

# THE LOGIC OF CAUSATION

## COUNTERFACTUAL THINKING

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**WHAT IS A  
COUNTERFACTUAL?**

**WHO CARES?**

**STRATEGIES**

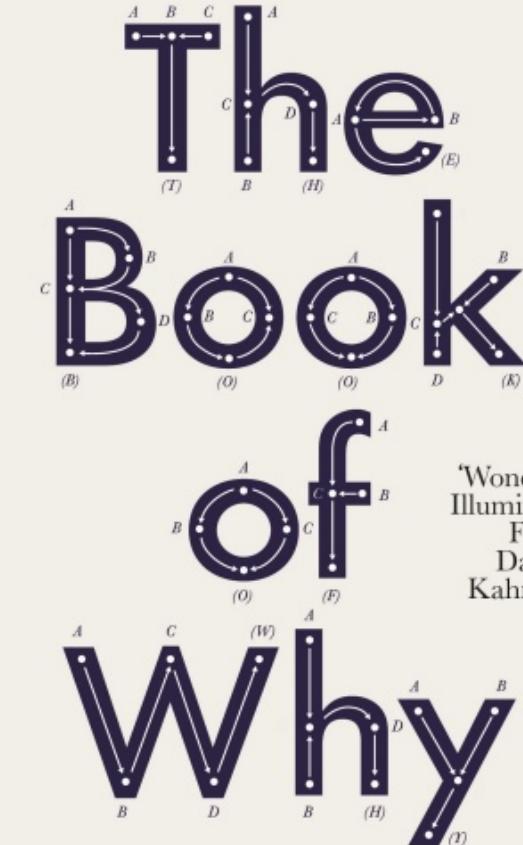
# **WHAT IS A COUNTERFACTUAL?**

They are “what-if X” scenarios & imply “but-for X” causation.

## **WHO CARES?**

## **STRATEGIES**

Judea Pearl  
& Dana Mackenzie

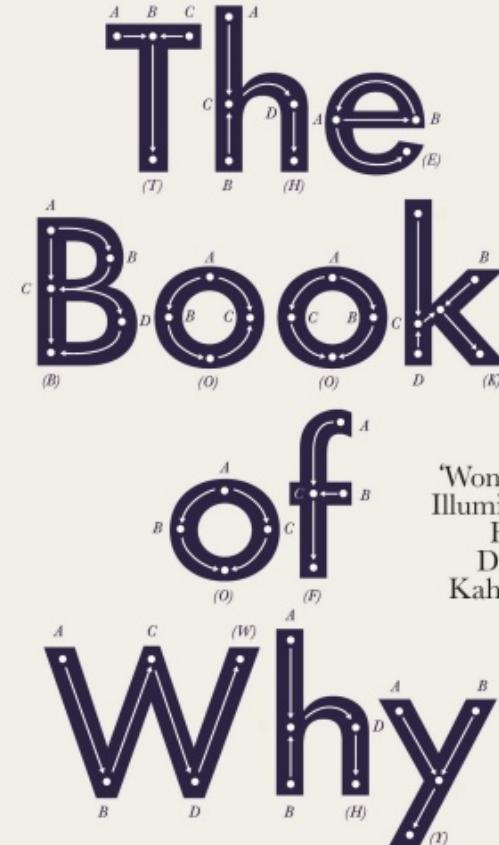


'Wonderful ...  
Illuminating ...  
Fun'  
Daniel  
Kahneman

The New Science  
of Cause and Effect

“In [Geog methods 1], every student learns to chant: “correlation is not causation.” With good reason! [Unfortunately,] this tells us that correlation is not causation, but it does not tell us what causation *is*.

Judea Pearl  
& Dana Mackenzie



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# COUNTERFACTUAL

/'kaʊntə'faktʃuəl/

characterizing what  
happens in a scenario  
that is not observed  
directly.

# **COUNTERFACTUAL (STATEMENT)**

/'kaʊntə'faktʃuəl/

characterizing what  
happens in a scenario  
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# COUNTERFACTUAL (SCENARIO)

/'kaʊntə'faktʃuəl/

an alternative present  
where the past has  
been changed.

