Antenna Arrays

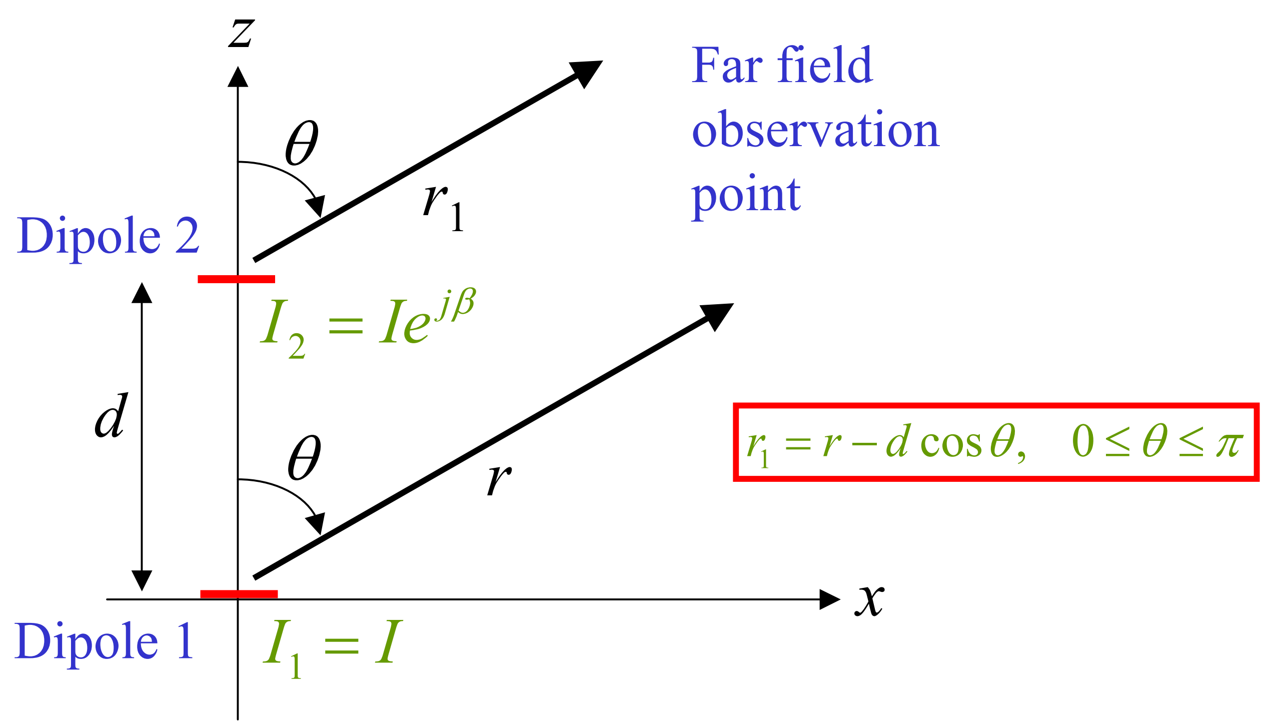
1 Introduction

Antenna arrays are becoming increasingly important in wireless communications. Advantages of using antenna arrays:

1. They can provide the capability of a steerable beam (radiation direction change) as in smart antennas.
2. They can provide a high gain (array gain) by using simple antenna elements.
3. They provide a diversity gain in multipath signal reception.
4. They enable array signal processing.

An important characteristic of an array is the change of its radiation pattern in response to different excitations of its antenna elements. Unlike a single antenna whose radiation pattern is fixed, an antenna array's radiation pattern, called the array pattern, can be changed upon exciting its elements with different currents (both current magnitudes and current phases). This gives us a freedom to choose (or design) a certain desired array pattern from an array, without changing its physical dimensions. Furthermore, by manipulating the received signals from the individual antenna elements in different ways, we can achieve many signal processing functions such as spatial filtering, interference suppression, gain enhancement, target tracking, etc.

2 Two Element Arrays



Two Hertzian dipoles of length $d\ell$ separated by a distance d and excited by currents with an equal amplitude I but a phase difference $\beta\,[0 \sim 2\pi)$.

$\mathbf{E}_1 =$ far-zone electric field produced by antenna 1 $= \hat{\mathbf{a}}_\theta E_1$

$\mathbf{E}_2 =$ far-zone electric field produced by antenna 2 $= \hat{\mathbf{a}}_\theta E_2$

$$\boxed{\begin{aligned} E_1 &= j\frac{\eta k I_1 d\ell}{4\pi}\left(\frac{e^{-jkr}}{r}\right)\left|\sin\left(\theta+\frac{\pi}{2}\right)\right| = j\frac{\eta k d\ell}{4\pi}\left(\frac{e^{-jkr}}{r}\right)\left|\cos\theta\right| I_1 \\ E_2 &= j\frac{\eta k I_2 d\ell}{4\pi}\left(\frac{e^{-jkr_1}}{r_1}\right)\left|\sin\left(\theta+\frac{\pi}{2}\right)\right| = j\frac{\eta k d\ell}{4\pi}\left(\frac{e^{-jkr_1}}{r_1}\right)\left|\cos\theta\right| I_2 \end{aligned}}$$

Use the following far-field approximations:

$$\boxed{0 \le \theta \le \pi}$$

$$\boxed{\begin{aligned} &\frac{1}{r_1} \approx \frac{1}{r} \\ &e^{-jkr_1} = e^{-jk(r-d\cos\theta)} \end{aligned}}$$

The total **E** field is:

$$
\begin{aligned}
\mathbf{E} &= \left(E_1 + E_2\right)\hat{\mathbf{a}}_\theta \\
&= \hat{\mathbf{a}}_\theta j \frac{\eta k d\ell}{4\pi}\left(\frac{e^{-jkr}}{r}\right)\left|\cos\theta\right|\left[I_1 + I_2 e^{jkd\cos\theta}\right] \\
&= \hat{\mathbf{a}}_\theta j \frac{\eta k I_1 d\ell}{4\pi}\left(\frac{e^{-jkr}}{r}\right)\left|\cos\theta\right|\left[1 + e^{j\beta}e^{jkd\cos\theta}\right] \\
&= \hat{\mathbf{a}}_\theta j \frac{\eta k I d\ell}{4\pi}\left(\frac{e^{-jkr}}{r}\right)\left|\cos\theta\right|\mathrm{AF}
\end{aligned}
$$

where

$$\boxed{\text{AF} = \text{Array Factor} = \left[1 + e^{j\beta} e^{jkd\cos\theta}\right]}$$

$$\left[1 + e^{j\beta} e^{jkd\cos\theta}\right] = 2e^{j\frac{1}{2}(\beta + kd\cos\theta)} \cos\left[\frac{1}{2}(\beta + kd\cos\theta)\right]$$

$$\boxed{= \frac{1}{I_1}\left[I_1 + I_2 e^{jkd\cos\theta}\right]}$$

The magnitude of the total **E** field is:

$$|\mathbf{E}| = \left|\hat{\mathbf{a}}_\theta j \frac{\eta k I d\ell}{4\pi}\left(\frac{e^{-jkr}}{r}\right)\right| |\cos\theta| |\text{AF}|$$

$$= \text{radiation pattern of a single Hertzian dipole} \times |\text{AF}|$$

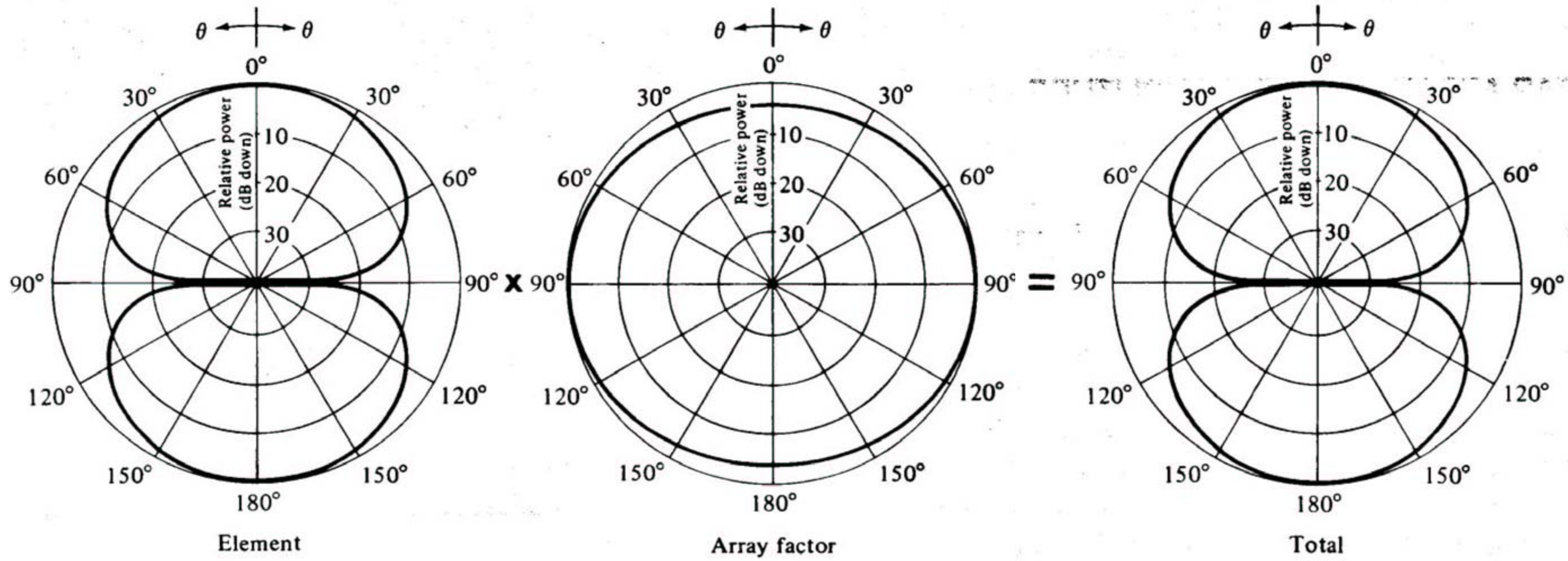
Hence we see the total far-field radiation pattern |**E**| of the array (array pattern) consists of the original radiation pattern of a single Hertzian dipole multiplying with the magnitude of the array factor |AF|. This is a general property of antenna arrays and is called the **principle of pattern multiplication**.

