A Modified IIN Algorithm for DOA Estimation Based on Sparse Representation

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Abstract— Recently, sparse representation has been widely used in localization and bearing estimation. The basic idea of the general sparse direction-of-arrival (DOA) estimation method is to divide the space into discrete grids. But the transmission signal's DOA doesn't always fall on the discrete network. This paper reformulates the above problem by exploiting the sparsity representation based on the modified iterative interpolation (IIN) algorithm. First, the linear combination of eigenvectors of the array covariance matrix is used in this paper. The multiple measurement vectors (MMV) can be converted to a single measurement vector (SMV) for sparse solution calculation in this way. And this method can reduce high computation of the MMV. Then, an on-grid DOA estimation can be got by the orthogonal matching pursuit (OMP) algorithm. In order to get an off-grid DOA which is closer to the real one, the modified IIN algorithm is simulated based on the on-grid DOA estimation which is obtained by the OMP algorithm. In the modified IIN algorithm, the most matched dictionary atom with the real signal DOA and the two neighboring atoms whose difference is semi gird resolution are chosen and the corresponding vectors are regarded as the measurement vectors. The smallest and sub-smallest Euclidean distances between the measurement vectors or its residual are applied to further improve the DOA estimation. The essential idea of the algorithm is a coarse-to-fine estimation. We demonstrate the effectiveness of the method on simulated data by comparing the estimator variance with the the L1-SVD algorithm. Simulation results show that our approach can reduce the DOA estimation error caused by grid effect and own low computation load.

1. INTRODUCTION

The problem of DOA estimation has been a major research topic in the field of array signal processing [1–4]. It has played a prominent role in the field of radar, sonar, communication, etc.. The traditional DOA estimation algorithm is mainly divided into three categories: The spatial spectrum estimation algorithm [5], the subspace-based algorithm [6] and the deterministic maximum likelihood (DML) algorithm [7]. However, the traditional subspace-based algorithm has many defects, some of which can't be improved by improving the algorithm. For example, the estimation accuracy is limited by the "Rayleigh limit" and the SNR can't be too low. How to solve the above problems effectively without increasing the complexity of the system is the key to the current DOA estimation. With the rise of the signal sparse representation technology, there is a new method for DOA estimation and some algorithms with excellent performance have emerged. The DOA estimation algorithm based on sparse representation converts the DOA estimation to reconstructing the sparse signal from MMV by utilizing the priori information on the sparse characteristic of the spatial domain of the transmitted signal [8]. The l_1-based singular value decomposition (L1-SVD) algorithm [9] constructs a joint sparse model based on signal singularity vector under the constraint of $l_{2,1}$ norm. Then the second order cone programming (SOCP) is applied to find the global optimal solution. Yin [10] proposed a noise suppression method based on asymptotic statistical characteristic. In this method, an array covariance matrix of the incident signal is used to estimate the signal DOA. However, these algorithms have to consider the correlation among the sampled data when the number of snapshots changes. Using the Toeplitz characteristic of the uniform linear array covariance matrix, Xu [11] simplified the DOA estimation as a SMV problem. Chen [12] proposed an algorithm based on weighted characteristic vector to convert MMV to SMV.

However, the sparse representation algorithms also face the problem of grid mismatch. This phenomenon will lead to the algorithm performance degradation. Of course, the accuracy of DOA estimation can be improved by diminishing the grid resolution. But it also can increase the amount of computation. On the contrary, if the number of grid points is reduced, the probability of grid mismatch will increase and the performance of DOA estimation will decrease. In order to balance the calculation and estimation precision, some algorithms such as iterative mesh refinement algorithm [13], sparse total least squares (STLA) algorithm [14] and coarse-to-fine multiple DOA estimation algorithm [15] were proposed. But the calculation of these algorithms is still great. In order to effectively reduce the computation, this paper proposes a sparse DOA estimation algorithm based on the modified iterative interpolation (IIN) algorithm. The proposed algorithm uses the modified IIN algorithm to further refine the result of DOA estimation and effectively improves the accuracy of DOA estimation.

