MUCH ADO ABOUT NULL THINGS

A replication issue in spatial stats

JEFF SAUER
SERGIO REY
TAYLOR OSHAN
LEVI JOHN WOLF

LOCAL STATISTICS

A REPLICATION FAILURE

"NULLS" IN LOCAL TESTS

The importance of null hypotheses in local statistics

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FORMALIZING INTUITION

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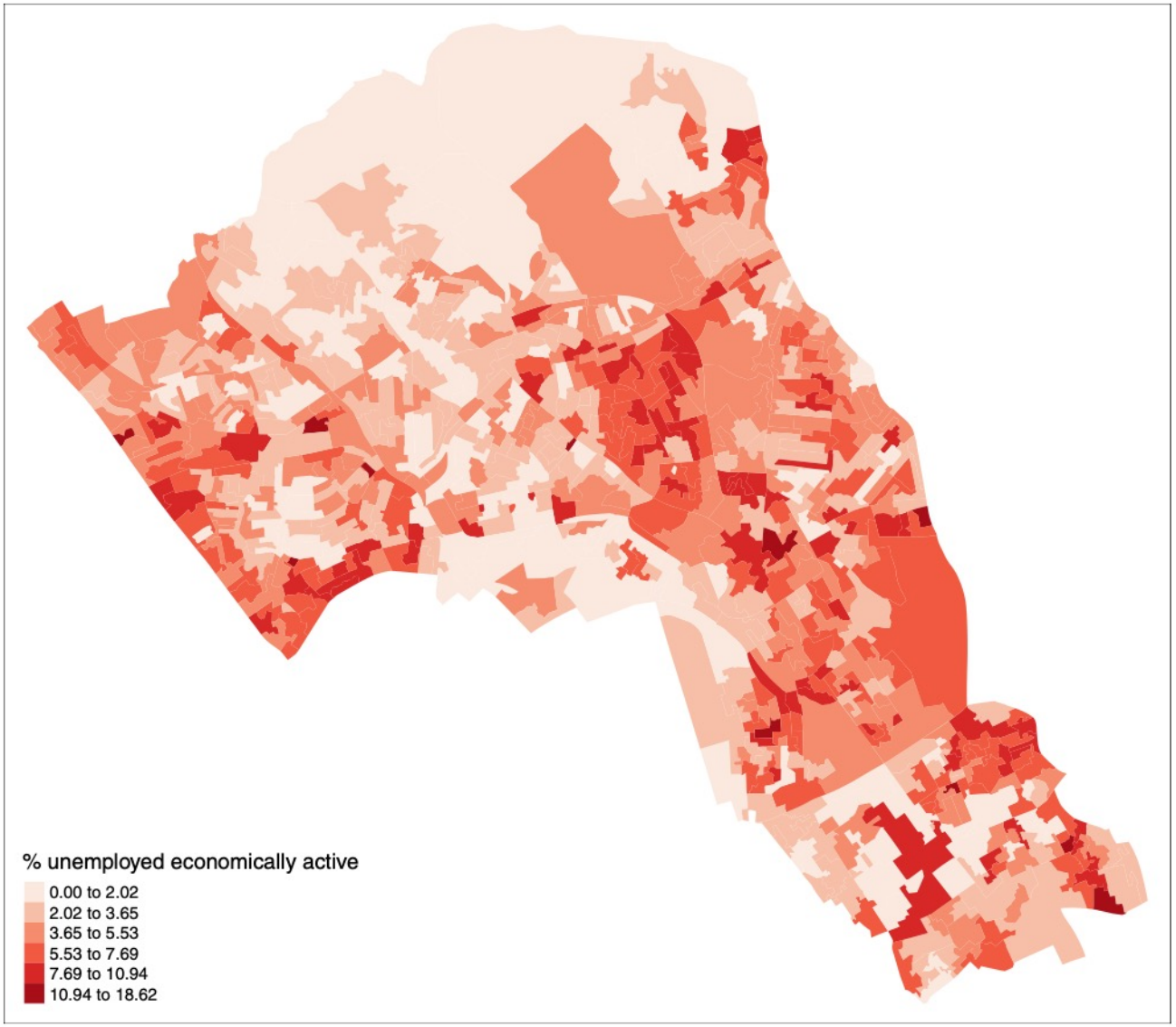
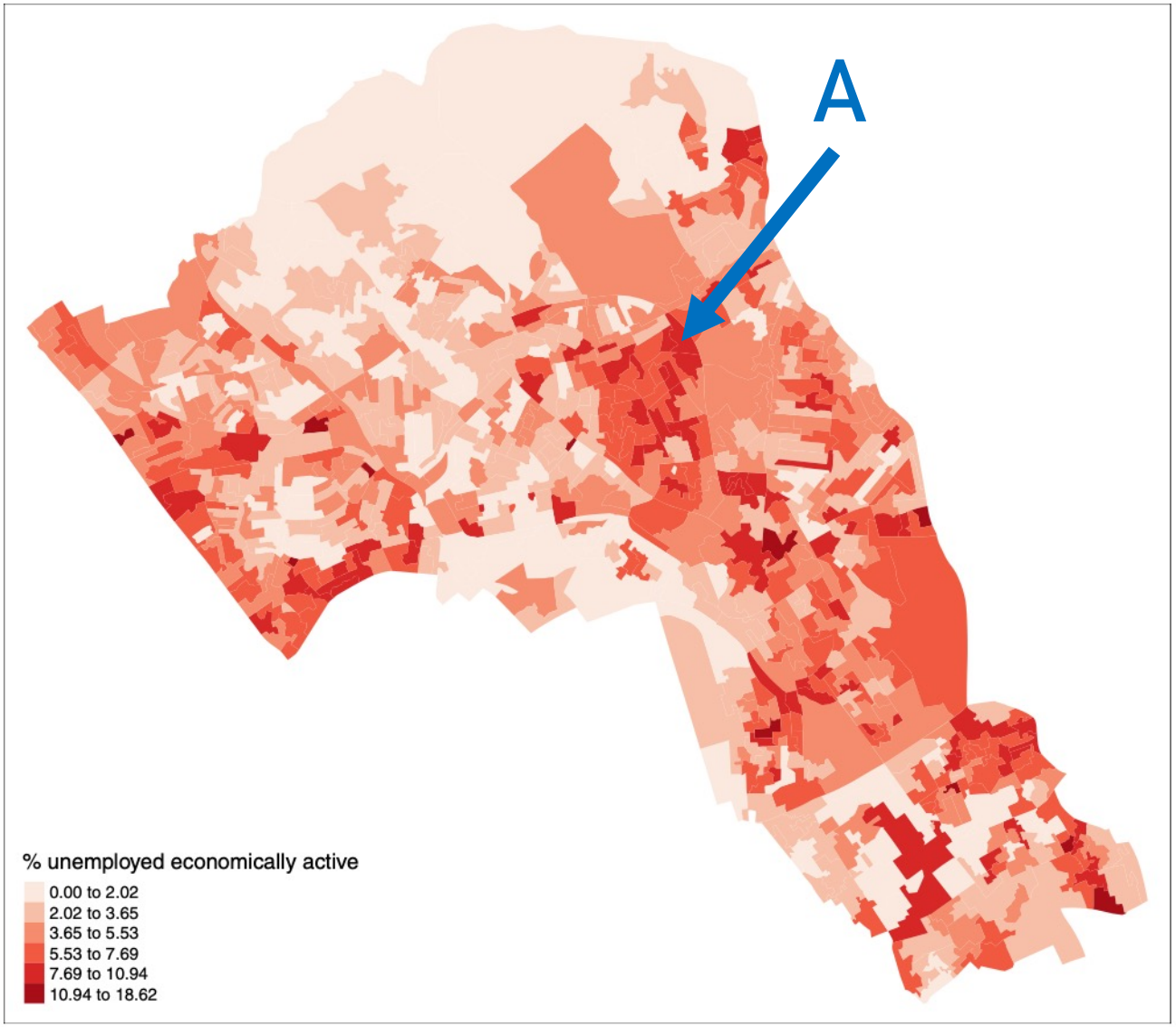


