MCMC

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1 Rejection sampling

2 Importance sampling

3 Markov Chain Monte Carlo (MCMC)

Instead of iid sampling from $f(x)$, draw from a Markov Chain with equilibrium density f.

M.C. $p(x_0), p(x_t|x_{t-1})$
equilibrium density: $p_{00}(x) : \int p(x_t|x_{t-1})p_{00}(x_{t-1})\partial x_{t-1}$ $Tp_{00} = p_{00}, Tf = f$. ergodicity, fast-mixing
$p_0 \to p_1 = Tp_0 \to p_2 = T^2 p_0 \to p_i = T^i p_0$
We want chains to mix as quickly as possibile and have the right invariant density.

Detailed balance: inflow = outflow
A chain is in detailed balance when inflow equals outflow between states. (doubly stochastic matrix) Flow rate from $i \to j = p(x_t = i)p(x_{t+1} = j|x_t = i)$
$j \to i = p(x_t = j)p(x_{t+1} = i|x_t = j)$.
$\Rightarrow p$ is invariant. The detailed balance condition is a sufficient, but not necessary condition for chains to have an invariant distribution.

3.1 Metropolis-Hastings

Goal: $p(x)p(x'|x) = p(x')p(x|x')\forall x, x'$
Approach: propose a sample from $q(x'|x)$. accept the sample with probability p = $\min\left(1, \frac{p(x)q(x|x'}{p(x)q(x'|x)}\right)$

We want:

- large p_{acc}
- large jump size

In the symmetric case where $q(x'|x) = q(x|x')$, the acceptance probability is $\min\left(1, \frac{p(x')}{p(x)}\right)$.

Independence sampler: $q(x'|x) = q(x')$. $p_{acc} = \min\left(1, \frac{p(x')q(x)}{p(x)q(x')}\right)$

3.2 Random Walk Metropolis Hastings

3.3 Gibbs sampler