The rest of the paper is organized as follows: Section 2 introduces a method to convert MMV into SMV. In Section 3, a new DOA estimation algorithm based on modified IIN algorithm is proposed. The simulation results are given in Section 4. In last Section, we summarize the work of this paper.

2. THE BASIC MODEL

Consider K narrowband far-field signals from distinct directions $\{\theta_k\}_{k=1}^{K}$ impinging on an uniform linear array (ULA) of M sensors with isotropic direction. The $M \times 1$ received data can be written as:

$$\mathbf{y}(t) = \mathbf{A}(\theta)\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{y}(t) = [y_1(t), y_2(t), \ldots, y_M(t)]^T$ is the received signal vector, $\mathbf{n}(t)$ is the complex Gaussian additive noise with zero mean and δ_n variance, $\mathbf{s}(t) = [s_1(t), s_2(t), \ldots, s_K(t)]^T$ is the source signal vector and $\mathbf{A}(\theta) = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2, \ldots, \mathbf{a}(\theta_K))]$ is the manifold array whose Kth steering vector is given by

$$\mathbf{a}(\theta_k) = \left[1, e^{j\phi(\theta_k)}, \ldots, e^{j(M-1)\phi(\theta_k)}\right] \tag{2}$$

where $\phi(\theta_k) = -2\pi\frac{d}{\lambda}\sin(\theta_k)$, $\theta_k \in (-\pi/2, \pi/2)$. λ and d represent the signal wavelength and the array element spacing, respectively.

The DOA estimation problem is to estimate the DOA of the transmit source signal by the received vector $\mathbf{y}(t)$. The single snapshot DOA estimation has its application value, but the DOA estimation of multiple snapshots data is more general in practical application. Considering the time sampling in the formula (1), the multiple snapshots narrowband signal DOA estimation problem can be expressed as:

$$\mathbf{Y} = \mathbf{A}(\theta)\mathbf{S} + \mathbf{N} \tag{3}$$

where $\mathbf{Y} = [\mathbf{Y}(t_1), \mathbf{Y}(t_2), \ldots \mathbf{Y}(t_L)]$, L is the number of snapshots and the definitions of $\mathbf{S}$ and $\mathbf{N}$ are the same as $\mathbf{Y}$.

The number of incident sources is limited relative to the whole angular space, that is to say, the transmission source has a sparse distribution in the angle space, so the signal can be represented by the over-complete atoms. The main idea of sparse representation DOA estimation is to expand the array manifold matrix $\mathbf{A}$ into an over-complete dictionary that contains all angles of transmission signals.

$$\mathbf{\Phi} = \left[\mathbf{a}(\bar{\theta}_1), \mathbf{a}(\bar{\theta}_2), \ldots, \mathbf{a}(\bar{\theta}_N)\right] (N \gg K) \tag{4}$$

where the over-complete dictionary $\mathbf{\Phi}$ is known and not related to the DOA of the transmission signal, which is composed by a series of discrete points set $\mathbf{\Omega} = \{\bar{\theta}_1, \bar{\theta}_2, \ldots, \bar{\theta}_N\}$ with uniformly spaced.

A $N \times 1$ dimensional vector is used to represent the unknown source signals. The element of $\mathbf{S}(t_j)$ is not equal to zero when $\bar{\theta}_n$ is equal to θ_k. Otherwise, $\mathbf{S}(t_j)$ is equal to zero. Therefore, the DOA information of source signals can be obtained from the non-zero position of $\mathbf{S}(t_j)$. However, real DOA may not always be equal to $\bar{\theta}_n (n = 1, 2, \ldots, N)$. If $\mathbf{\Omega}$ is dense enough, $\bar{\theta}_n$ approximately equals to θ_k. According to the above analysis, we can expand the spatial domain of the formula (3) and get the sparse data model of the array space:

$$\mathbf{Y} = \mathbf{\Phi}\mathbf{S} + \mathbf{N} \tag{5}$$

This paper only considers the signal in the static situation, so the signal DOA is a time-invariant vector during the whole measurement process. The non-zero values for each column in $\mathbf{S}$ appear in the same row. We can seek the joint sparse matrix $\mathbf{S}$ appear in the same row. We can seek the joint sparse matrix $\mathbf{Y}$. Therefore, the DOA estimation can be simplified as a MMV problem.