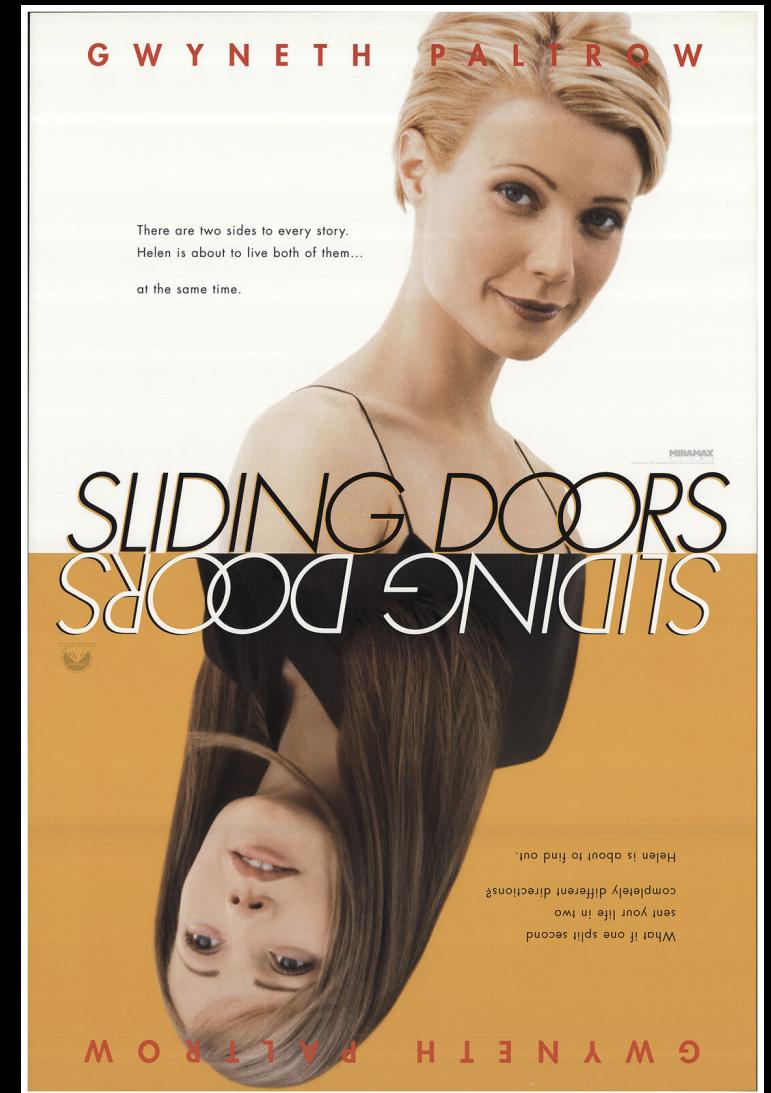
# COUNTERFACTUAL (SCENARIO)?

/'kaʊntə'faktʃnəl/

**IF THINGS HAD  
GONE DIFFERENTLY,  
WOULD THE  
OUTCOME HAVE  
BEEN DIFFERENT?**

# SLIDING DOORS

Gwyneth tries to catch the tube at Waterloo. In one timeline she fails, and in the other she succeeds. When she succeeds, she dies. When she fails, she lives.

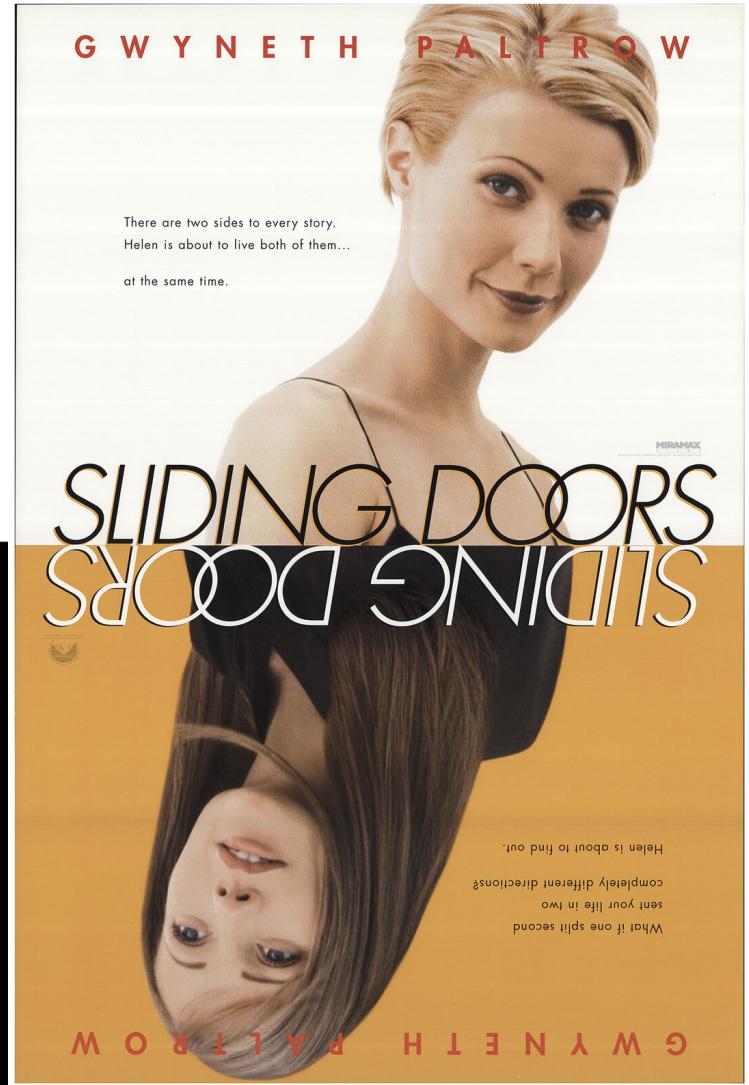


We observe an outcome  $y$   
And some conditions  $x$

We'd like to make statements about  
how likely it is that  $y$  happens, given that  
we saw  $x$

In one timeline she fails,  
and in the other she  
succeeds. When she  
succeeds, she dies.

