mathbf{s}}^{(l_2)}=\left[\bar{s}_1^{(l_2)},\ldots,\bar{s}_{\bar{K}}^{(l_2)}\right]^T$, $\bar{\mathbf{s}}_i=\left[\bar{s}_i^{\rm SV}(1),\ldots,\bar{s}_i^{\rm SV}(K)\right]^T$, $\bar{s}_i^{(l_2)}=\|\bar{\mathbf{s}}_i\|_2$, $\bar{s}_i^{\rm SV}(k)$ denotes the (i,k)th element of matrix $\mathbf{S}_{\rm SV}$, $\mathbf{X}_{{\rm SV}W}=\bar{\mathbf{R}}_n^{-1/2}\mathbf{X}_{\rm SV}$, $\mathbf{B}_W=\bar{\mathbf{R}}_n^{-1/2}\mathbf{B}$, $\bar{\mathbf{R}}_n=\sigma_n^2\mathbf{I}_M$, $\|\cdot\|_2$ denotes the l_2-norm, and $\|\cdot\|_0$ denotes the l_0-norm. Here we exploit the property that $\mathbf{n}_{\rm SV}=\mathbf{x}_{\rm SV}-\mathbf{Bs}_{\rm SV}$ is an additive complex Gaussian random vector with mean $\mathbf{0}$ and covariance matrix $\bar{\mathbf{R}}_n$ which is denoted as $\mathbf{n}_{\rm SV}\sim\mathcal{N}(\mathbf{0},\bar{\mathbf{R}}_n)$, where $\mathbf{x}_{\rm SV},\mathbf{s}_{\rm SV},\mathbf{n}_{\rm SV}$ corresponds to the same column in $\mathbf{X}_{\rm SV}$, $\mathbf{S}_{\rm SV}$, $\mathbf{N}_{\rm SV}$, respectively. The regularization parameter h controls the tradeoff between l_2 term and l_0 term. Note that (6) is nonconvex and very hard to solve. An alternative is to use l_1-norm instead of l_0-norm to enforce sparsity [4], which leads to

$$\min\frac{\|\mathbf{X}_{SVW}-\mathbf{B}_W\mathbf{S}_{\rm SV}\|_2^2}{2}+h\left\|\bar{\mathbf{s}}^{(l_2)}\right\|_1 \tag{7}$$

However, in most cases, the entries in $\bar{\mathbf{s}}^{(l_2)}$ does not satisfy $\bar{s}_i^{(l_2)}\le 1$, which means that the l_1-norm is not the convex envelope of the l_0-norm function. Thus, the criterion (7) is not a good convex approximation of (6).

As is known to all, we have $\|\bar{\mathbf{s}}^{(l_2)}\|_1=\|\bar{\mathbf{s}}^{(l_2)}\|_0$ when the absolute value of every entry of $\bar{\mathbf{s}}^{(l_2)}$ is either 0 or 1. Also, if we know that $\bar{s}_i^{(l_2)}\le u_i$ where u_i is a constant, based on the Boolean interpretation in [11], we can convert (6) into a mixed Boolean problem.

$$\begin{aligned}&\min\frac{\|\mathbf{X}_{SVW}-\mathbf{B}_W\mathbf{S}_{\rm SV}\|_2^2}{2}+h\mathbf{l}^T\mathbf{z}\\&s.t.\ \bar{s}_i^{(l_2)}\le u_iz_i, z_i\in\{0,1\}, i=1,\ldots,\bar{K}\end{aligned} \tag{8}$$

It is also not a convex problem. By relaxing $z_i\in\{0,1\}$ to $z_i\in[0,1]$, we obtain its convex approximation

$$\begin{aligned}&\min\frac{\|\mathbf{X}_{SVW}-\mathbf{B}_W\mathbf{S}_{\rm SV}\|_2^2}{2}+h\mathbf{1}^T\mathbf{z}\\&s.t.\ \bar{s}_i^{(l_2)}\le u_iz_i, 0\le z_i\le 1, i=1,\ldots,\bar{K}\end{aligned} \tag{9}$$

which is equivalent to

$$\min\frac{\|\mathbf{X}_{SVW}-\mathbf{B}_W\mathbf{S}_{\rm SV}\|_2^2}{2}+h\sum_{i=1}^{\bar{K}}w_i\bar{s}_i^{(l_2)} \tag{10}$$

where $w_i=1/u_i$. Note that (10) is a standard form of weighted l_1-norm directly derived from canonical l_0-norm, thus, (10) is a better approximation of (6) than the traditional l_1-norm penalty. In fact, a similar form of weighted l_1-norm has already been presented in [12] using sequential convex optimization.

