HW3

1 Exercise

1.1

The spectrum of F(u,v) is $\sqrt{\mathrm{R}^2(\mathrm{u},\mathrm{v})+\mathrm{I}^2(\mathrm{u},\mathrm{v})}$ and it relates to the gray values of the picture. After taking the complex conjugate of F(u,v), the spectrum won't change and the gray values of the picture neither. So the color of the picture remains. But the phase angle which is $\tan^{-1}[\frac{I(u,v)}{R(u,v)}]$ will change and increase 180°. So the picture rotates 180° after processing.

1.2

A Fourier spectrum shows us how sharply the gray values change in a picture. The more sharply the gray values change, there are more bright points in the spectrum. After padding zeros to the picture, the edges are much more obvious and the intensity of how the gray values change increases. As a result, the signal strength increases in the spectrum.

1.3

1. g(x,y)=f(x-1,y-1)+2f(x-1,y)+f(x-1,y+1)-f(x+1,y-1)-2f(x+1,y)-f(x+1,y+1)

   According to $f(x-x_0,y-y_0)\Leftrightarrow F(u,v)e^{-2j*pi*(ux0/M+vy0/N)}$,we have

   $G(u,v)=F(u,v)(e^{-2j*pi*(u/M+v/N)}+2e^{-2j*pi*u/M}+ e^{-2j*pi*(u/M-v/N)}- e^{-2j*pi*(-u/M+v/N)}-2e^{-2j*pi*(-u/M)}- e^{-2j*pi*(-u/M-v/N)})$

   =F(u,v)(-2jsin(2*pi*(u/M+v/N))-4jsin(2*pi*u/M)-2jsin(2*pi*(u/M-v/N))).

Then,G(u,v)=F(u,v)H(u,v) and H(u,v)=-2jsin(2*pi*(u/M+v/N))-4jsin(2*pi*u/M)-2jsin(2*pi*(u/M-v/N)).

2. The filter is a high-pass filter.

1.4

The high-frequency-emphasis filtering and the histogram equalization are both linear operations. If we have an image f(x,y), we can use h1(f(x,y)) to represent the filtering operation and h2(f(x,y)) to represent the histogram equalization. Since they are both linear operations, we have h1(h2(f(x,y)))=h2(h1(f(x,y))). That is to say, the result is the same no matter how these two operations are ordered.

2 Programming

2.1 I make use of the openVC library to load the picture and store it.

```
#include <cv.h>
#include <highgui.h>
```

```
Mat img = imread("./81.png");
```

```
imwrite("spec", spec);
```

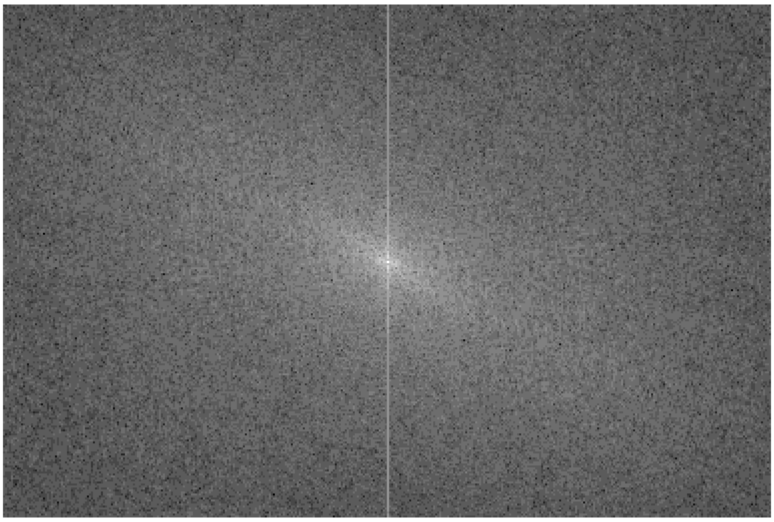
```
imwrite("res", res);
```

```
imshow("filtered res", res);
```

This is my source picture----81.png.