Local statistics: find outliers and/or clusters



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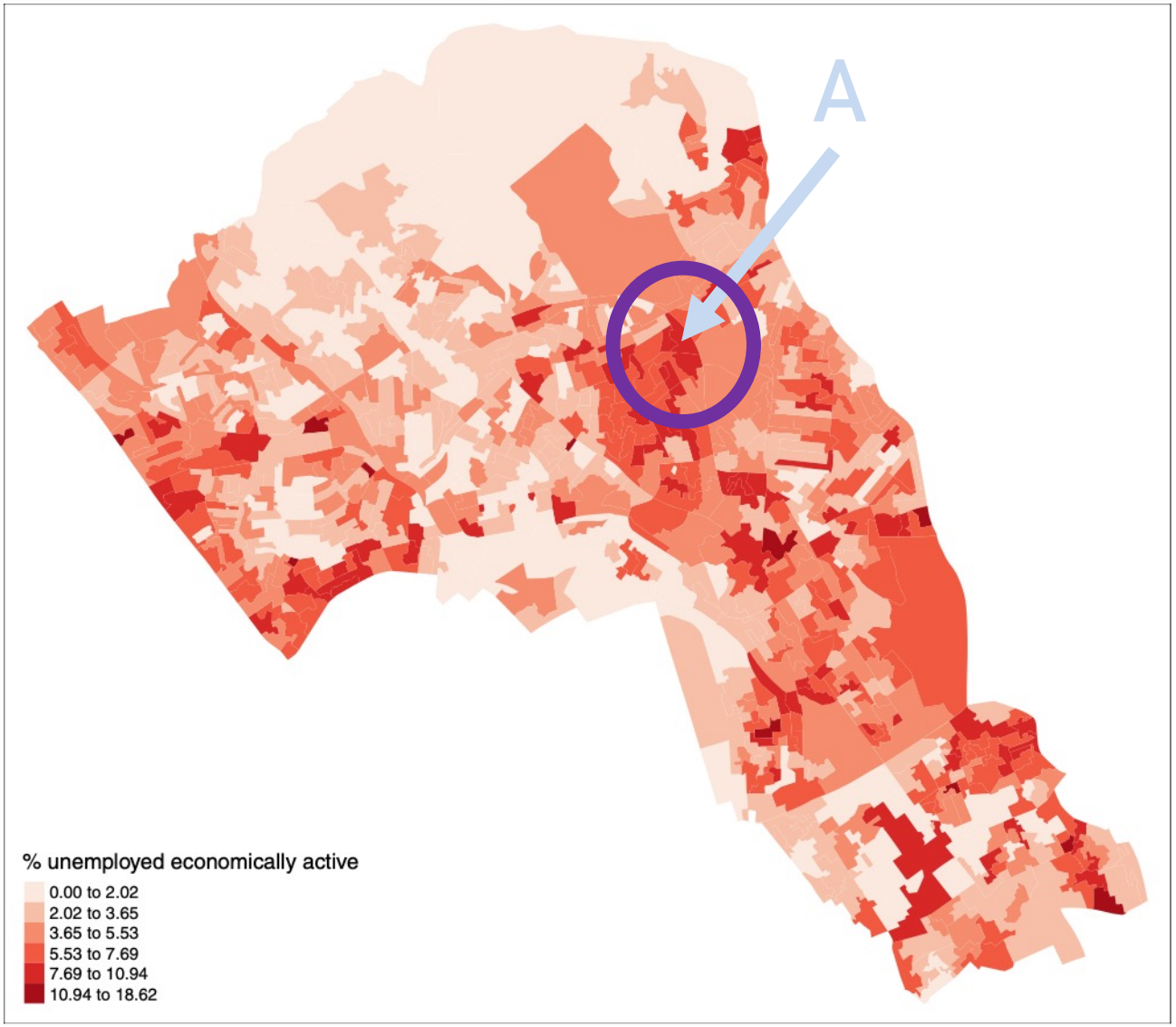

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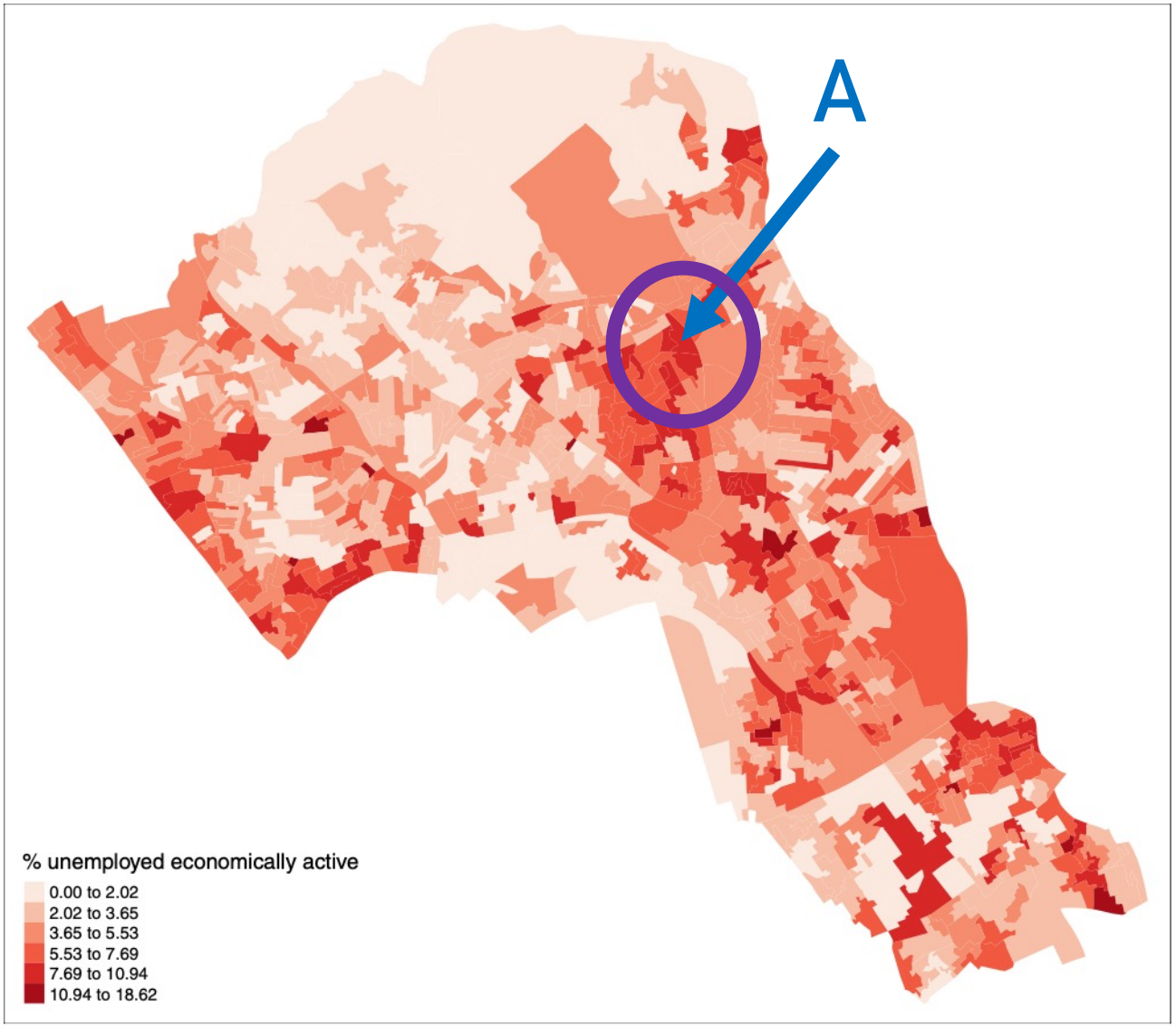
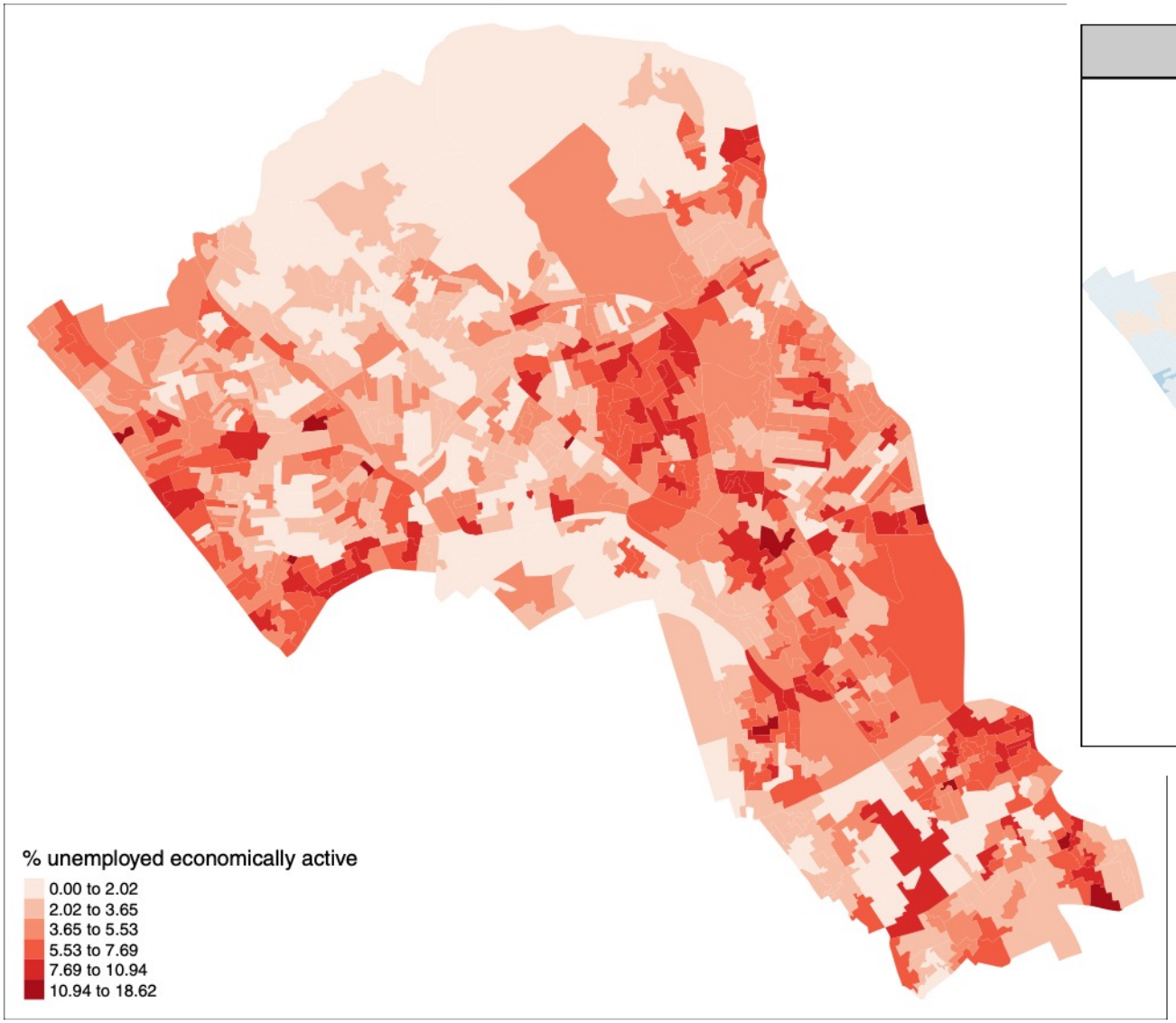
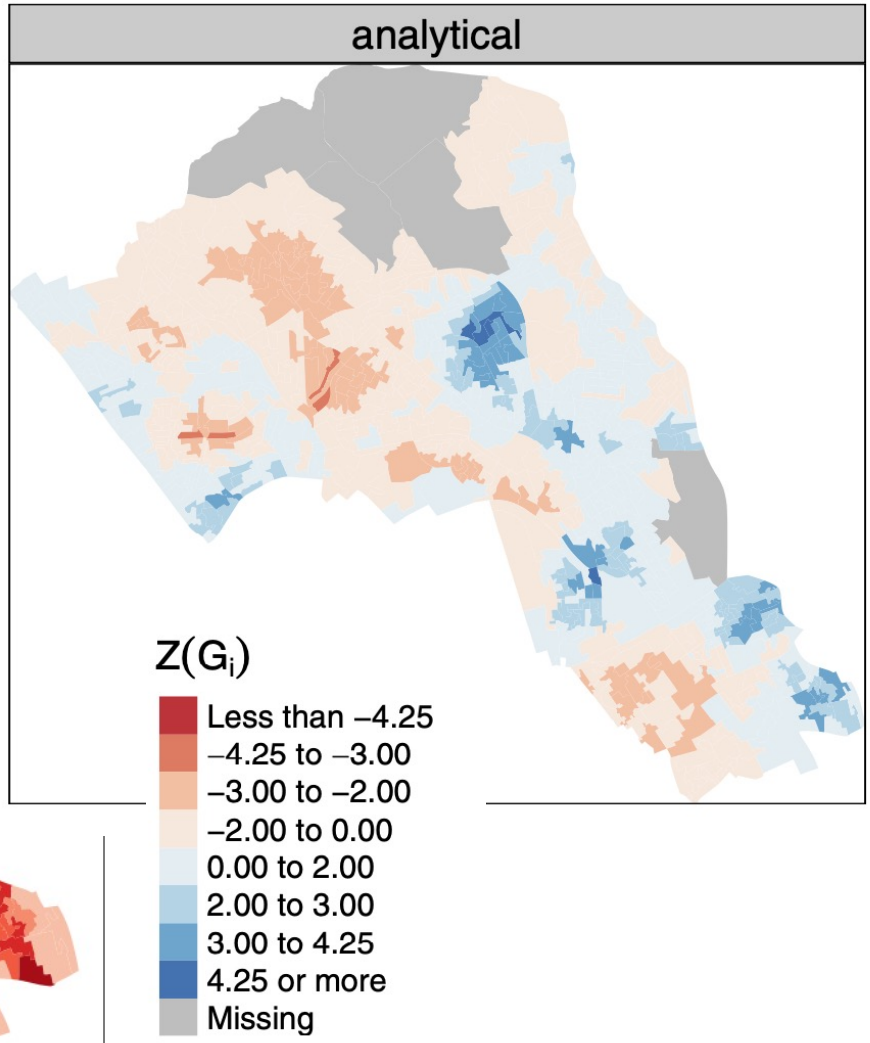