When we plot the array pattern, we usually use the normalized array factor which is:

$$\left|AF_n\right| = \frac{1}{\Gamma}\left|AF\right| = \frac{1}{\Gamma}\left|2\cos\left[\frac{1}{2}\left(\beta + kd\cos\theta\right)\right]\right|$$

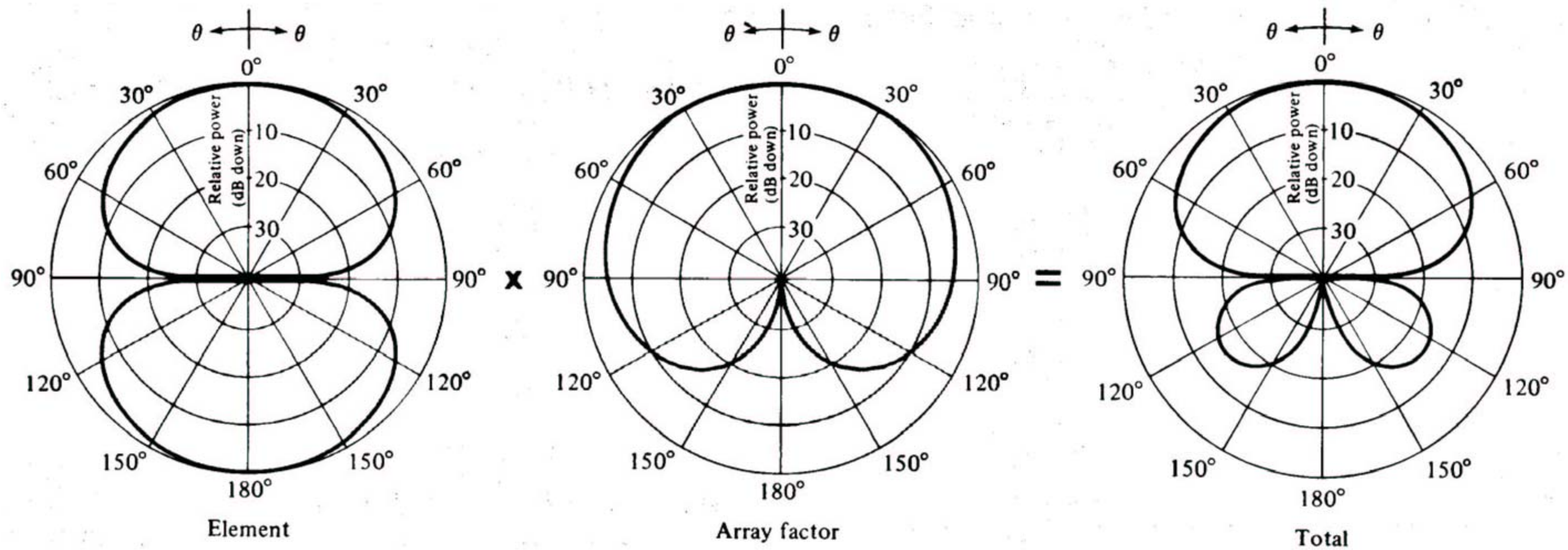
where Γ is a constant to make the maximum value of $|AF_n|$ equal to one.

Examples of array patterns using pattern multiplication:



Array pattern of a two-element array of Hertzian dipoles ($\beta = 0°$, and $d = \lambda/4$)

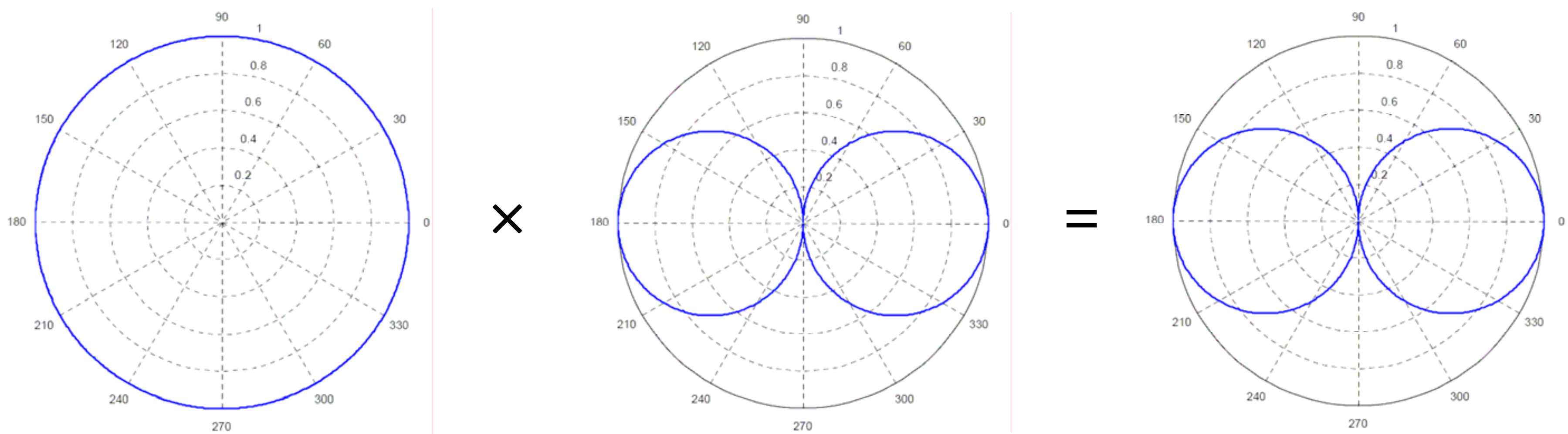
$$\left|\mathrm{AF_n}\right| = \frac{1}{\Gamma}\left|2\cos\left[\frac{1}{2}\left(\beta + kd\cos\theta\right)\right]\right| = \frac{1}{\Gamma}\left|2\cos\left[\frac{1}{2}\left(\frac{\pi}{2}\cos\theta\right)\right]\right|$$



Array pattern of a two-element array of Hertzian dipoles ($\beta = -90°$, and $d = \lambda/4$)

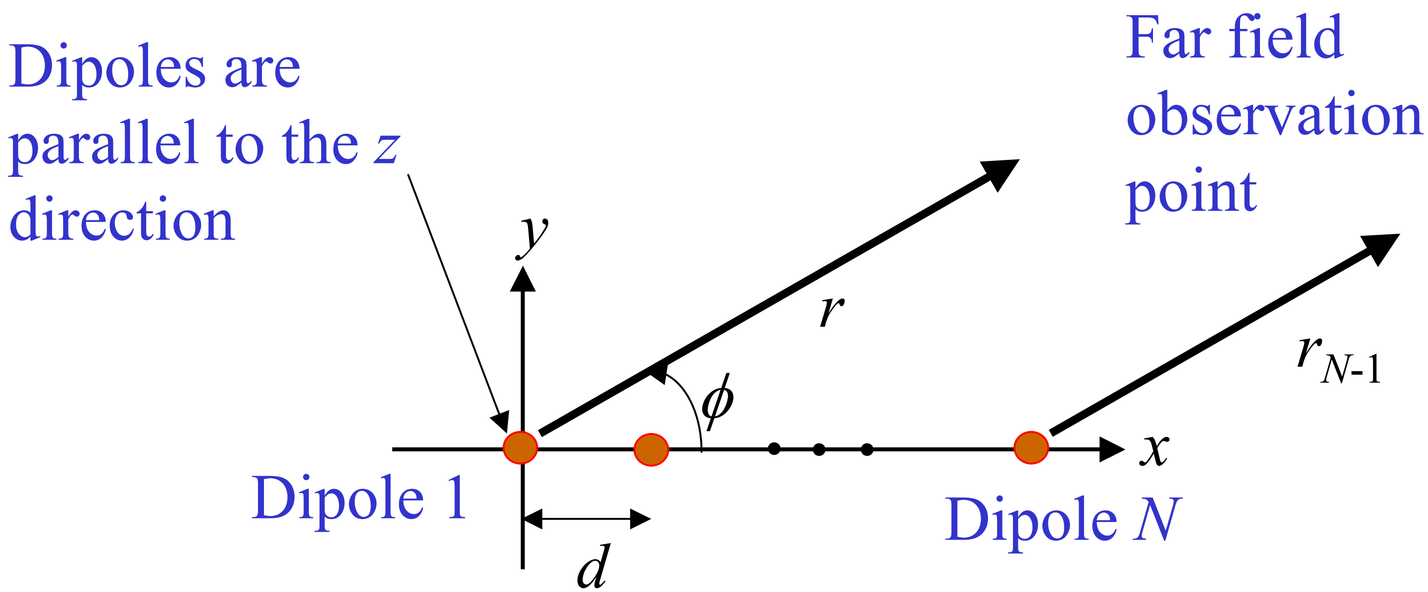
$$\left|\mathrm{AF_n}\right| = \frac{1}{\Gamma}\left|2\cos\left[\frac{1}{2}(\beta + kd\cos\theta)\right]\right| = \frac{1}{\Gamma}\left|2\cos\left[\frac{1}{2}\left(-\frac{\pi}{2} + \frac{\pi}{2}\cos\theta\right)\right]\right|$$