When she fails, she lives.



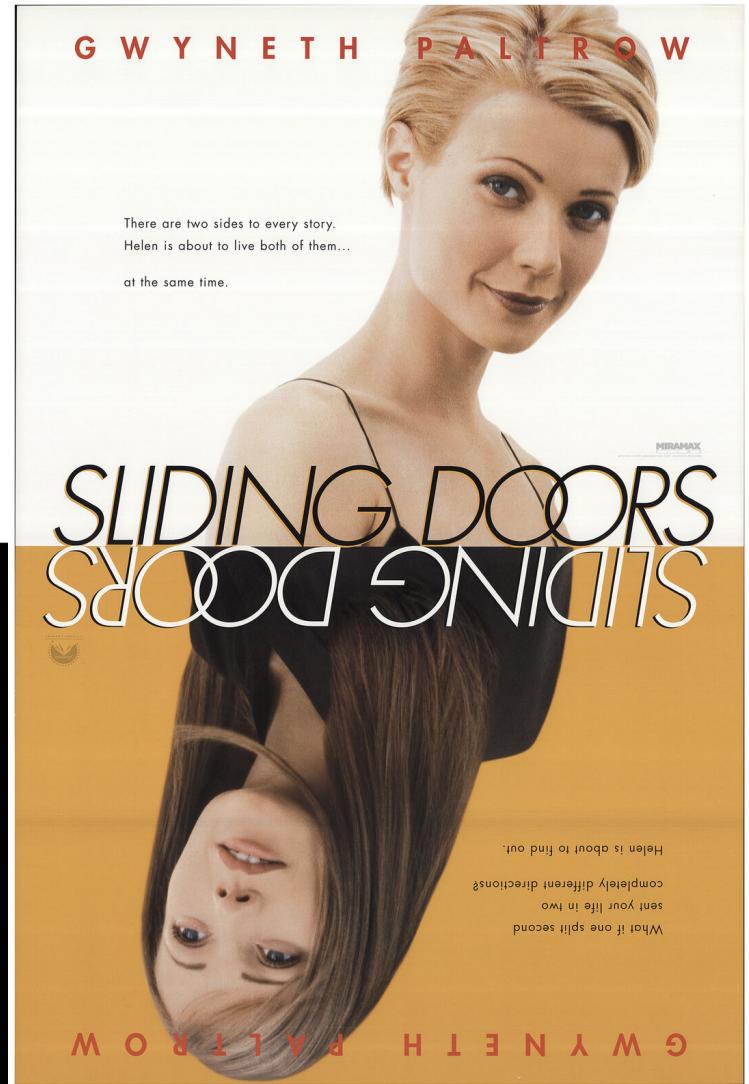
We observe an outcome       $y$   
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$$p(y \mid x)$$

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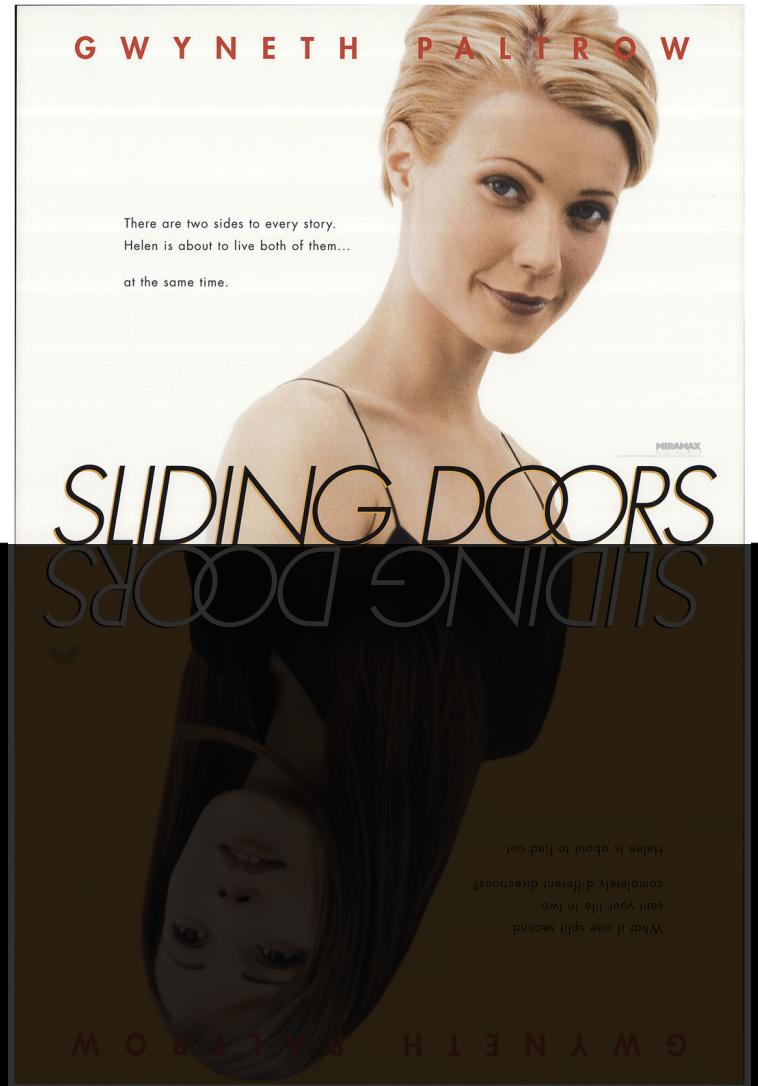