Is the distribution around this site what we'd expect?

Local statistics: find outliers and/or clusters





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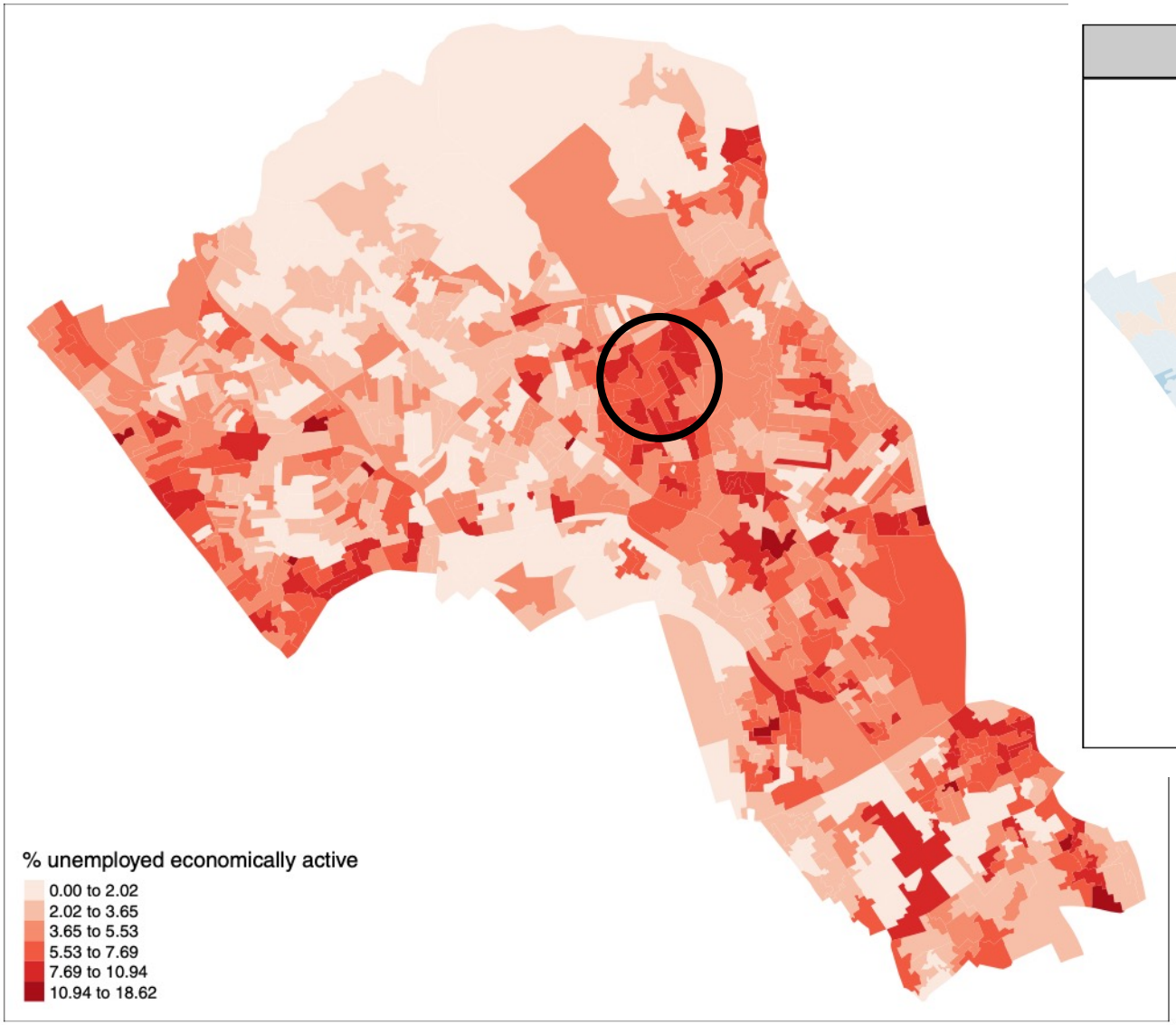