In many practical arrays, the element radiation pattern is usually chosen to be non-directional, for example the ϕ-plane pattern of a Hertzian dipole or a half-wave dipole. Then in this case, the array radiation pattern will be totally determined by the array factor AF alone, as shown in the example below:



Element pattern $\times$ $|F(\phi)|$ Array factor $|A(\phi)|$ $=$ Array pattern (normalized)

3 *N*-Element Uniform Liner Arrays (ULAs)



An N-element uniform antenna array with an element separation d

The principle of pattern multiplication can be extended to N-element arrays with identical antenna elements and equal inter-element separation (ULAs). If the excitation currents have the same amplitude but the phase difference between adjacent elements is β (the progressive phase difference), the array factor for this array is:

$$\mathrm{AF} = 1 + e^{j(kd\cos\phi+\beta)} + e^{j2(kd\cos\phi+\beta)} + \cdots + e^{j(N-1)(kd\cos\phi+\beta)}$$

$$= \sum_{n=1}^{N} e^{j(n-1)\psi} = \frac{\sin\left(N\dfrac{\psi}{2}\right)}{\sin\left(\dfrac{\psi}{2}\right)} e^{j(N-1)\frac{\psi}{2}}$$

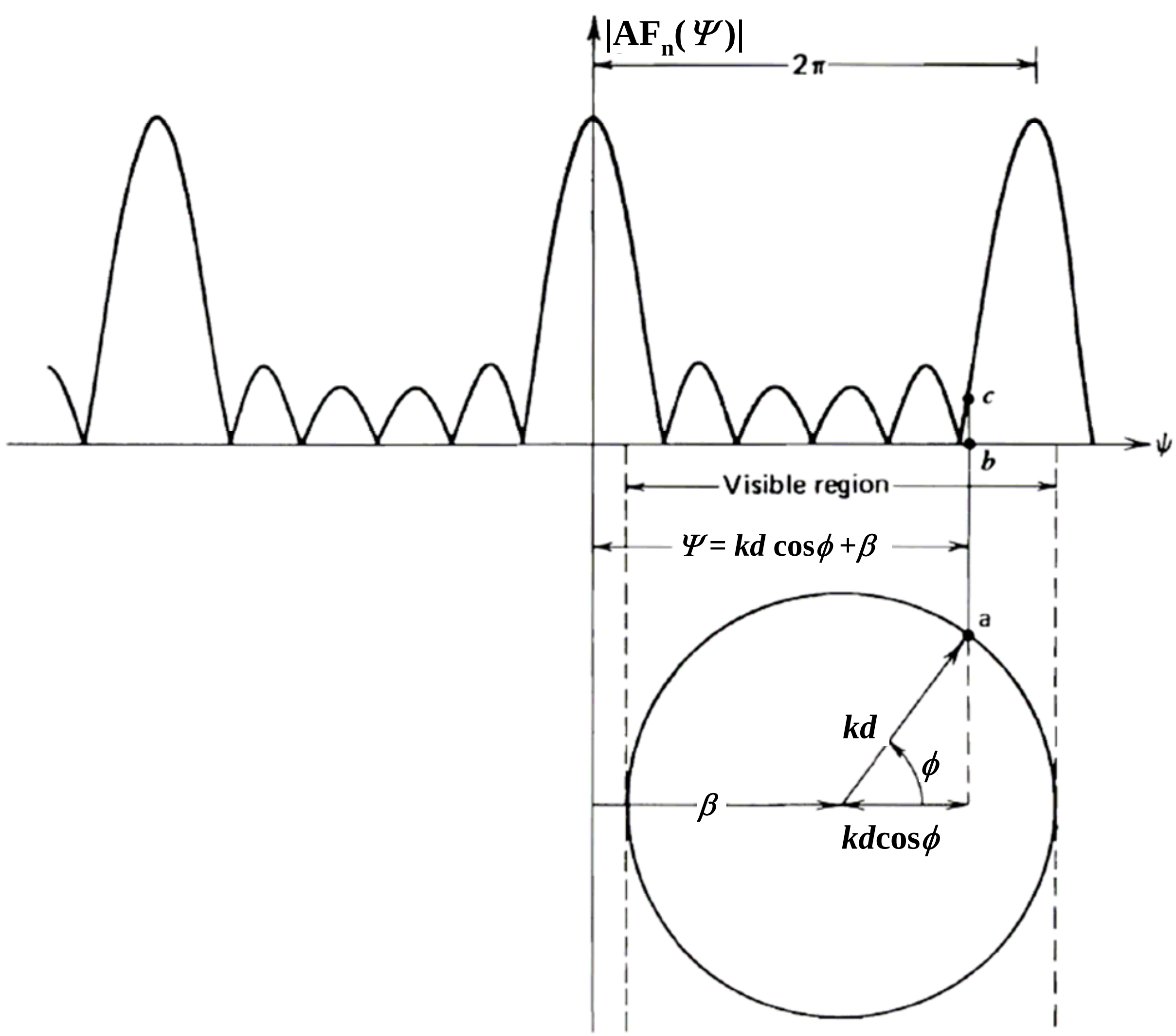
where $\psi = kd\cos\phi + \beta$ and $0 \le \phi, \beta \le 2\pi$

The normalized array factor is:

$$\boxed{\left|\mathrm{AF_n}(\psi)\right| = \frac{1}{\Gamma}\left|\frac{\sin\left(N\dfrac{\psi}{2}\right)}{\sin\left(\dfrac{\psi}{2}\right)}\right|}$$

where Γ is a constant to make the largest value of $|\mathrm{AF_n}|$ equal to one. Note that Γ is not necessarily equal to N.

The relation between $|\mathrm{AF_n}|$, Ψ, d, and β is shown graphically on next page. Note that $|\mathrm{AF_n}|$ is a period function of Ψ, which is in turn a function of ϕ. The angle ϕ is in the real space and its range is 0 to 2π. However, Ψ is not in the real space and its range can be greater than or smaller than 0 to 2π, leading to the problem of **grating lobes** or not achieving the maximum values of the $|\mathrm{AF_n}|$ expression.



