Assignment 1 of Algorithm Design and Analysis

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1 Stable Matching

Based on my implementation(details in the appendix), the statement is **false**.

My test instance is:

If men propose to women, we can get a stable matching as shown in fig-

	1st (favorite)	2nd	3rd (least favorite)
Xavier	Amy	Bertha	Clare
Yancey	Bertha	Amy	Clare
Zeus	Amy	Bertha	Clare

Men's Preference Profile

	1st (favorite)	2nd	3rd (least favorite)
Amy	Yancey	Xavier	Zeus
Bertha	Xavier	Yancey	Zeus
Clare	Xavier	Yancey	Zeus

Women's Preference Profile

Figure 1: Preference Table

ure1. On the contrary, if women propose to men, another stable matching is obtained as shown in figure2.

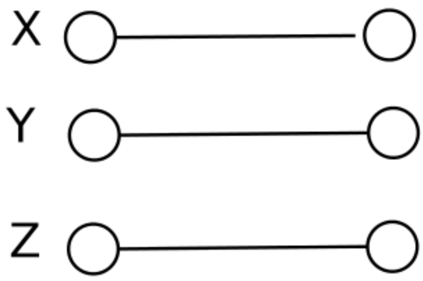

Figure 2: Stable Matching 1

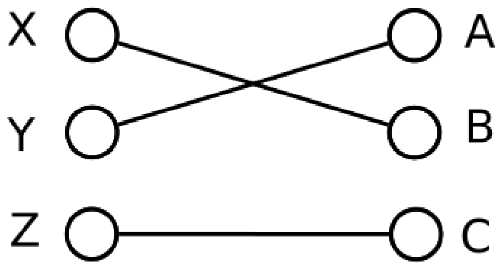

Figure 3: Stable Matching 2

2 Complexity Analysis

QUESTION:

A sequence of n operations is performed on a data structure. The ith operation costs i if i is an exact power of 2, and 1 otherwise. Use a potential method to determine the amortized cost per operation.

Definition: Potential function $\Phi_i = 2i - 2^{k+1}$, where k is the largest integer subject to $2^k <= i$.
Obviously $\Phi_i >= 0$.

$$\hat{c_i} = c_i + \Phi_i - \Phi_{i-1} = \begin{cases} i + (2i - 2i) - [2(i-1) - 2 \times \frac{i}{2}] = 2; & \text{if } i = 2^k, k \text{ is an integer} \\ 1 + (2i - 2^{k+1}) - [2(i-1) - 2^{k+1}] = 1 + 2 = 3; & \text{otherwise} \end{cases} \quad (1)$$

Thus, $T(n) = \sum_{i=1}^{n} c_i <= \sum_{i=1}^{n} \hat{c_i} <= \sum_{i=1}^{n} 3 = 3n$