Notes on atomic physics

Anton Ljungdahl,
anton.ljungdahl@fysik.su.se

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1 Angular momentum

1.1 Commutation relations

We consider a point particle (say an electron) whose motion can be described by some coordinate system. For simplicity let's say cartesian coordinates, so we can describe the particle as having position $\mathbf{r} = (x, y, z)$ and linear momentum $\mathbf{p} = (p_x, p_y, p_z)$ at any given time. We define orbital angular momentum in quantum mechanics in the same way we do in classical physics. The orbital angular momentum $\mathbf{l}$ about the origin of the chosen coordinate system is given by the cross-product

$$\mathbf{l} = \mathbf{r} \times \mathbf{p}. \tag{1.1}$$

In quantum mechanics we consider $\mathbf{r}$ and $\mathbf{p}$ as operators (and each of their components is an operator), and the linear momentum operator can be replaced by

$$\mathbf{p} \to -\mathrm{i}\nabla. \tag{1.2}$$

So we write the angular momentum operator by components as

$$
\begin{aligned}
l_x &= y p_z - z p_y = & -\mathrm{i}\left(y\frac{\mathrm{d}}{\mathrm{d}z} - z\frac{\mathrm{d}}{\mathrm{d}y} \right), \\
l_y &= z p_x - x p_z = & -\mathrm{i}\left(z\frac{\mathrm{d}}{\mathrm{d}x} - x\frac{\mathrm{d}}{\mathrm{d}z} \right), \\
l_z &= x p_y - y p_x = & -\mathrm{i}\left(x\frac{\mathrm{d}}{\mathrm{d}y} - y\frac{\mathrm{d}}{\mathrm{d}x} \right).
\end{aligned}
\tag{1.3}
$$

In quantum mechanics we know that the components of $\mathbf{r}$ and $\mathbf{p}$ does not commute in general, in fact we have a defining commutation relation of quantum mechanics:

$$
\left[x_i, p_j \right] = \mathrm{i}\hbar\delta_{ij}, \tag{1.4}
$$

and $\left[x_i, x_j \right] = \left[p_i, p_j \right] = 0$. In words: The operators x_i and p_j does *not* commute "along the same axis" (but they do along different). One can suspect that this has consequences for the commutation relations of the angular momentum operators l_x, l_y, l_z. One such relation would for example be

$$
\left[l_x, l_y \right] = l_x l_y - l_y l_x = \left(y p_z - z p_y \right)(z p_x - x p_z) - (z p_x - x p_z)\left(y p_z - z p_y \right). \tag{1.5}
$$