

# **CITY SYSTEMS AND THE CAUSE OF INEQUALITY**

**A CITY SCIENCE APPROACH**

**Levi John Wolf**

**University of Bristol**

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# Causality in City Science

## Regularity and Contextuality

## Moving to generative value

Talk outline

# Causality in City Science

What do we mean by it?

Regularity and Contextuality

Moving to generative value

Talk outline

*regularity value*

*How often is this true?*

*contextual value*

*Where/when is this true?*

*generative value*

*How is this true?*

Three kinds of “value”  
for law-like statements

*regularity value*

*How often is this true?*

*contextual value*

*Where/when is this true?*

*generative value*

*How is this true?*

Three kinds of “value”  
for law-like statements

Causal reasoning can  
play a role in each

*regularity value*

*How often is this true?*

*contextual value*

*Where/when is this true?*

*generative value*

*How is this true?*

City size distributions often follow Zipf's Law...

... in “coherent” urban systems ...

... because of within-city inequality.

# Beyond open science: Data, code, and causality

Levi John Wolf

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*Indeed, city science is relatively unusual in the social sciences, in that one can still often find work seeking ‘laws’*



# A COMPUTER MOVIE SIMULATING URBAN GROWTH IN THE DETROIT REGION

W. R. TOBLER

*University of Michigan*

**The LAWS of MIGRATION.**

*By E. G. RAVENSTEIN, Esq., F.R.G.S.*

HUMAN BEHAVIOR

---

AND

THE PRINCIPLE  
OF LEAST EFFORT



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*An Introduction to Human Ecology*

GEORGE KINGSLEY ZIPF, Ph.D.

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# Tobler's First Law

# Gravity Law (of flows)

# Zipf's Law (power law)

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Near things are more similar than distant things

Separation decreases interaction between cities

The sizes of cities in a “coherent” system will decay in a regular way

*regularity value*

*Statements  
we make that  
derive their value  
from how often  
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# Causal Values in GIScience

What do we mean by it?

## Regularity and Contextuality

City scaling as an example

## Moving to generative value

*Talk outline*

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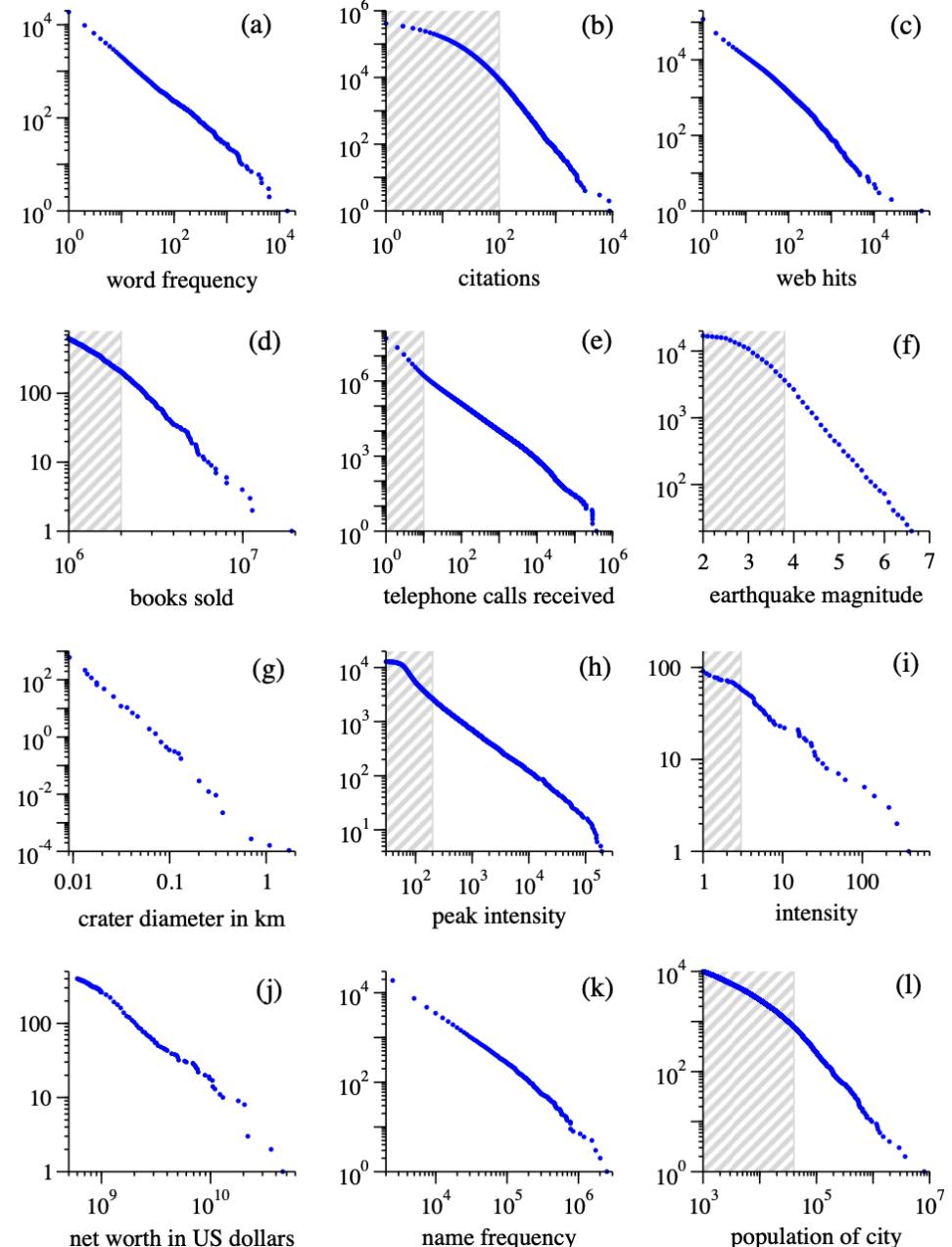
*regularity value*

# Power laws in plain language

Starting from the biggest item, the next item gets smaller at a consistent ratio

$$rank \propto size^{-decay}$$

Newman, (2005)



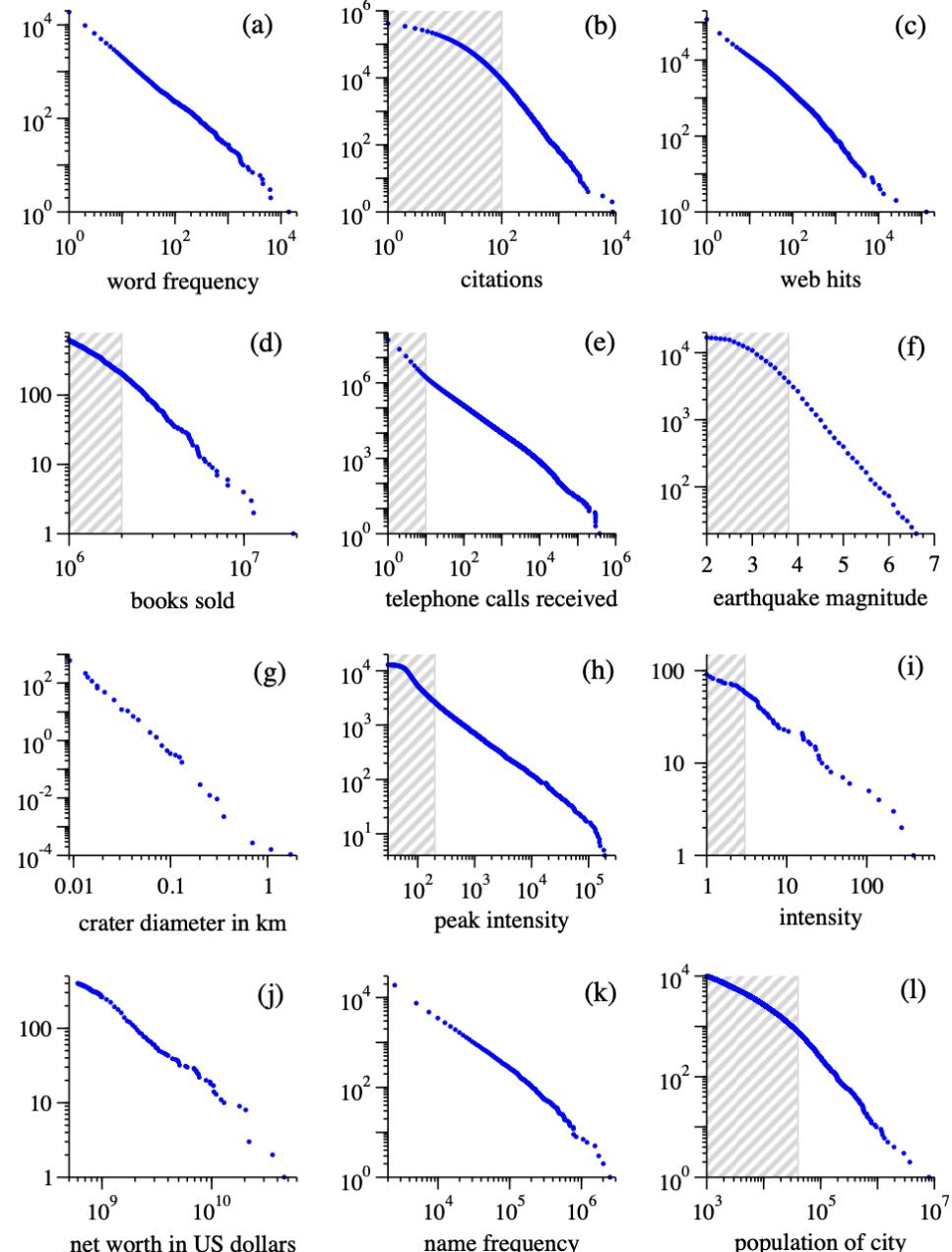
# Power laws in plain language

Starting from the biggest item, the next item gets smaller at a consistent ratio

$$rank \propto size^{-decay} \quad \text{primal}$$

$$size \propto rank^{-decay} \quad \text{dual}$$

Gabaix & Ibragimov (2005)

