Reduced SVD

- Reduced SVD is applicable when $m \gg n$. E.g. $A \in \mathbb{R}^{10^6 \times 3}$
- Recall that the SVD $A\vec{v}_i = \vec{u}_i \sigma_i$
- Recall that directly estimating U requires eigen calculations of $B = AA^T \in \mathbb{R}^{10^6 \times 10^6}$
- For such large matrices B, we may not have required memory and CPU cycles. Therefore, direct method of estimating U is not practical.
- Recall that when $m \gg n$, eigen calculations of $C = A^T A \in \mathbb{R}^{3x3}$ is feasible
- Key Idea: Estimate $\{v_i, \sigma_i\}_{i=1}^{n}$ first. Then estimate $\{u_i\}_{i=1}^{m}$ from $A\vec{v}_i = \vec{u}_i \sigma_i$

Reduced SVD Algorithm

Input : $A \in \mathbb{R}^{m \times n}, m \gg n$
Output: $U \in \mathbb{R}^{m \times n}; \Sigma \in \mathbb{R}^{n \times n}; V \in \mathbb{R}^{n \times n}$

1 $C \leftarrow A^{\mathsf{T}} A$;
2 $[v_i, \sigma_i] \leftarrow \texttt{Eigen}\,(C)$;
3 **foreach** right singular vector v_i **do** $u_i \leftarrow \frac{A\,\vec{v}_i}{\sigma_i}$;

Algorithm 0: Reduced SVD Algorithm for $A = U \Sigma V^{\mathsf{T}}$