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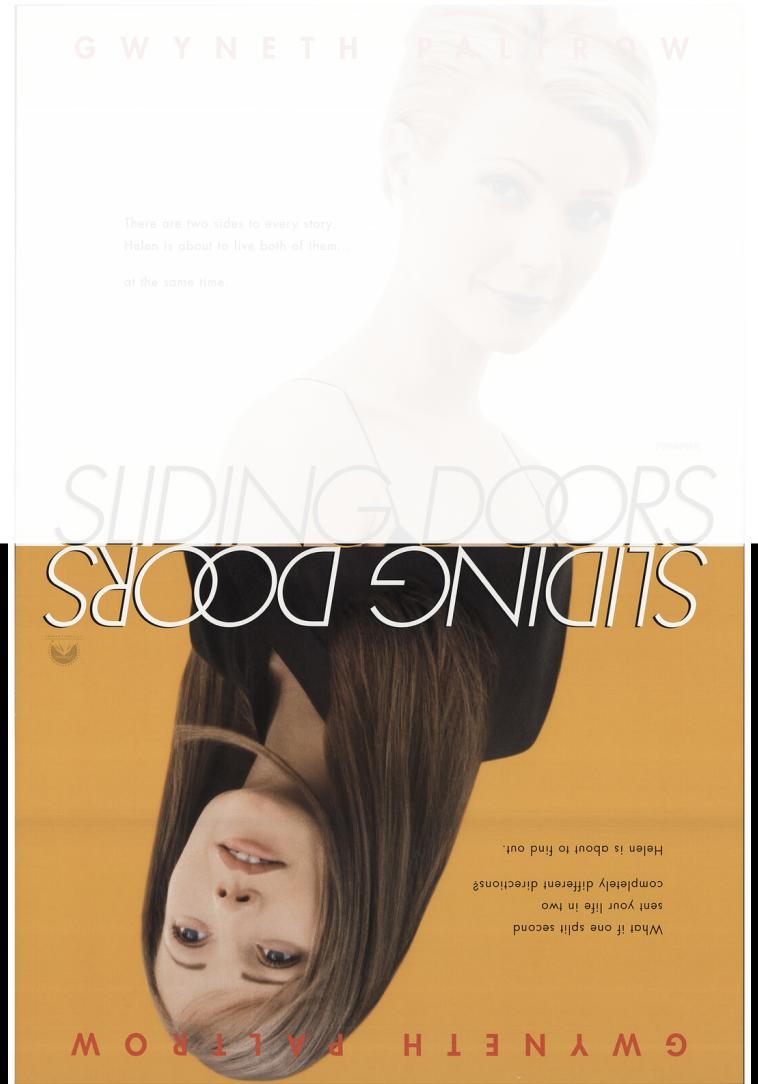
We observe an outcome       $y$   
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We'd like to make statements about

$$p(y' \mid x')$$

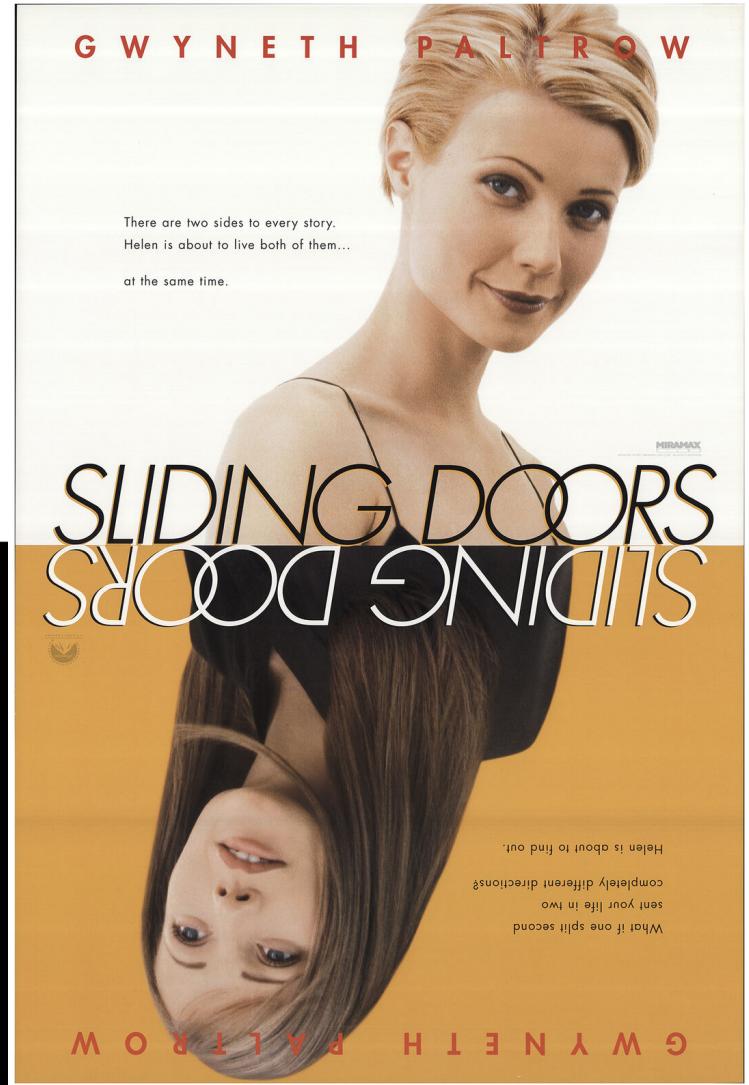
In one timeline she fails,  
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When she fails, she lives.