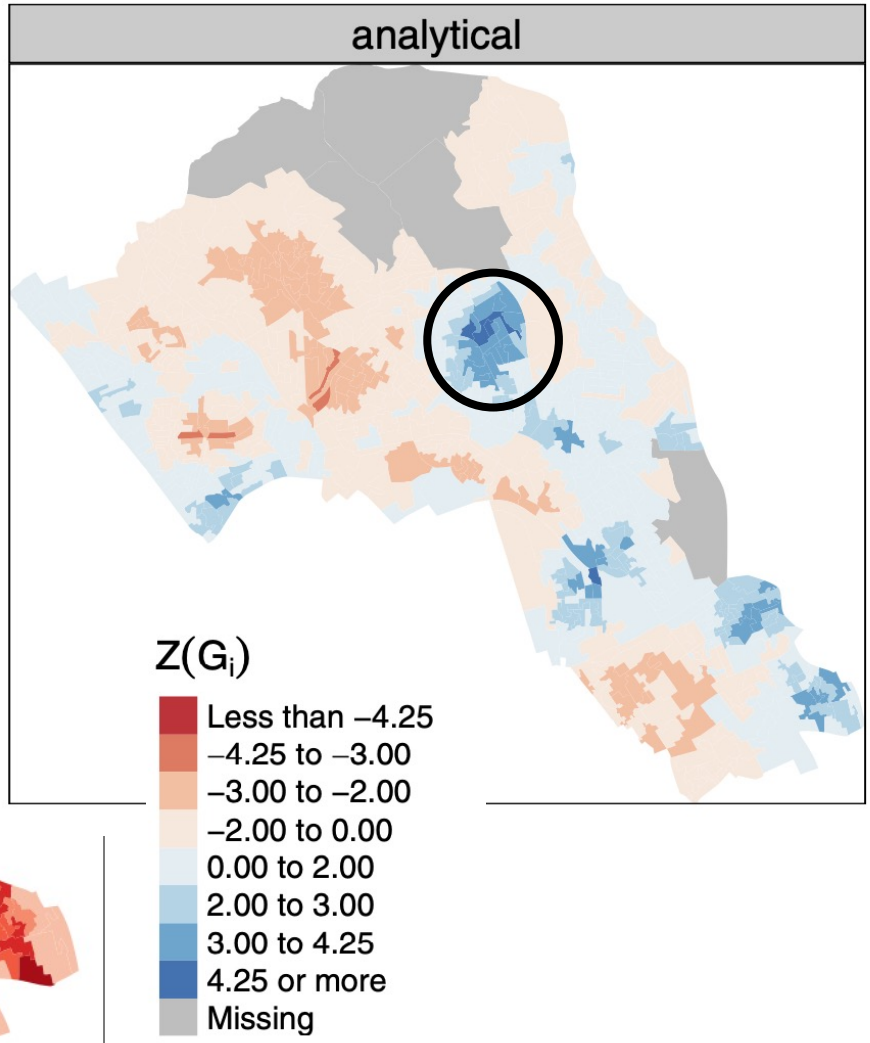

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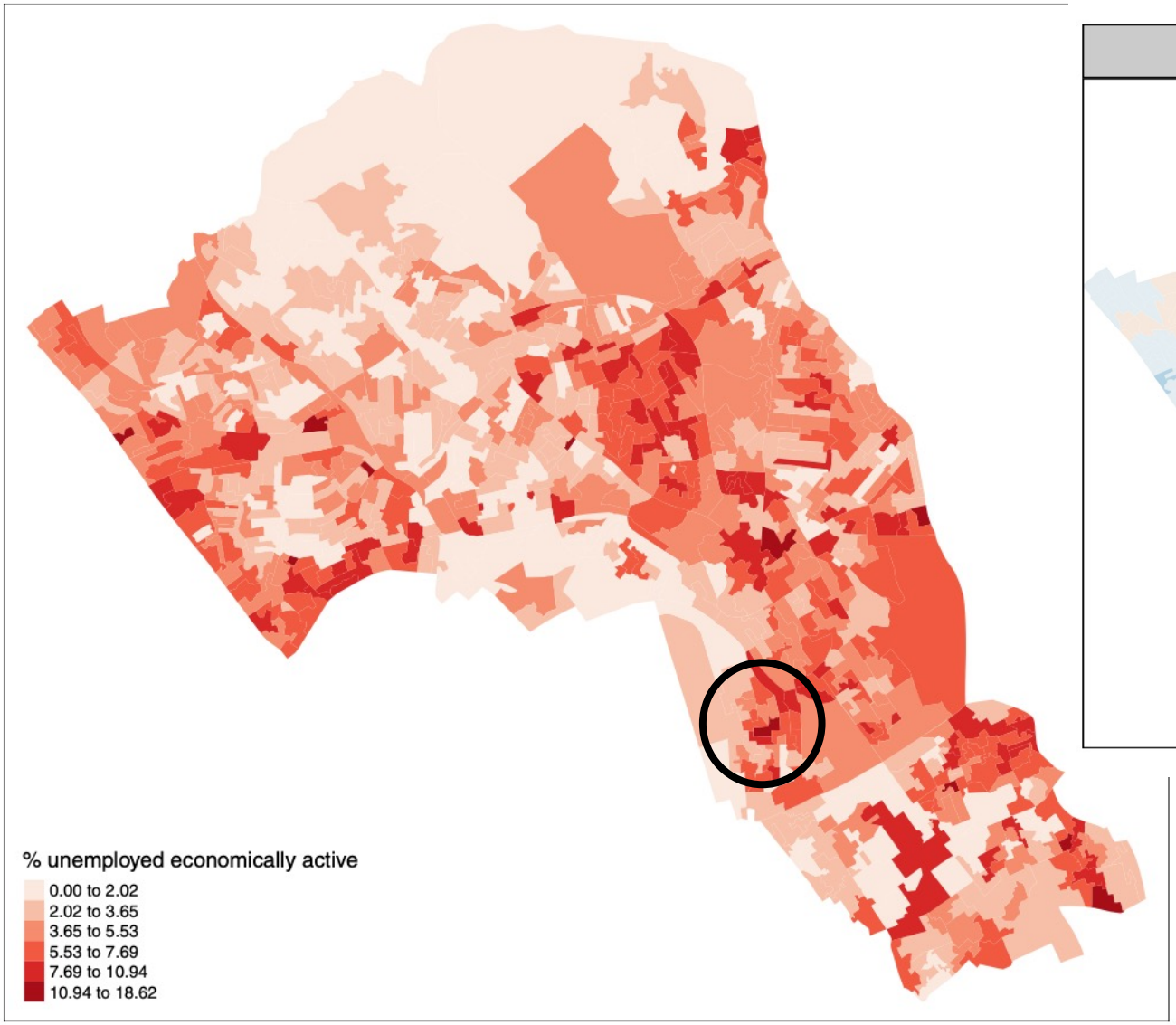

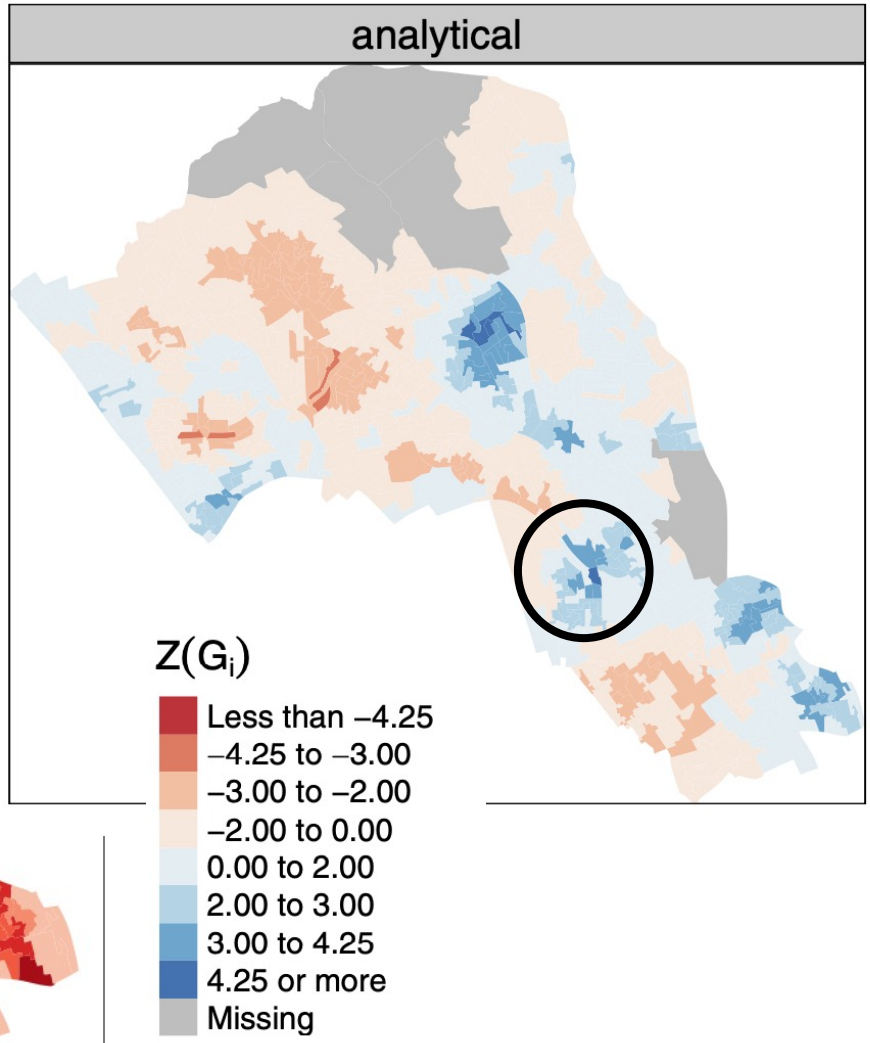

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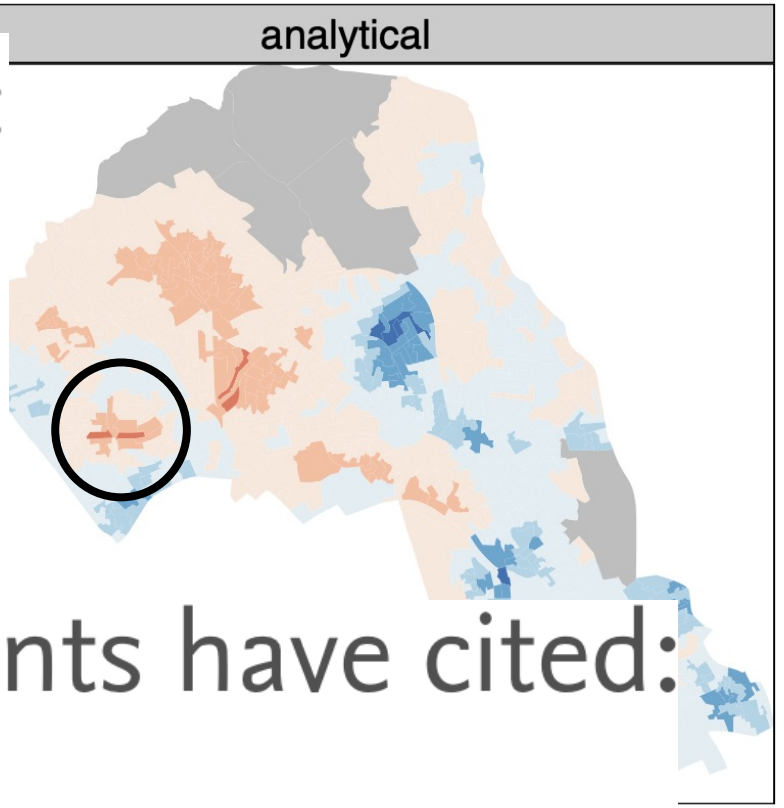
5,679 documents have cited:

Local Indicators of Spatial Association—LISA

Anselin L.

(1995) Geographical Analysis, 27 (2) , pp. 93-115.

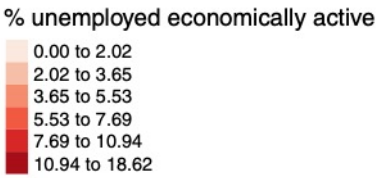


2,893 documents have cited:

The Analysis of Spatial Association by Use of Distance Statistics

Getis A., Ord J.K.

(1992) Geographical Analysis, 24 (3) , pp. 189-206.



