Notes on atomic physics

Anton Ljungdahl,
anton.ljungdahl@fysik.su.se

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1 Angular momentum

1.1 Commutation relations

We consider a point particle (say an electron) whose motion can be described by some coordinate system. For simplicity let's say cartesian coordinates, so we can describe the particle as having position $\mathbf{r} = (x_1, x_2, x_3)$ and linear momentum $\mathbf{p} = (p_1, p_2, p_3)$ at any given time. We define orbital angular momentum in quantum mechanics in the same way we do in classical physics. The orbital angular momentum $\mathbf{l}$ about the origin of the chosen coordinate system is given by the cross-product

$$\mathbf{l} = \mathbf{r} \times \mathbf{p}. \tag{1.1}$$

Looking at this definition component-wise we would write

$$l_i = x_j p_k - x_k p_j, \tag{1.2}$$

where i, j, k are cyclic permutations of $1, 2, 3$. We will now quickly see the big difference from classical physics, since in quantum mechanics we have the canonical commutation relation (ref maybe?)

$$\left[x_i, p_j\right] = \mathrm{i}\hbar\delta_{ij}, \tag{1.3}$$

between the position and momentum operators. While x_j and p_k certainly commute for $j \neq k$ we will see how the *different components of* $\mathbf{l}$ do not! Let us calculate the commutator between components l_i and l_k:

$$[l_i, l_k] = l_i l_k - l_k l_i = \big(x_j p_k - x_k p_j\big)\big(x_i p_j - x_j p_i\big). \tag{1.4}$$