# NEITHER GWEN CAN KNOW HOW THE OTHER GWEN TURNED OUT

In one timeline she fails,  
and in the other she  
succeeds. When she  
succeeds, she dies.  
When she fails, she lives.



We observe an outcome  $y$   
And some conditions  $x$

We'd like to make statements about

$$p(y_x \mid x', y')$$

*Would I be alive if I had  
caught the tube that day?  
Given I didn't catch the tube  
and am definitely alive!*

G W Y N E T H P A L F R O W

There are two sides to every story.  
Helen is about to live both of them...  
at the same time.

SLIDING DOORS

G W Y N E T H P A L F R O W

Helen is about to find out,  
completely different directions  
send your life in two  
What if one split second

# WHAT IS A COUNTERFACTUAL?

They are “what-if X” scenarios & imply “but-for X” causation.

# WHO CARES?

You should! They are fundamental to human reasoning.

# STRATEGIES

“It’s one thing to say ‘smoking causes cancer’ but another to say that my uncle Joe, who smoked a pack a day for thirty years, would have been alive had he not smoked. The difference is both obvious and profound: none of the people who, like Uncle Joe, smoked for thirty years and died can ever be observed in the alternate world where they did not smoke for thirty years.”

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# HOW CAN WE “KNOW” THINGS?

THE LADDER OF CAUSATION

PEARL (2016)

# HOW CAN WE “KNOW” THINGS?

**SEEING** Observed relationship between effect and cause

Is Gwen alive when she misses her train?

Do unequal societies have cities with really different sizes?

# HOW CAN WE “KNOW” THINGS?

**SEEING**  $p(y \mid x)$

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# HOW CAN WE “KNOW” THINGS?

**SEEING**  $p(y \mid x)$

Is Gwen alive when she misses her train?

Do unequal societies have cities with really different sizes?

**DOING** Predictive relationship between effect and cause

If Gwen catches the train, will she live?

If we reduce inequality in society, will city size converge?

# HOW CAN WE “KNOW” THINGS?

**SEEING**  $p(y \mid x)$

Is Gwen alive when she misses her train?

Do unequal societies have cities with really different sizes?

**DOING**  $p(y \mid \text{do}(x))$

If Gwen catches the train, will she live?

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**SEEING**  $p(y \mid x)$

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Do unequal societies have cities with really different sizes?

**DOING**  $p(y \mid \text{do}(x))$

If Gwen catches the train, will she live?

If we reduce inequality in society, will city size converge?

**IMAGINING** Predictive relationship between alternative cause/effect

If Gwen had caught the train, would she be alive today?

If society had not been so unequal, would city size converge?

# HOW CAN WE “KNOW” THINGS?

**SEEING**  $p(y \mid x)$

Is Gwen alive when she misses her train?

Do unequal societies have cities with really different sizes?

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**IMAGINING**  $p(y_x \mid x', y')$

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# WHY WE “KNOW” THINGS? FOCUS ON IMAGINING?

**IMAGINING**  $p(y_x \mid x', y')$

If Gwen had caught the train, would she **be alive today**?

If society had not been so unequal, would **city size converge**?

# WHY FOCUS ON IMAGINING?

"If we have a model that can answer counterfactual questions, we can also answer questions about interventions and observations."

**IMAGINING**  $p(y_x \mid x', y')$

If Gwen had caught the train, would she be alive today?

If society had not been so unequal, would city size converge?

*“If we have a model that can answer counterfactual questions, we can also answer questions about interventions and observations.”*

$$p(y_x \mid x', y')$$

We saw: Gwen missed the train, she's alive.  
We model: If Gwen had caught the train,  
would she have lived?

$$p(y \mid \text{do}(x))$$

We saw: Gwen missed the train, she's alive.  
We model: If Gwen catches the train from now on,  
will she live?

*“If we have a model that can answer counterfactual questions, we can also answer questions about interventions and observations.”*

$p(y_x \mid x', y')$

We saw: Gwen missed the train, she's alive.