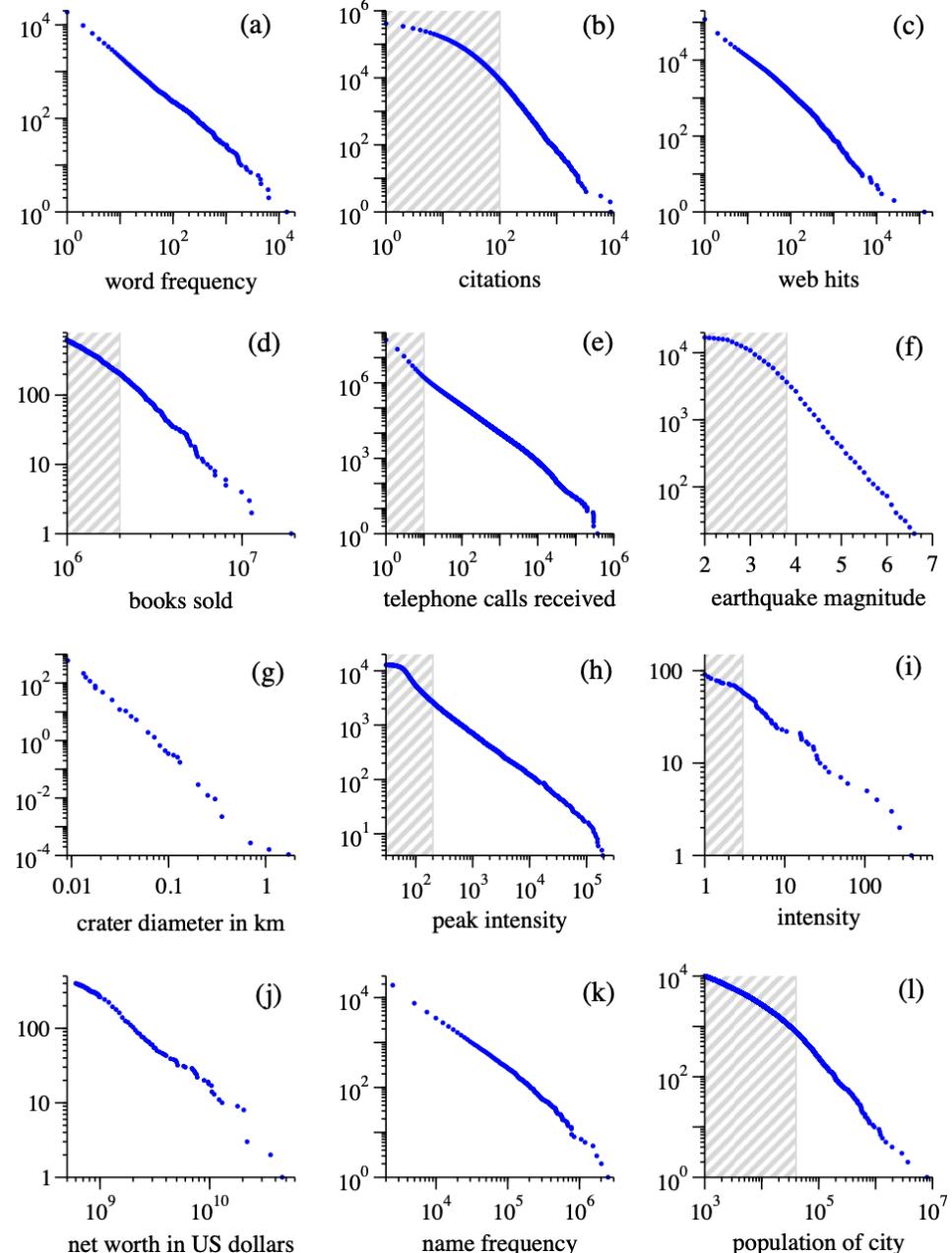


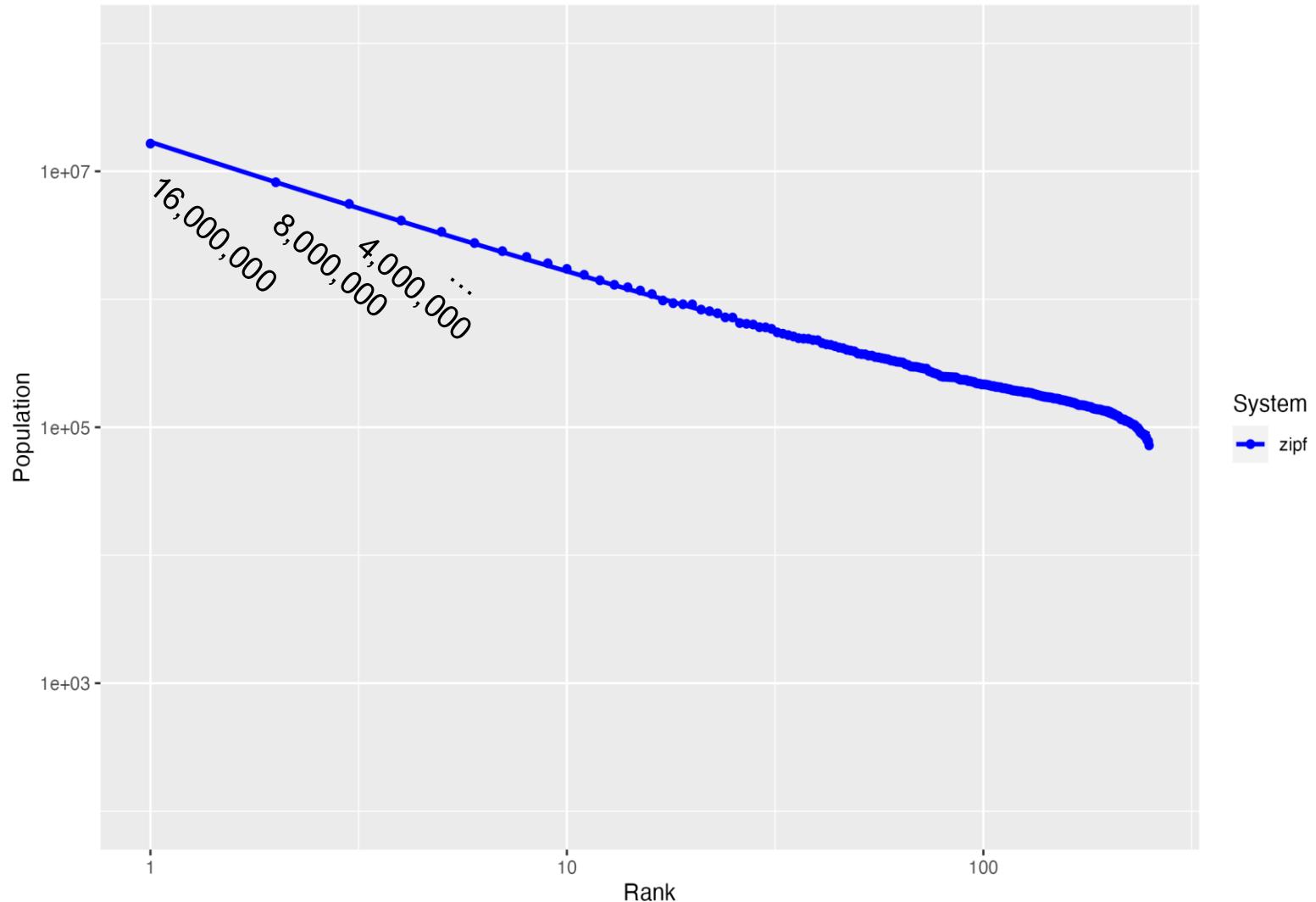
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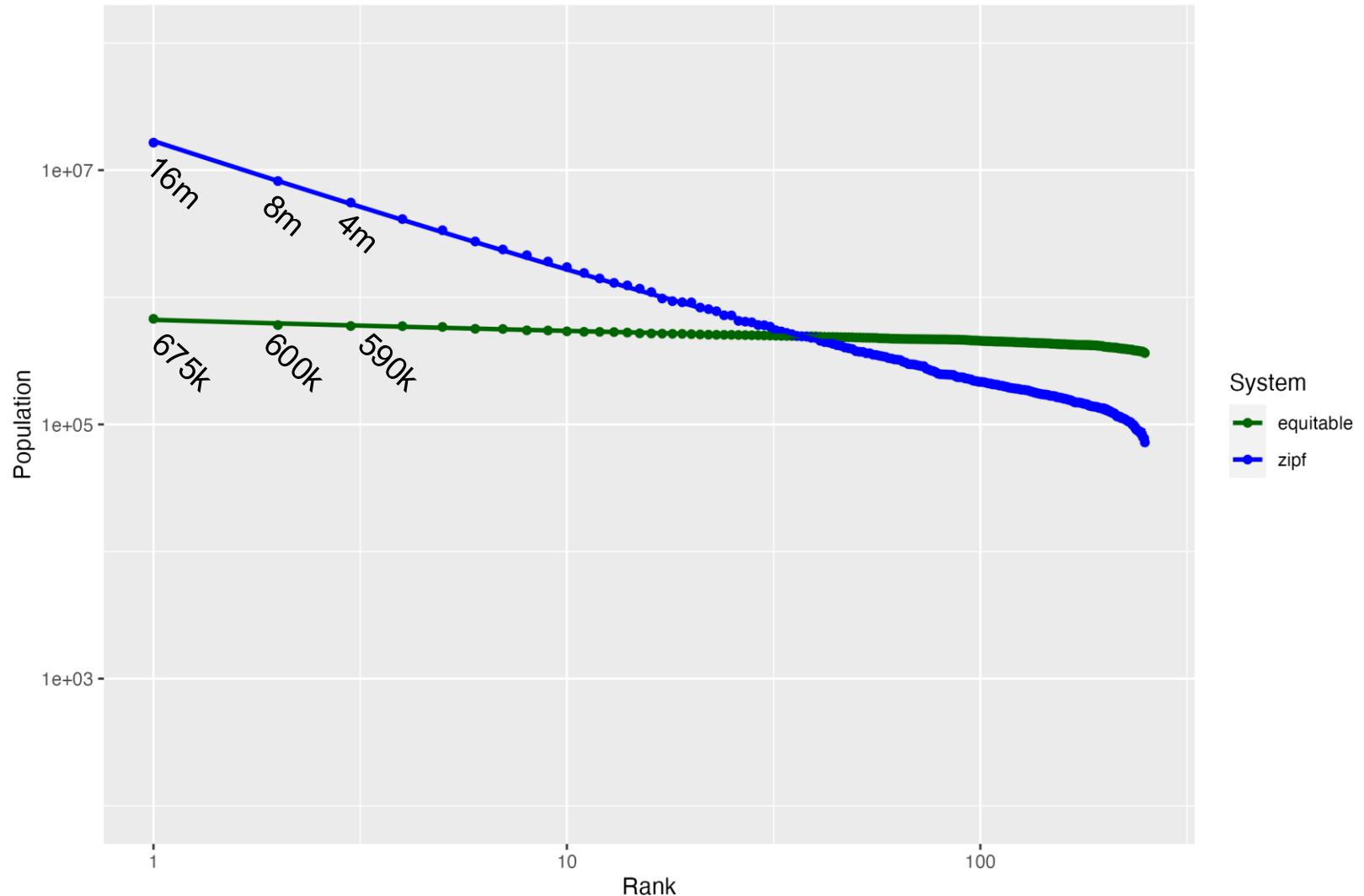
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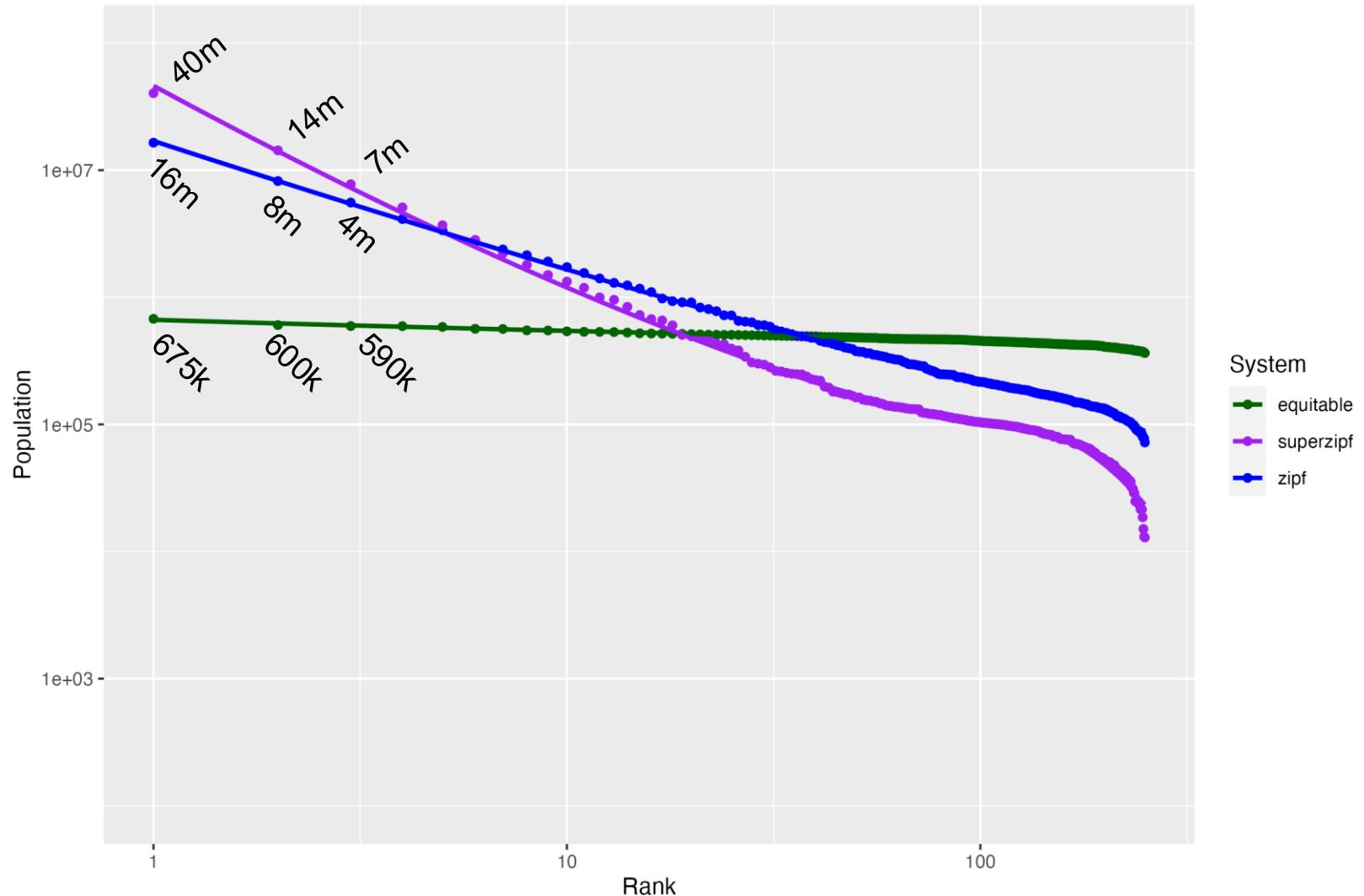
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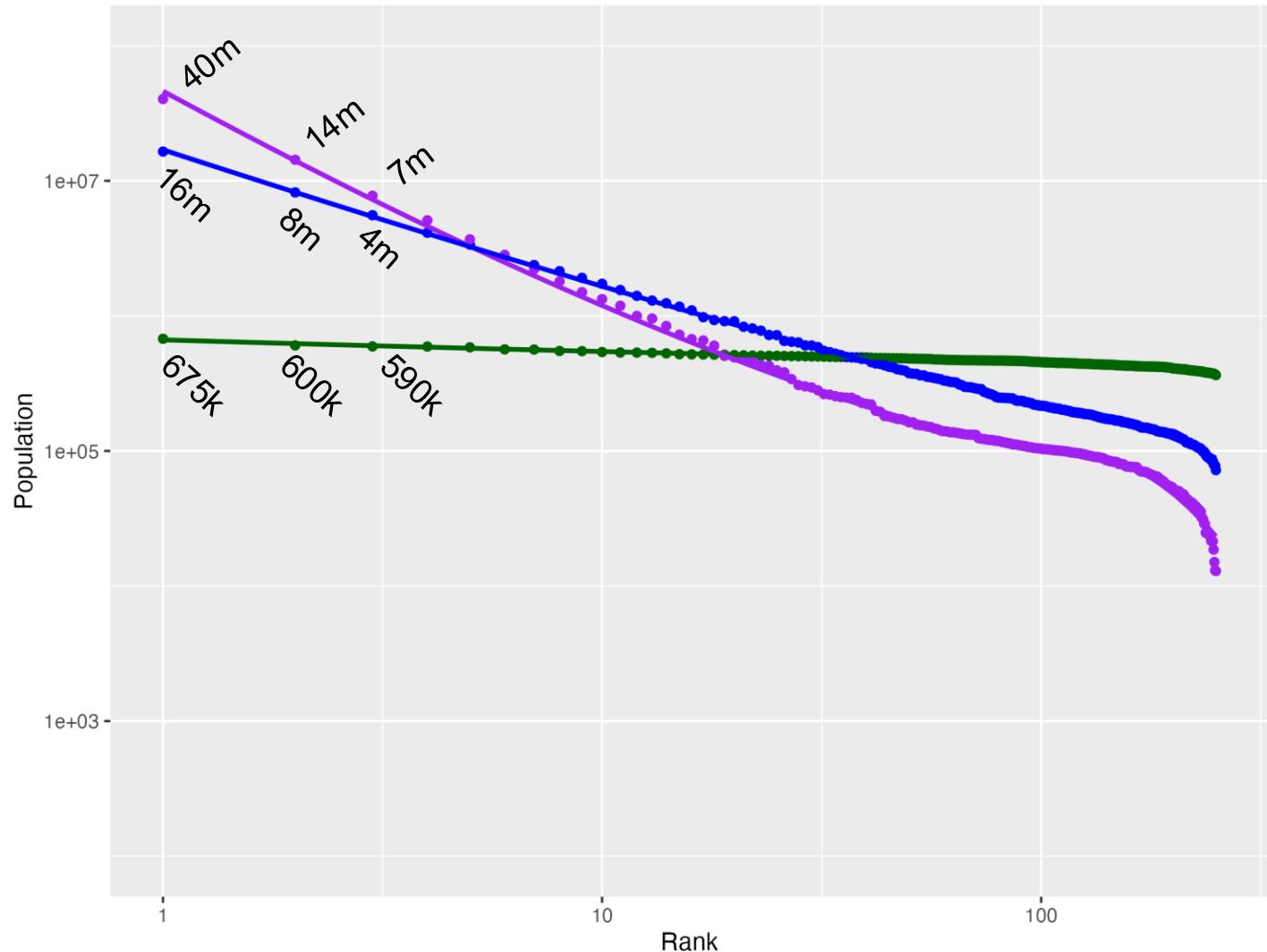
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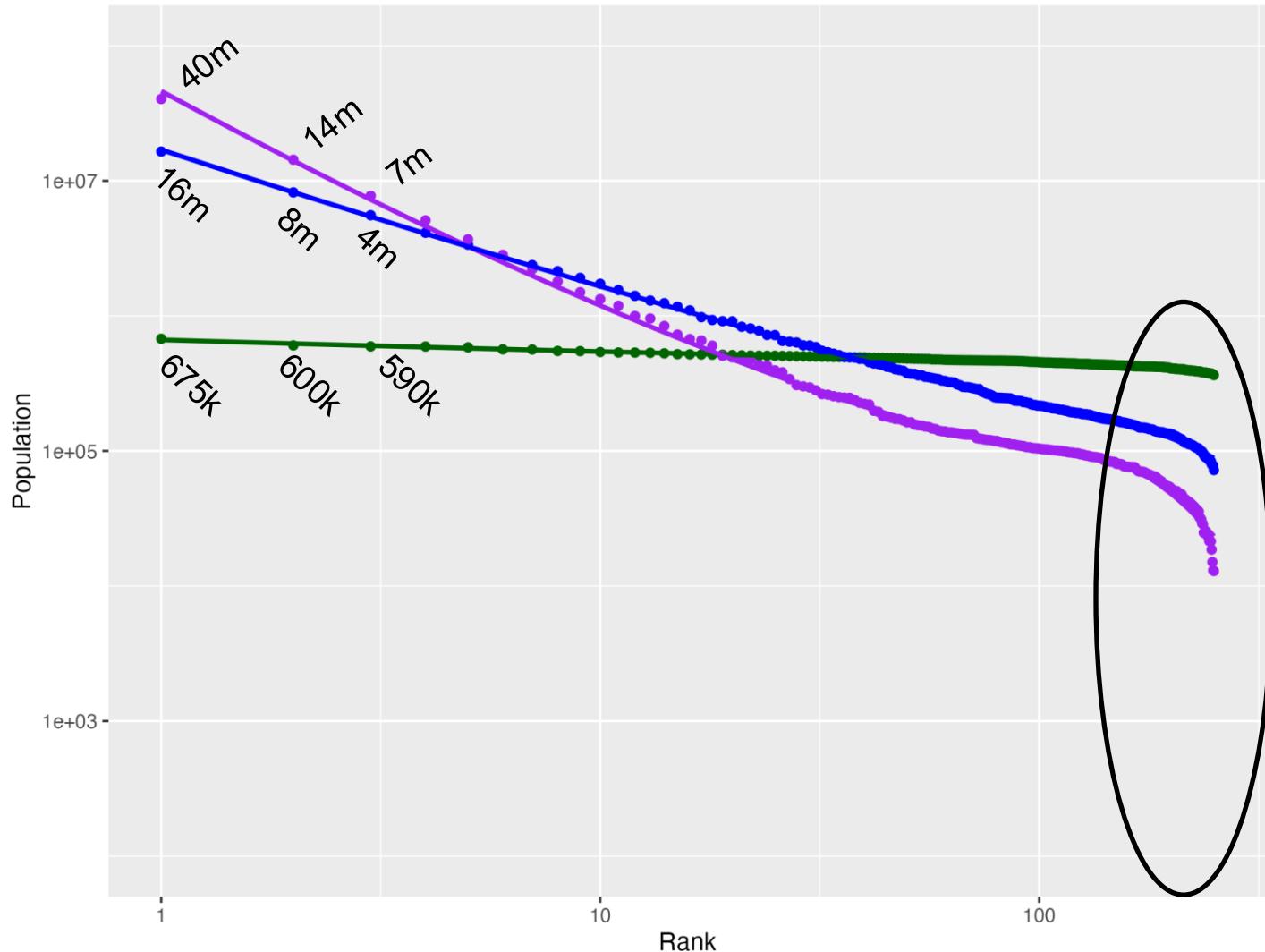




System

- equitable ( $\lambda = .1$ )
- superzipf ( $\lambda = 1.5$ )
- zipf ( $\lambda = 1$ )