In order to reduce the computation of the MMV model, a sparse DOA estimation algorithm based on the weighted characteristic vector is used to convert the MMV into a SMV. First of all, this algorithm needs to carry out eigenvalue decomposition of the array covariance matrix. Then, the linear combination of the feature vectors in the signal subspace is regarded as a new observation vector, so that the MMV model can be simplified. Finally, the incident signal DOA can be obtained by a SMV reconstruction algorithm.

In the case of ideal Gauss white noises, the array covariance matrix can be expressed as:

$$\mathbf{R_Y} = E[\mathbf{YY}^H] = \mathbf{\Phi} E[\mathbf{SS}^H]\mathbf{\Phi}^H + \sigma^2\mathbf{I_N} = \mathbf{\Phi R_S \Phi}^H + \sigma^2\mathbf{I_N} = \mathbf{R} + \mathbf{R_N} \tag{6}$$

where $\mathbf{R_S}$ is the correlation matrix of the spatial signal and σ^2 is the power of Gauss white noises. $\mathbf{R} = \mathbf{\Phi R_S \Phi}^H$, $\mathbf{R_N} = \sigma^2\mathbf{I_N}$.

As one can easily confirm, $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_K > \lambda_K + 1 = \ldots = \lambda_M = 0$ are the characteristic values of R. $\mu_1 \geq \mu_2 \geq \ldots \geq \mu_K \geq \mu_{K+1} \geq \ldots \geq \mu_M \geq 0$ are the characteristic values of $\mathbf{R_Y}$. There is:

$$\mu_1 \approx \lambda_1 + \sigma^2, \quad \mu_2 \approx \lambda_2 + \sigma^2, \ldots, \quad \mu_M \approx \lambda_M + \sigma^2, \tag{7}$$

The number of the principal eigenvalues of the array covariance matrix $\mathbf{R_Y}$ is equal to the number K of the source signal in the case of high SNR. Let us suppose that $\gamma_k = \mu_k/\mu_k{+}1(k = 1, 2, \ldots, M-2)$. Then the number K of the source signal should be satisfy $\gamma_k = \max(\gamma_1, \gamma_2, \ldots, \gamma_{M-2})$. This method is simple in operation and high accuracy.

The formula (6) is utilized to carry out the Eigen-Decomposition of $\mathbf{R_Y}$:

$$\mathbf{R_Y} = \mathbf{U_S \Sigma_S U_S^H} + \mathbf{U_N \Sigma_N U_N^H} \tag{8}$$

where $\mathbf{U_S} = [\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_K]$ represents the signal subspace. $\mathbf{e}_k(k = 1, 2, \ldots, K)$ is an eigenvector corresponding to the kth dominant eigenvalue.

Use a linear combination of the principal eigenvectors to transform MMV into SMV:

$$\mathbf{e} = \sum_{i=1}^{K} \frac{\gamma_i}{\gamma}\mathbf{e}_i \tag{9}$$

where $\gamma = \gamma_1 + \ldots + \gamma_1 + \ldots + \gamma_K$.

Then the OMP algorithm is utilized to search the support set in the atomic dictionary to get the estimated DOA.

3. THE PROPOSED ALGORITHM

The computation of the algorithm can be greatly reduced by converting MMV to SMV. Considering that the angular space is divided into discrete grids and the real DOA is often thought to fall on the grid, the performance of the algorithm will be decreased when real DOA is off-grid. We can improve the performance of sparse representation DOA estimation algorithm by refining the resolution of the grid.At the same time, it will lead to a large amount of computation. In order to improve the DOA estimation accuracy without increasing the computation, this paper proposes a modified IIN algorithm. The proposed algorithm estimates the DOAs by using the Euclidean distance between the neighboring atoms.