We model: If Gwen had caught the train,  
would she have lived?

$p(y \mid \text{do}(x))$

We saw: Gwen missed the train, she's alive.

We model: If Gwen catches the train from now on,  
will she live?

If we know the counterfactual, we can answer the intervention:

When we set conditions for the counterfactual, we **set them to right now!**

*“If we have a model that can answer counterfactual questions, we can also answer questions about interventions and observations.”*

$$p(y_x \mid x', y')$$

We saw: Gwen missed the train, she's alive.  
We model: If Gwen had caught the train,  
would she have lived?

$$p(y \mid x)$$

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We model: When Gwen caught the train, did she live?

*“If we have a model that can answer counterfactual questions, we can also answer questions about interventions and observations.”*

$p(y_x \mid x', y')$	We saw: Gwen missed the train, she's alive. We model: If Gwen had caught the train, would she have lived?
$p(y \mid x)$	We saw: Gwen missed the train, she's alive. We model: When Gwen caught the train, did she live?

If we know the counterfactual, we can answer the observation:

The observational conditions are the **grounds for the counterfactual!**

# **WHAT IS A COUNTERFACTUAL?**

They are “what-if X” scenarios & imply “but-for X” causation.

# **WHO CARES?**

You should! They are fundamental to human reasoning.

# **STRATEGIES**

Causality is about theory, so get theoretical before empirical.

# HEURISTIC

Use counterfactual thinking to make assumptions about processes clear.

# ANALYTIC

Use counterfactual analysis to do causal inference.

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# HEURISTIC

Asthma is caused by exposure to air pollution.

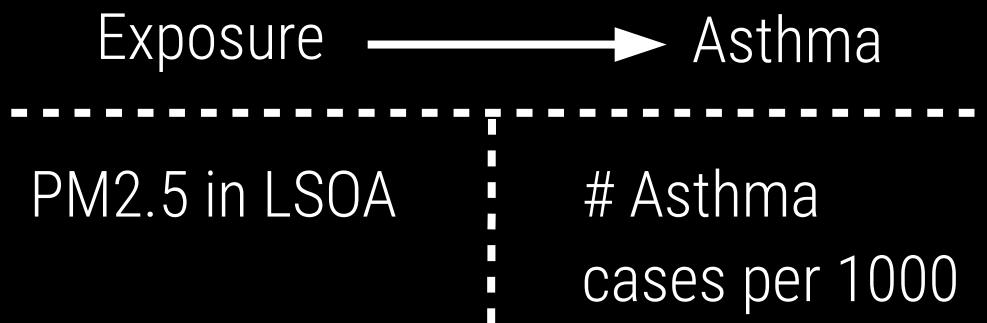
# HEURISTIC

Asthma is caused by exposure to air pollution.

Exposure → Asthma

# HEURISTIC

Asthma is caused by exposure to air pollution.



# IF THIS IS TRUE

ARE THERE ANY PLACES  
WITH IDENTICAL EXPOSURE  
BUT DIFFERENT ASTHMA?  
WHY WOULD THEY DIFFER?

Asthma is caused by exposure to air pollution.



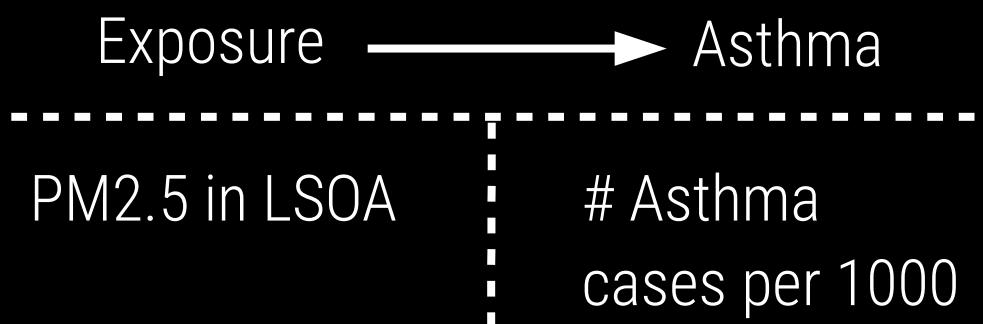
# IF THIS IS TRUE

ARE THERE ANY PLACES  
WITH IDENTICAL EXPOSURE  
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**PM2.5 may not be exposure!**

- Ozone, PM10, NO<sub>x</sub>

Asthma is caused by exposure to air pollution.



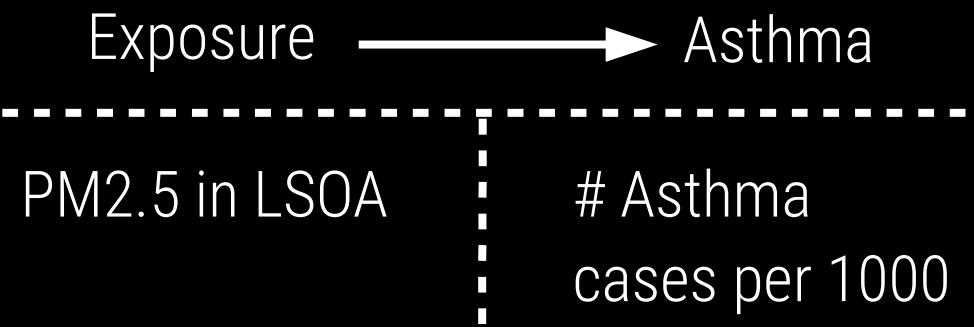
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## LSOA may not be exposure!