$$x \propto r(x)^{-\lambda}$$

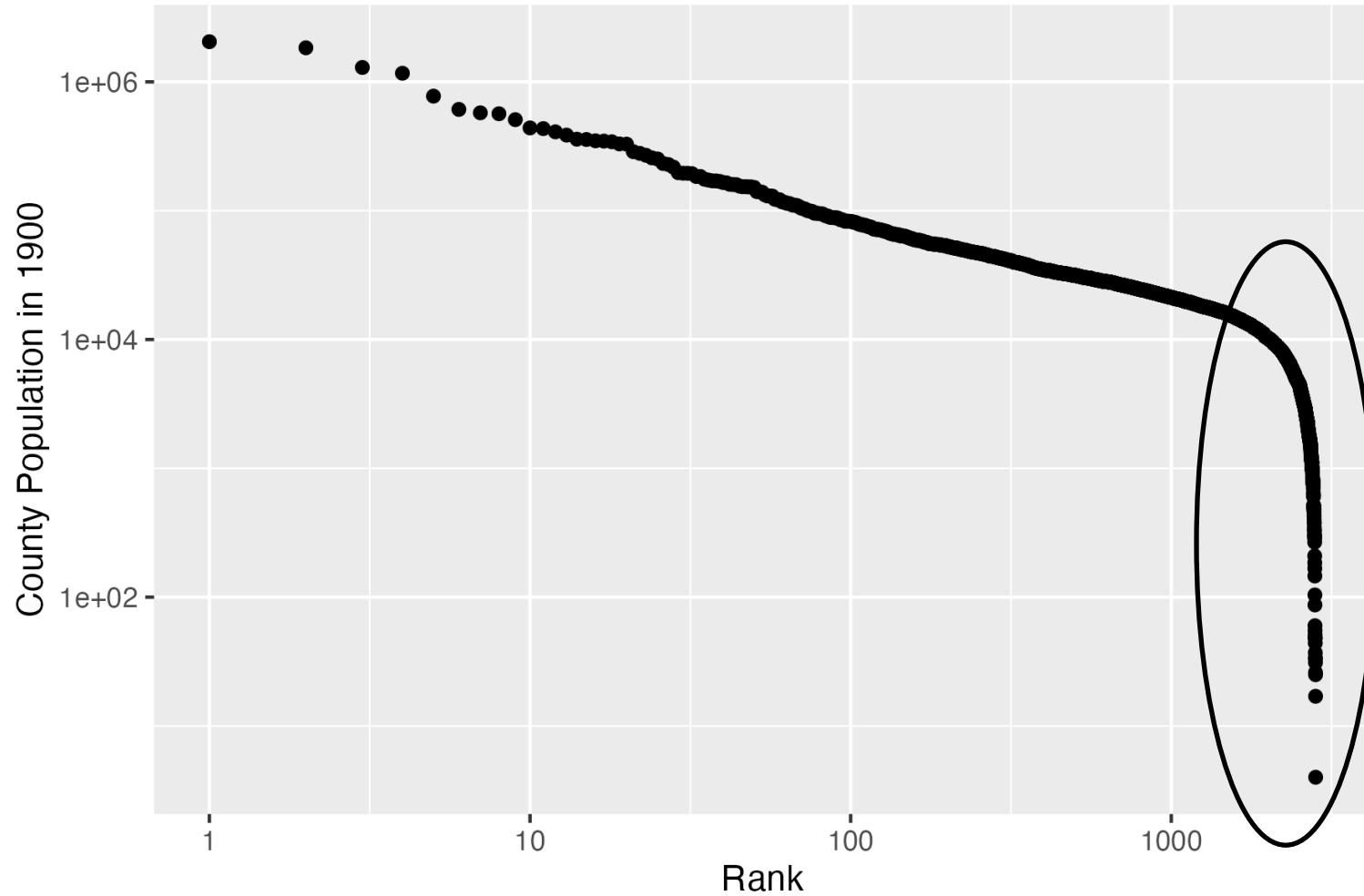


“fall off”  
common to many  
real-world things with  
more small values

System

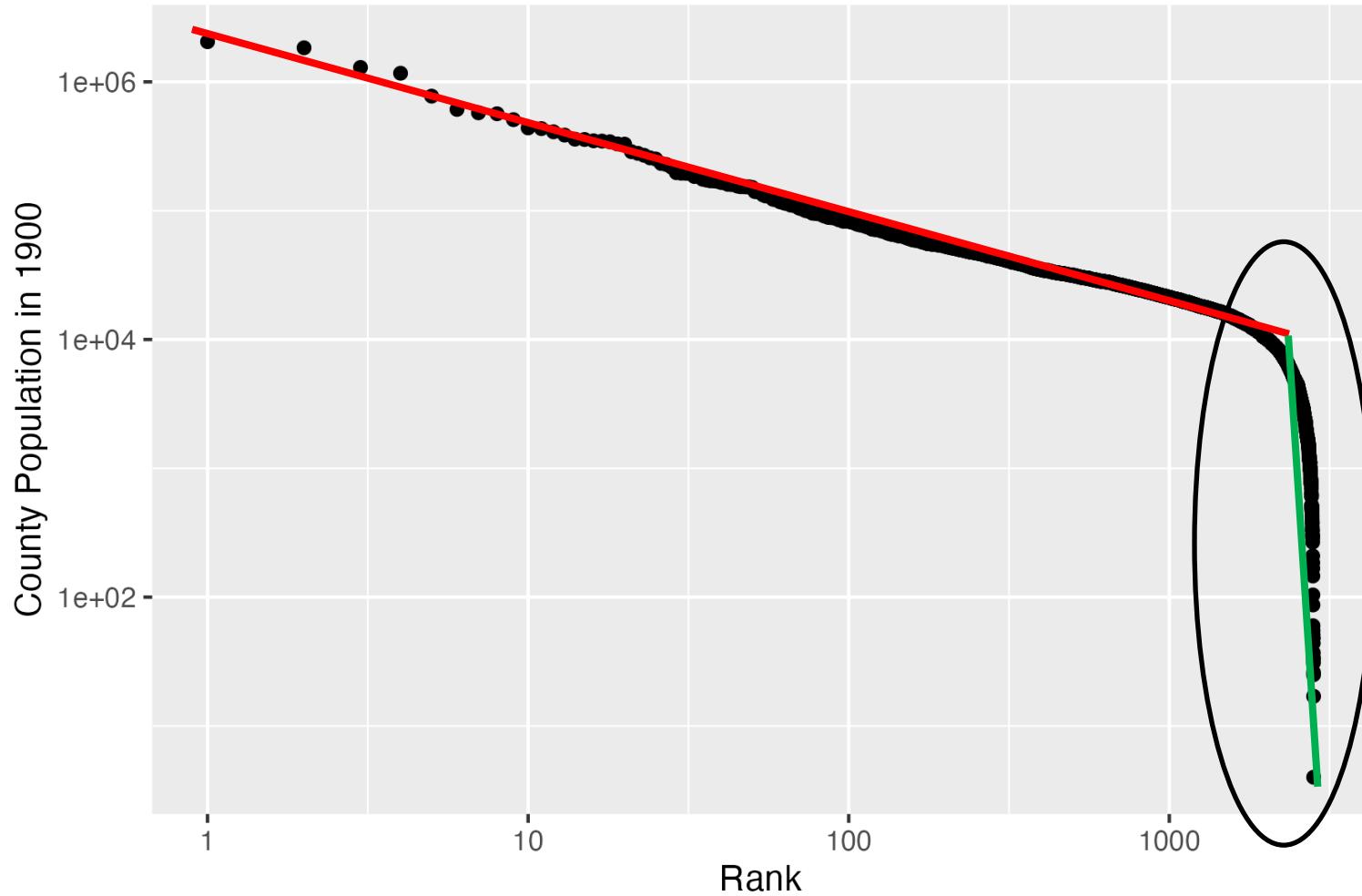
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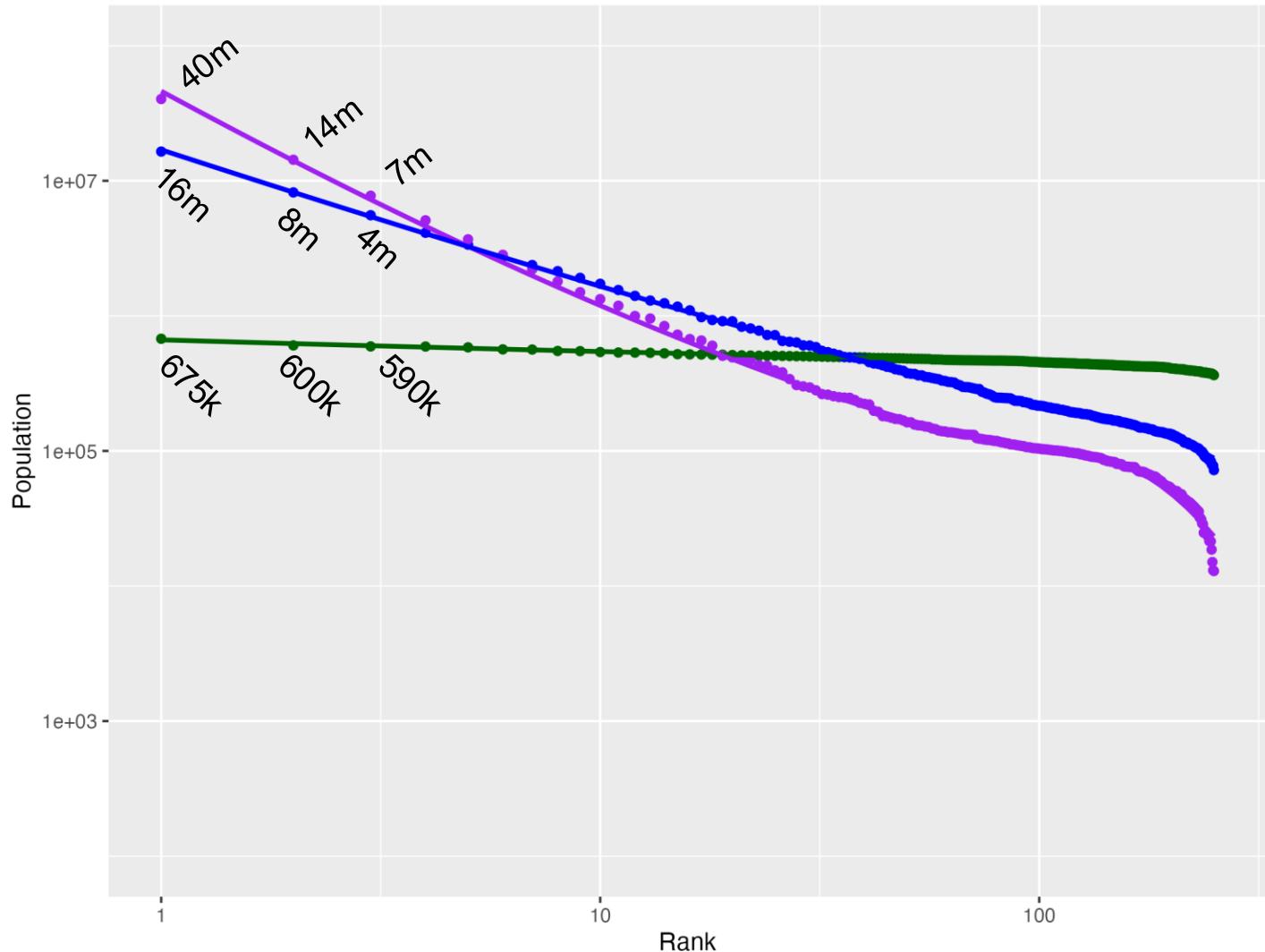
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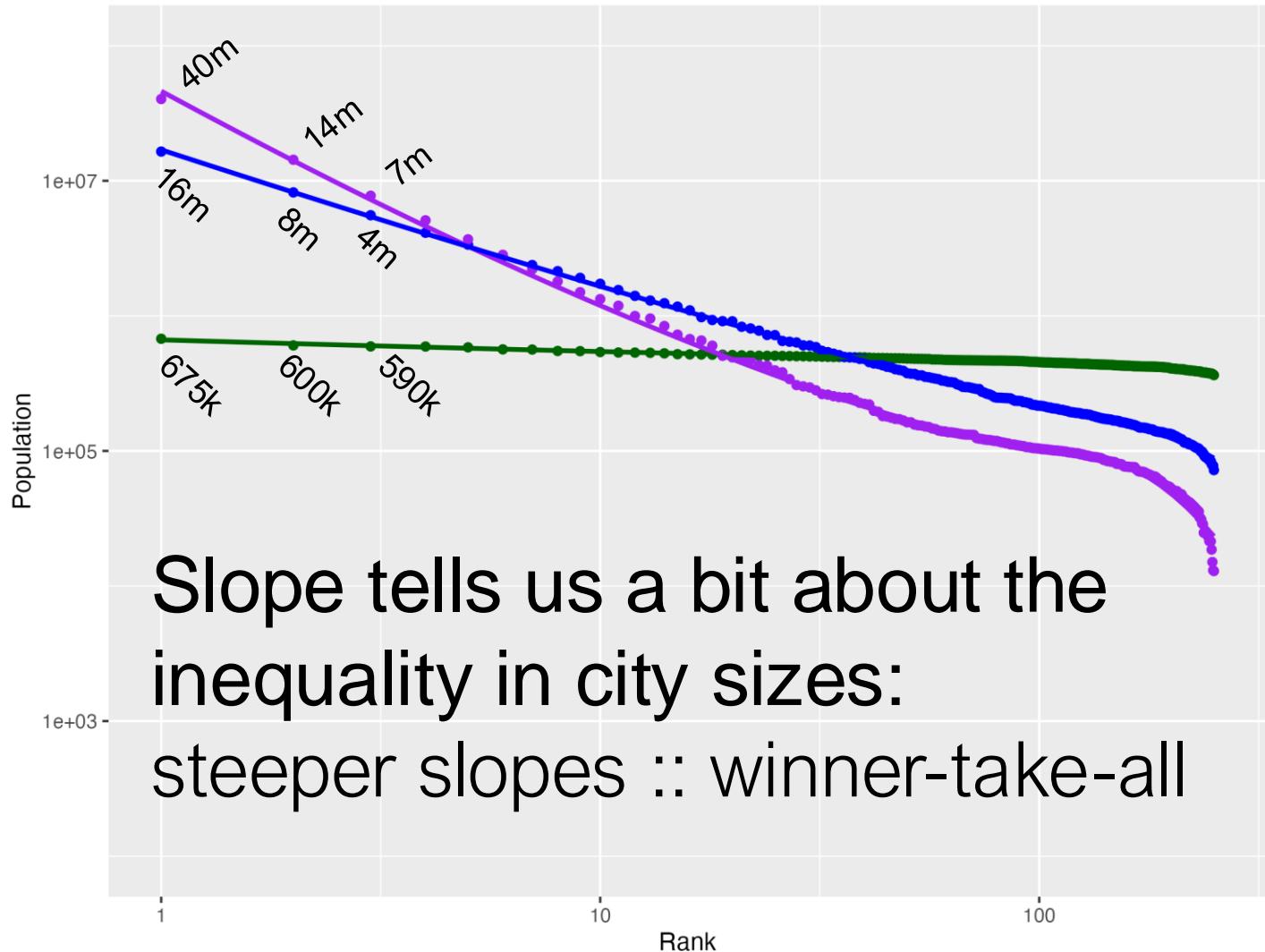
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# Critical Truths About Power Laws

Michael P. H. Stumpf<sup>1</sup> and Mason A. Porter<sup>2</sup>

*What genuinely new insights have been gained by having found a robust, mechanistically-supported, and in-all-other-ways superb power law? We believe that such insights are very rare.*



Slope tells us a bit about the  
inequality in city sizes:  
steeper slopes :: winner-take-all

$$x \propto r(x)^{-\lambda}$$

# *Rethinking Causality in Quantitative Human Geography*

Mirah Zhang      Levi John Wolf

*“Weak replicability” should instead be framed as “strong contextuality”: our specific explanations of why context counts should be sufficient to understand when a given causal mechanism might operate.*



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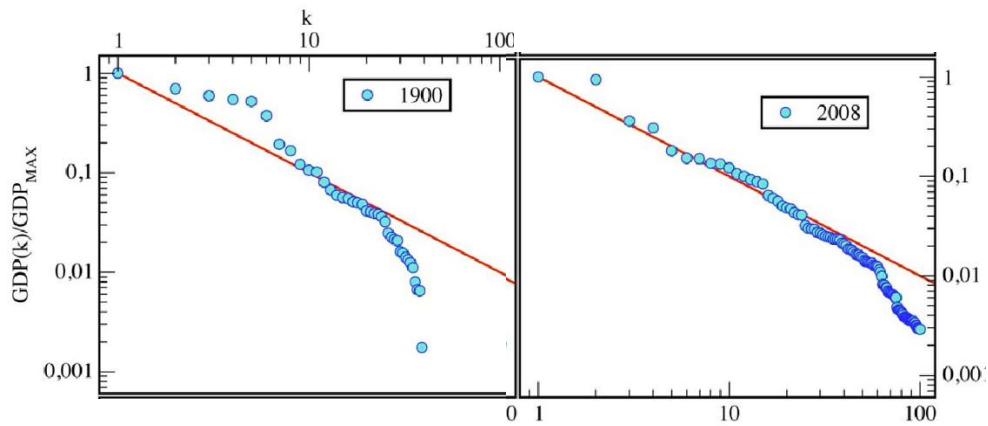
Separation decreases flow between cities

The sizes of cities in a “**coherent**” system will decay in a regular way

*While Zipf's Law holds approximately for the city sizes of each European country ... it completely fails in the aggregated sets, that is in the EU.*

## There is More than a Power Law in Zipf

Matthieu Cristelli<sup>1,2</sup>, Michael Batty<sup>3,4</sup> & Luciano Pietronero<sup>1,2,5</sup>



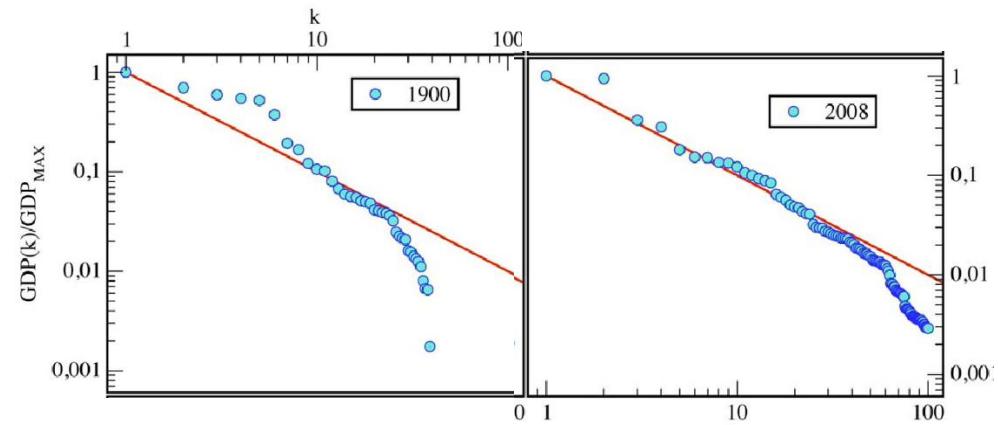
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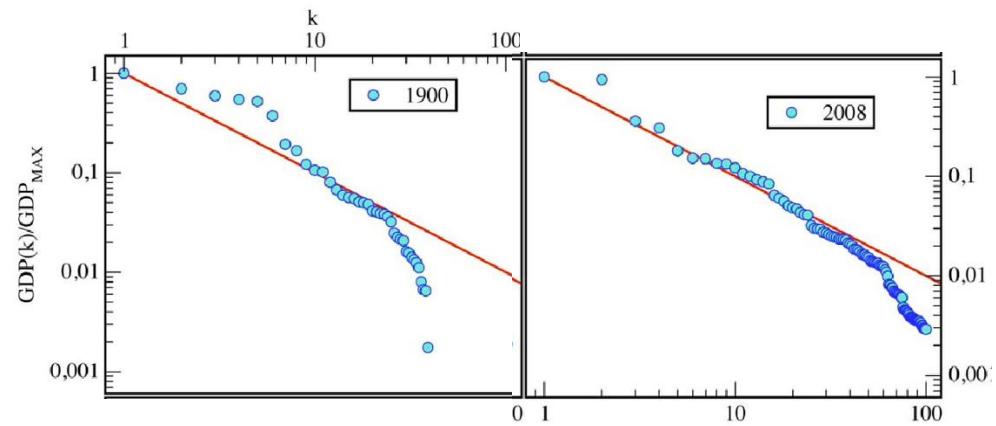
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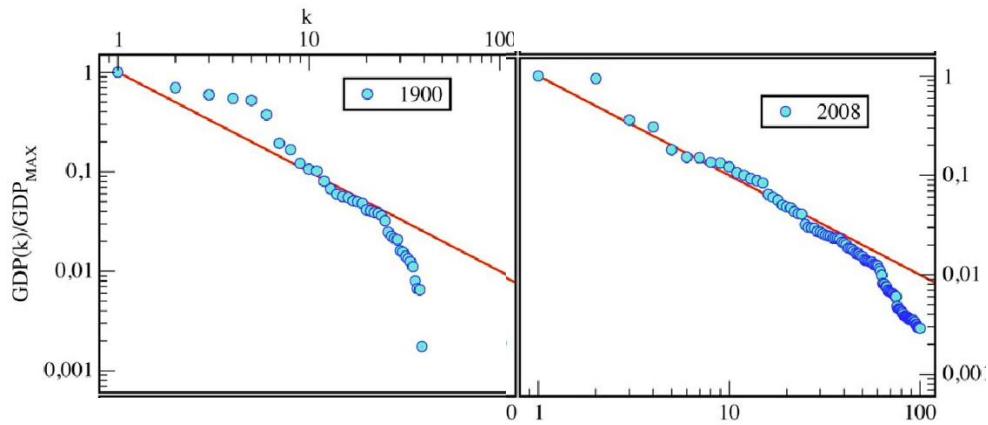
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*Contextual value*

*Statements  
we make that  
derive their value  
from when/where  
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(or are not!)*

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*Contextual value*

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*Where/when is this true?*

*generative value*

*How is this true?*

City size distributions often  
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... in “coherent” urban  
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... because of within-city  
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**Winner-take-all urbanity**

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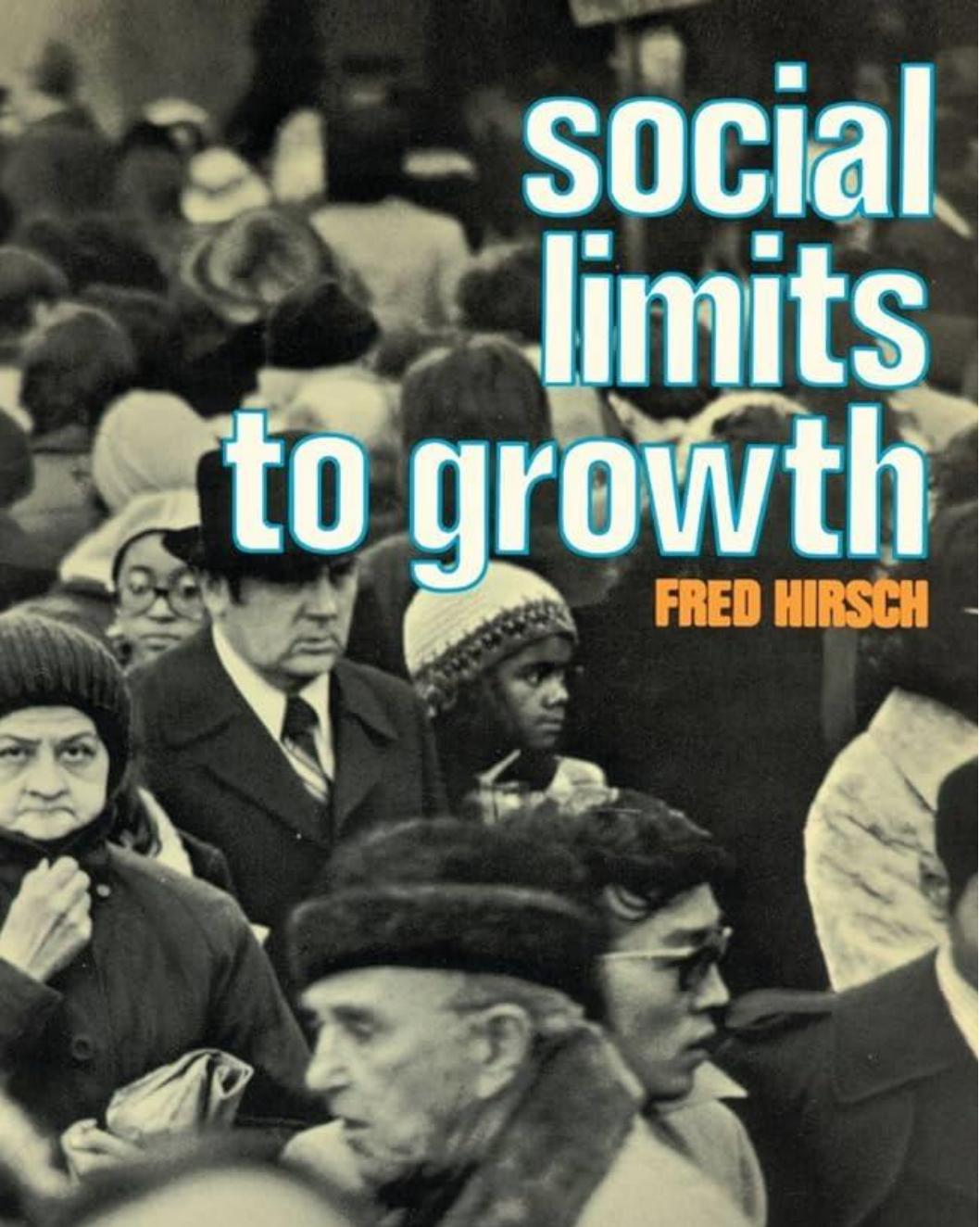
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*Positional goods* derive their value from their relation to other consumers' goods.

*“What each of us can achieve, all cannot.”*

*Examples include luxuries, education, and real estate.*

*Are cities themselves positional?*



# Winner-take-all Urbanism: Geographic Divisions in the Modern Era

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RICHARD FLORIDA  
University Professor  
University of Toronto

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An Interview with Gray Brakke

*The first dimension [of the New Urban Crisis] is the growing divide between the winner cities ... and the rest. I call this winner-take-all urbanism.*

## Urban scaling laws arise from within-city inequalities

Arvidsson et al. (2023)

## City size-dependent cumulative advantage

- (1) Cities benefit their inhabitants
- (2) Benefits are unevenly distributed
- (3) In big cities, “winners” win more from more people
- (4) Disparity compounds over time due to generational transfer and migration
- (5) These winners drive migration, increasing city system scaling

