Assignment 8 of Algorithm Design and Analysis

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2 Bin Packing

Bin Packing problem is as follows: Given n items with sizes$a_1, ..., a_n \in (0, 1]$, find a packing in unit-sized bins that minimizes the number of bins used. Give a 2-approximation algorithm for this problem.
解：
新建一个 Bin b 和一个大顶堆 H, Bin 的值定义为其剩余容量 c. 将 b 插入 H 中。 **for i=0 to n do**
t = H 的堆顶元素。
if ai 能够放入 H 的堆顶的 Bin b 中 then
将 ai 放入 b 中;
c(b) = c(b) - ai;
else
新建一个 Bin t 将 ai 放入 t 中
c(t) = 1 - ai;
将 t 插入 H 中;
end if
end for
首先不会有两个罐的剩余容量同时少于 1/2,因为否则, 第二个罐的物品会直接放入每一个罐中。根据算法,不会新建一个罐。

若每个罐的容量都大于或等于 1/2,
则 $\sum_{i=1}^{k} \geq k/2$. **故**$k \leq 2\sum_{i=1}^{n} a_i \leq 2[\sum_{i=1}^{n} a_i] \leq 2B^*$
若每个罐的容量小于1/2,则
$\sum_{i=1}^{k} a_i = \sum_{i=1}^{k-1} v_i + v_k \geq \sum_{i=1}^{k-1}(1 - v_k) + v_k \geq (k-1) - (k-2)v_k \geq k/2.$

4 Approximation Algorithm

Consider the following maximization version of the 3-Dimensional Matching Problem. Given disjoint sets X, Y , Z, and given a set T X × Y × Z of ordered triples, a subset M T is a 3 dimensional matching if each element of X ∪ Y ∪ Z is contained in at most one of these triples. The Maximum 3 Dimensional M atching P roblem is to find a 3-dimensional matching M of maximum size. (You may assume $|X| = |Y| = |Z|$ **if you want.)**
Give a polynomial-time algorithm that finds a 3-dimensional matching of size at least 1 times the maximum possible size.

Proof:

$T = M; M = \emptyset;$
while $T \neq \emptyset$ **do**
t **= any element in** T**;** $M = M \cup t;$
$T = T\{t$**and all its neighbors in**$T\}$**;**
end while;