Answers to the exercises for chapter: Splines

1. Define $G_H = [q_0, q'_0, q_1]$. That gives

$$T_H = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}, \qquad M_H = T_H^{-1} = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}.$$

The derivative in the other end point is

$$q'_1 = Q'_H(1) = G_H M_H \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} = G_H \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix} = 2q_1 - 2q_0 - q'_0.$$

The control point satisfies

$$q_2 = \begin{cases} q'_0 u + q_0 \\ q'_1 v + q_1 \end{cases}.$$

Substituting q'_1, we get

$$Q_H \left[\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ 1 & 1 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} \right] = 0$$

which is satisfied for $u = 1/2$, $v = -1/2$. This gives for the Bezier geometry matrix

$$G_B = [q_0, q_2, q_1] = G_H \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad G_H = G_B \begin{pmatrix} 1 & -2 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and for the generating equation

$$Q = G_H M_H T = G_B \begin{pmatrix} 1 & -2 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} T = G_B \begin{pmatrix} 1 & -2 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$