2.2

1. Fourier spectrum:



2. Image after IDFT:

3. Implementation:

DFT:

I use 1-D DFT along y-axis and then 1-D DFT along x-axis. I store the real part and the imaginary part in two images and then merge them into a two-channel image as the return image. To calculate the spectrum and display it, I use the formula $\sqrt{\mathrm{R}^2(\mathrm{u},\mathrm{v})+\mathrm{I}^2(\mathrm{u},\mathrm{v})}$ to calculate and log the result of $\sqrt{\mathrm{R}^2(\mathrm{u},\mathrm{v})+\mathrm{I}^2(\mathrm{u},\mathrm{v})}+1$ to make it visual. Then I scale the gray values to 0-255 and centralize the picture.

IDFT:

At first, I split the two-channel picture into two one-channel images to store the real part and the imaginary part. Then, I calculate the result using the algorithm of forwarding DFT mentioned above.

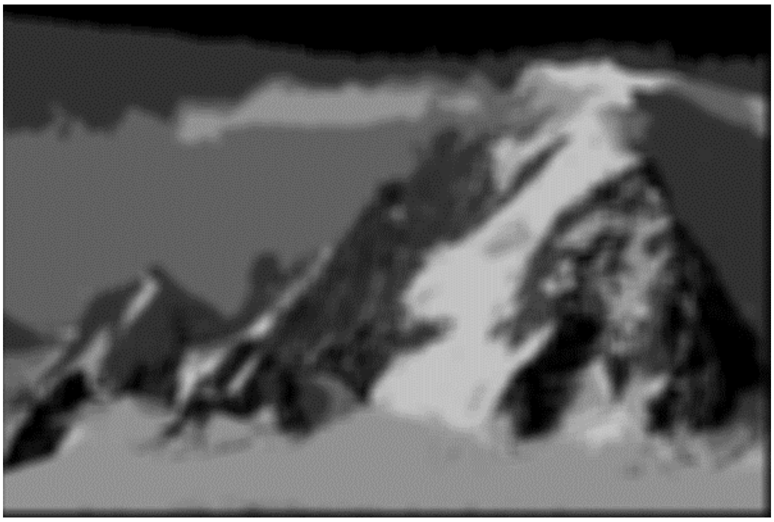
I store the real part of the result and scale it to 0-255. This is the result image of IDFT.

2.3

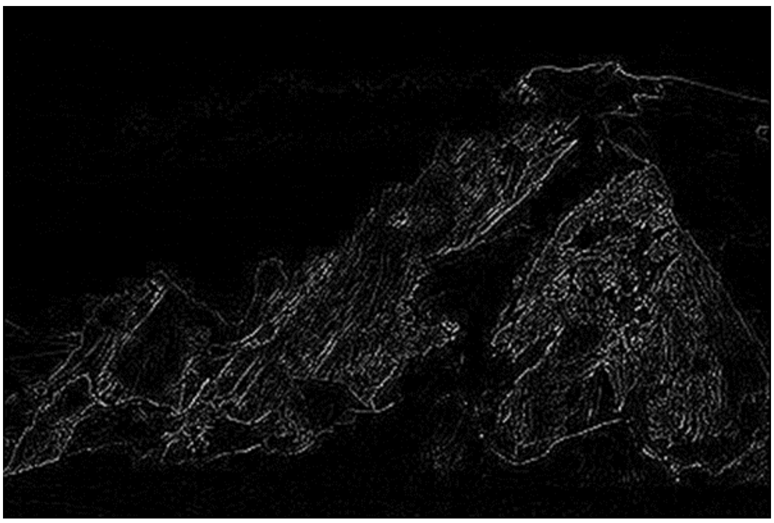
I don't have enough time to finish the FFT.

2.4

1. Smoothed image:



2. Sharpened image:



3. Implementation:

Firstly, I pad the source image f(x,y) with zeros to make it becomes a 2M*2N image $f_p(x,y)$. The source image locates at the top-left. Then I pad the spatial filter h(x,y) with zeros to make it have the same size as the $f_p(x,y)$. And I use the dft2d to transform them. According to G(u,v)=F(u,v)H(u,v), I calculate the G(u,v) and use the inverse dft2d to get the g(x,y). The g(x,y) is the image after filtering. I don't try to centralize the f(x,y) and h(x,y) though the textbook suggests us to do.