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TEST (2018) 27:716–748
https://doi.org/10.1007/s11749-018-0599-x

ORIGINAL PAPER

Comparing implementations of global and local indicators of spatial association

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Received: 4 April 2018 / Accepted: 14 July 2018 / Published online: 27 July 2018

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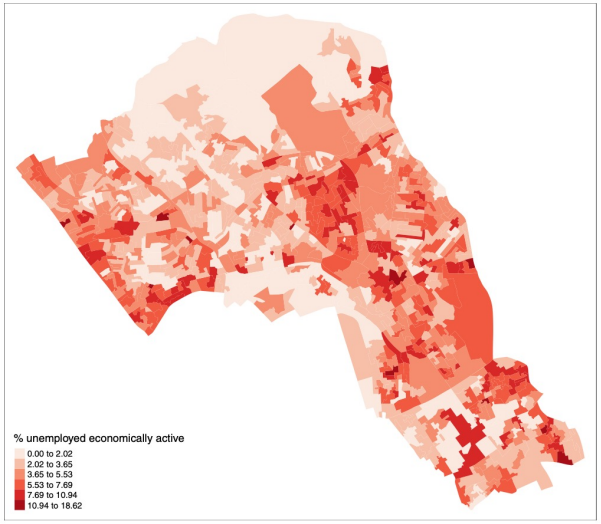

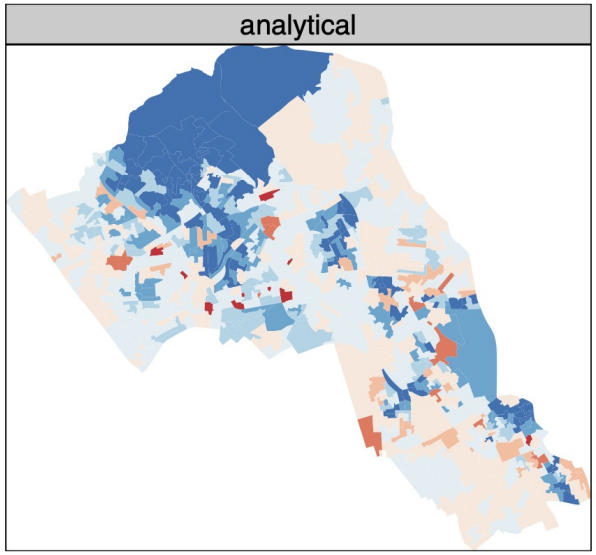

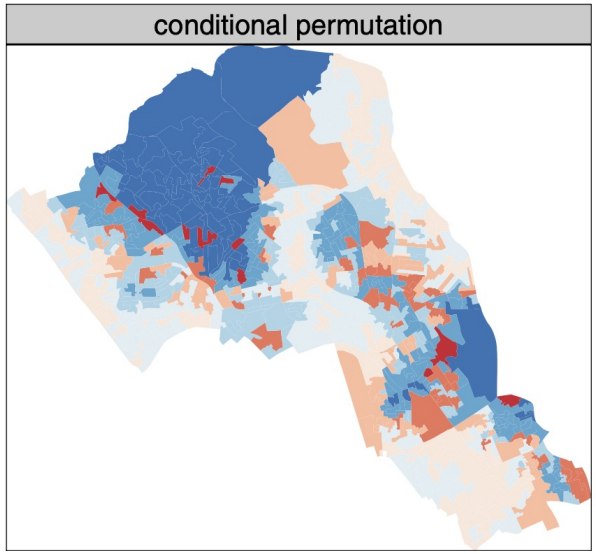

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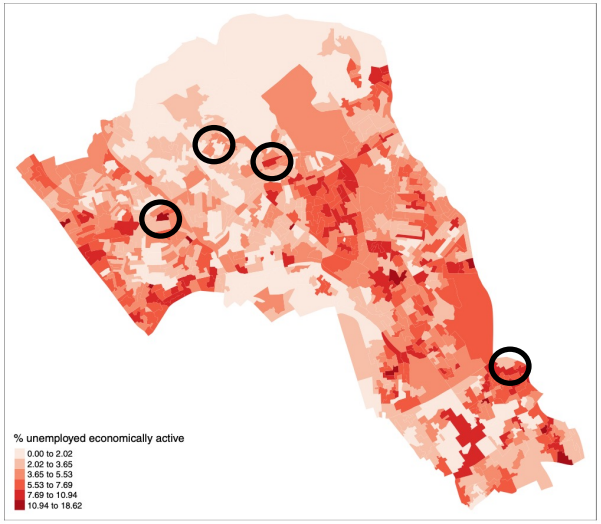
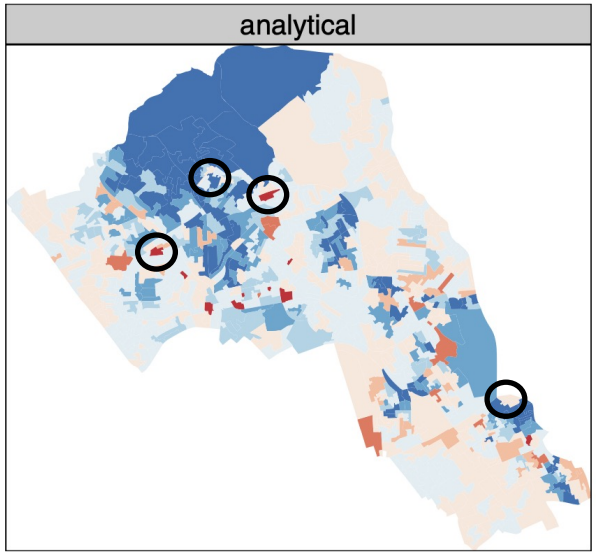
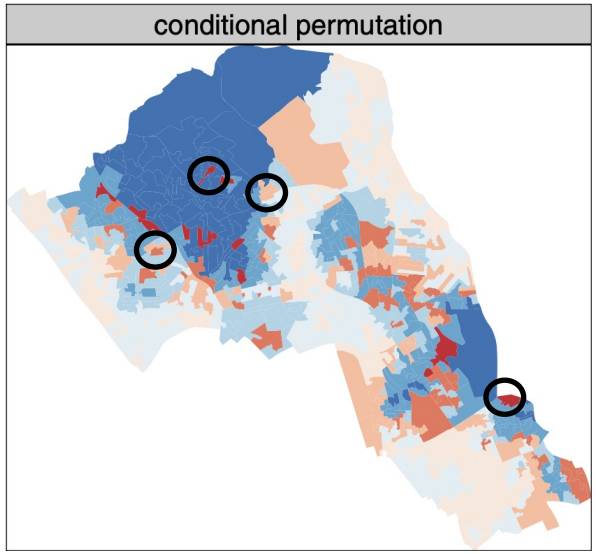
Sauer et al. (2021), p3







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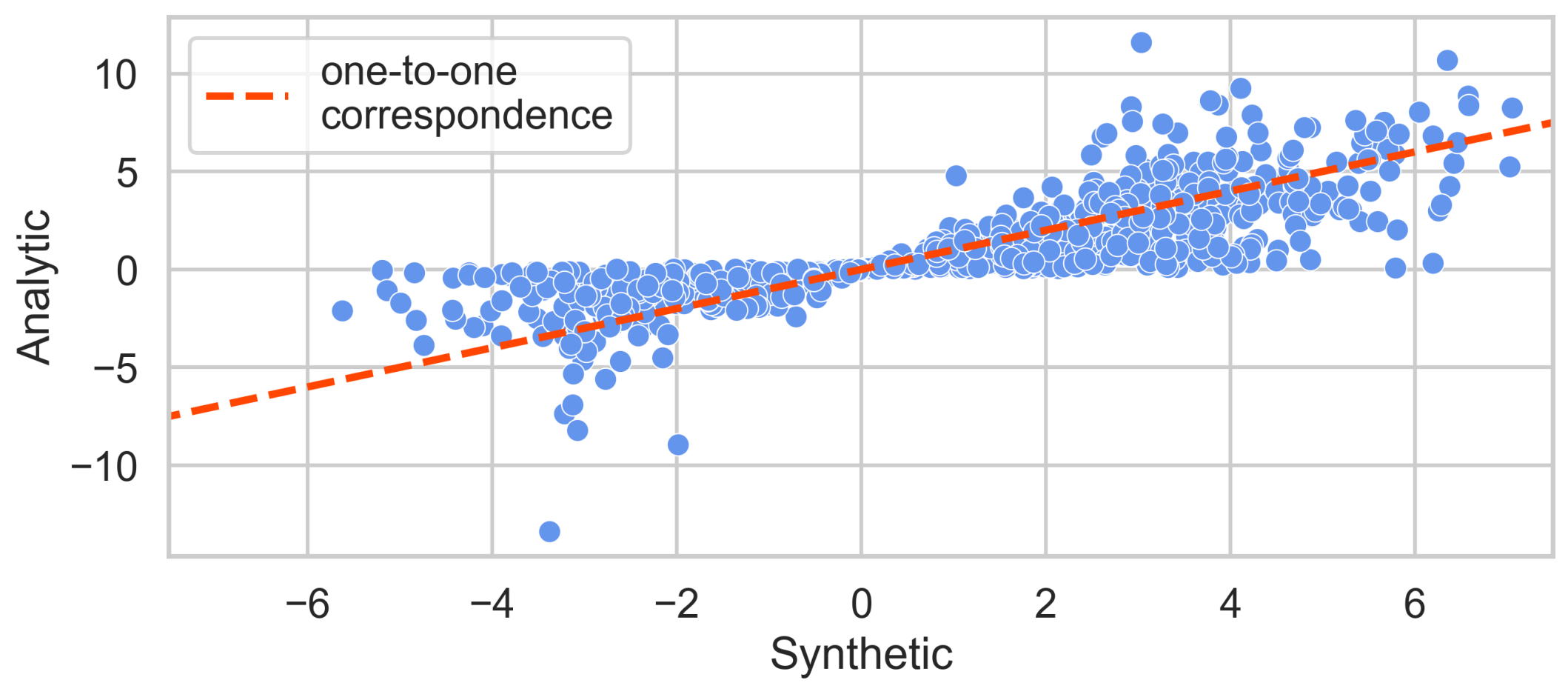