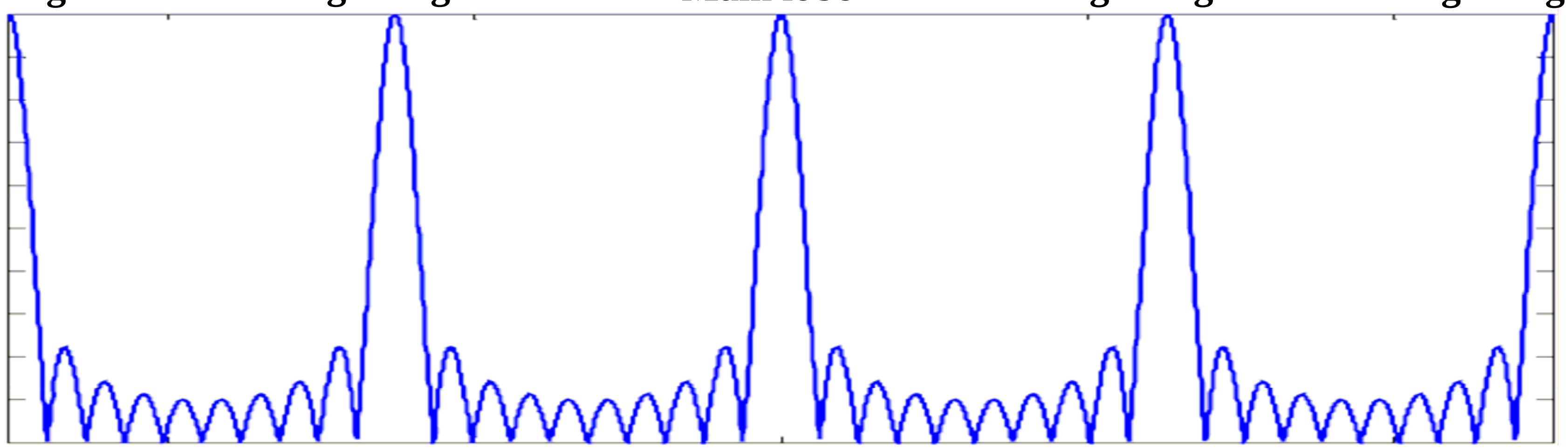
The relation between AF_n, Ψ, d, and β

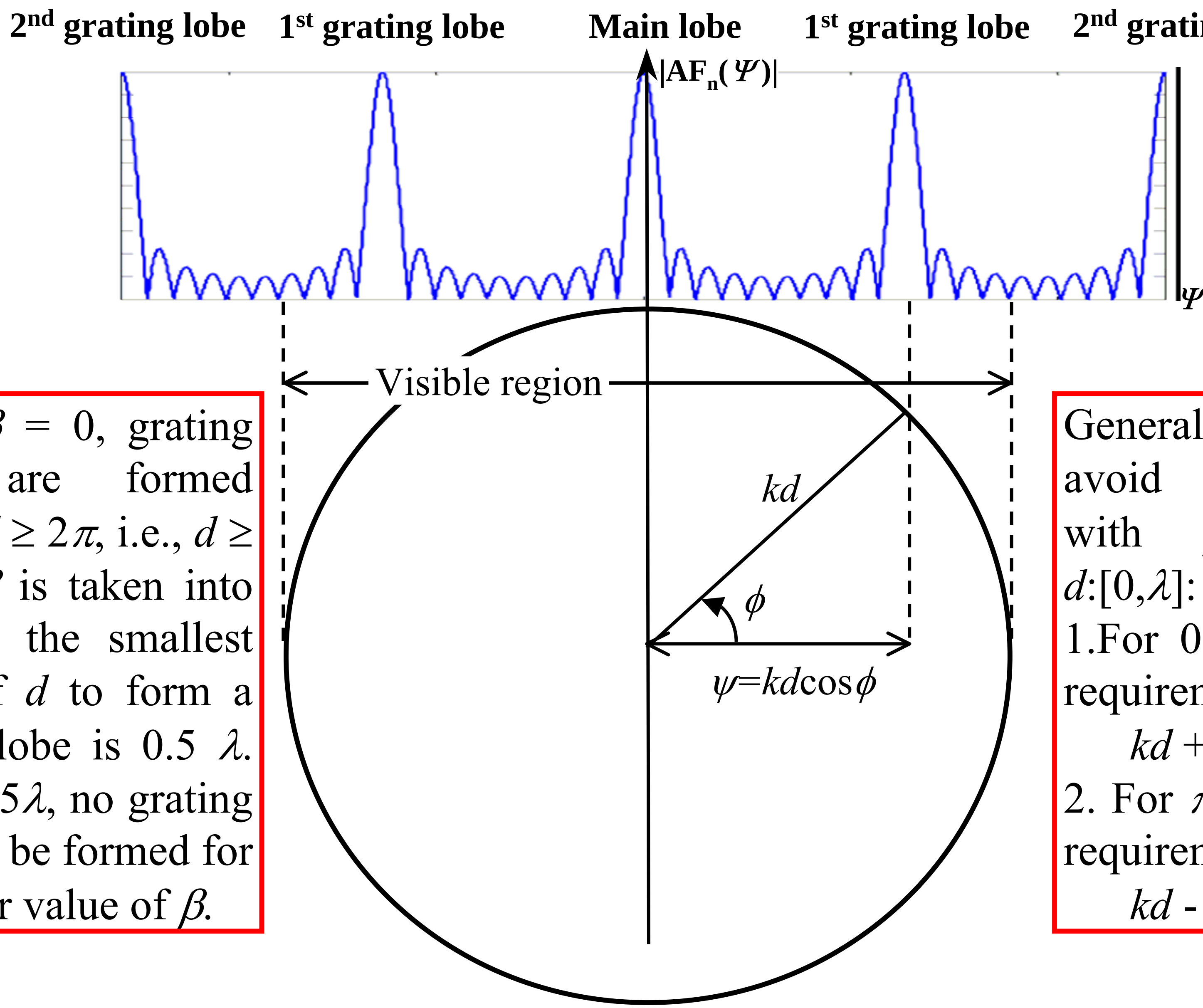
Properties of the normalized array factor AF_n:

1. $|AF_n|$ is a periodic function of ψ, with a period of 2π. This is because
$$|AF_n(\psi + 2\pi)| = |AF_n(\psi)|.$$
2. As $\cos(\phi) = \cos(-\phi)$, $|AF_n|$ is symmetric about the line of the array, i.e., $\phi = 0$ & π. Hence it is enough to know $|AF_n|$ for $0 \le \phi \le \pi$.
3. The maximum values of $|AF_n|$ occur when (see Supplementary Notes):
$$\frac{\psi}{2} = \frac{1}{2}(kd\cos\phi + \beta) = \pm m\pi, \qquad m = 0,1,2,\cdots$$

$$\boxed{\Rightarrow \quad \phi_{\max} = \text{main beam directions} = \cos^{-1}\left[\frac{\lambda}{2\pi d}(-\beta \pm 2m\pi)\right]}$$

Note that there may be more than one angles ϕ_{max} corresponding to the same value of m because $\cos^{-1}(x)$ is a multi-value function. If there are more than one maximum angles ϕ_{max}, the second and the subsequent maximum angles give rise to the phenomenon of grating lobes. The condition for grating lobes to occur is that $d \geq \lambda$ (disregarding the value of β) as shown below:





When $\beta = 0$, grating lobes are formed when $kd \geq 2\pi$, i.e., $d \geq \lambda$. If β is taken into account, the smallest value of d to form a grating lobe is 0.5 λ. If $d < 0.5\lambda$, no grating lobe can be formed for whatever value of β.

General conditions to avoid grating lobes with β:[0,2π] and d:[0,λ]:

1. For $0 \leq \beta < \pi$, the requirement is:

$$kd + \beta \leq 2\pi$$

2. For $\pi \leq \beta < 2\pi$, the requirement is:

$$kd - \beta \leq 0$$

4. There are other angles corresponding to the maximum values for the minor lobes (minor beams) but these angles cannot be found from the formula in no. 3 above.
5. When β and d are fixed, it is possible that ψ can never be equal to $2m\pi$. In that case, the maximum values of $|AF_n|$ cannot be determined by the formula in no. 3.
6. The main beam directions $\phi_{\max}$ are not related to N. They are functions of β and d only.
7. The nulls of $|AF_n|$ occur when:

$$\frac{\psi}{2} = \pm\frac{n\pi}{N}, \qquad \begin{cases} n = 1, 2, 3, \cdots \\ n \neq N, 2N, 3N, \cdots \end{cases}$$

$$\boxed{\Rightarrow \qquad \phi_{\text{null}} = \text{null directions} = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2n}{N}\pi\right)\right]}$$

Note that there may be more than one angles ϕ_{null} corresponding to a single value of n because $\cos^{-1}(x)$ is a multi-value function.

8. The null directions ϕ_{null} are dependent on N.
9. The larger the number N, the closer is the first null (n = 1) to the first maximum (m = 0). This means a narrower main beam and an increase in the directivity or gain of the array.
10. The angle for the main beam direction (m = 0) can be controlled by varying β or d.

Example 1

A uniform linear array consists of 10 half-wave dipoles with an inter-element separation $d = \lambda/4$ and equal current amplitude. Find the excitation current phase difference β such that the main beam direction is at 60° ($\phi_{max} = 60°$).