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*Statements  
we make that  
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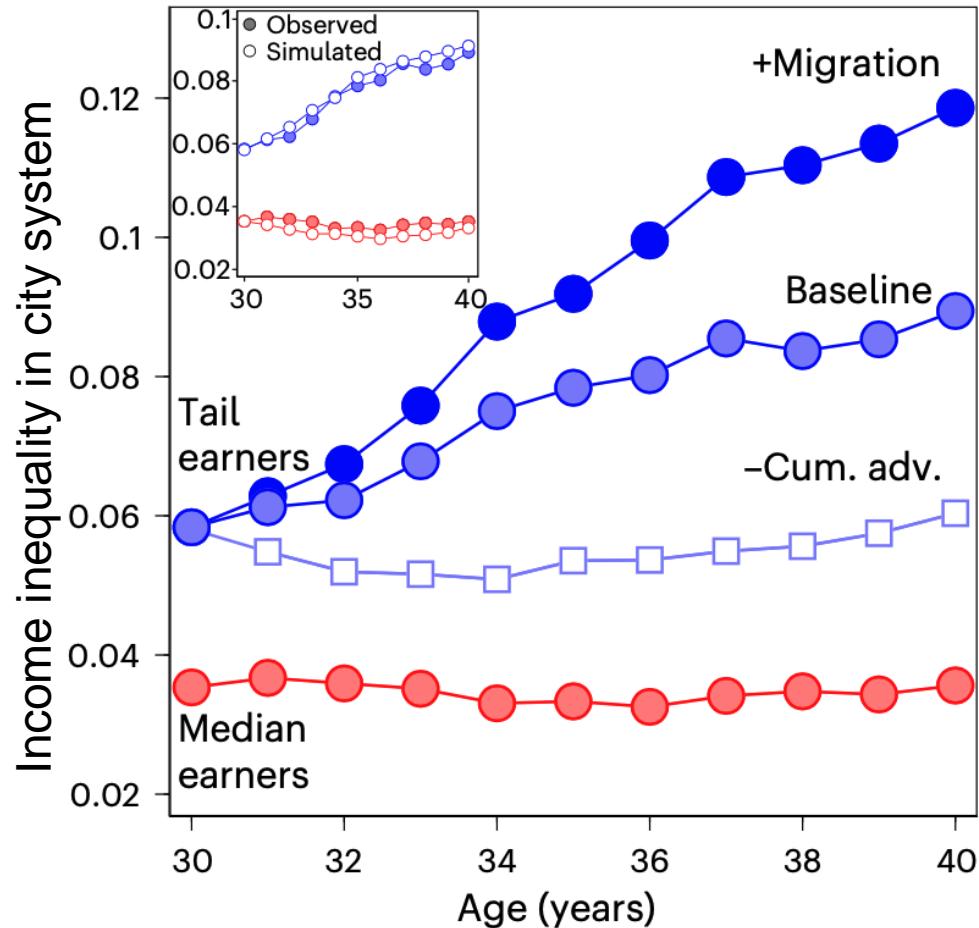
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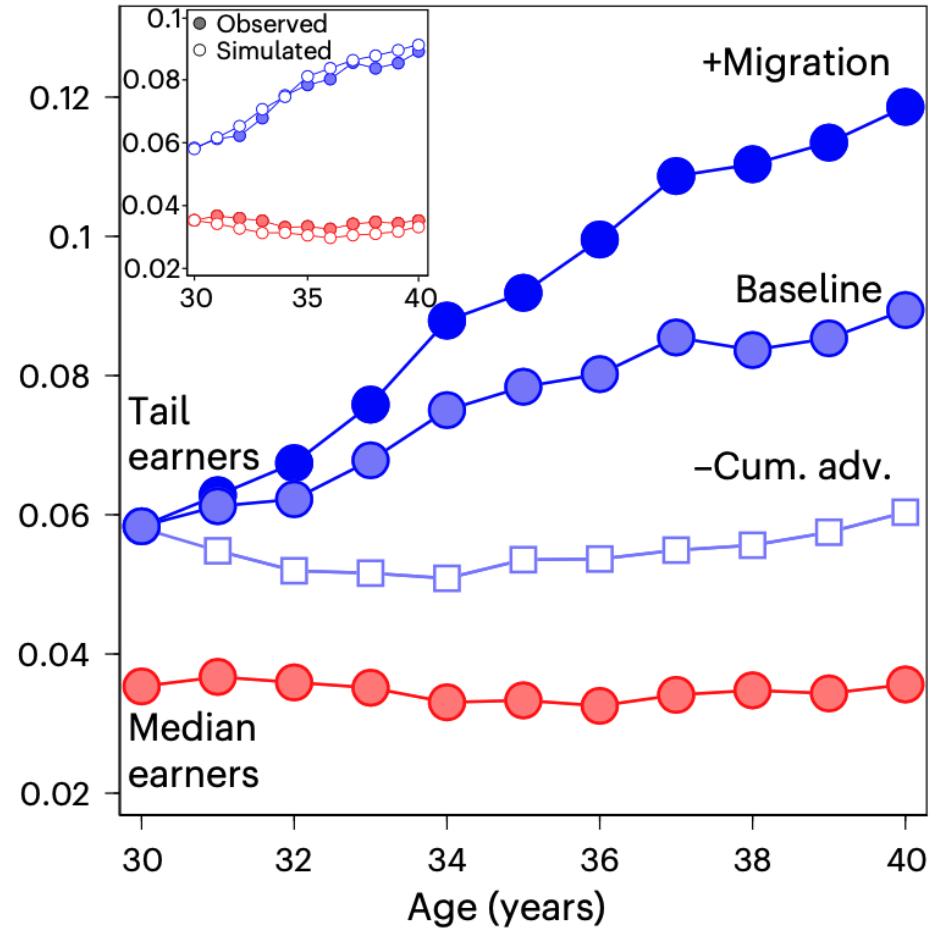
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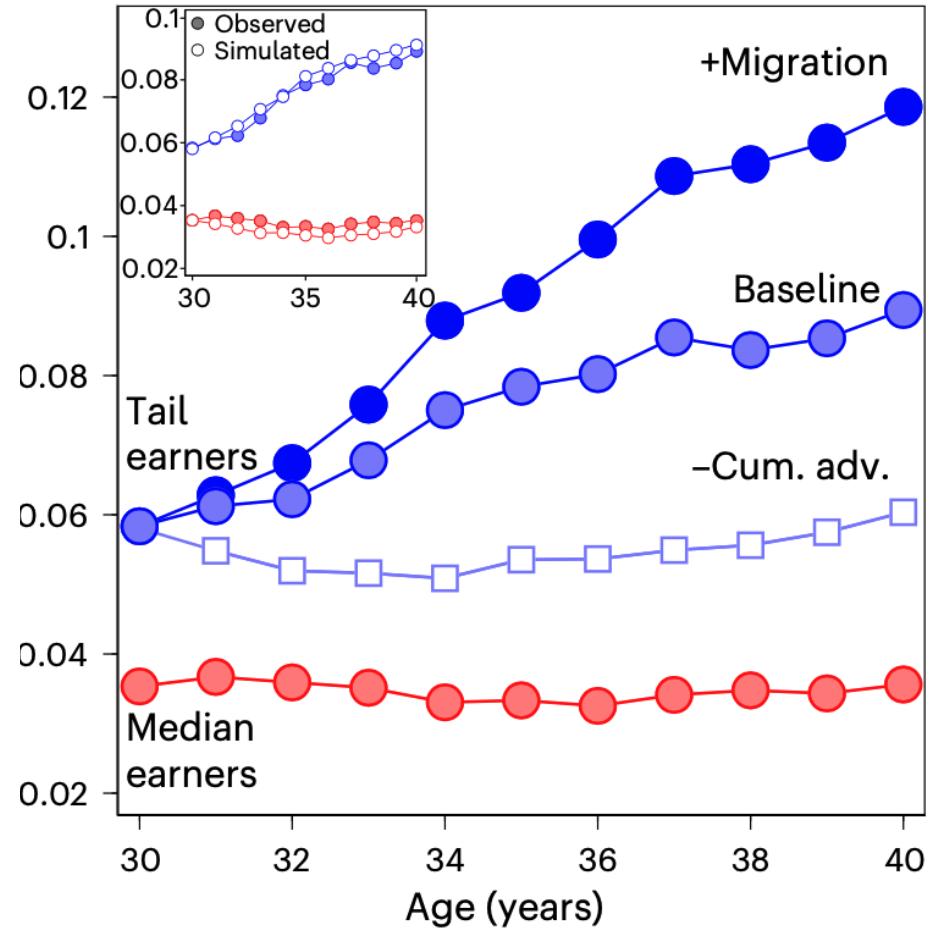
Empirical results show the contribution of each mechanism

- As below, including “move to opportunity” effects
  - As below, *including* inheritance effects over generations
  - Scaling for wealthy incomes, minus inheritance effects
- 
- Scaling (inequality) in median wage across the city system



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Wealth scaling in city system  
increases as the wealthy inherit, move

## Urban scaling laws arise from within-city inequalities

Arvidsson et al. (2023)

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So, if this is true:

**Within-city inequality** should go hand-in-hand with pop & growth

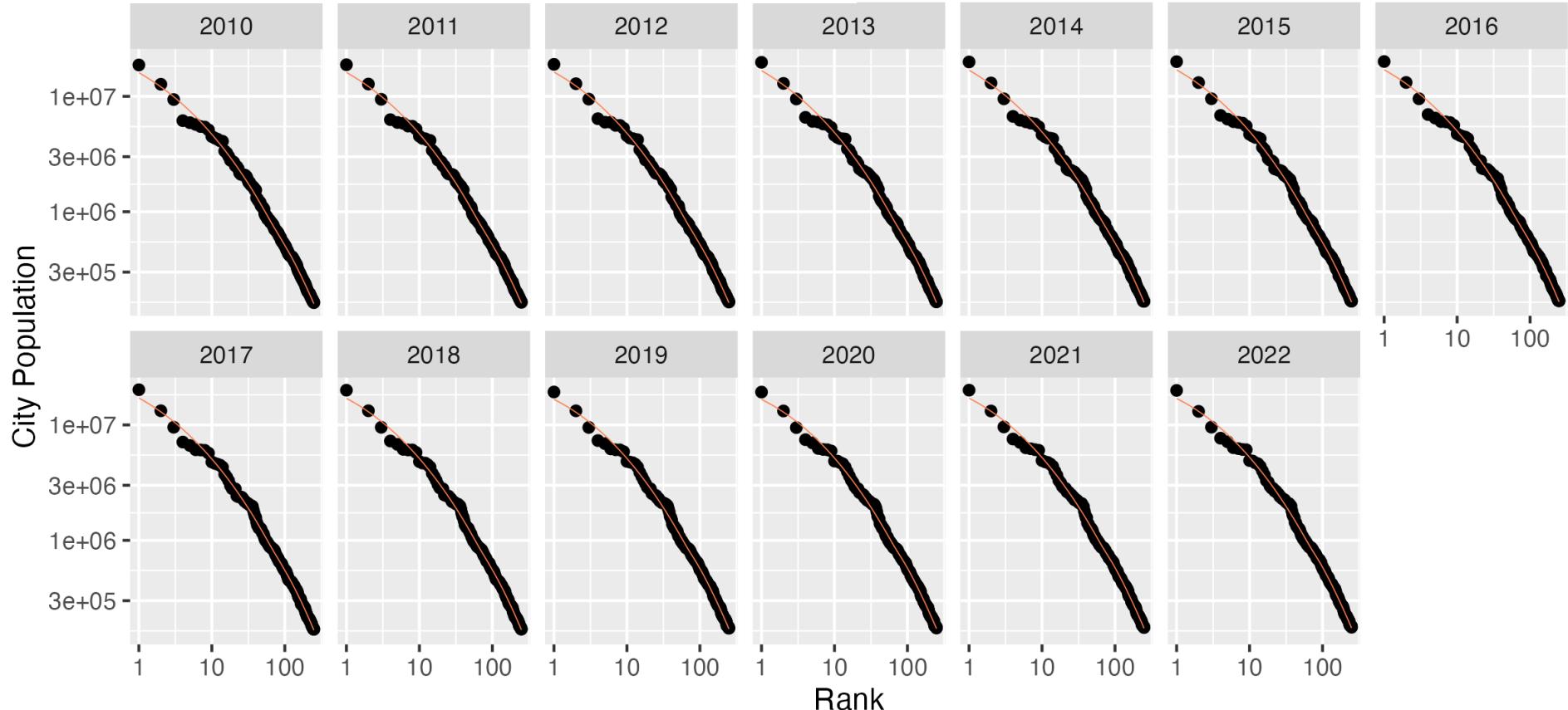
*Cities grow by attracting people at all levels of the income distribution*

**Between-city inequality** should rise as within-city inequality rises

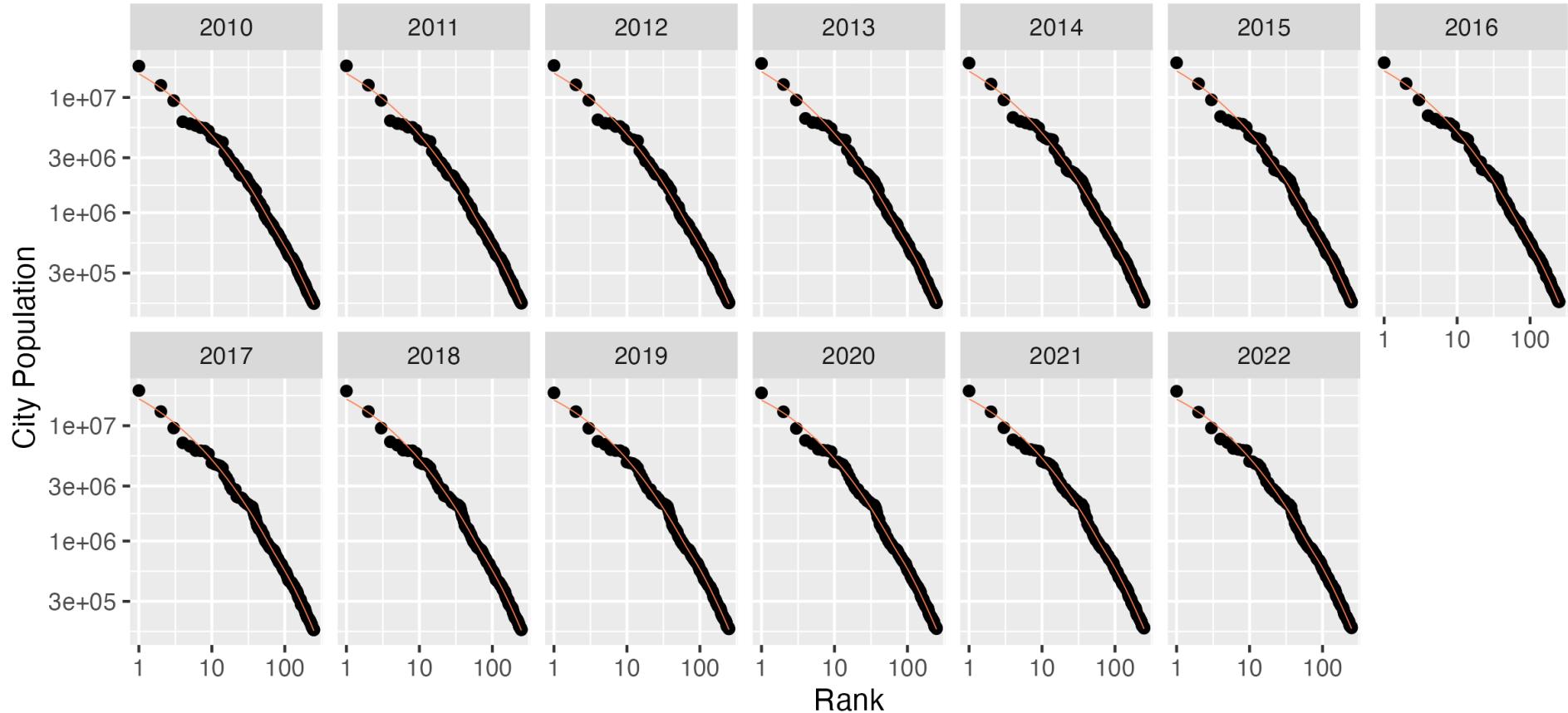
*Unequal cities will attract more “winners” with more to “win”*

*generative value*

# Size-Rank plot of cities by year (ACS)

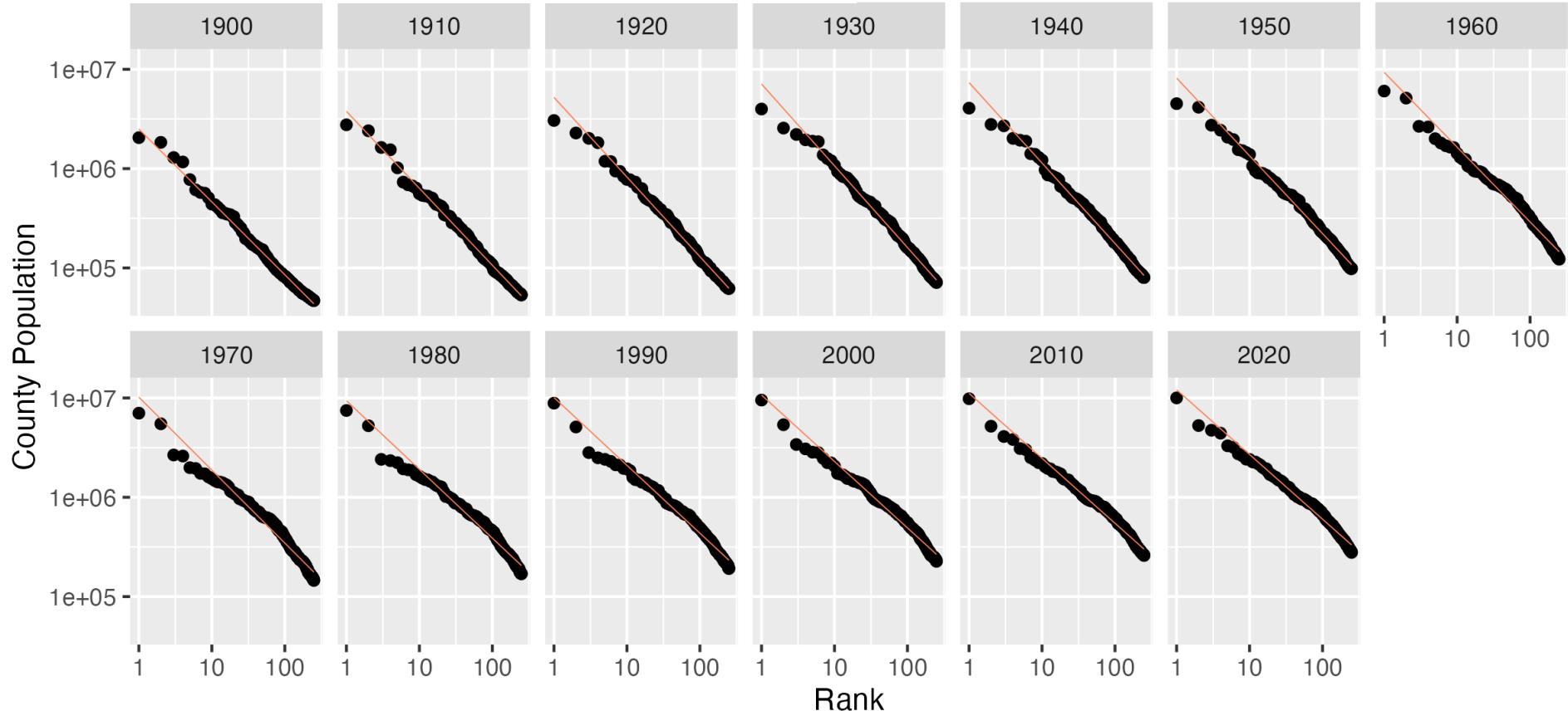


## Size-Rank plot of cities by year (ACS)



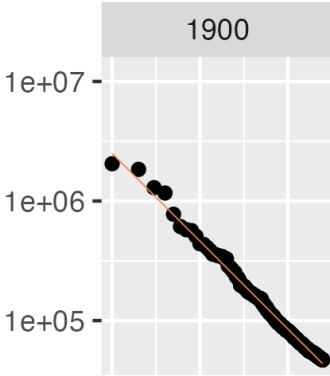
Cities don't really change  
*substantially* on this timescale

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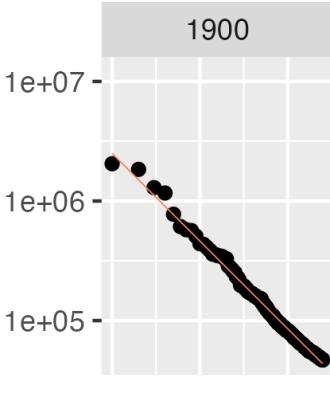
County Population



- NY (Manhattan)
- Cook (Chicago)
- Philadelphia
- Kings (Brooklyn)
- Allagheny (Pittsburgh)
- Suffolk (Central Boston)
- St. Louis
- Middlesex (Boston-Cambridge)
- Baltimore
- Cuyahoga (Cleveland)
- Erie (Buffalo)
- Hamilton (Cincinnati)
- Hudson (Jersey City)
- Essex (Newark)
- Essex (Boston-Salem)
- Wayne (Detroit)
- Worcester
- San Francisco
- Milwaukee