Following the idea above, our goal is to find a kind of weights $w_i=1/u_i$ that satisfy $\bar{s}_i^{(l_2)}\le u_iz_i$ in (9). Also, these weights should change with the signal environment. The greater the signal powers, the smaller the corresponding weights. We propose to make w_i the following form.

$$w_i=\sqrt{\frac{\mathbf{b}_i^H\mathbf{R}_x^{-1}\mathbf{b}_i}{LK}} \tag{11}$$

We can see that $1/\mathbf{b}_i^H\mathbf{R}_x^{-1}\mathbf{b}_i$ corresponds to the Capon spectrum and reflects the average power of the signal from $\bar{\theta}_i$ plus the noise. Note that $\bar{s}_i^{(l_2)^2}$ reflects LK times the average power of the signal from $\bar{\theta}_i$. Then, we have the inequality

$$\bar{s}_i^{(l_2)}\le\sqrt{\frac{LK}{\mathbf{b}_i^H\mathbf{R}_x^{-1}\mathbf{b}_i}} \tag{12}$$

which can be easily proved. Then the constraint in (9) holds. Further, if $\bar{s}_i^{(l_2)}$ corresponds to the actual signal, w_i is relatively small and $w_i\bar{s}_i^{(l_2)}$ will approach to 1, otherwise, w_i is relatively large and $w_i\bar{s}_i^{(l_2)}$ will approach to 0.

B. Selecting the Regularization Parameter

The value h plays an important role in the final performance. A large h emphasizes the role of the l_1-term, which may cause wrong DOA estimation. A small h emphasizes the role of the l_2-term, which may produce many spurious peaks in spatial spectrum. Then a theoretical guidance of selecting it is very necessary. By vectorizing the matrices in (10), we can get its equivalent form.

$$\begin{aligned}&\min_{\bar{\mathbf{s}}}\frac{\|\mathbf{y}\|_2^2}{2}+h\sum_{i=1}^{\bar{K}}w_i\bar{s}_i^{(l_2)}\\&s.t.\ \mathbf{y}=\bar{\mathbf{x}}_W-\bar{\mathbf{B}}_W\bar{\mathbf{s}}\end{aligned} \tag{13}$$

where $\bar{\mathbf{x}}_W = \mathrm{vec}(\mathbf{X}_{SVW})$, $\bar{\mathbf{s}} = \mathrm{vec}(\mathbf{S}_{\mathrm{SV}})$, $\mathrm{vec}(\cdot)$ denotes the vectorization operator by stacking the columns of a matrix one underneath the other, and $\bar{\mathbf{B}}_W = \mathrm{blkdiag}\{\mathbf{B}_W, \ldots, \mathbf{B}_W\}$ is an $M\bar{K} \times K\bar{K}$ block diagonal matrix. Thus, the Lagurangian dual criterion can be written as

$$\min_{\bar{\mathbf{s}},\mathbf{y}} \frac{\|\mathbf{y}\|_2^2}{2} + h\sum_{i=1}^{\bar{K}} w_i \bar{s}_i^{(l_2)} + \boldsymbol{\mu}^T\left(\mathbf{y} - \bar{\mathbf{x}}_W + \bar{\mathbf{B}}_W\bar{\mathbf{s}}\right) \quad (14)$$

This is a separable optimization problem. We first minimize (14) over $\mathbf{y}$, which achieves the minimum when

$$\mathbf{y} = -\boldsymbol{\mu}. \quad (15)$$

Substituting (15) back into (14) and we get the following criterion

$$\min_{\bar{\mathbf{s}}} -\frac{\|\boldsymbol{\mu}\|_2^2}{2} - \boldsymbol{\mu}^T\bar{\mathbf{x}}_W + h\sum_{i=1}^{\bar{K}} w_i\bar{s}_i^{(l_2)} + \boldsymbol{\mu}^T\bar{\mathbf{B}}_W\bar{\mathbf{s}} \quad (16)$$

Then, we minimize (16) respect to $\bar{\mathbf{s}}$, which is equivalent to minimizing $h\sum_{i=1}^{\bar{K}} w_i\bar{s}_i^{(l_2)} + \boldsymbol{\mu}^T\bar{\mathbf{B}}_W\bar{\mathbf{s}}$. Rewrite the function according to the rows of $\mathbf{S}_{\mathrm{SV}}$ as the following form.