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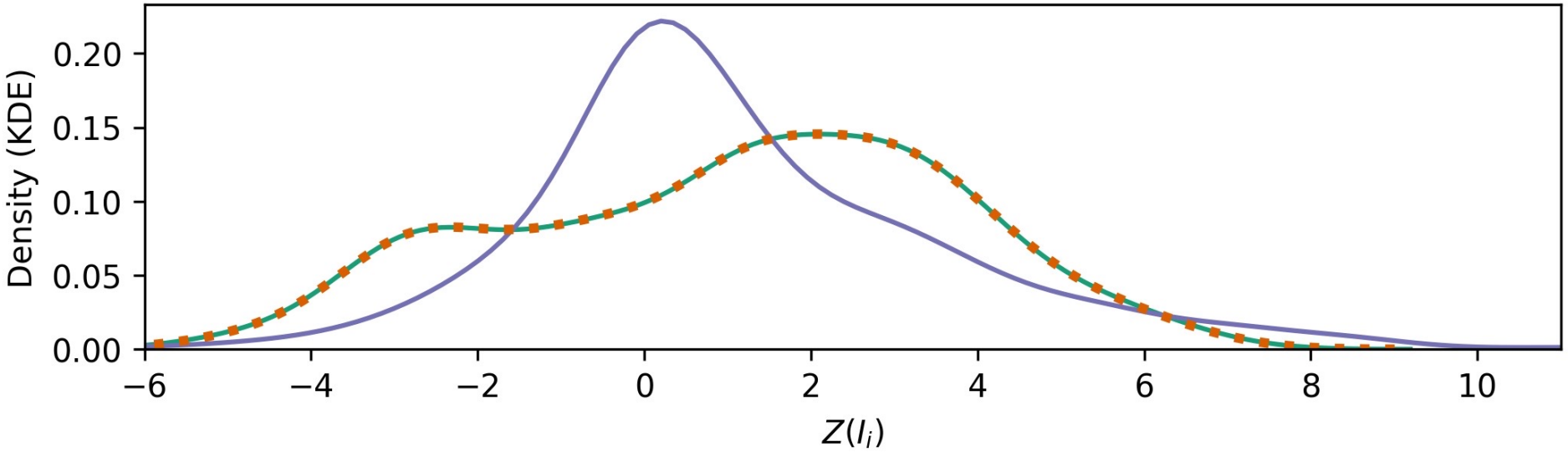

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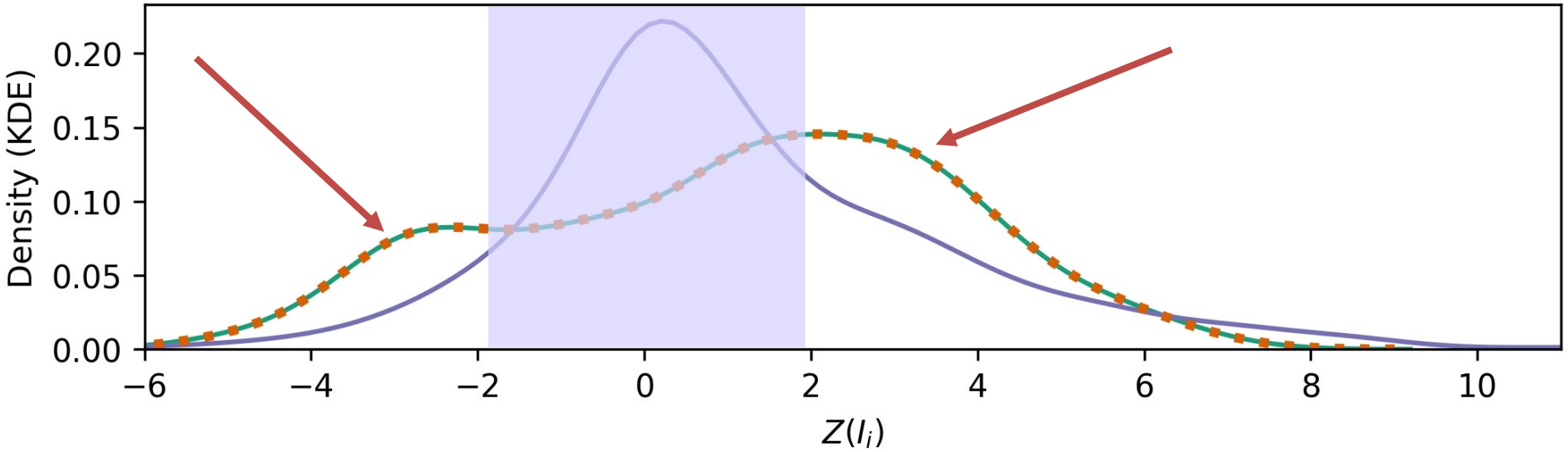

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geographical analysis

Geographical Analysis (2021) **0**, 1–17

The Importance of Null Hypotheses: Understanding Differences in Local Moran's I_i under Heteroskedasticity

Jeffery Sauer[1], Taylor Oshan[1], Sergio Rey[2], Levi John Wolf[3]

[1]Center for Geospatial Information Science, Department of Geographical Sciences, University of Maryland, College Park, MD USA, [2]Center for Geospatial Sciences, University of California Riverside, Riverside, CA USA, [3]School of Geographical Sciences, University of Bristol, Bristol, U.K.

doi.org/10.1111/gean.12304

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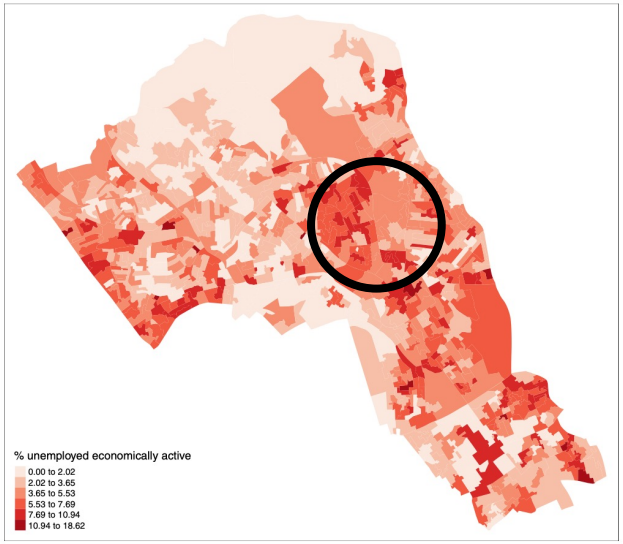
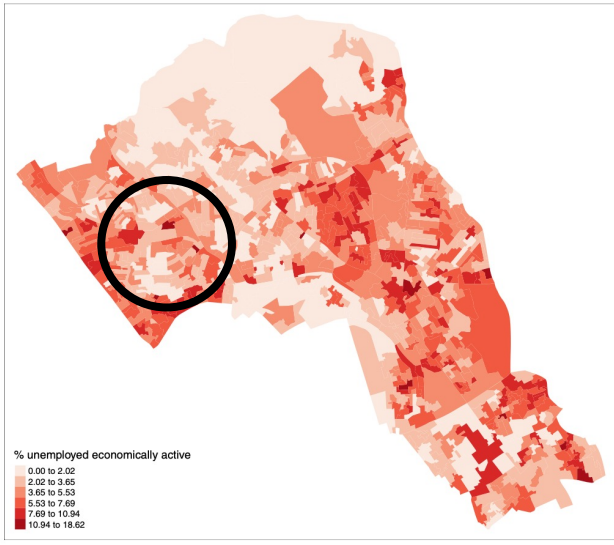


LOW
σ

HIGH
σ



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ALL DEPENDS ON THEORY

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ANALTYIC

SYNTHETIC

ANALTYIC

Assume normal observations distributed over a regular grid

Derive $\mathbf{E}[\Gamma]$ while assuming “all values are equally likely at any site”

Use to define $\mathbf{Var}[\Gamma]$

Construct z-score

Assess against a standard normal distribution: $|z_i| > 1.96$

Hope it generalizes!

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Assume the data that you *do have* is like data you *could have*

Hold the value at site i constant

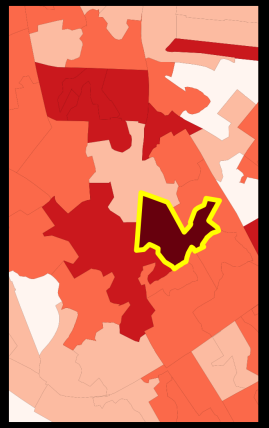
Shuffle the rest of the map

Compute a new local statistic

Repeat k - 1 times to obtain distribution of replicates, $\{\Gamma'\}$

Assess Γ against $\{\Gamma'\}$, so p is % of $\{\Gamma'\}$ at least as extreme as Γ

REAL



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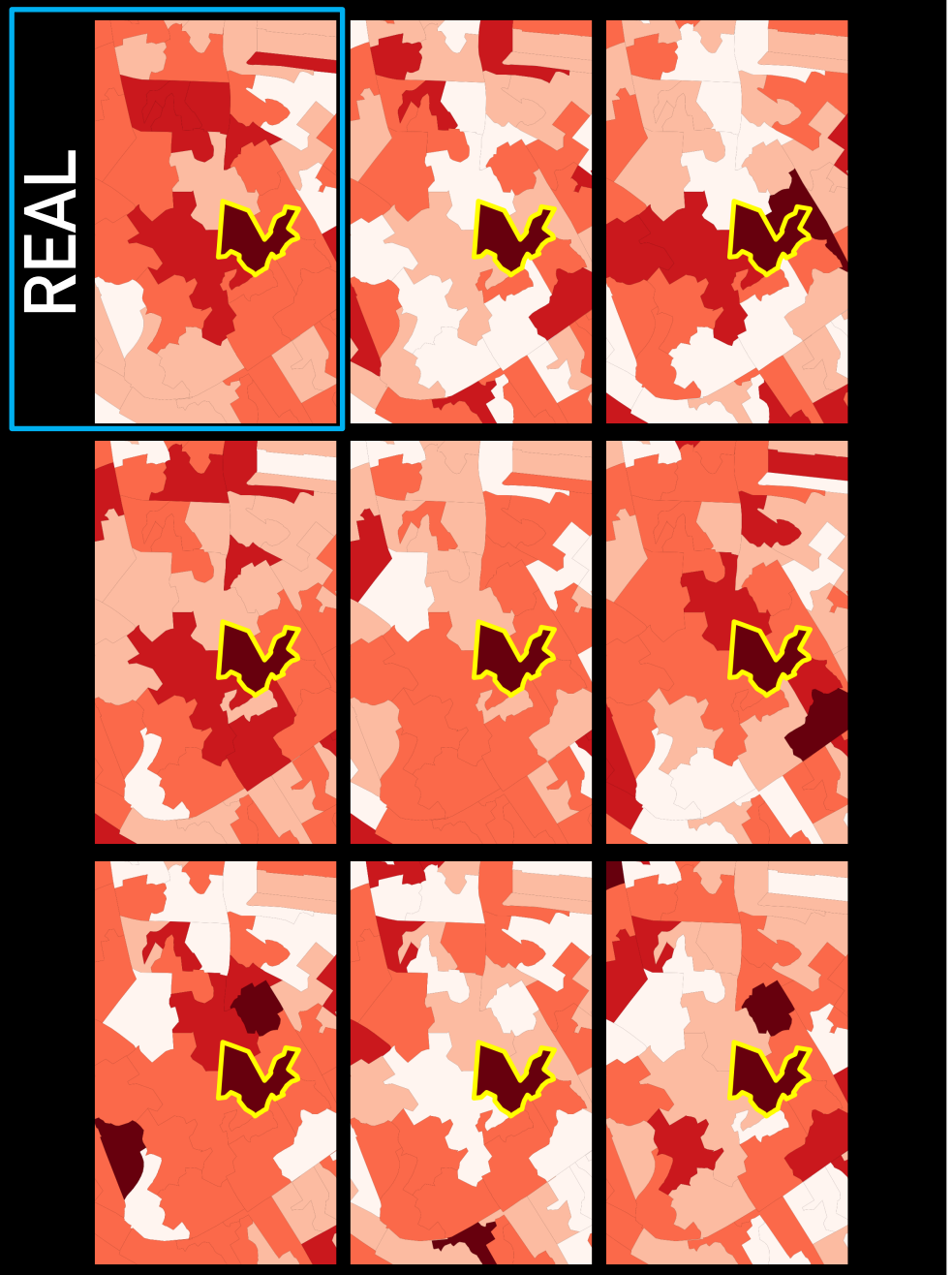