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County Population

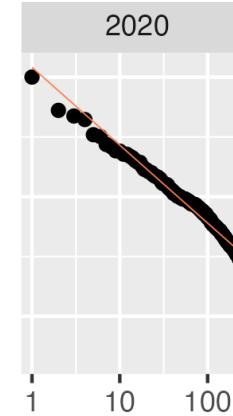


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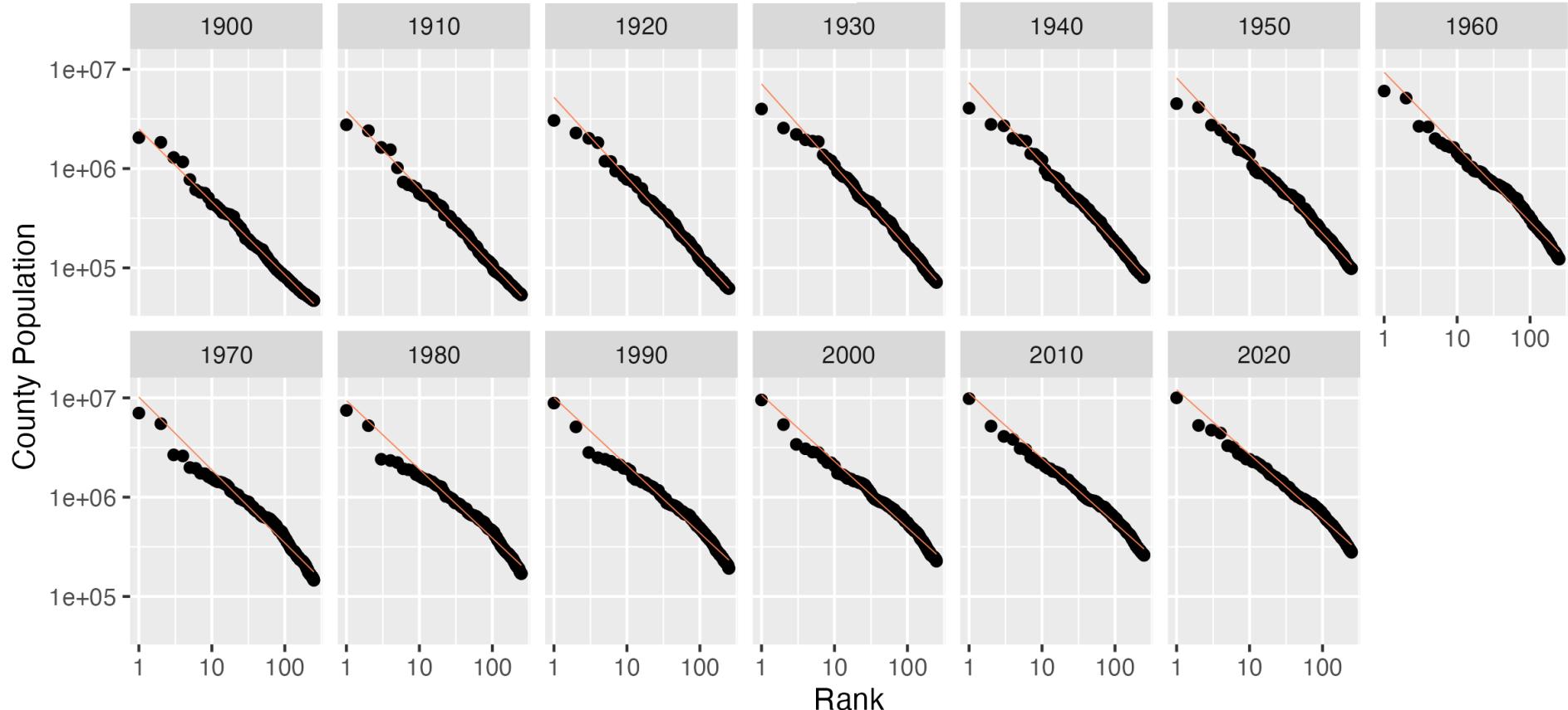
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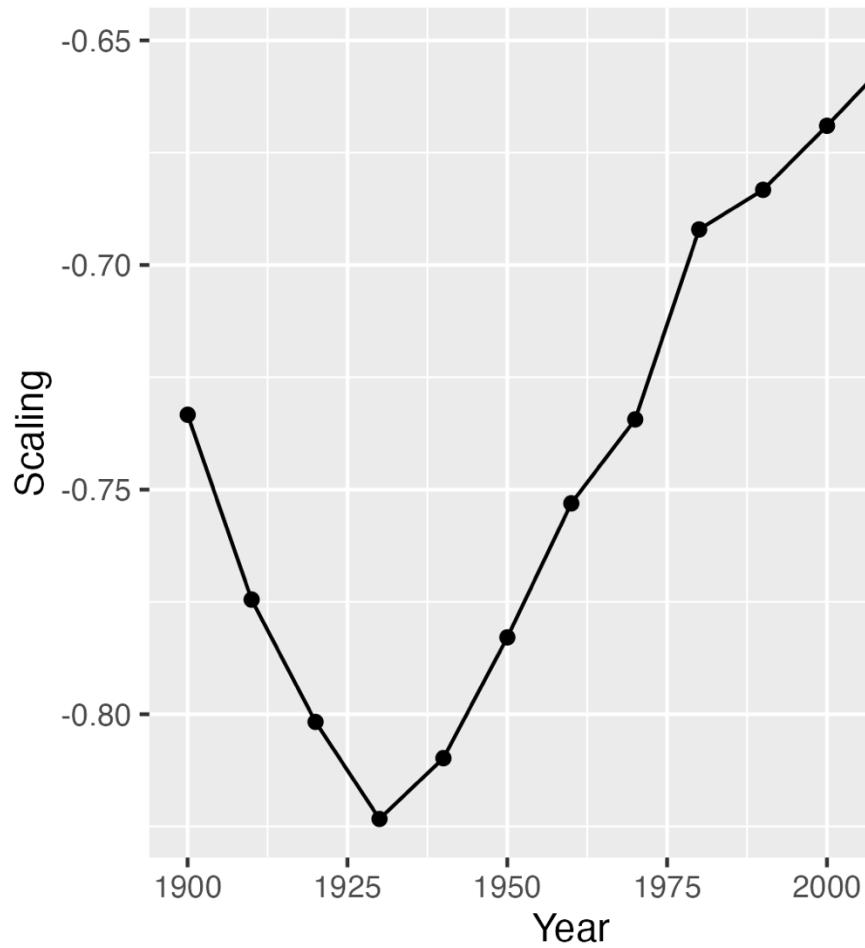
- Los Angeles
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- Harris (Houston)
- Maricopa (Phoenix)
- San Diego
- Orange (Anaheim/Irvine)
- Kings (Brooklyn)
- Dallas
- Riverside (Inland Empire LA)
- Queens
- King (Seattle)
- Clark (Las Vegas)
- San Bernardino
- Tarrant (Ft. Worth)
- Bexar (San Antonio)
- Broward (Miami)
- Santa Clara (San Jose/South Bay)
- Wayne (Detroit)
- NY (Manhattan)



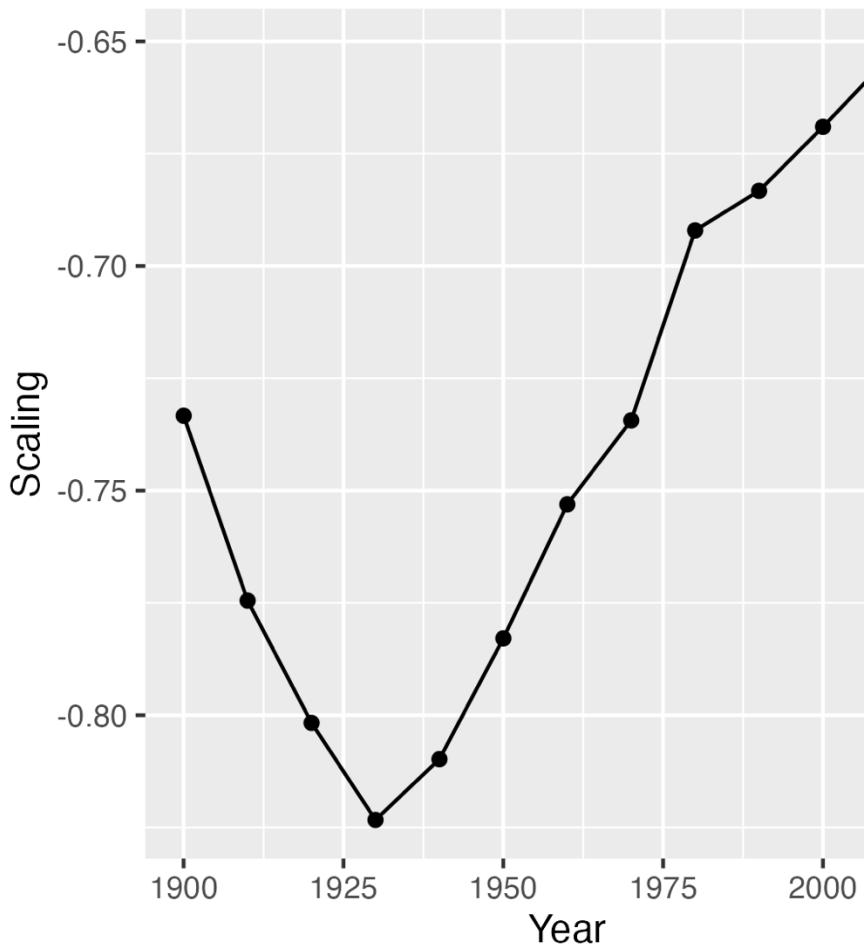
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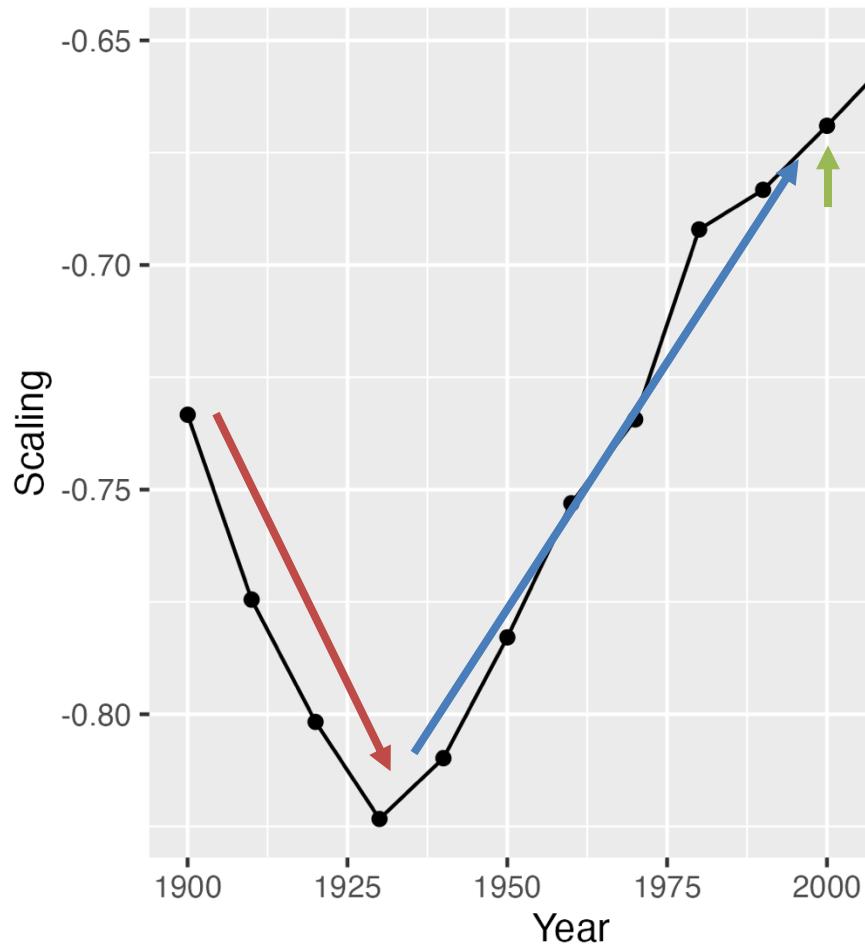


Size-Rank plot of cities by year (ACS)



# US city population scaling in the 20<sup>th</sup> C.

- Increases during the “Gilded Age”
- Rapid reversal during the mid-20<sup>th</sup>
  - Population transfer west & south
  - Massive growth overall
  - Mass sub-urbanization
- Now, population distribution across cities is more equitable than ever

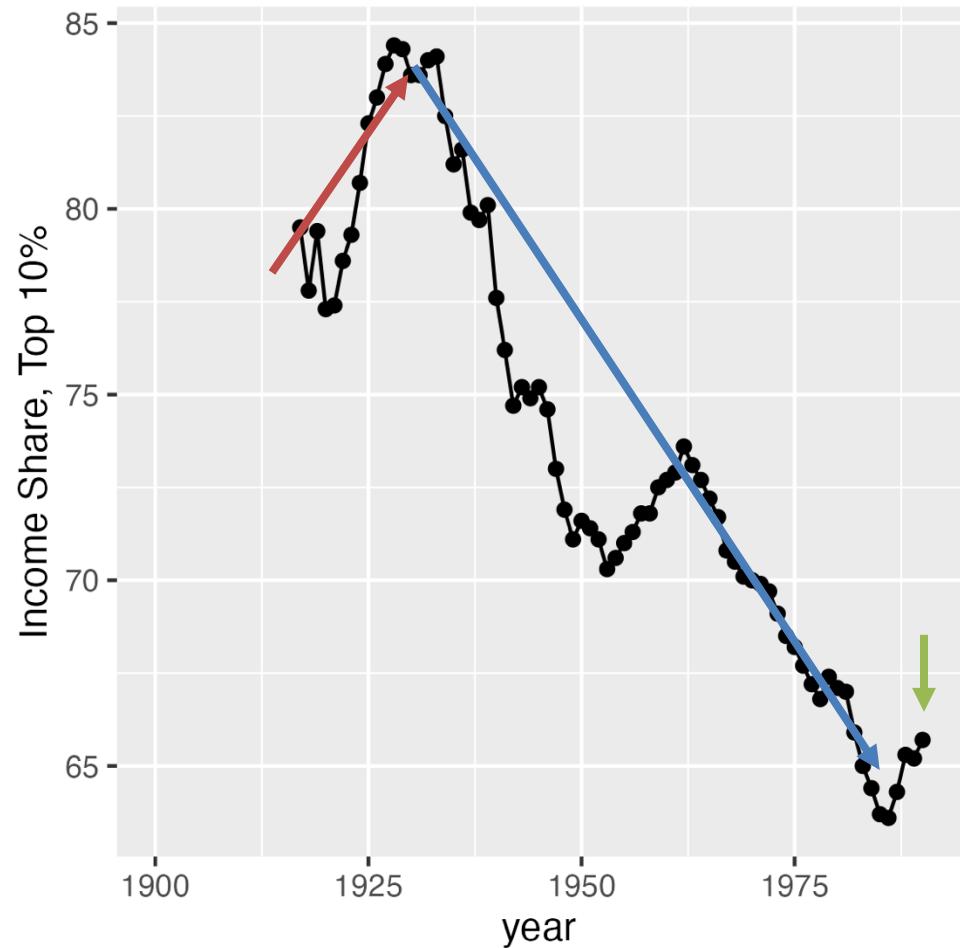


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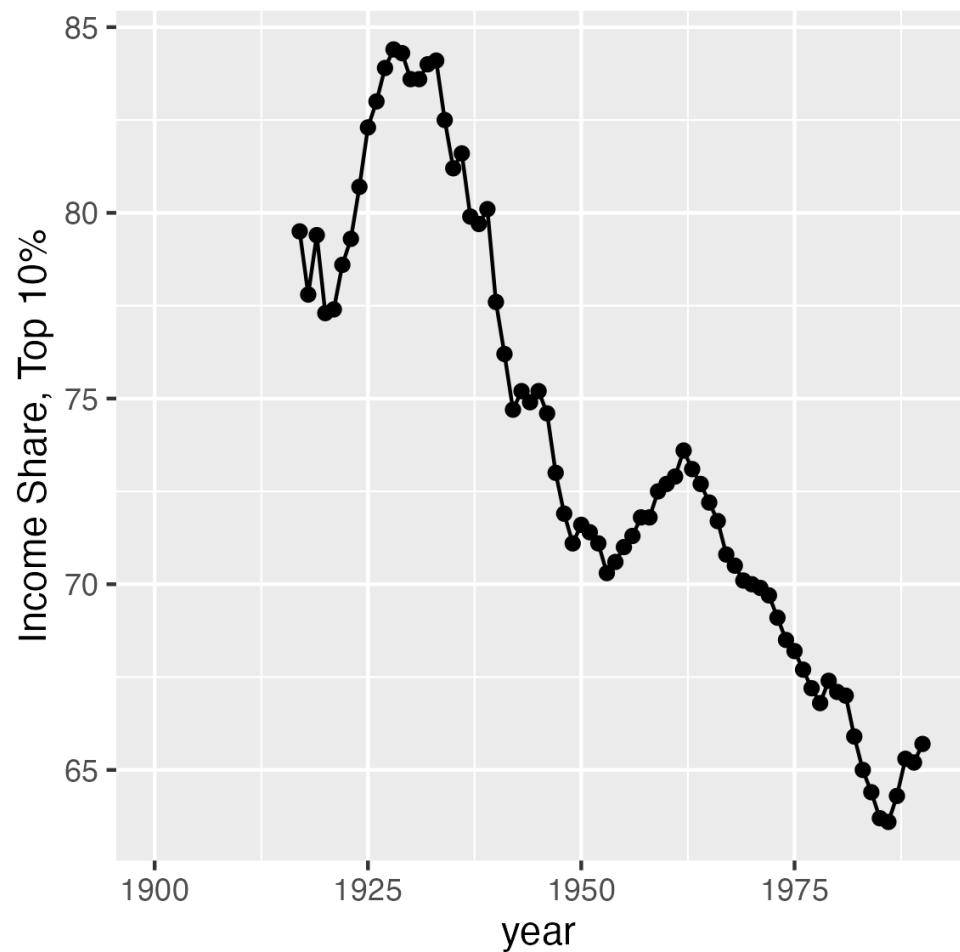
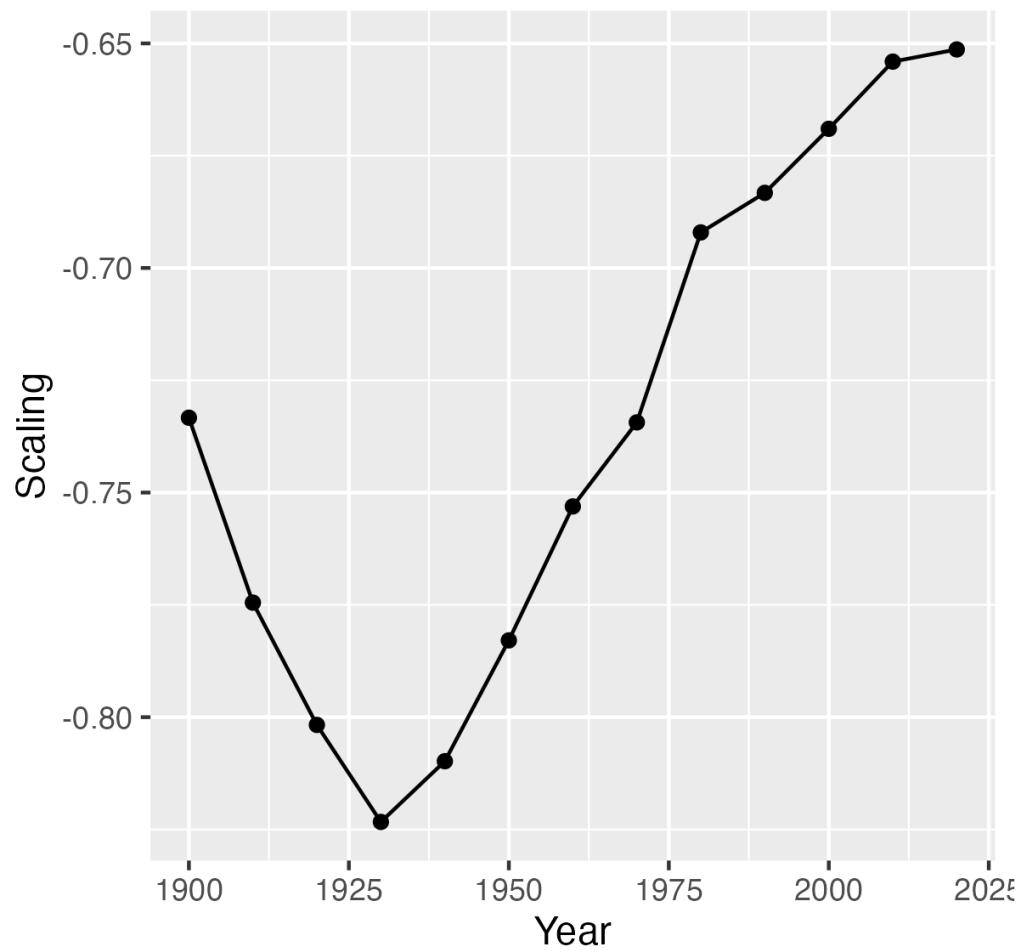
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# US Inequality in the 20<sup>th</sup> Century