$$\begin{aligned} h\sum_{i=1}^{\bar{K}} w_i\bar{s}_i^{(l_2)} + \boldsymbol{\mu}^T\bar{\mathbf{B}}_W\bar{\mathbf{s}} &= \sum_{i=1}^{\bar{K}}\left(hw_i\bar{s}_i^{(l_2)} + \bar{\boldsymbol{\beta}}_i^T\bar{\mathbf{s}}_i\right) \\ &\triangleq \sum_{i=1}^{\bar{K}} f_i(\bar{\mathbf{s}}_i) \end{aligned} \quad (17)$$

where $\bar{\boldsymbol{\beta}}_i = [\beta_{i1}, \ldots, \beta_{iK}]^T$, and β_{ij} is an element in $\boldsymbol{\mu}^T\bar{\mathbf{B}}_W$ which corresponds to $\bar{s}_i^{\mathrm{SV}}(j)$. It is obvious that (17) reaches its minimum if each $f_i(\bar{\mathbf{s}}_i)$ is minimized respectively. It can be easily proved by the contradiction that the necessary condition for $f_i(\bar{\mathbf{s}}_i)$ to get its minimum is $\bar{\mathbf{s}}_i = \mathbf{0}$. As $f_i(\bar{\mathbf{s}}_i)$ is nondifferentiable at $\bar{\mathbf{s}}_i = \mathbf{0}$, according to [11], the necessary and sufficient condition for $\bar{\mathbf{s}}_i = \mathbf{0}$ being a global minimum of $f_i(\bar{\mathbf{s}}_i)$ is that $\mathbf{0}$ is a subgradient of $f_i(\bar{\mathbf{s}}_i)$ at zero. A vector $\mathbf{g}$ is a subgradient of $f_i(\bar{\mathbf{s}}_i)$ at zero if $f_i(\bar{\mathbf{s}}_i) \geq f_i(\mathbf{0}) + \mathbf{g}^T\bar{\mathbf{s}}_i$ for any $\bar{\mathbf{s}}_i$. When $\mathbf{g} = \mathbf{0}$, we have $f_i(\bar{\mathbf{s}}_i) = hw_i\bar{s}_i^{(l_2)} + \bar{\boldsymbol{\beta}}_i^T\bar{\mathbf{s}}_i \geq 0$. Using Cauchy inequality $\left\|\bar{\boldsymbol{\beta}}_i\right\|_2\|\bar{\mathbf{s}}_i\|_2 \geq \left|\bar{\boldsymbol{\beta}}_i^T\bar{\mathbf{s}}_i\right|$, we only need $hw_i\bar{s}_i^{(l_2)} \geq \left\|\bar{\boldsymbol{\beta}}_i\right\|_2\|\bar{\mathbf{s}}_i\|_2 = \left\|\bar{\boldsymbol{\beta}}_i\right\|_2\bar{s}_i^{(l_2)}$. Thus, the dual criterion of (13) can be written as

$$\begin{aligned} &\min_{\boldsymbol{\mu}} \frac{\|\boldsymbol{\mu}\|_2^2}{2} + \boldsymbol{\mu}^T\bar{\mathbf{x}}_W \\ &s.t.\ \left\|\bar{\boldsymbol{\beta}}_i\right\|_2 \leq hw_i,\ i = 1, \ldots, \bar{K}. \end{aligned} \quad (18)$$

Using the KKT condition [11], the primal constraint $\mathbf{y} = \bar{\mathbf{x}}_W - \bar{\mathbf{B}}_W\bar{\mathbf{s}}$ must be satisfied. Combining with (15), we get

$$\begin{aligned} &\min_{\bar{\mathbf{s}}} \left\|\bar{\mathbf{B}}_W\bar{\mathbf{s}}\right\|_2^2 \\ &s.t. \sqrt{\sum_{j=1}^{K}\left(\bar{\mathbf{b}}_{ij}^T(\bar{\mathbf{x}}_W - \bar{\mathbf{B}}_W\bar{\mathbf{s}})\right)^2} \\ &\quad\leq hw_i\ i = 1, \ldots, \bar{K} \end{aligned} \quad (19)$$