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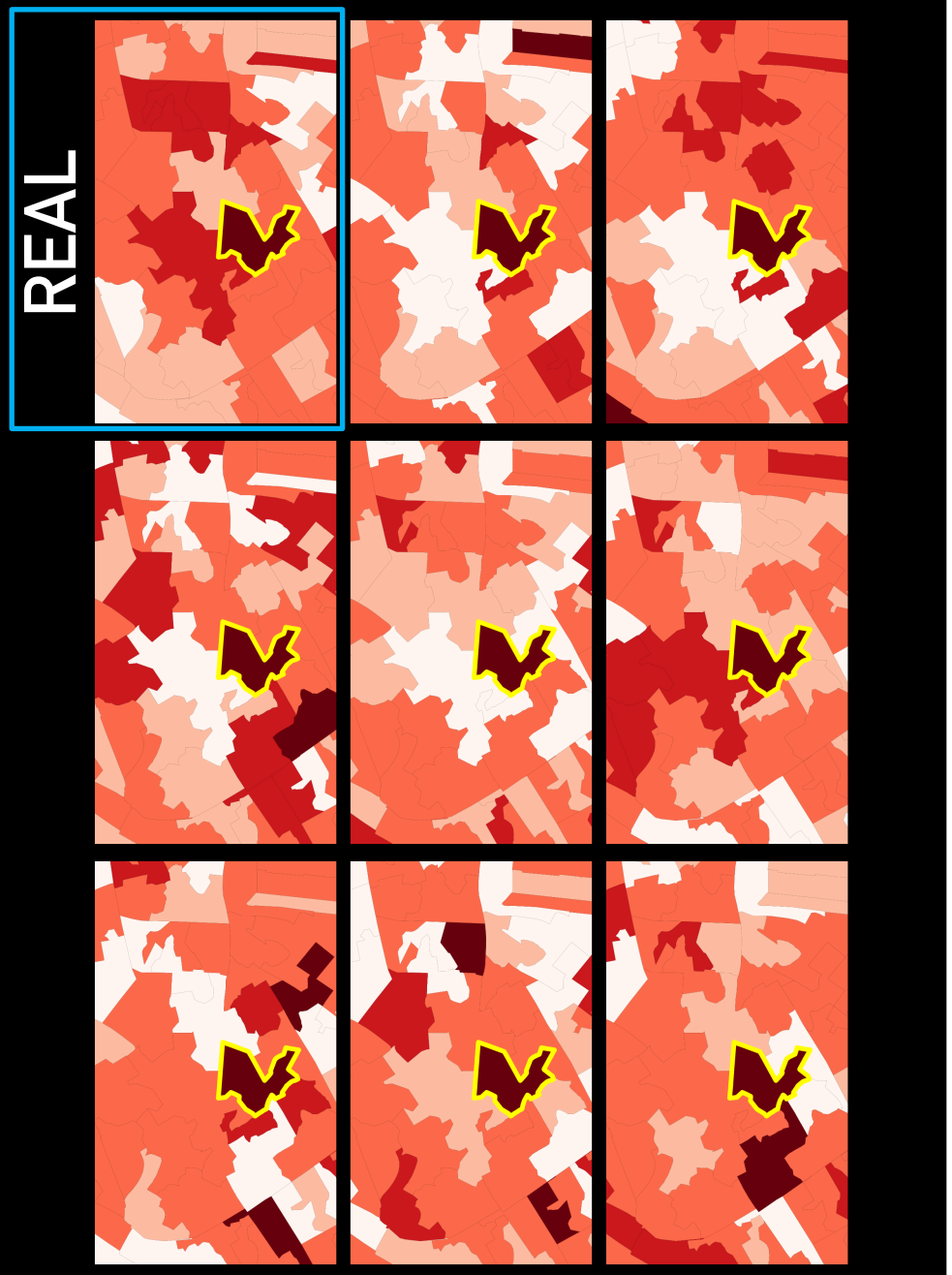

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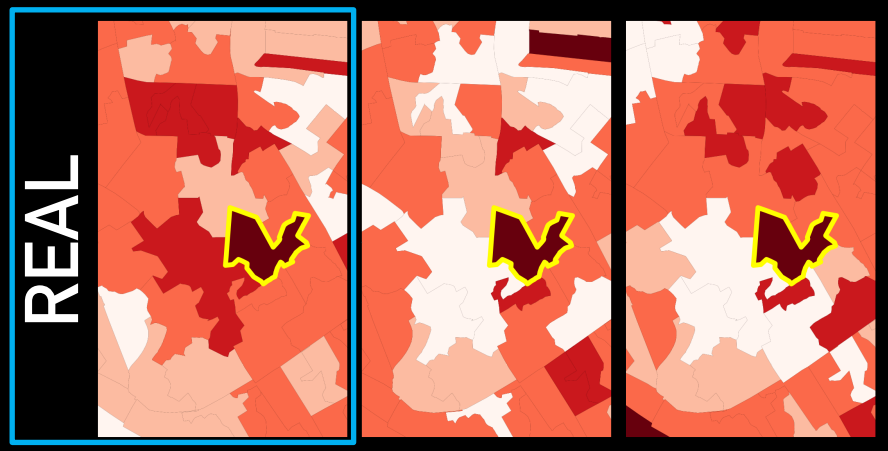

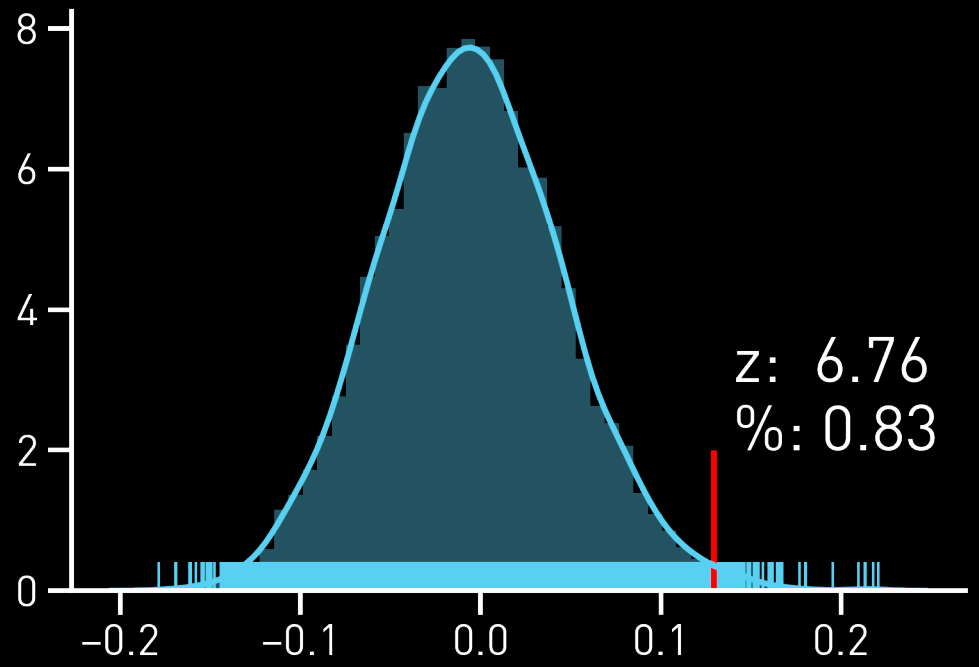

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Analytical estimator *closed-form mathematical expressions for test statistics*	Originally in Anselin (1995). Implemented in **`pysal`** and **`spdep`**. Used in Bivand & Wong (2018). Denoted here as $E[I_i]$ and $Var[I_i]$	Originally in Sokal (1998). Not implemented in **`pysal`** and **`spdep`** or considered by Bivand & Wong (2018). Denoted here as $E_c[I_i]$ and $Var_c[I_i]$
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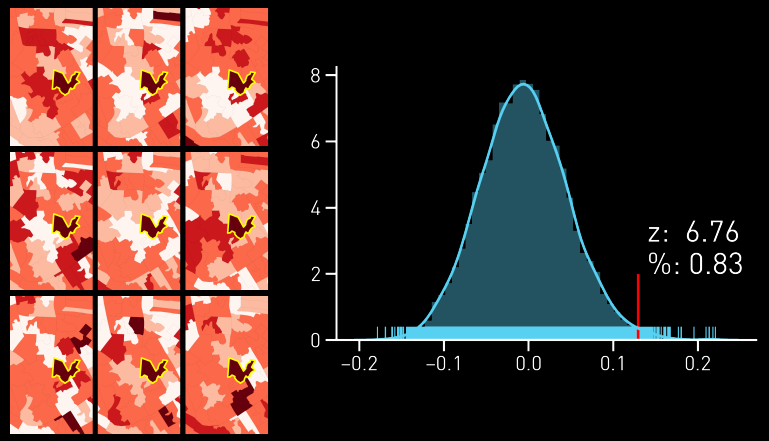
Permutation estimator *test statistics computed from set of simulated maps*	Not considered or implemented.	Originally in Anselin (1995). Implemented in **`pysal`**. Used in Bivand and Wong (2018). Denoted here as $E_p[I_i]$ and $Var_p[I_i]$