The IIN algorithm [16] originally proposed is used to estimate the frequency of the signal. First,the received signal is processed by Fourier transformation. Then, we find out two spectrums adjacent to the maximum spectrum and carry out interpolation processing so as to estimate the frequency. The distance curve of the adjacent atoms in an over-complete dictionary is similar to the spectrums of the IIN algorithm. Therefore,this paper utilizes the idea of the IIN algorithm to eliminate the influence of off-grid on the performance of DOA estimation.

The sparse representation DOA estimation algorithm generally chooses the atom closest to the real DOA as a supporting set when the signal DOA is off-grid. The performance of the algorithm decreases due to the mismatch of the grid. The modified IIN algorithm is an off-grid algorithm. First, select the most matched atom with the real DOA and two neighboring atoms whose difference is semi gird resolution as target atoms. Then utilize the IIN idea to estimate the sparse signal DOA.

According to the idea of IIN algorithm, the formula of DOA can be expressed as:

$$\widehat{\alpha}_c = \alpha_0 + \frac{1}{2}\alpha_{res}\frac{\rho(\alpha + 0.5\alpha_{res}) + \rho(\alpha - 0.5\alpha_{res})}{\rho(\alpha + 0.5\alpha_{res}) - \rho(\alpha - 0.5\alpha_{res})} \tag{10}$$

where α_0 is an on-grid DOA obtained by the above sparse representation algorithm, α_{res} is the grid resolution and $\rho(\alpha) = \|\mathbf{a}(\alpha) - \mathbf{r}\|_2$ is the Euclidean distance between steering vector $\mathbf{a}(\alpha)$ which corresponds to α and r is the residual of the OMP algorithm.

The steps of the M-IIN algorithm are shown in Table 1.

Table 1. M-IIN algorithm.

Input: array received signal matrix $\mathbf{Y}$, over complete dictionary $\mathbf{\Phi}$, grid resolution α_{res}:
1.Use the weighted eigenvector method to convert MMV into SMV.
2.Use the OMP algorithm to get the on-grid DOA α_0 and $\alpha_0 \pm 0.5\alpha_{res}$.
3.Calculate the Euclidean distance between $\mathbf{a}(\alpha_0 \pm 0.5\alpha_{res})$ and the residual $\mathbf{r}$, respectively.
4.Use the IIN algorithm to modify the on-grid DOA.
Output: the estimated DOA.

4. ALGORITHM SIMULATION AND ANALYSIS

In this section, we carry out a number of experiments to prove the rationality of the theoretical derivation. In the following experiments, we consider a ULA with eight antennas and antenna distance of $\lambda/2$.

In Figure 1 and Figure 2, the initial angle α_p is equal to $0°$ or $-50°$. From the curve of Euclidean distance between two atoms, we can see with the decrease of the distance, the Euclidean distance between two atoms decreases til it is close to the minimum 0.

In Figure 3, the number L of snapshots is equal to 150, SNR varies from -10 dB to 10 dB with

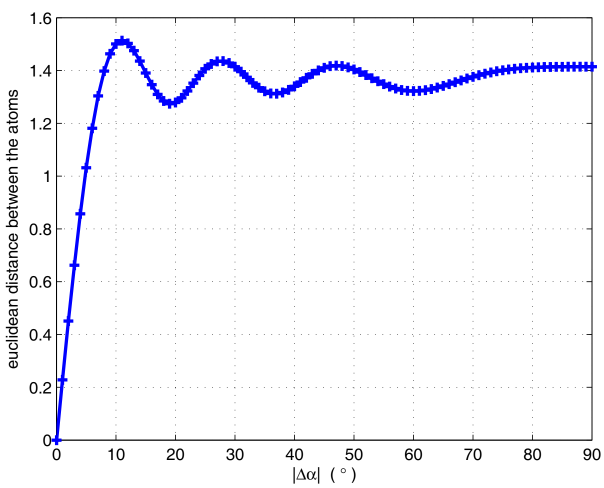

Figure 1. Euclidean distance between two atoms with $\alpha_p = 0°$.