- a cyclist commuting daily from a “clean” location is exposed on their ride daily. And, the exposure is intense.
- a builder on a construction site is exposed at their workplace, independently of env. pollutants



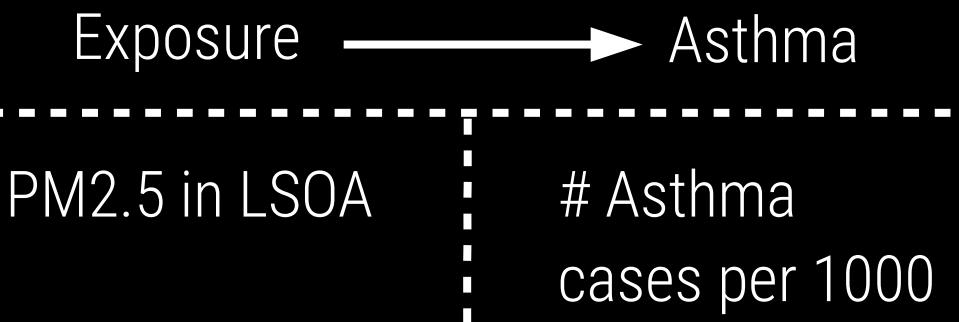
# IF THIS IS TRUE

ARE THERE ANY PLACES  
WITH IDENTICAL EXPOSURE  
BUT DIFFERENT ASTHMA?  
WHY WOULD THEY DIFFER?

Asthma is caused by exposure to air pollution.

## Effect of exposure might vary!

- Rich LSOAs might have “protection”



# HEURISTIC

**IF THIS THEORY IS TRUE,**  
What would prove me wrong?

# ANALYTIC

Use counterfactual analysis  
to do causal inference.

Air pollution —————→ asthma

# ANALYTIC

Use counterfactual analysis  
to do causal inference.

Air pollution

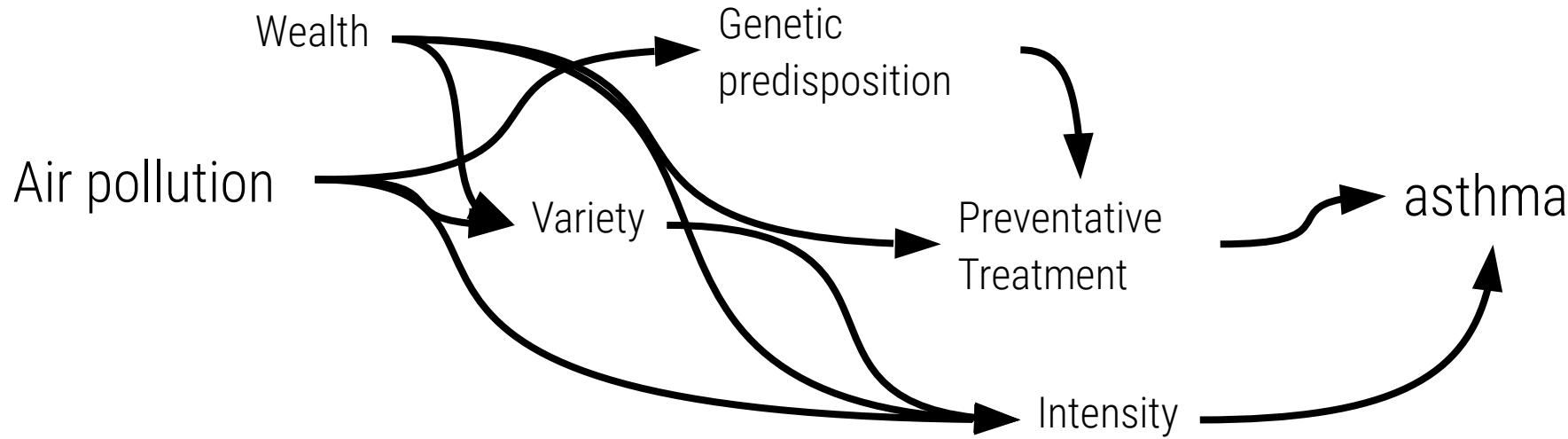
## EXPOSURE PROCESS MODEL

Wealth & genetic predisposition?  
Intensity of exposure varies  
Variety of pollution may impact