Solutions $\boxed{d = \lambda/4,\ \theta_{max} = 60^\circ,\ N = 10}$

$$\text{main beam dirction} = \phi_{max} = 60° = \cos^{-1}\left[-\frac{\lambda}{2\pi d}(\beta \pm m2\pi)\right]$$

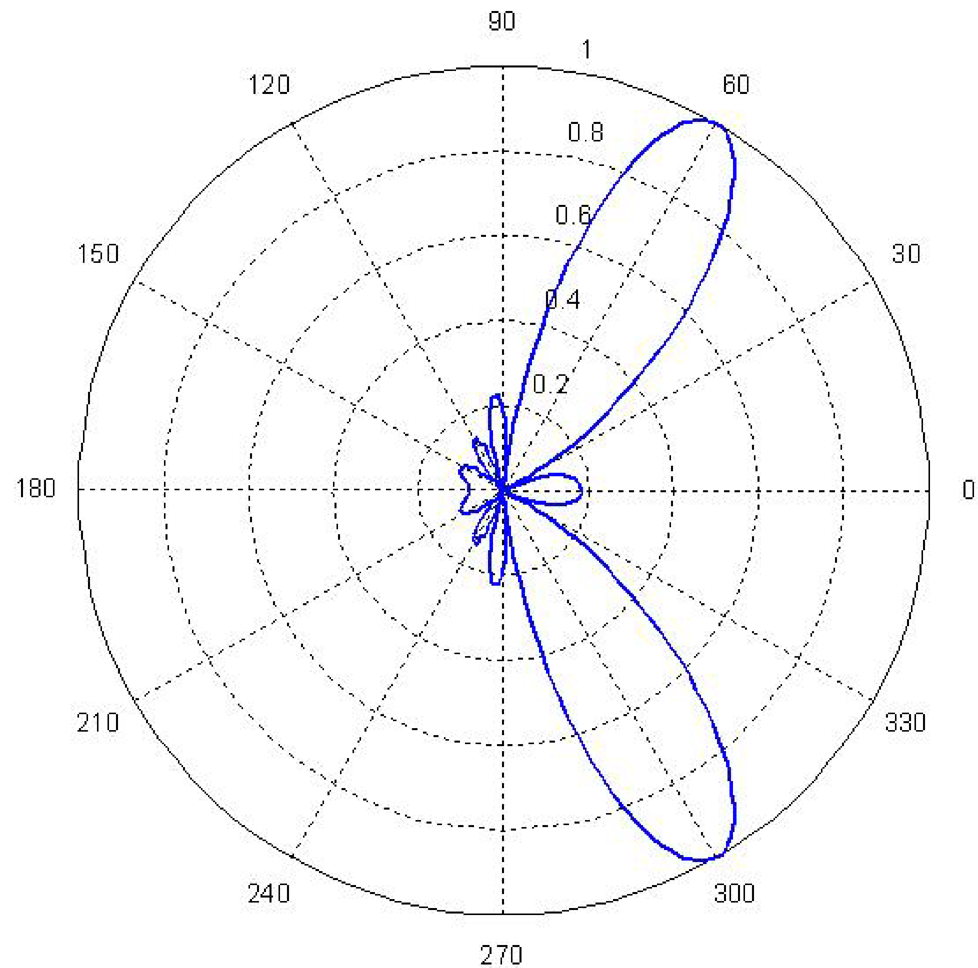
$$\Rightarrow \quad -\frac{2}{\pi}(\beta \pm m2\pi) = \cos(60°) = 0.5$$

$$\Rightarrow \quad \beta = -\frac{\pi}{4} \mp m2\pi = -45° + 360° = 315°, \quad \text{when } m = 1$$

Other values of β corresponding to other values of m are outside the range of $0 \le \beta \le 2\pi$ and are not included.

$$\left|AF_n\right| = \frac{1}{\Gamma}\left|\frac{\sin\left[5\left(\frac{\pi}{2}\cos\phi+\beta\right)\right]}{\sin\left[\frac{1}{2}\left(\frac{\pi}{2}\cos\phi+\beta\right)\right]}\right|$$

$$\Gamma = 10$$

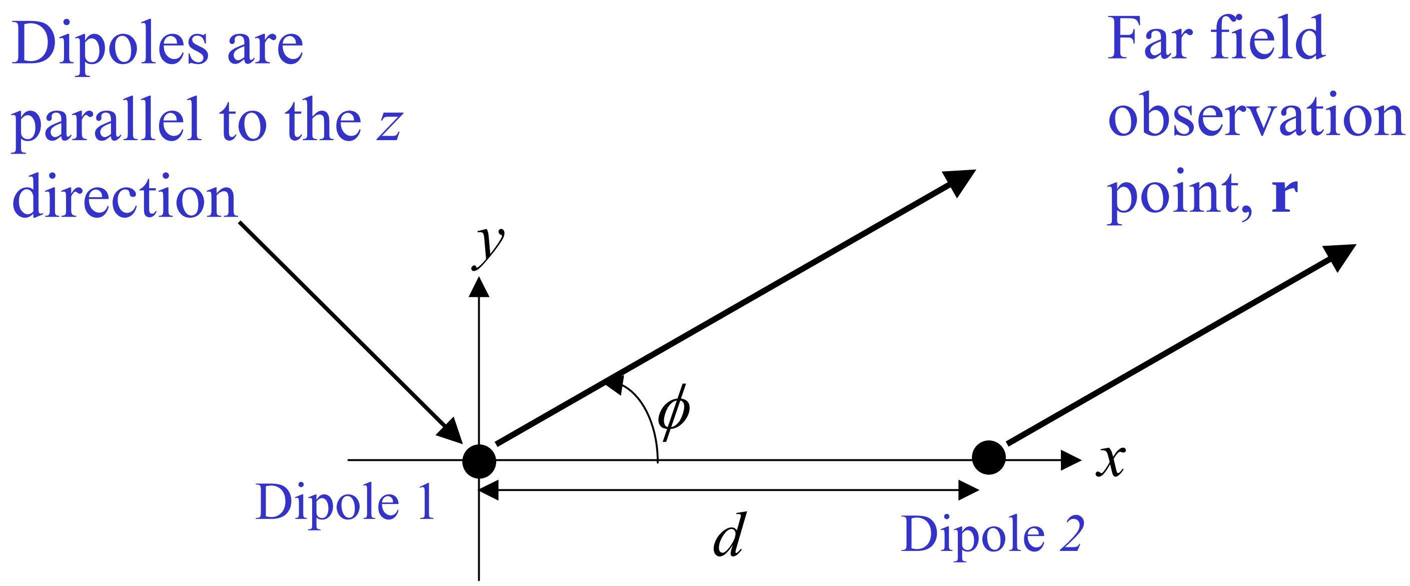


4 Mutual Coupling in Transmitting Antenna Arrays

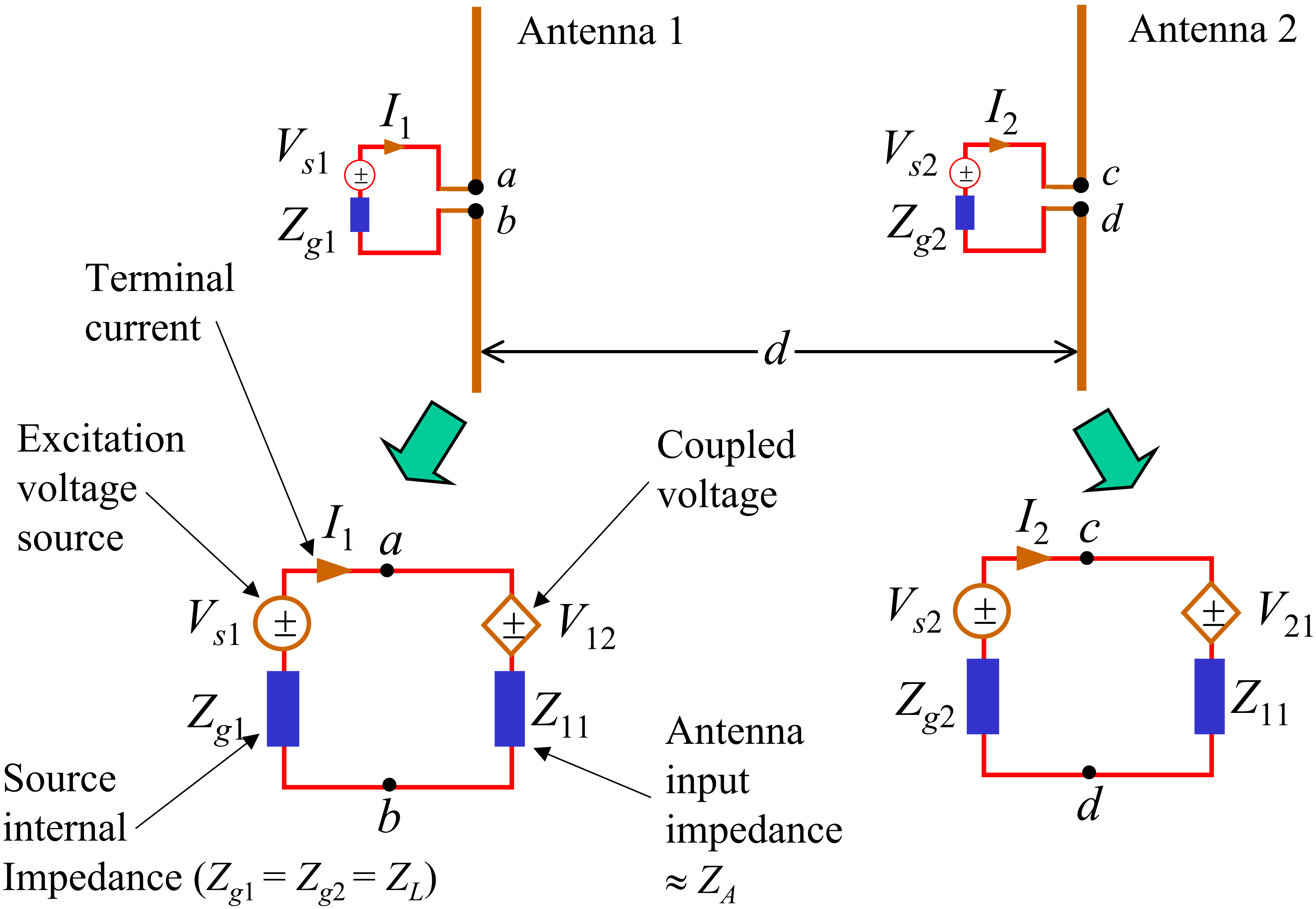
What we studies before about antenna arrays has assumed that the antenna elements operate independently. In reality, antennas placed in close proximity to each other interact strongly. This interaction is called **mutual coupling effect** and it will distort the array characteristics, such as the array pattern, from those predicted based on the **pattern multiplication principle**. We need to consider the mutual coupling effect in order to apply the pattern multiplication principle.