	"Total" Randomization Null *shuffle all values, so each value is equally likely at any site*	**"Conditional" Randomization Null** *for each site, the site's value is fixed and remaining sites are shuffled*
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TOTAL

$$\mathbf{E}[I_i] = -\sum \frac{w_{ij}}{n-1}$$

$$\mathbf{Var}[I_i] = \frac{w_{i(2)}(n-b_2)}{n-1} + \frac{2w_{i(kh)}2b_2-n}{(n-1)(n-2)} + \left[\sum_{i\neq j} \frac{w_{ij}}{n-1}\right]^2$$

CONDITIONAL

$$\mathbf{E}_c[I_i] = -z_i^2 \sum_{i\neq j} \frac{w_{ij}}{n-1}$$

$$\mathbf{Var}_c[I_i] = \left[\frac{z_i}{m_2}\right]^2 \left[\frac{n}{n-2}\right] \left[w_{i(2)} - \frac{\left(\sum_{i\neq j} w_{ij}\right)^2}{n-1}\right] \left[m_2 - \frac{z_i^2}{n-1}\right]$$

TOTAL

$$\mathbf{E}[I_i] = -\sum \frac{w_{ij}}{n-1}$$

$$\mathbf{Var}[I_i] = \frac{w_{i(2)}(n-b_2)}{n-1} + \frac{2w_{i(kh)}2b_2-n}{(n-1)(n-2)} + \left[\sum_{i\neq j} \frac{w_{ij}}{n-1}\right]^2$$

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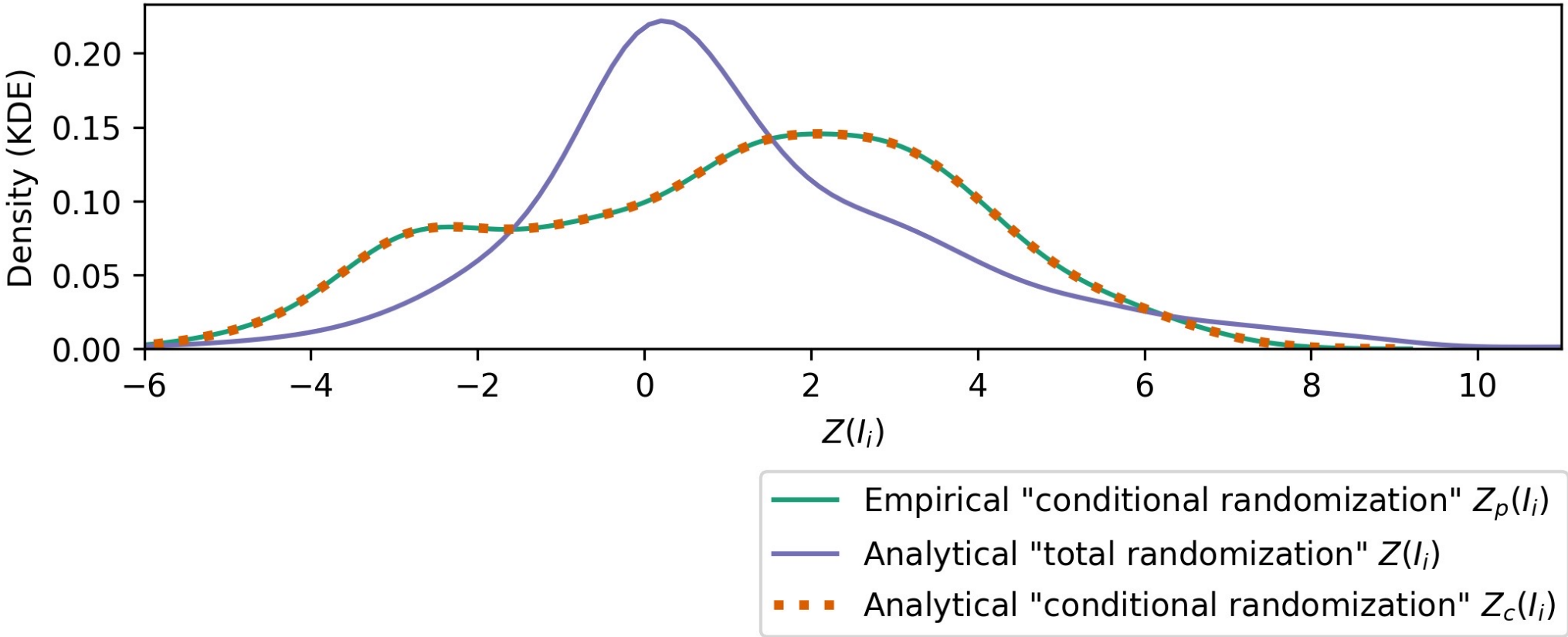
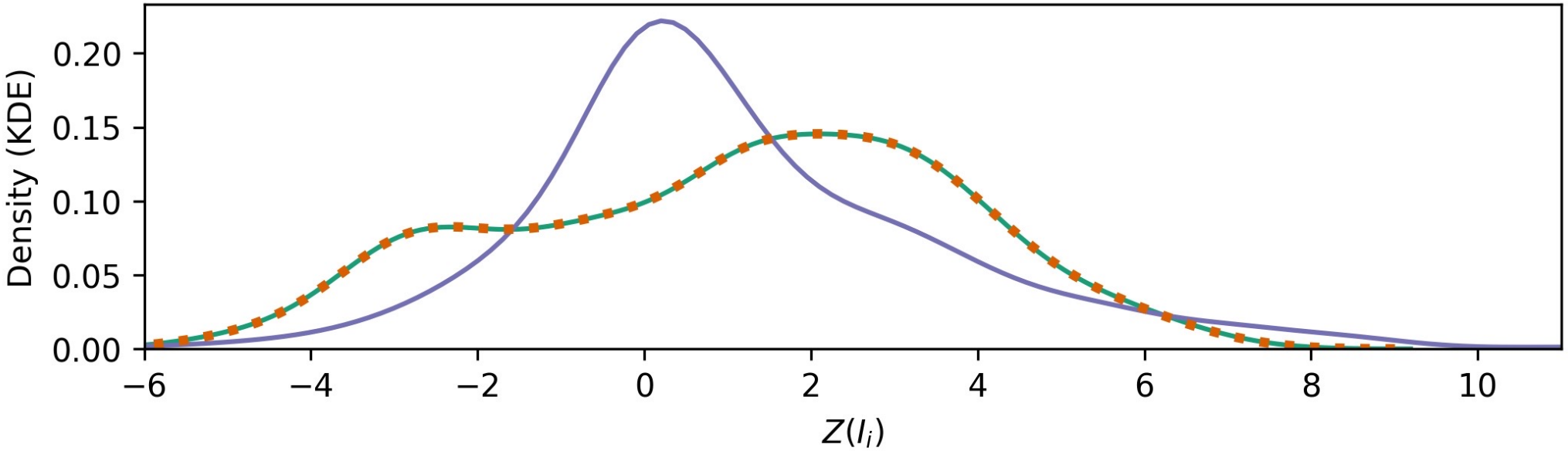


Perfect match for Bivand & Wong (2018)'s data



	$Z(I_i)$	$Z_p(I_i)$	$Z_c(I_i)$
$Z(I_i)$	1.00	0.85	0.85
$Z_p(I_i)$	0.85	1.00	1.00
$Z_c(I_i)$	0.85	1.00	1.00

Empirical "conditional randomization" $Z_p(I_i)$

Analytical "total randomization" $Z(I_i)$

Analytical "conditional randomization" $Z_c(I_i)$

And practically never differ for random simulations

The Importance of Null Hypotheses: Understanding Differences in Local Moran's I_i under Heteroskedasticity

Jeffery Sauer[1], Taylor Oshan[1], Sergio Rey[2], Levi John Wolf[3]

A vast body of work has expanded upon Anselin (1995). But, we [still] tend to distinguish the "total" and "conditional" null by computation, not conceptualization. Our null hypotheses should be more explicitly acknowledged, however, because they are themselves design decisions.

LOCAL STATISTICS

FORMALIZING INTUITION

A REPLICATION FAILURE

FINDING "REAL" CLUSTERS

"NULLS" IN LOCAL TESTS

ALL DEPENDS ON THEORY

The importance of null hypotheses in local statistics

WHERE TO?

The importance of null hypotheses in local statistics

We do not have the right balance

a. Expecting something to replicate &
b. Reflexively arguing about its "complexity" when it doesn't

Replication across space and time must be weak in the social and environmental sciences

Michael F Goodchild & Wenwen Li
doi.org/10.1073/pnas.2015759118

"Democratizing" methods
makes our defaults "political!"

- Stats is a "science of defaults"
- These issues weren't ever solved, just supplanted by newer shinier problems

TESTING FOR LOCAL SPATIAL AUTOCORRELATION IN THE PRESENCE OF GLOBAL AUTOCORRELATION

J. Keith Ord & Arthur Getis
doi.org/10.1111/0022-4146.00224

MUCH ADO ABOUT NULL THINGS

A replication issue in spatial stats

JEFF SAUER
SERGIO REY
TAYLOR OSHAN
LEVI JOHN WOLF