where $\bar{\mathbf{b}}_{ij}$ is a column in $\bar{\mathbf{B}}_W$ which corresponds to $\bar{s}_i^{\mathrm{SV}}(j)$. It is known that $\bar{\mathbf{n}} = \bar{\mathbf{B}}_W\bar{\mathbf{s}} - \bar{\mathbf{x}}_W \sim \mathcal{N}(0, \mathbf{I}_{MK})$. Then $\sqrt{2\sigma_n^2/M}\bar{\mathbf{b}}_{ij}^T\bar{\mathbf{n}} \sim \mathcal{N}(0,2)$ and $2\sigma_n^2\left|\bar{\mathbf{b}}_{ij}^T\bar{\mathbf{n}}\right|^2/M$ will satisfy a χ^2 distribution with 2 degrees of freedom. Thus, $2\sigma_n^2\sum_{j=1}^{K}\left(\bar{\mathbf{b}}_{ij}^T(\bar{\mathbf{x}}_W - \bar{\mathbf{B}}_W\bar{\mathbf{s}})\right)^2/M$ satisfies a χ^2 distribution with $2K$ degrees of freedom. Now, the task of choosing a regularization parameter h properly is considerably more transparent. We choose h so that the probability of satisfying all $\bar{K}$ inequalities $\sqrt{\sum_{j=1}^{K}\left(\bar{\mathbf{b}}_{ij}^T(\bar{\mathbf{x}}_W - \bar{\mathbf{B}}_W\bar{\mathbf{s}})\right)^2} \leq h\min\{\mathbf{w}\}, i = 1, \ldots, \bar{K}$, which is defined as p, is very large, where $\min\{\mathbf{w}\}$ denotes the minimum element of $\mathbf{w} = [w_1, \ldots, w_{\bar{K}}]^T$. Define the probability that a single inequality is satisfied is p_0. For simplicity, we set $p_0 = \sqrt[\bar{K}]{p}$ and then have

$$h\min\{\mathbf{w}\} = \sqrt{\frac{M\gamma}{2\sigma_n^2}}, \quad \Pr(\chi_{2K}^2 \leq \gamma) = p_0. \quad (20)$$

Now $h = \sqrt{M\gamma}/\left(\sqrt{2\sigma_n^2}\min\{\mathbf{w}\}\right)$. We will see that the regularization parameter h can lead to quite good performance in the following simulations. Substituting h into (10), we can directly use an optimization package called SeDuMi [13] to solve the optimization problem in (10).

IV. Simulation Experiments and Discussions

In this section, we will present several simulation results which illustrate the effectiveness of the proposed method. The method l_1-SVD in [4], the NSW-l_1 in [6], and CRLB are selected to be compared methods. The input SNR of the kth signal is defined as $10\log_{10}(\sigma_k^2/\sigma_n^2)$. Assume $\sigma_1^2 = \cdots = \sigma_K^2$. The covariance matrix $\mathbf{R}_x$ is estimated through L snapshots as $\hat{\mathbf{R}}_x = \sum_{t=1}^{L}\mathbf{x}(t)\mathbf{x}^H(t)/L$, and the noise power is estimated as the average of $M - K$ smallest eigenvalues of $\hat{\mathbf{R}}_x$. The number of array elements is set to be 11. The probability p is set to be 0.99. The regularization parameter in [4] and [6] is selected to be 0.99 confidence interval. Define the average root mean square error (RMSE) of the estimates from 500 Monte Carlo trials as the performance index:

$$RMSE = \sqrt{\sum_{m=1}^{500}\sum_{k=1}^{K}\frac{\left(\hat{\theta}_k(m) - \theta_k\right)^2}{500K}} \quad (21)$$

where $\hat{\theta}_k(m)$ is the estimate of θ_k in the mth trial.

The estimation precision of DOA is restricted by the resolution of the grid set Θ, but too fine grid will increase the computational complexity. In the simulations, we first use a coarse grid in the range of 0° to 180° with 1° spacing and then set a finer grid around the estimated angles to improve the estimation precision.