We study an example of a two-element dipole array. We characterize the mutual coupling effect using the **mutual impedance**.

Consider two *transmitting* antennas as shown on next page. They are separated by a distance of d and the excitation voltage sources, V_{s1} and V_{s2}, have a phase difference of β but an equal magnitude. Hence if there is no mutual coupling effect, the excitation currents also differ by a phase difference of β and have an equal magnitude. When the mutual coupling effect is taken into account, the two coupled antennas can be modelled as two equivalent circuits. Now because of the mutual coupling effect, there is another excitation source (the controlled voltage source) in the equivalent circuit. This controlled voltage source is to model the coupled voltage from the other antenna.



Two coupled dipoles

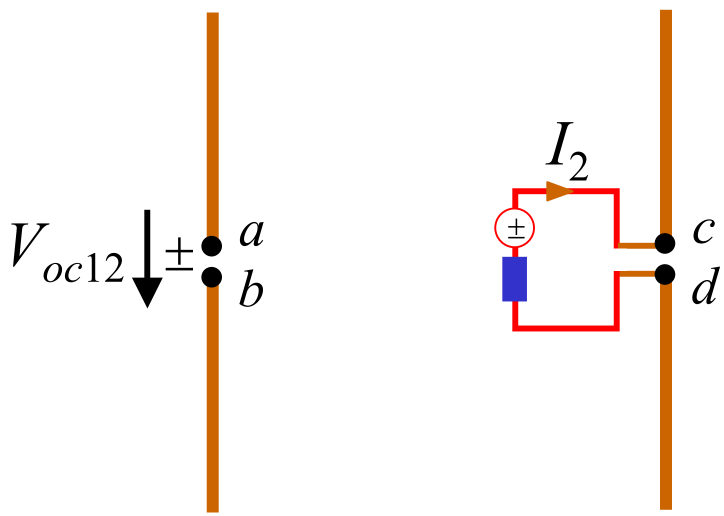

Antenna 1
Antenna 2
I_1
V_{s1}
Z_{g1}
a
b
I_2
V_{s2}
Z_{g2}
c
d
Terminal current
d
Excitation voltage source
Coupled voltage
I_1 a
V_{s1}
V_{12}
Z_{g1}
Z_{11}
b
I_2 c
V_{s2}
V_{21}
Z_{g2}
Z_{11}
d
Source internal Impedance ($Z_{g1} = Z_{g2} = Z_L$)
Antenna input impedance $\approx Z_A$

$$Z_{12} = \text{mutual impedance with antenna 2 excited}$$

$$= \frac{V_{12}}{I_2}$$

$$= \frac{\text{coupled voltage across antenna 1's open-circuit terminal}}{\text{terminal current through antenna 2's terminal load}}$$

$$= \left.\frac{V_{oc12}}{I_2}\right|_{I_1=0, V_{s1}=0}$$

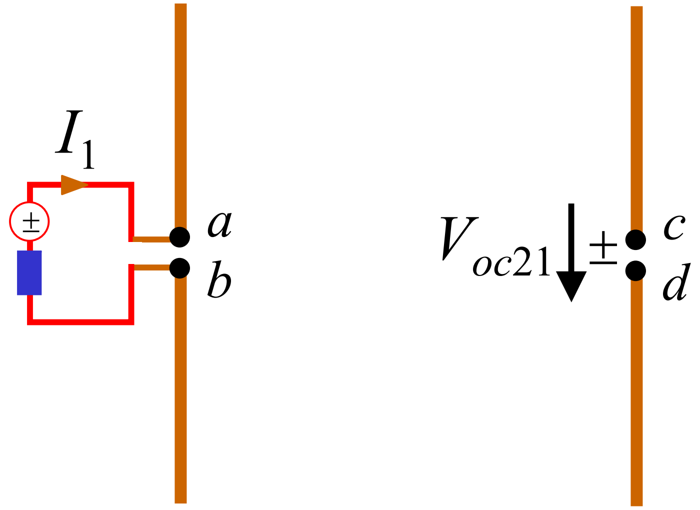


$$Z_{21} = \text{mutual impedance with antenna 1 excited}$$

$$= \frac{V_{21}}{I_1}$$

$$= \frac{\text{coupled voltage across antenna 2's open-circuit terminal}}{\text{terminal current through antenna 1's terminal load}}$$

$$= \left.\frac{V_{oc21}}{I_1}\right|_{I_2=0, V_{s2}=0}$$



Note that for passive antennas, $Z_{12} = Z_{21}$

Using the mutual impedance, the **coupled voltages** V_{12} and V_{21} can be expressed as follows:

$$V_{12} = Z_{12} I_2 \qquad\qquad V_{21} = Z_{21} I_1$$

I_1 and I_2 are the **actual terminal currents** at the antennas when there is mutual coupling effect. From the antenna equivalent circuits on pp. 25,

$$I_1 = \frac{V_{s1} - V_{12}}{Z_L + Z_A} \qquad\qquad I_2 = \frac{V_{s2} - V_{21}}{Z_L + Z_A}$$

I_{s1} and I_{s2} are the terminal currents at the antennas when there is no mutual coupling effect.

$$I_{s1} = \frac{V_{s1}}{Z_L + Z_A} \qquad I_{s2} = \frac{V_{s2}}{Z_L + Z_A}$$

Our aim is to express I_1 and I_2 in terms of I_{s1} and I_{s2}.

$$I_1 = \frac{V_{s1} - V_{12}}{Z_L + Z_A} \qquad I_2 = \frac{V_{s2} - V_{21}}{Z_L + Z_A}$$

$$= I_{s1} - \frac{I_2 Z_{12}}{Z_L + Z_A} \qquad = I_{s2} - \frac{I_1 Z_{21}}{Z_L + Z_A}$$

From these two relations, we can find:

$$I_1 = \frac{I_{s1} - \dfrac{Z_{12}}{Z_A + Z_L} I_{s2}}{\left[1 - \dfrac{Z_{12}Z_{21}}{\left(Z_A + Z_L\right)^2}\right]} \qquad I_2 = \frac{I_{s2} - \dfrac{Z_{21}}{Z_A + Z_L} I_{s1}}{\left[1 - \dfrac{Z_{12}Z_{21}}{\left(Z_A + Z_L\right)^2}\right]}$$

That is:

$$I_1 = \frac{1}{D}\left(I_{s1} - Z'_{12} I_{s2}\right) \qquad I_2 = \frac{1}{D}\left(I_{s2} - Z'_{21} I_{s1}\right)$$

where

$$D = 1 - \frac{Z_{12} Z_{21}}{(Z_A + Z_L)^2}$$

$$Z'_{12} = \frac{Z_{12}}{Z_A + Z_L}$$

$$Z'_{21} = \frac{Z_{21}}{Z_A + Z_L}$$

Now if we want to find the array pattern **E** on the horizontal plane ($\theta = \pi/2$) with mutual coupling effect, then **E** is just equal to the array factor (see pages 10 and 6).

Vector magnitude, not absolute value

$$\|\mathbf{E}\| = \mathrm{AF} = \frac{1}{I_1}\left[I_1 + I_2 e^{jkd\cos\phi} \right]$$

$$\left\|\mathbf{E}\right\| = \frac{1}{I_1}\left[I_1 + I_2 e^{jkd\cos\phi}\right]$$

$$= \frac{1}{I_1 D}\left[\left(I_{s1} - Z'_{12} I_{s2}\right) + \left(I_{s2} - Z'_{21} I_{s1}\right) e^{jkd\cos\phi}\right]$$

$$= \frac{1}{I_1 D}\left[\left(I_{s1} + I_{s2} e^{jkd\cos\phi}\right) - Z'_{12}\left(I_{s2} + I_{s1} e^{jkd\cos\phi}\right)\right] \quad \left(\text{with } Z'_{12} = Z'_{21}\right)$$

$$= \frac{I_{s1}}{I_1 D}\left[\left(1 + e^{j\beta} e^{jkd\cos\phi}\right) - Z'_{12}\left(e^{j\beta} + e^{jkd\cos\phi}\right)\right] \quad \left(\text{with } \frac{I_{s2}}{I_{s1}} = e^{j\beta}\right)$$

$$= \frac{I_{s1}}{I_1 D}\left\{\underbrace{\left[1 + e^{j(kd\cos\phi+\beta)}\right]}_{\text{original pattern}} - Z'_{12} e^{j\beta} \underbrace{\left[1 + e^{j(kd\cos\phi-\beta)}\right]}_{\text{additional pattern}}\right\}$$

We see that the array pattern now consists of two parts: the original array pattern plus an additional pattern:

$$Z'_{12} e^{j\beta} \left[1 + e^{j(kd\cos\phi - \beta)} \right]$$

which has a reverse current phase -β and a modified amplitude with a multiplication of a complex number $Z'_{12}e^{j\beta}$. Note that all parameters in the above formula can be calculated except I_1 which will be removed after normalization. Normalization of the above formula can only be done when its maximum value is known, for example by numerical calculation.