asthma

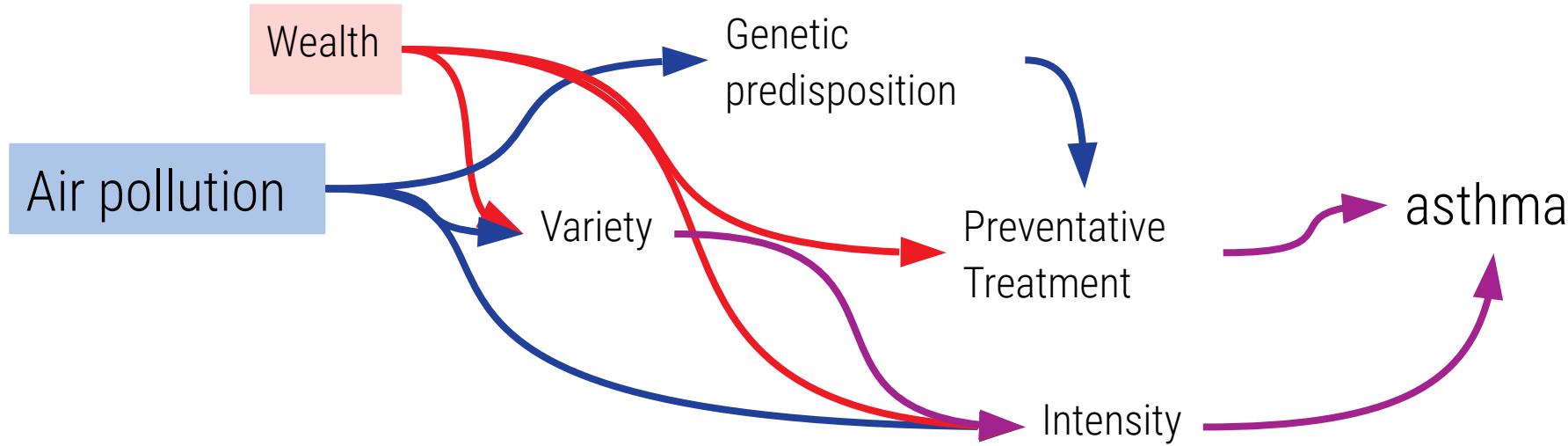
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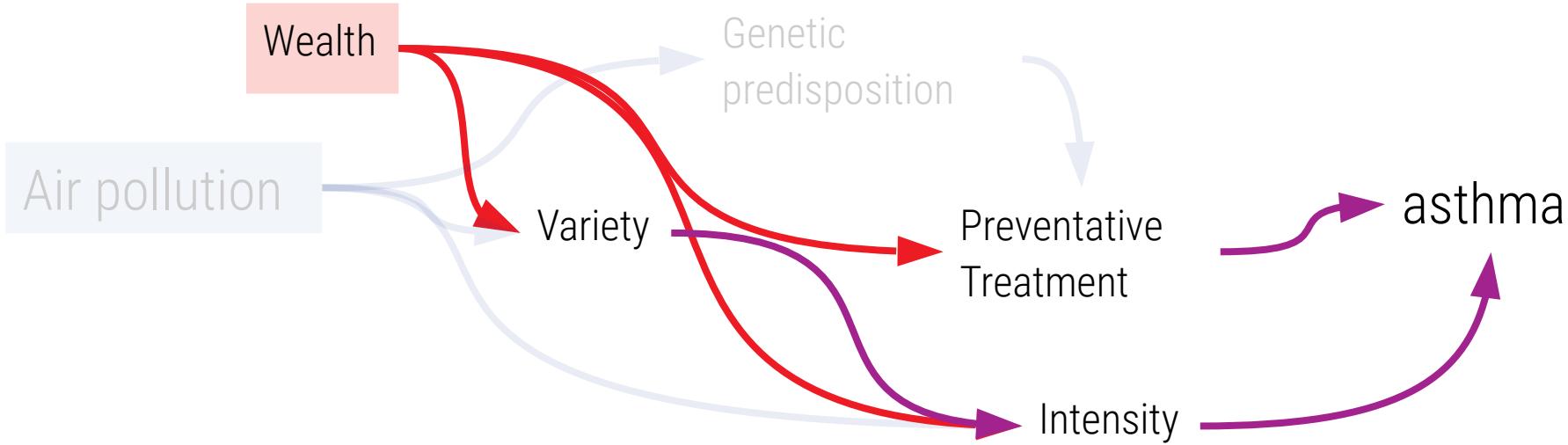
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Use counterfactual analysis  
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Changing wealth affects *more than just asthma outcomes!*

If individual A were £20kpa richer, she might get treatment, but also might change her commute or residence! **Regression misses this!**

# ANALYTIC

Use counterfactual analysis  
to do causal inference.

# HEURISTIC

**IF THIS THEORY IS TRUE,**  
What would prove me wrong?

# ANALYTIC

**DIRECT & INDIRECT**  
Do causes interact/interfere?

# **WHAT IS A COUNTERFACTUAL?**

They are “what-if X” scenarios & imply “but-for X” causation.

# **WHO CARES?**

You should! They are fundamental to human reasoning.

# **STRATEGIES**

Informal heuristics & formal analysis can use counterfactuals!

# QUESTIONS ABOUT

# THE LOGIC OF CAUSATION

## COUNTERFACTUAL THINKING

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