The first simulation considers the scenario that there are six uncorrelated signals impinging from $[30.5°, 65°, 75°, 90°, 110.4°, 140.6°]$, respectively. Note that there are two closely spaced angles (10° separation). The RMSE of the DOA estimates versus input SNR is shown in Fig. 1 with 200 snapshots, whereas the RMSE of the DOA estimates versus number of snapshots is shown in Fig. 2 with -5 dB SNR. We can see that the proposed method has the best performance especially when SNR is low or the number of snapshots is small. The reason is that the weighted l_1-norm we use in (10) is a better approximation of l_0-norm than those

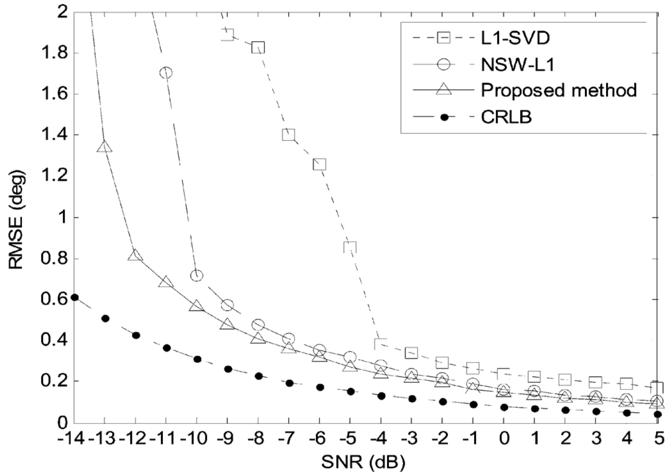

Fig. 1. RMSE of the DOA estimates versus input SNR with 200 snapshots.

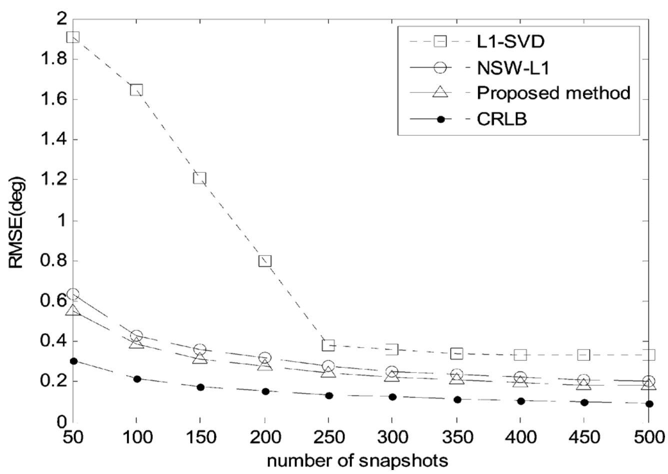

Fig. 2. RMSE of the DOA estimates versus number of snapshots with -5 dB SNR.

in [4] and [6], also, the regularization parameter h is selected properly according to a theoretical guidance.

The second simulation considers the resolving ability of those methods. Two uncorrelated signals impinge on the array from $90°, 90° + \Delta\theta$. The SNR is -5 dB and the number of snapshots is 200. The RMSE of the DOA estimates versus angle separation $\Delta\theta$ is shown in Fig. 3. It is presented that the proposed method still yields the best performance when the angle separation is no less than 3°.

V. Conclusions

In this letter, we present a new formulation of DOA estimation problem in a sparse signal representation framework, where a weighted l_1-norm penalty is used based on Capon spectrum. A theoretical guidance for choosing the regularization parameter is also given. Simulation results validate the effectiveness of the new method and illustrate that it has higher estimation precision and resolution ability than the compared methods especially when SNR is low or the number of snapshots is small.

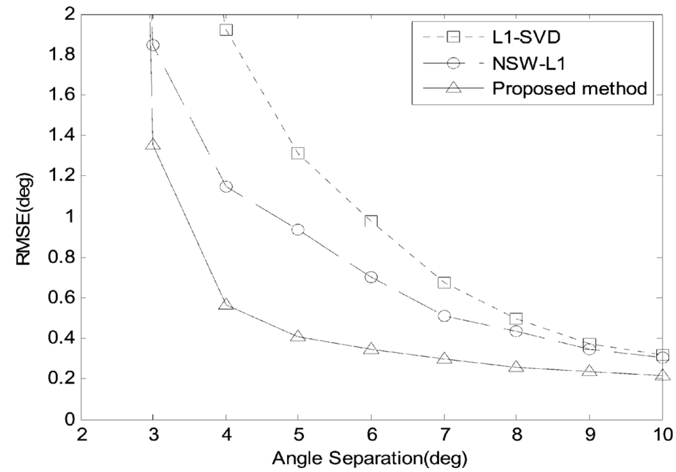

Fig. 3. RMSE of the DOA estimates versus angle separation with -5 dB SNR and 200 snapshots.

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