Example 2

Absolute value

Find the normalized array pattern $|\mathbf{E}_n|$ on the horizontal plane ($\theta=\pi/2$) of a two-monopole array with the following parameters with mutual coupling taken into account:

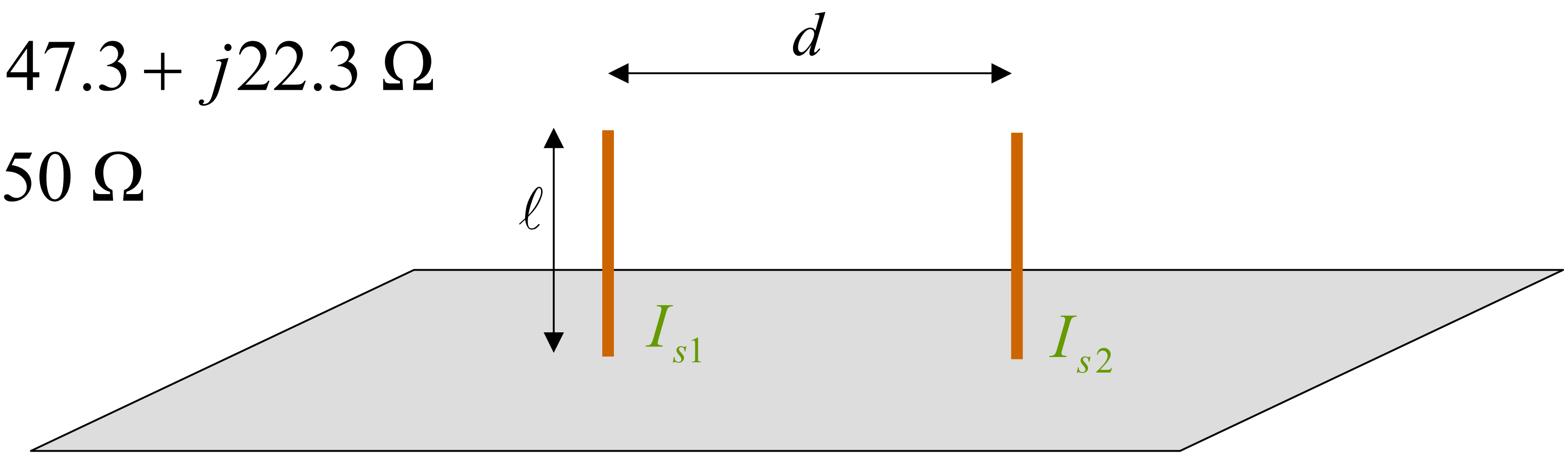
$$I_{s1}=1, \quad I_{s2}=e^{j\beta}, \quad |I_{s1}|=|I_{s2}|=1, \quad \beta=150°$$

$$d=\lambda/4, \quad \ell=\lambda/4$$

$$Z_{12}=Z_{21}=21.8 - j21.9\ \Omega$$

$$Z_A=47.3+j22.3\ \Omega$$

$$Z_L=50\ \Omega$$



Solution

As the required array pattern $|E_n|$ is on the horizontal plane, it is equal to the normalized array factor $|AF_n|$.

$$I_{s1} = 1, \quad I_{s2} = e^{j\beta}$$

$$|I_{s1}| = |I_{s2}| = 1$$

$$\angle I_{s1} = 0°, \quad \angle I_{s2} = \beta = 150° = 2.62 \text{ rad}$$

$$kd = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$$

$$Z'_{12} = \frac{Z_{12}}{Z_A + Z_L} = \frac{Z_{21}}{Z_A + Z_L} = 0.16 - j0.26$$

$$D = 1 - \frac{Z_{12}Z_{21}}{(Z_A + Z_L)^2} = 1.042 + j0.09$$

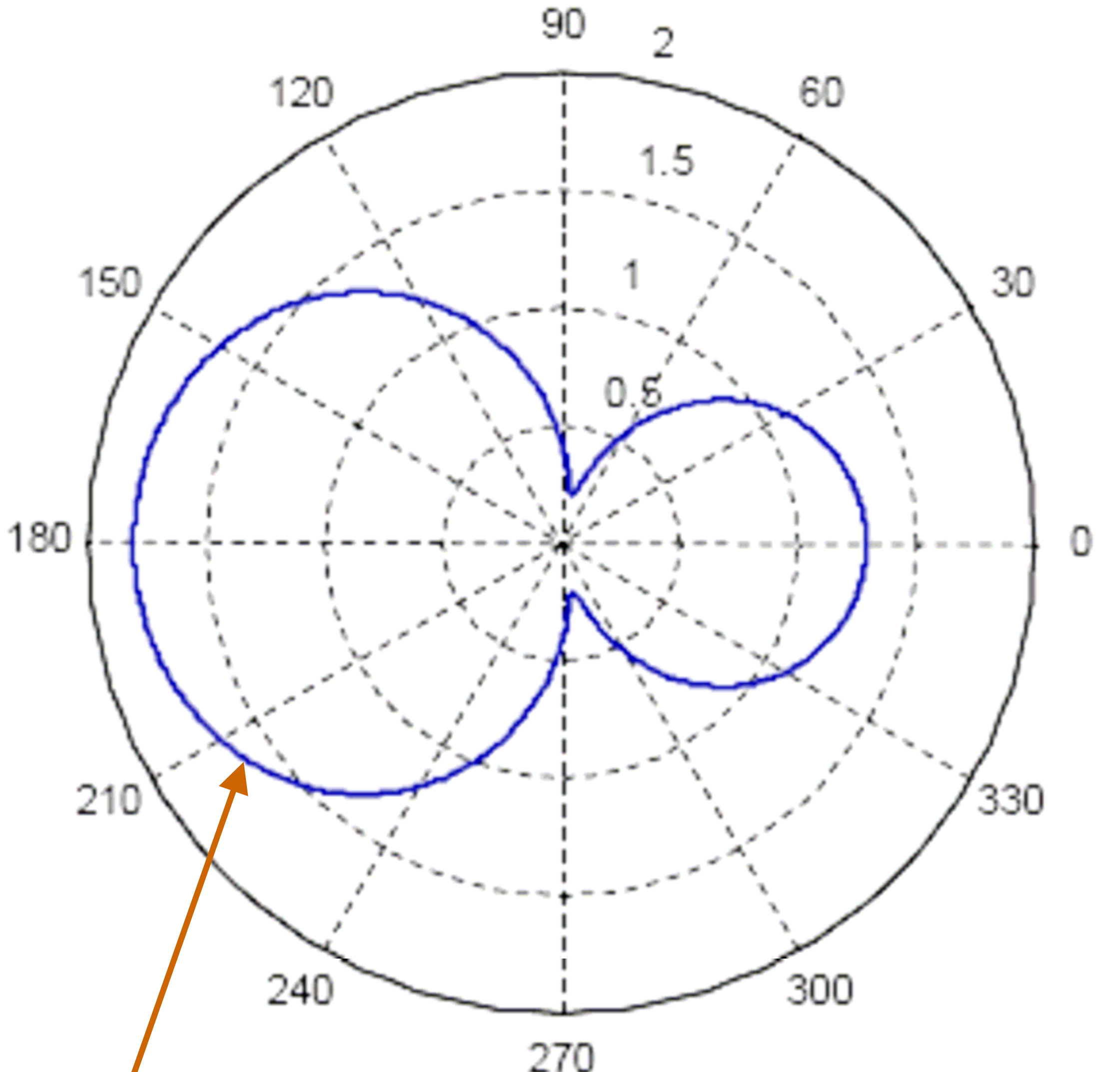
$$|\mathbf{E}| = |\mathrm{AF}| = \left| \frac{I_{s1}}{I_1 D} \left\{ \left[1 + e^{j(kd\cos\phi+\beta)} \right] - Z'_{12} e^{j\beta} \left[1 + e^{j(kd\cos\phi-\beta)} \right] \right\} \right|$$

$$= \left| \frac{0.95 - j0.08}{I_1} \left[1 + e^{j2.62} e^{j(\pi/2)\cos\phi} \right. \right.$$

$$\left. \left. -(0.16 - j0.26) e^{j2.62} \left(1 + e^{-j2.62} e^{j(\pi/2)\cos\phi} \right) \right] \right|$$

$$= \left| \frac{0.94 - j0.37}{I_1} \right| \left| 1 + (-1.14 + j0.40) e^{j(\pi/2)\cos\phi} \right|$$