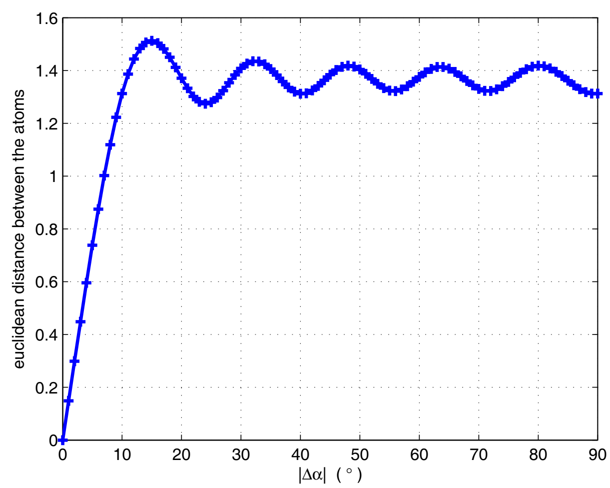

Figure 2. Euclidean distance between two atoms with $\alpha_p = -50°$.

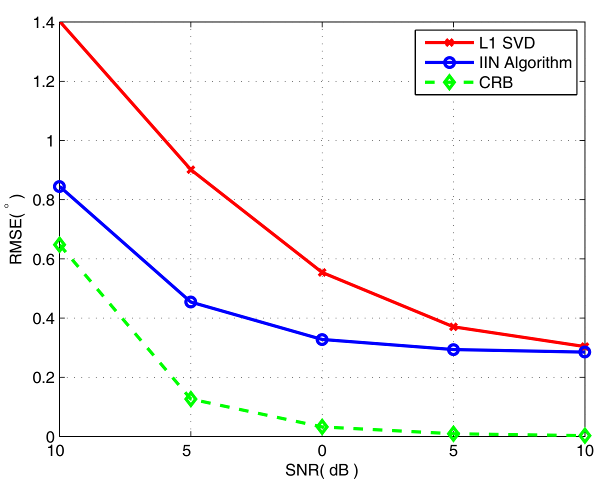

Figure 3. The RMSE of DOA estimation with SNR.

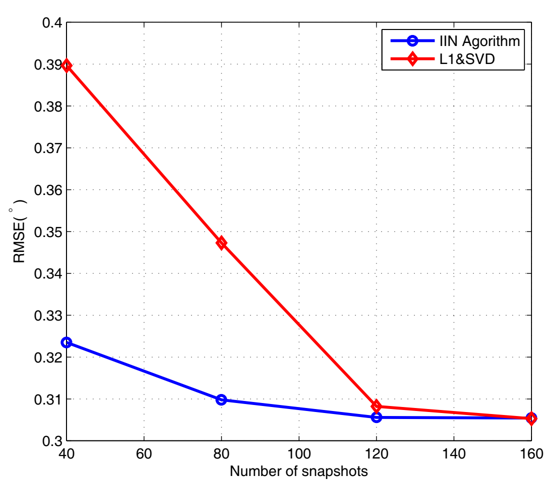

Figure 4. RMSE of DOA estimation versus number of snapshots.

the 5 dB step. 500 independent Monte Carlo simulations are carried out in this paper. In each Carlo Monte experiment, the angle of the incident signal is random. As shown in Figure 3, the RMSE of the estimated DOA with the SNR of the curve, the estimation performance of the method proposed in paper is significantly better than the L1-SVD method.

In Figure 4, the SNR is equal to 10 dB and the number of snapshots varies from 40 to 160 with the 40 step. 400 independent Monte Carlo simulations are carried out. As shown in Figure 4, the RMSE of the DOA estimation with the number of snapshots of the curve, the estimation performance of the method prosed in this paper is significantly better than the L1-SVD method in the case of small number of snapshots.

5. CONCLUSION

In this paper, a modified IIN algorithm for DOA estimation based on sparse representation is proposed. This algorithm using the IIN idea modifies the on-grid angle obtained by the OMP algorithm and gets a more accurate off-gird estimated angle. Both theoretical analysis and simulation results show that the proposed method has a better estimation performance than the L1-SVD algorithm and the computation is lower.

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