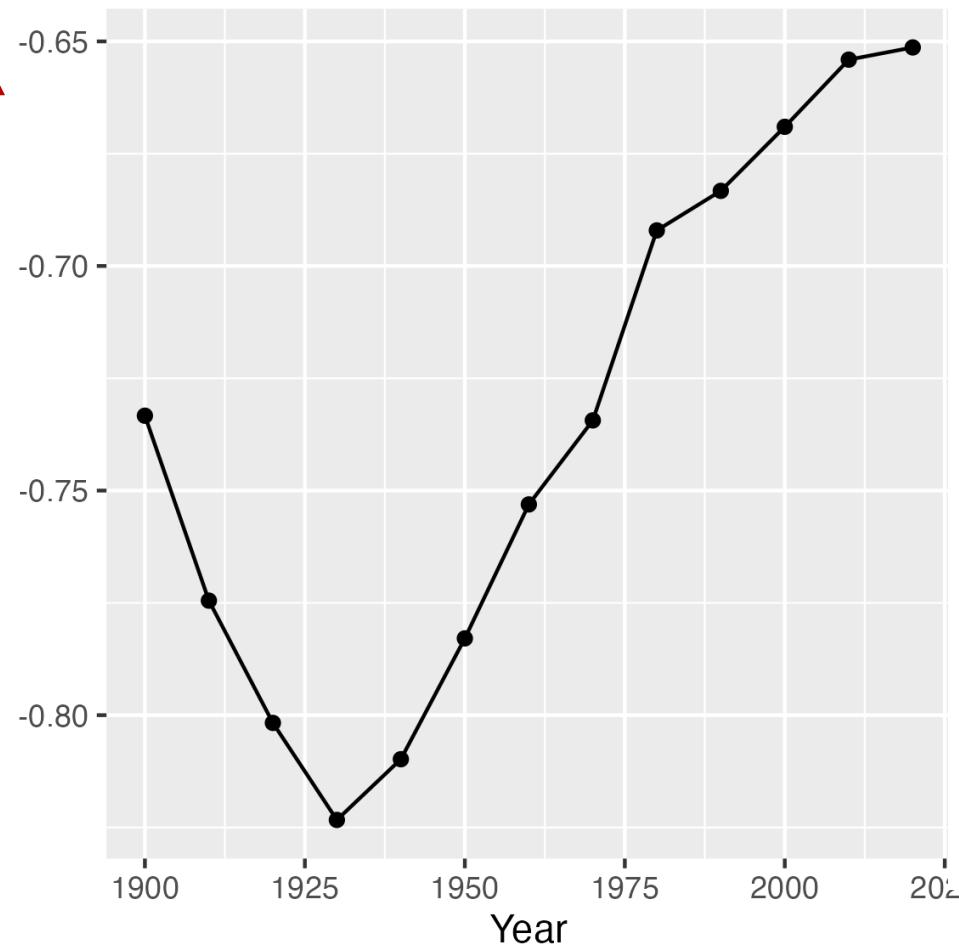
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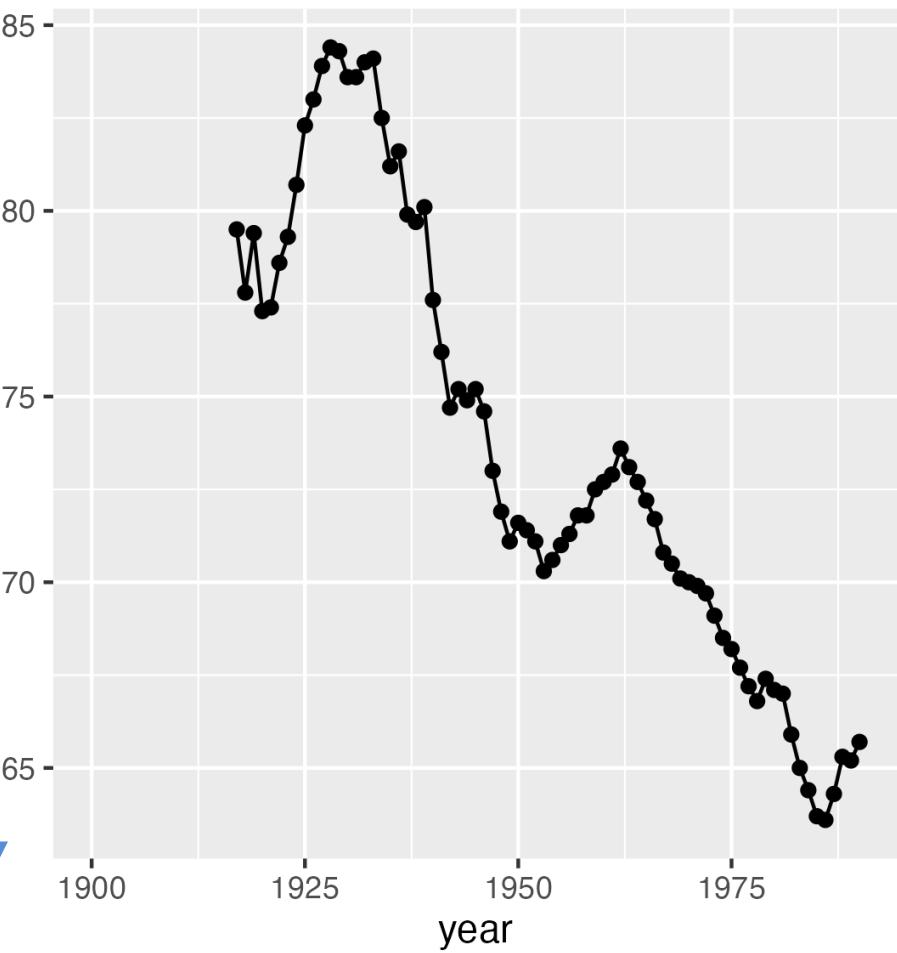
Saez-Zucman (2015)

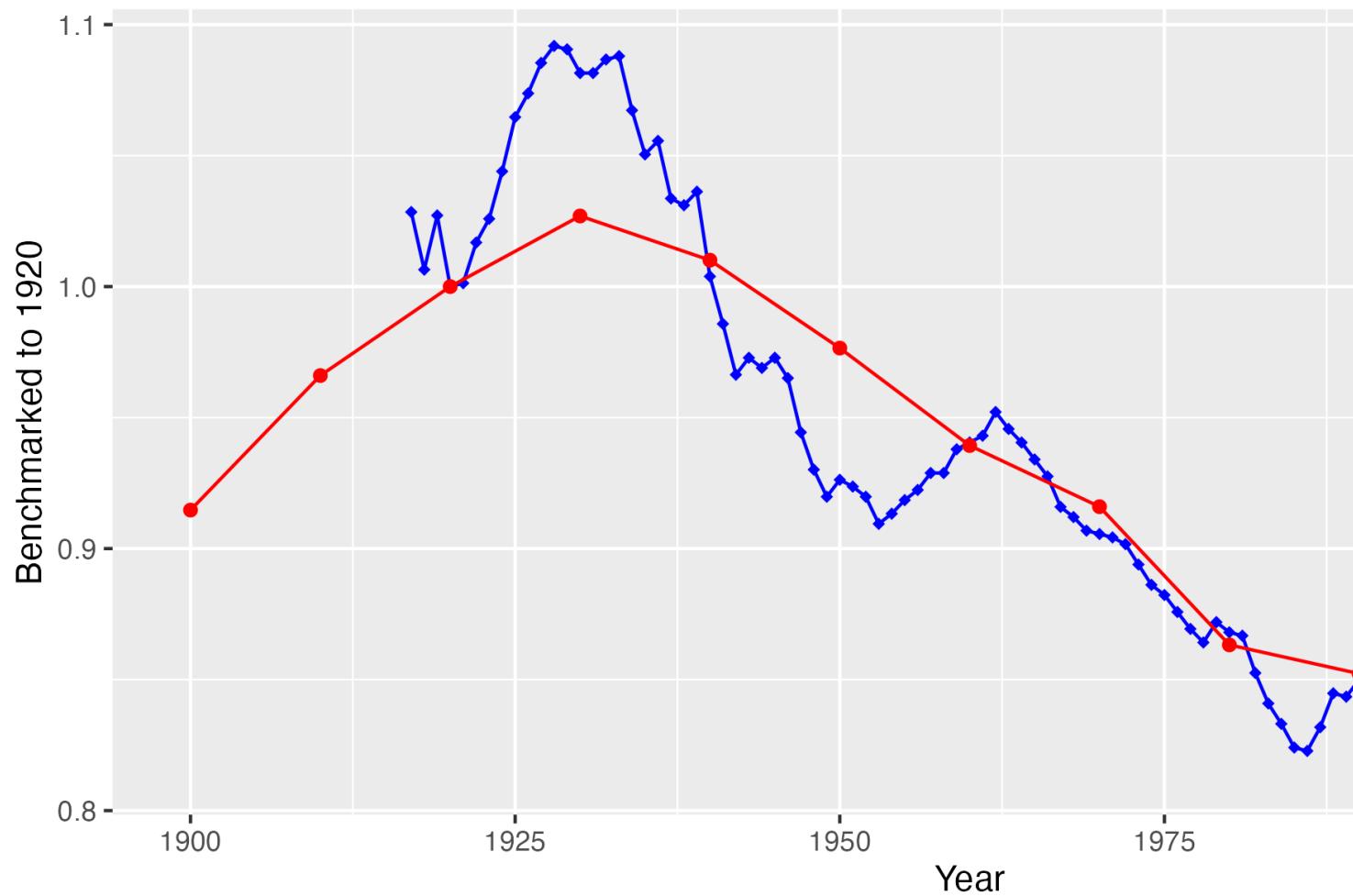


Inequality in city population decreases

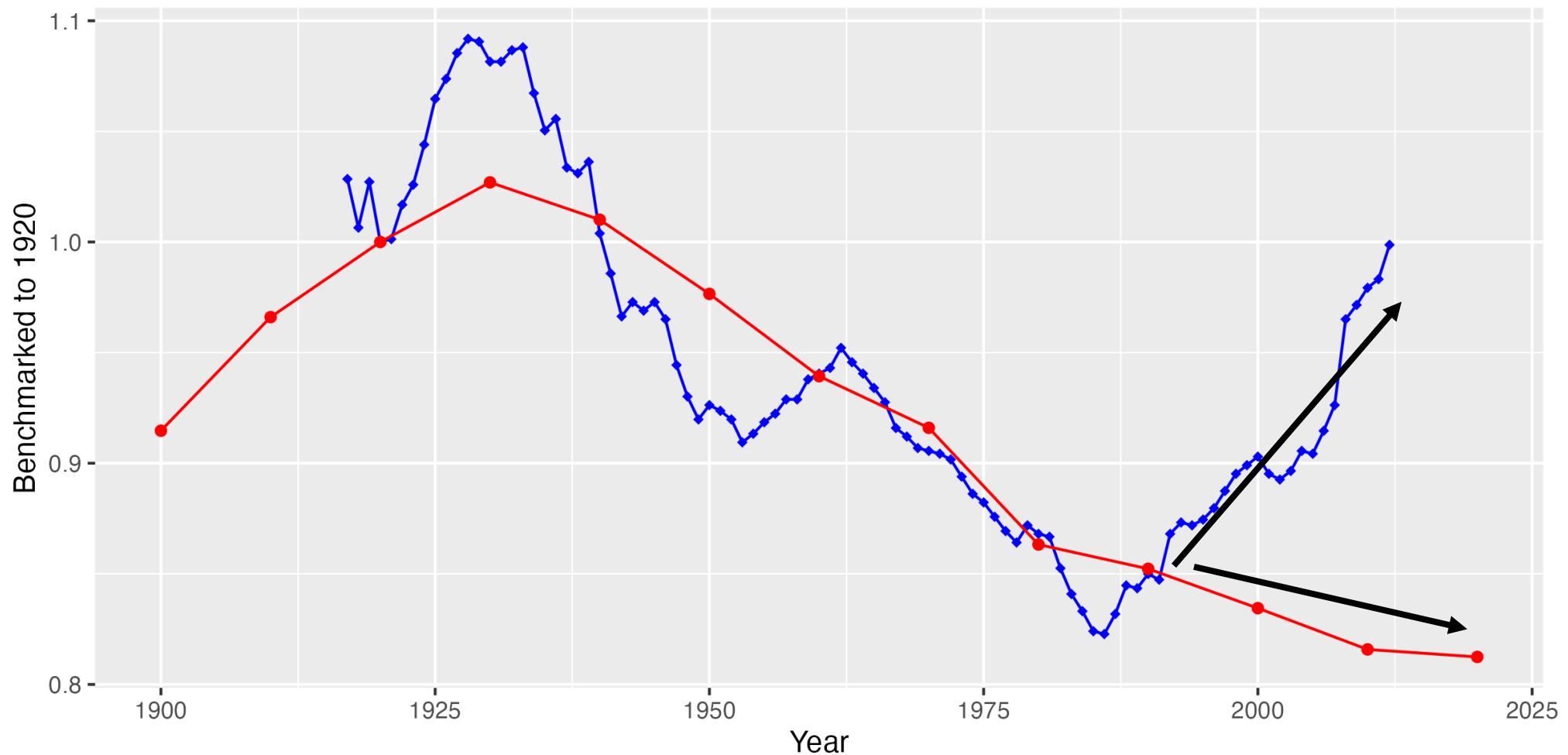


Inequality in population income decreases

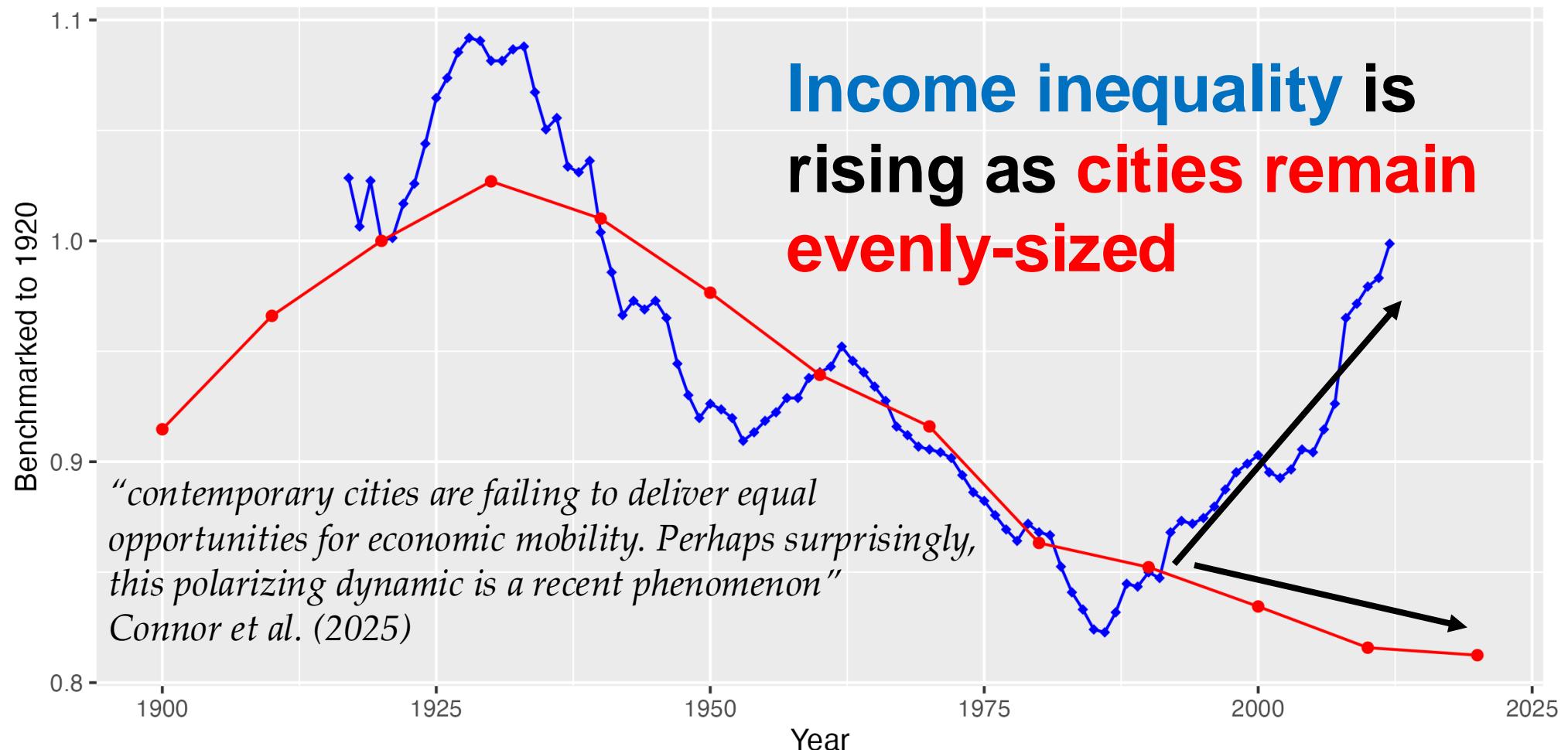




Between-city population inequality moved with macro-level income inequality

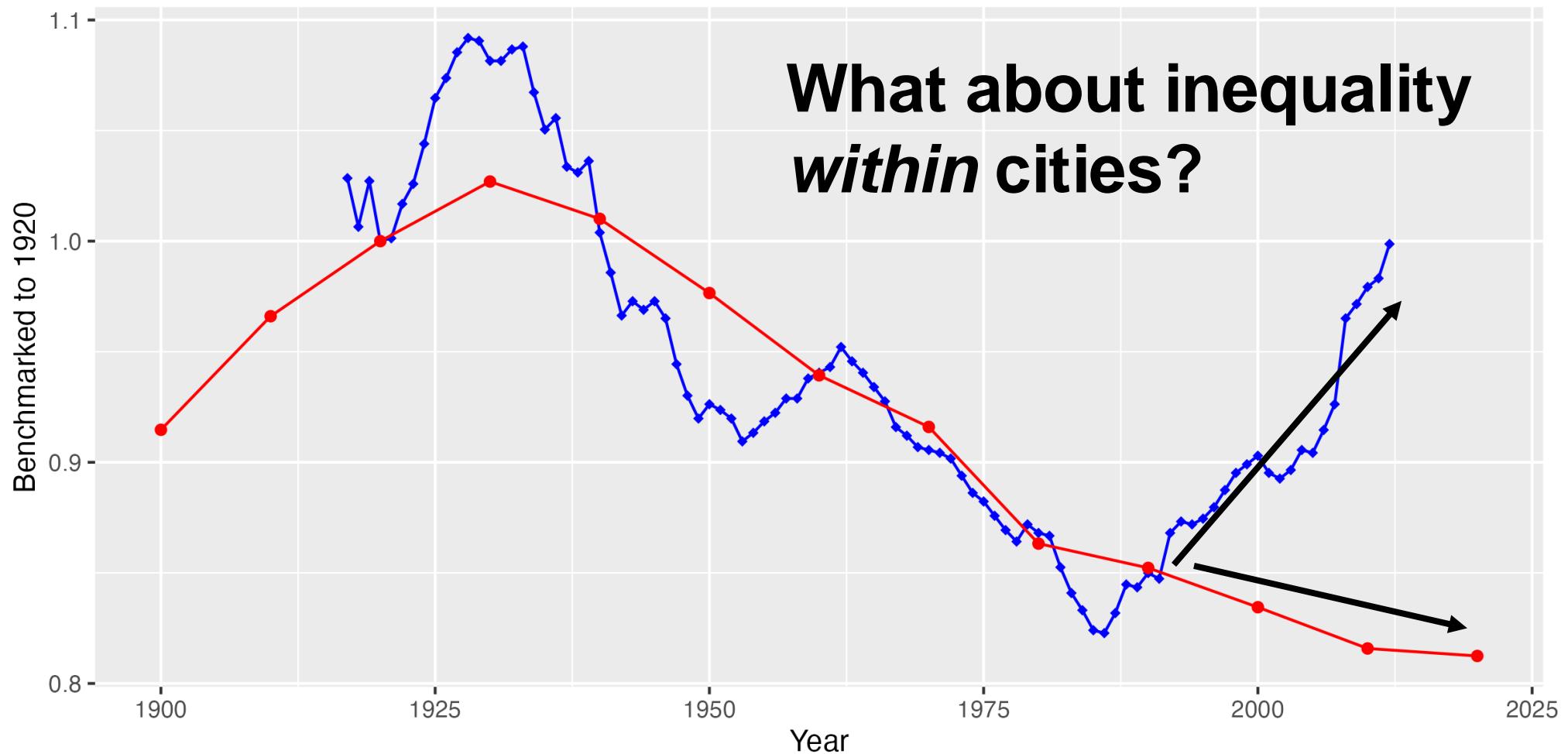


Between-city population inequality moved with macro-level income inequality until the 1990s...