The pattern of $f = \left| 1 + (-1.14 + j0.40) e^{j(\pi/2)\cos\phi} \right|$ is shown on next page.



$$f = \left| 1 + (-1.14 + j0.40)\, e^{j(\pi/2)\cos\phi} \right|$$

Normalization

The pattern of f attains the maximum value when $\phi = 180°$. When $\phi = 180°$,

$$\left|\mathbf{E}\right|_{\phi=180°}$$

$$= \left|\frac{0.94 - j0.37}{I_1}\right| \left|1 + (-1.14 + j0.40) e^{j(\pi/2)\cos\phi}\right|_{\phi=180°}$$

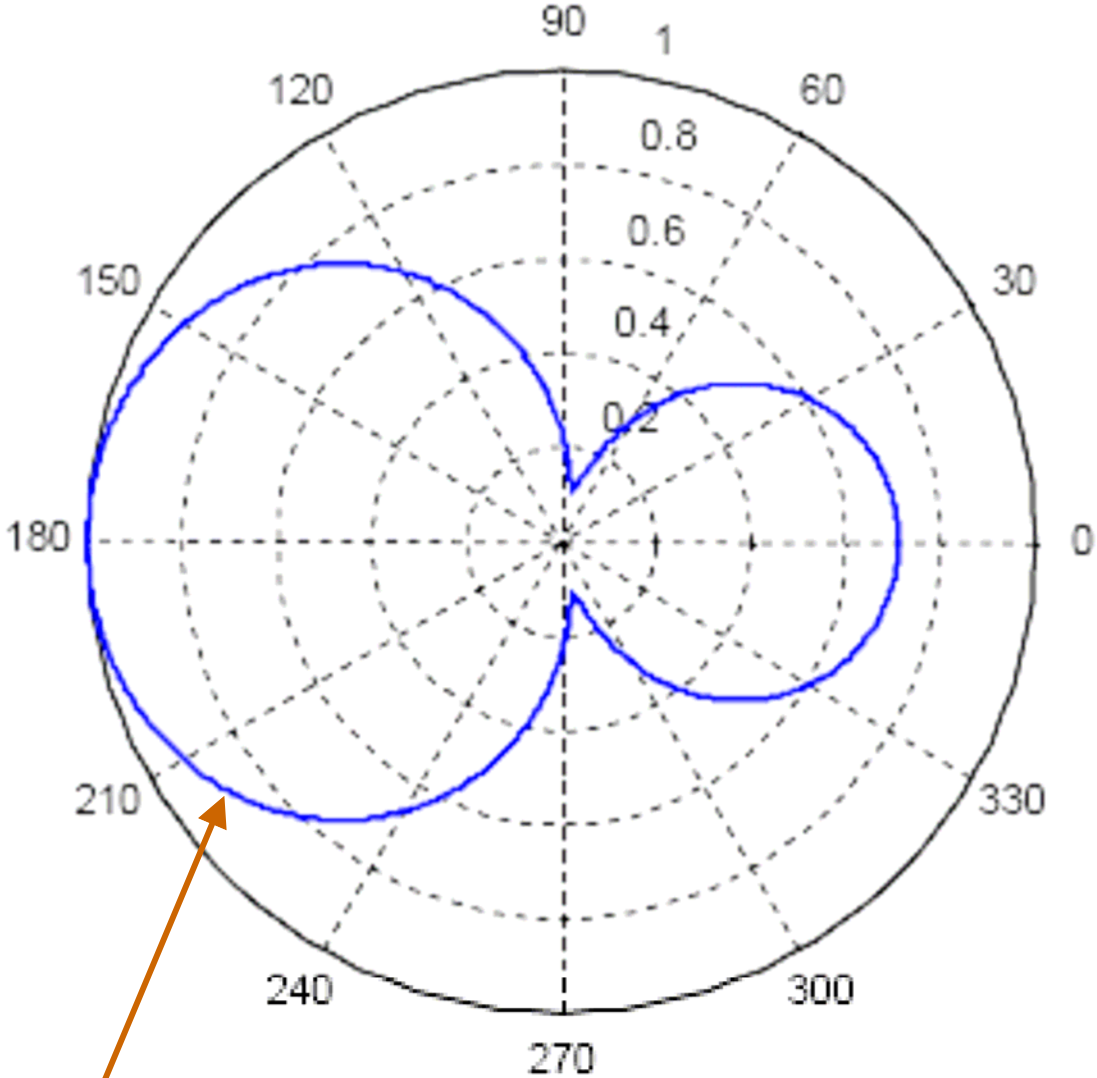
$$= \frac{1.83}{\left|I_1\right|}$$

Hence we normalize $|\mathbf{E}|$ by this factor $(1.83/|I_1|)$ to get:

$$\left|\mathbf{E}_n\right| = \frac{\left|\dfrac{0.94 - j0.37}{I_1}\left[1 + \left(-1.14 + j0.40\right)e^{j(\pi/2)\cos\phi}\right]\right|}{\dfrac{1.83}{\left|I_1\right|}}$$

$$= 0.52\left|1 + \left(-1.14 + j0.40\right)e^{j(\pi/2)\cos\phi}\right|$$

The polar plot of $|\mathbf{E}_n|$ is shown on next page.

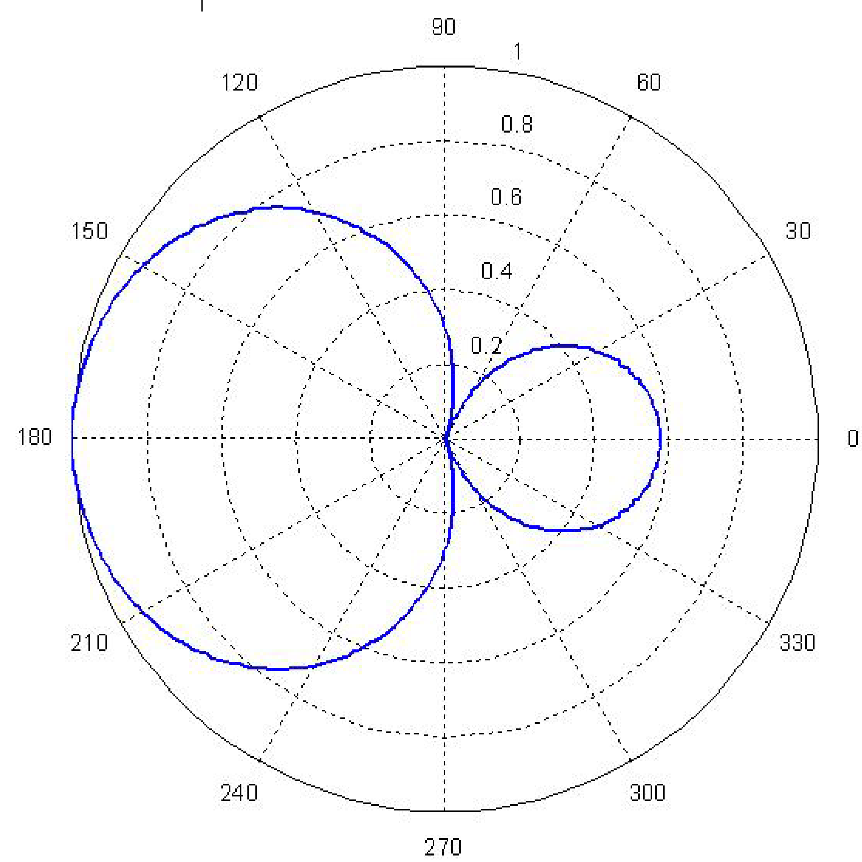


$$\left|\mathbf{E}_n\right| = 0.52\left|1+\left(-1.14+j0.40\right)e^{j(\pi/2)\cos\phi}\right|$$

The case when there is no mutual coupling is shown below for comparison.

$$\left|\mathbf{E}_n\right|_{\text{no mutual coupling effect}} = \frac{1}{\Gamma}\left|1 + e^{j\beta} e^{jkd\cos\phi}\right|$$

where Γ is a constant to make the largest value of $|AF_n|$ equal to one ($\Gamma = 1.73$).



References:

1. C. A. Balanis, *Antenna Theory, Analysis and Design*, John Wiley & Sons, Inc., New Jersey, 2005.
2. W. L. Stutzman and G. A. Thiele, *Antenna Theory and Design*, Wiley, New York, 1998.
3. David K. Cheng, *Field and Wave Electromagnetic*, Addison-Wesley Pub. Co., New York, 1989.
4. John D. Kraus, *Antennas*, McGraw-Hill, New York, 1988.
5. Fawwaz T. Ulaby, *Applied Electromagnetics*, Prentice-Hall, Inc., New Jersey, 2007.
6. Joseph A. Edminister, *Schaum's Outline of Theory and Problems of Electromagnetics*, McGraw-Hill, Singapore, 1993.
7. Yung-kuo Lim (Editor), *Problems and solutions on electromagnetism*, World Scientific, Singapore, 1993.