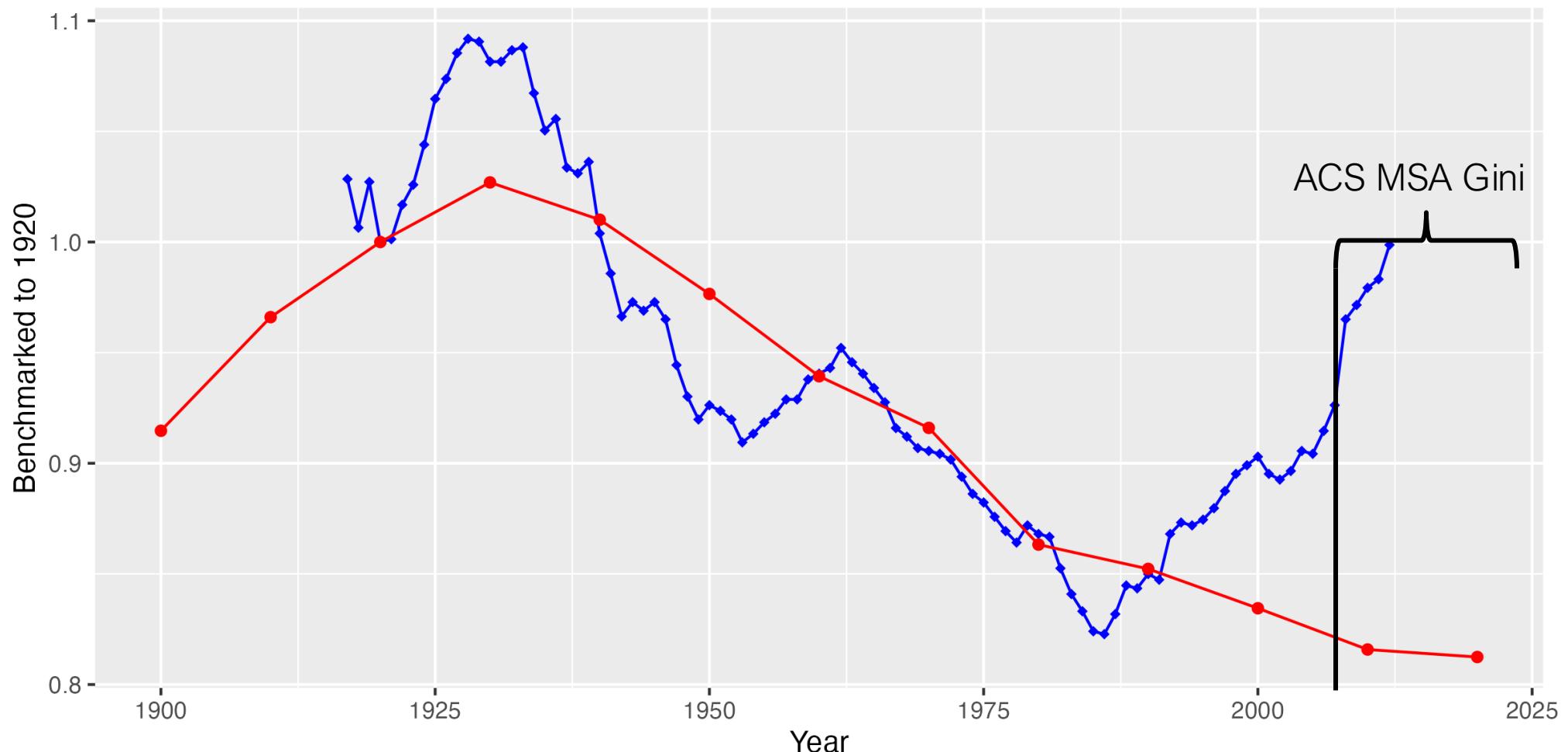


Between-city population inequality moved with macro-level income inequality until the 1990s...

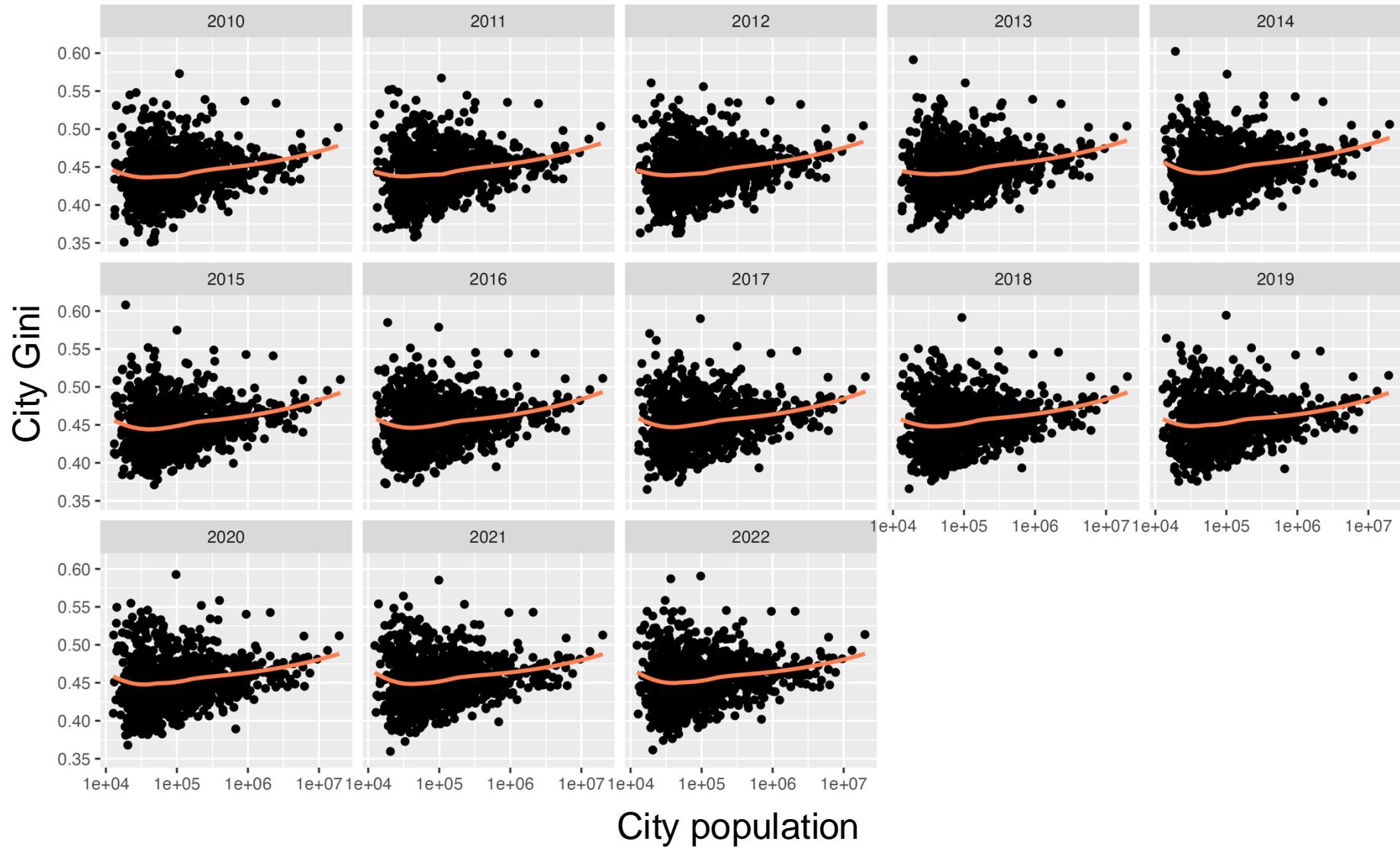
# What about inequality *within* cities?

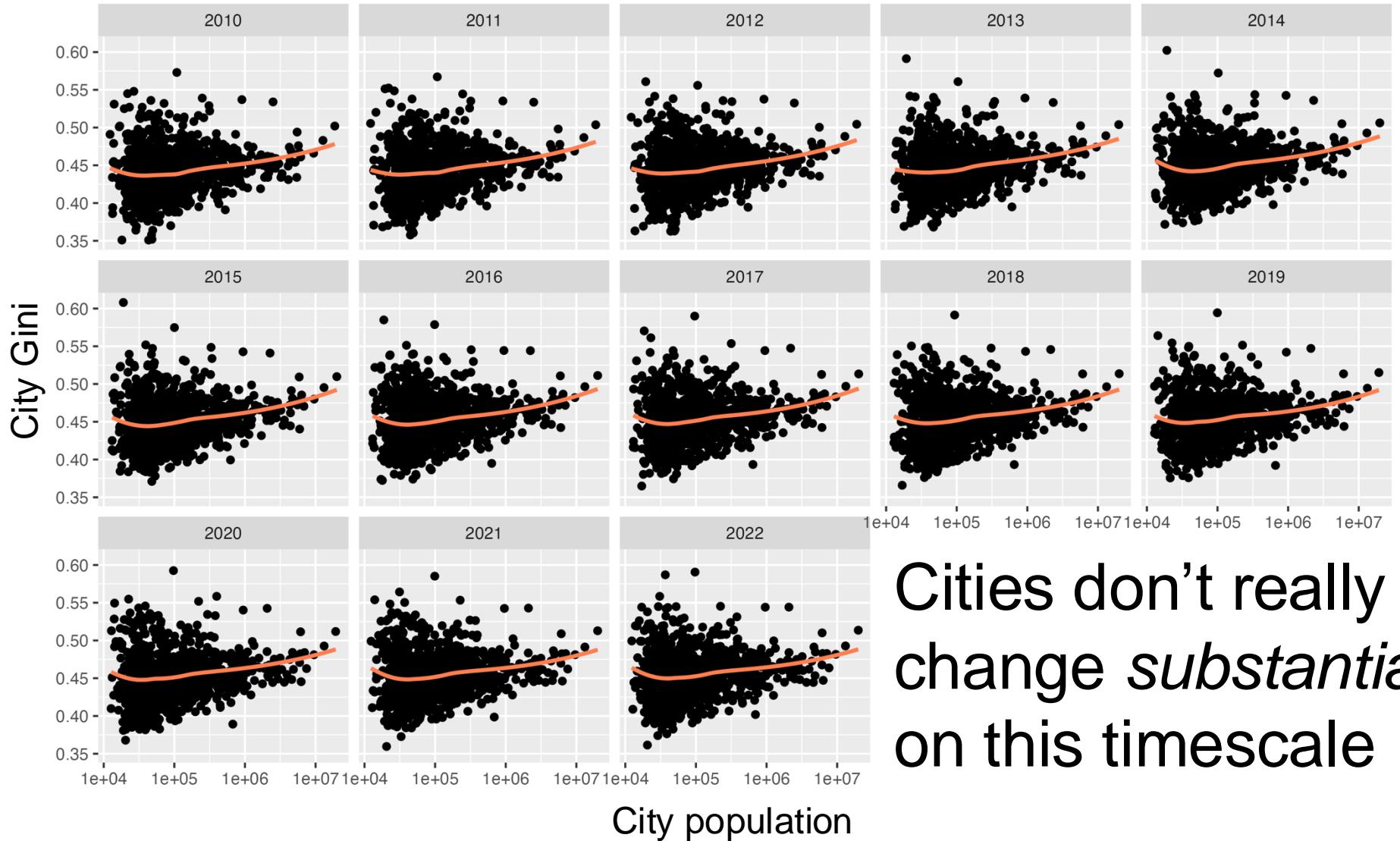


Between-city population inequality moved with macro-level income inequality until the 1990s...

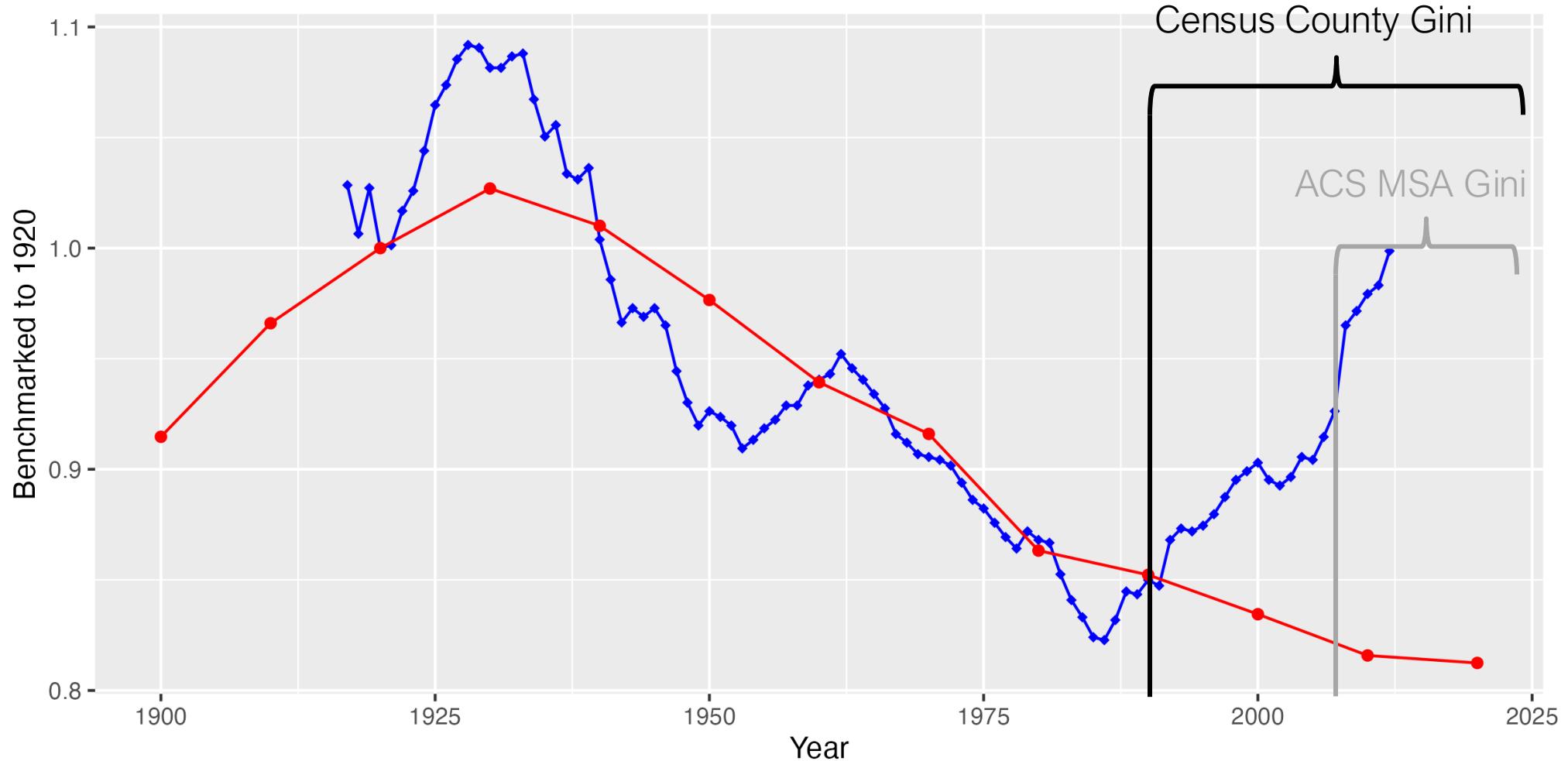


Between-city population inequality moved with macro-level income inequality until the 1990s...

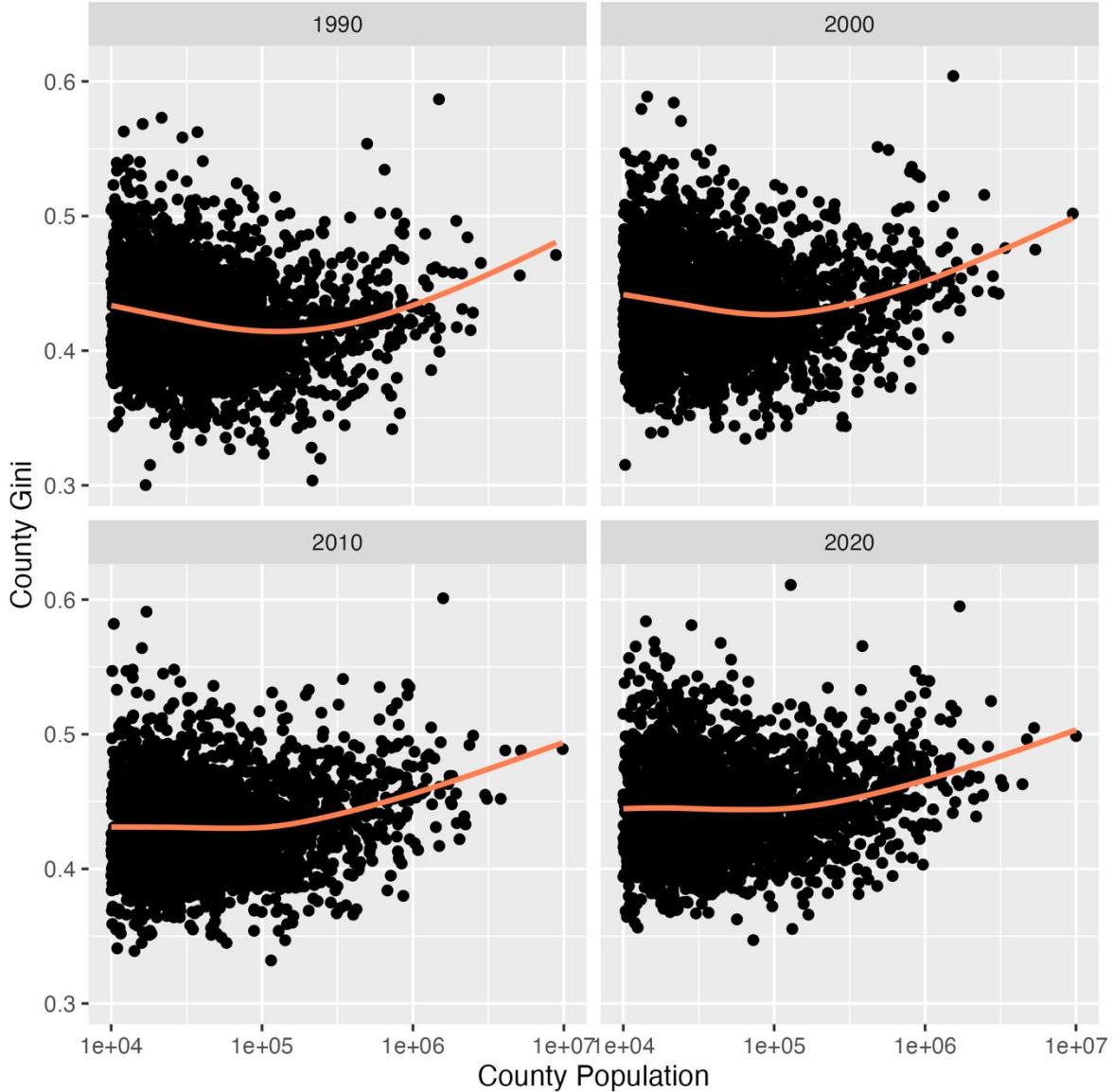


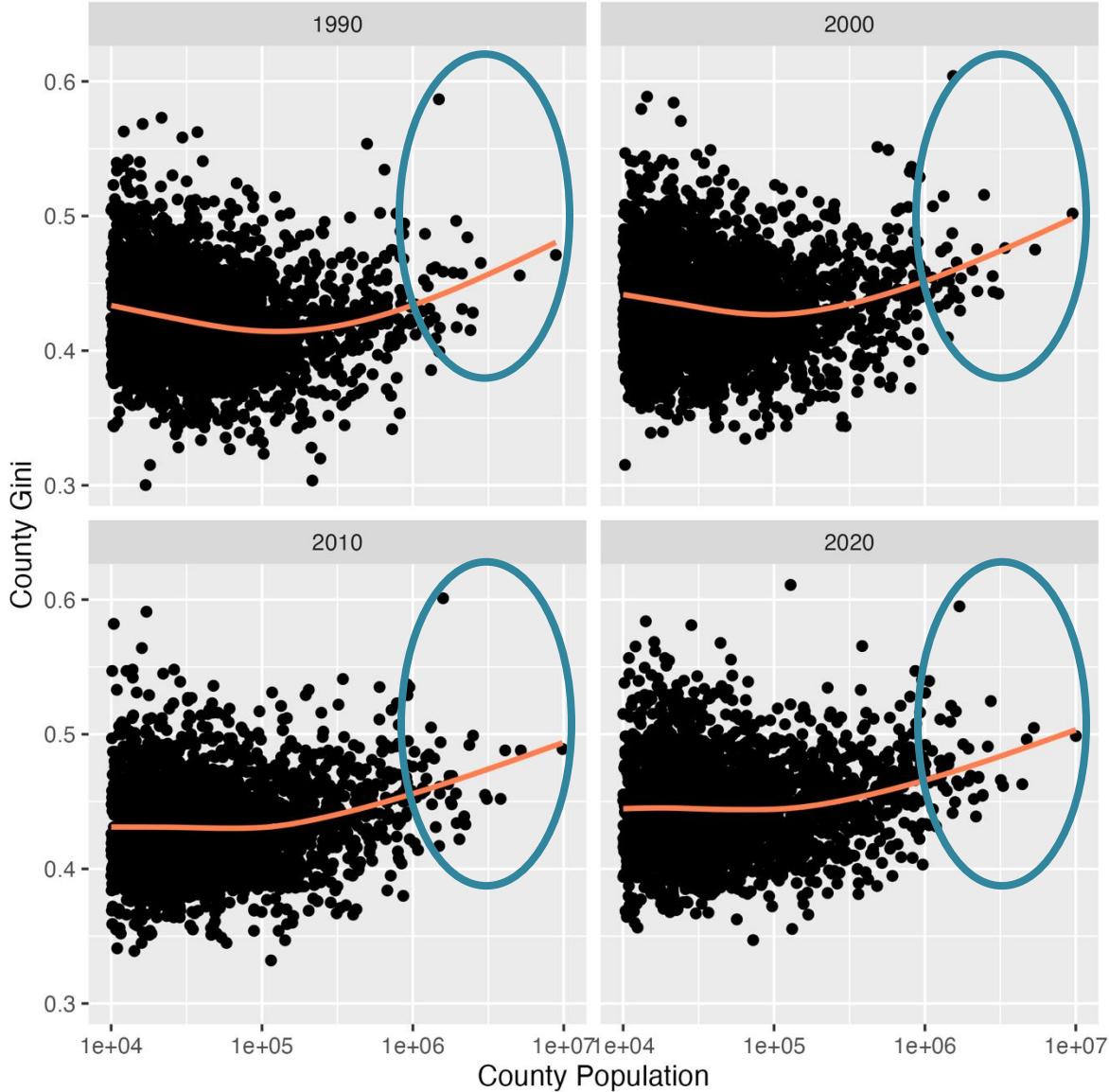


Cities don't really  
change substantially  
on this timescale

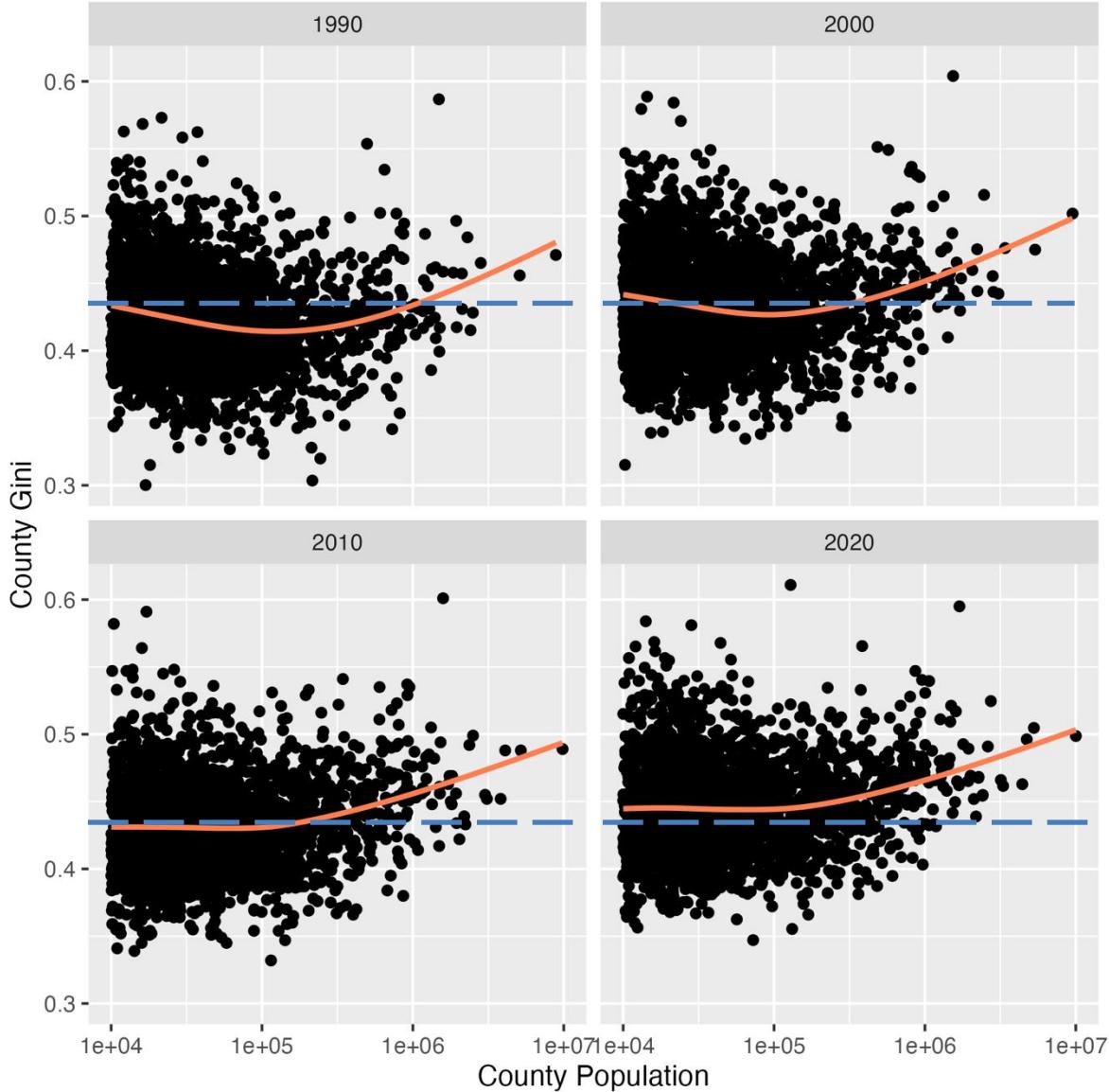


Between-city population inequality moved with macro-level income inequality until the 21<sup>st</sup> Century...

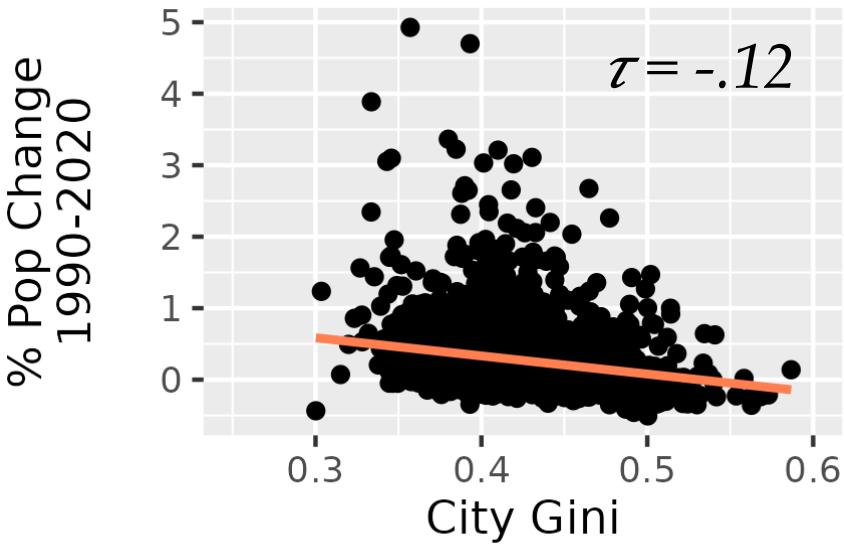
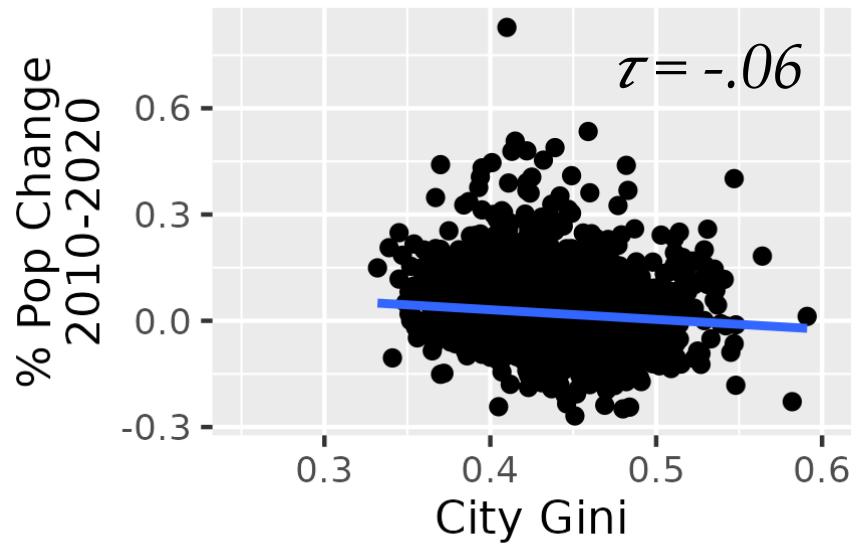
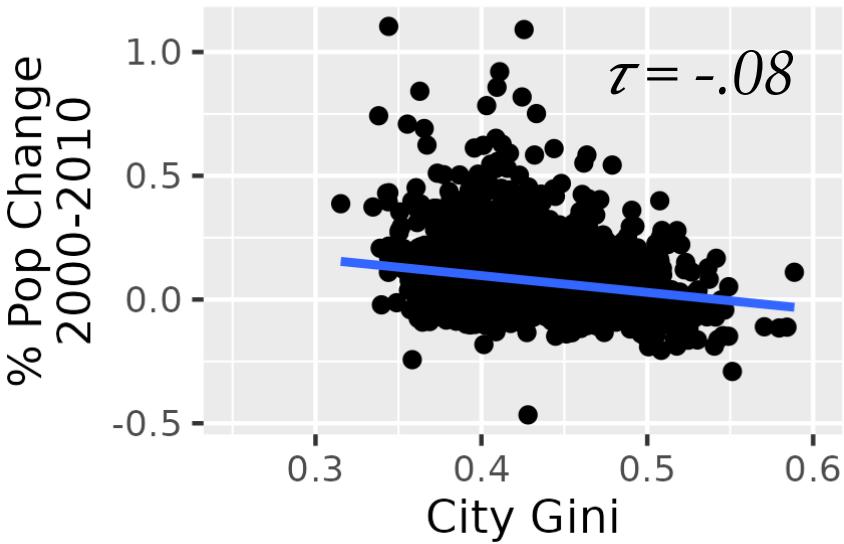
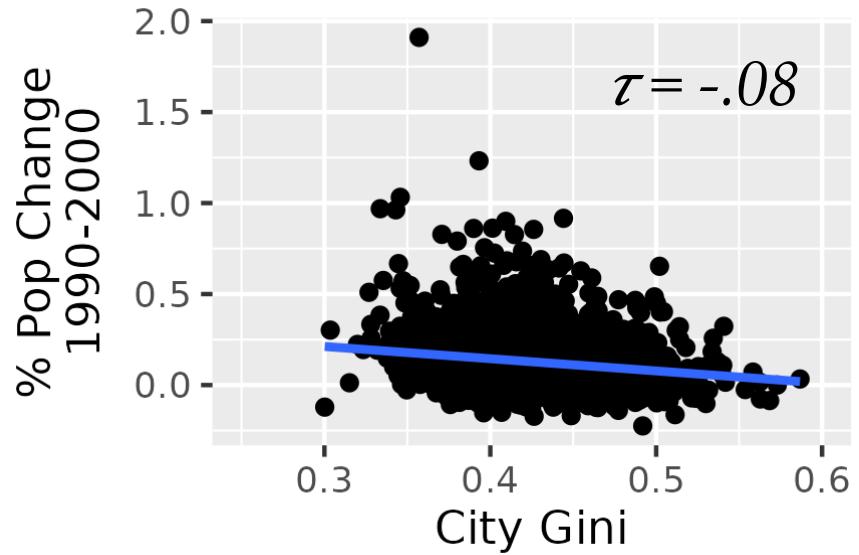




Bigger cities  
have tended to  
have higher  
income  
inequality

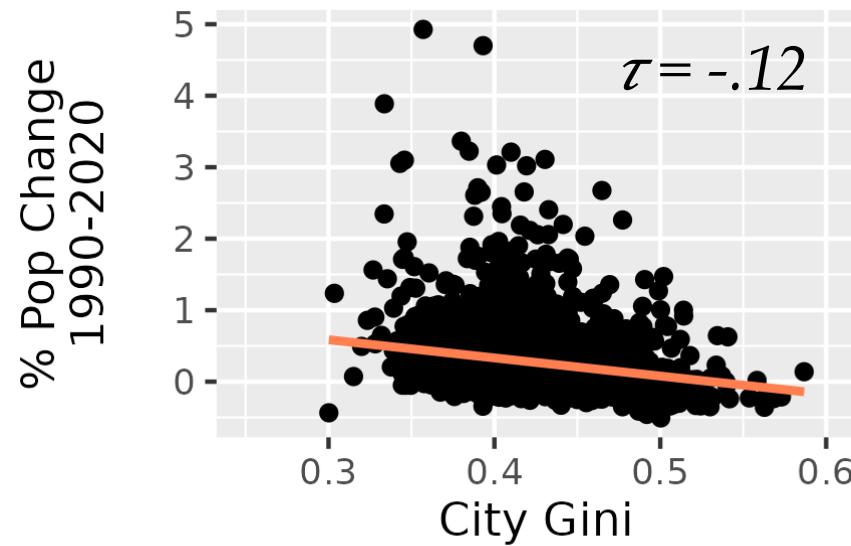
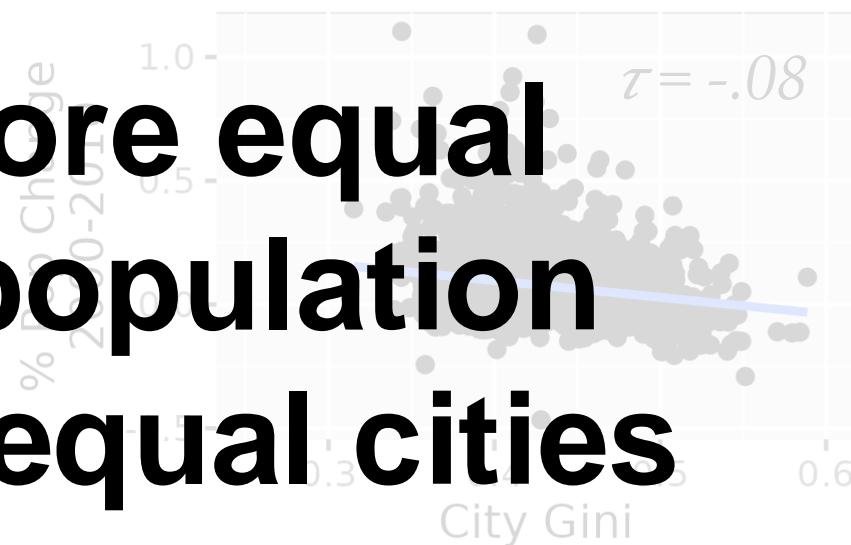
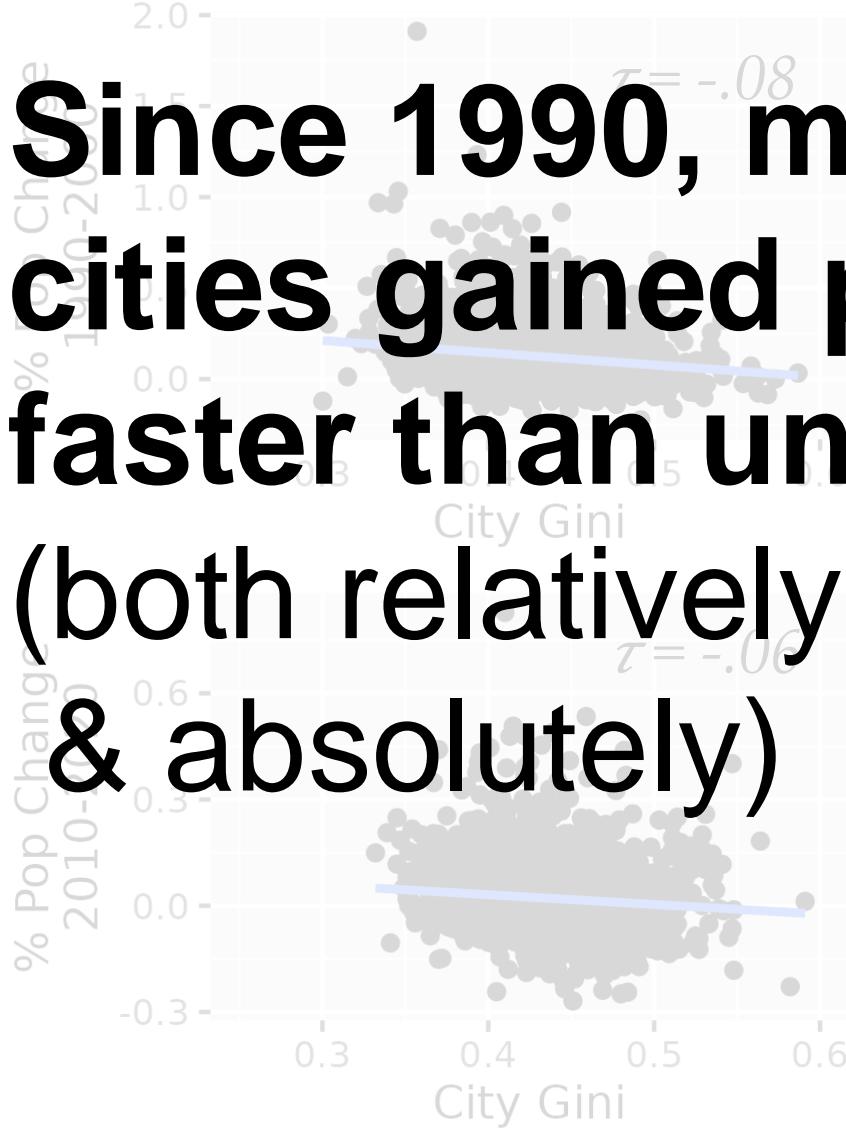


But, mid-sized cities have become more unequal recently

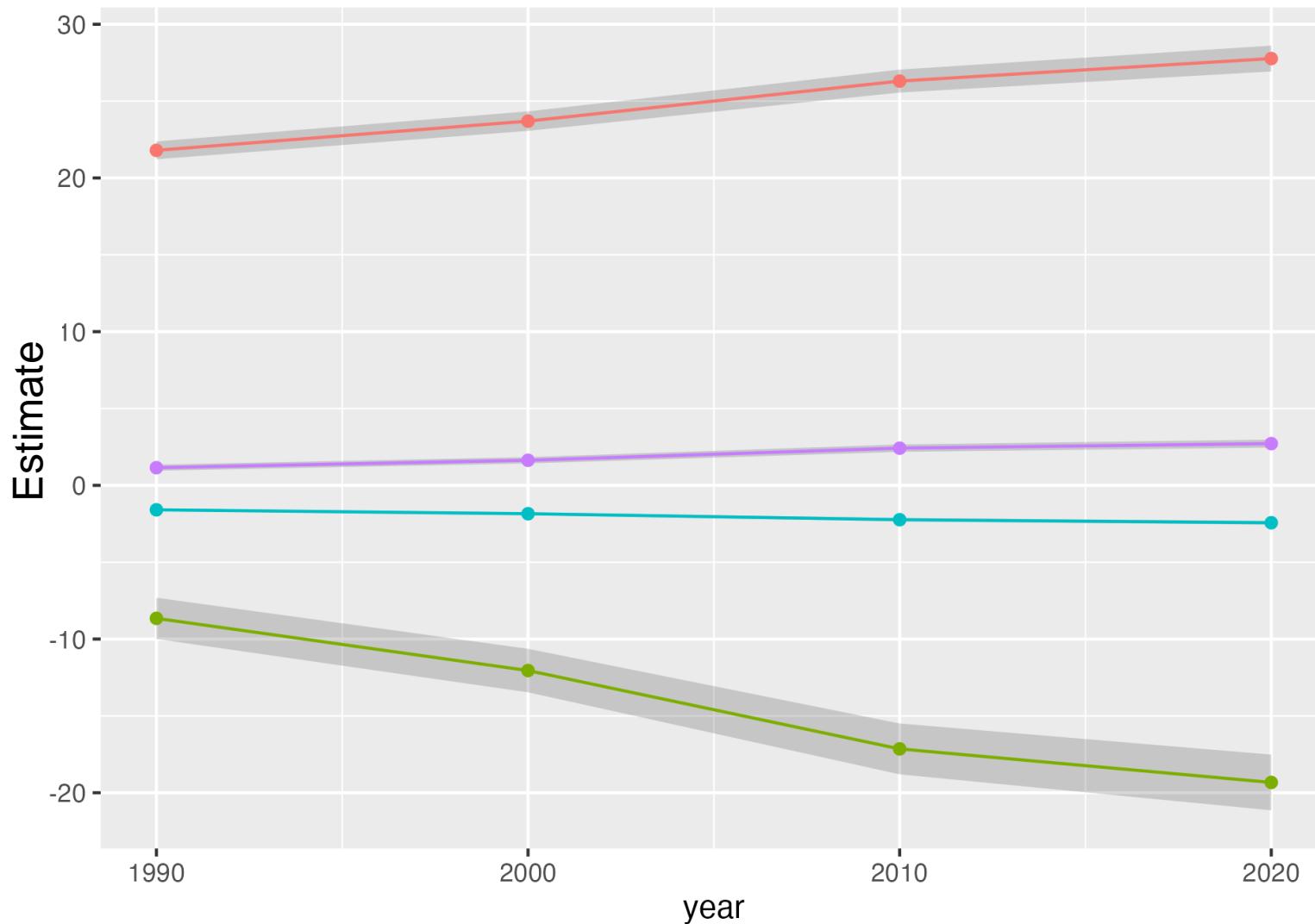


**Since 1990, more equal cities gained population faster than unequal cities**

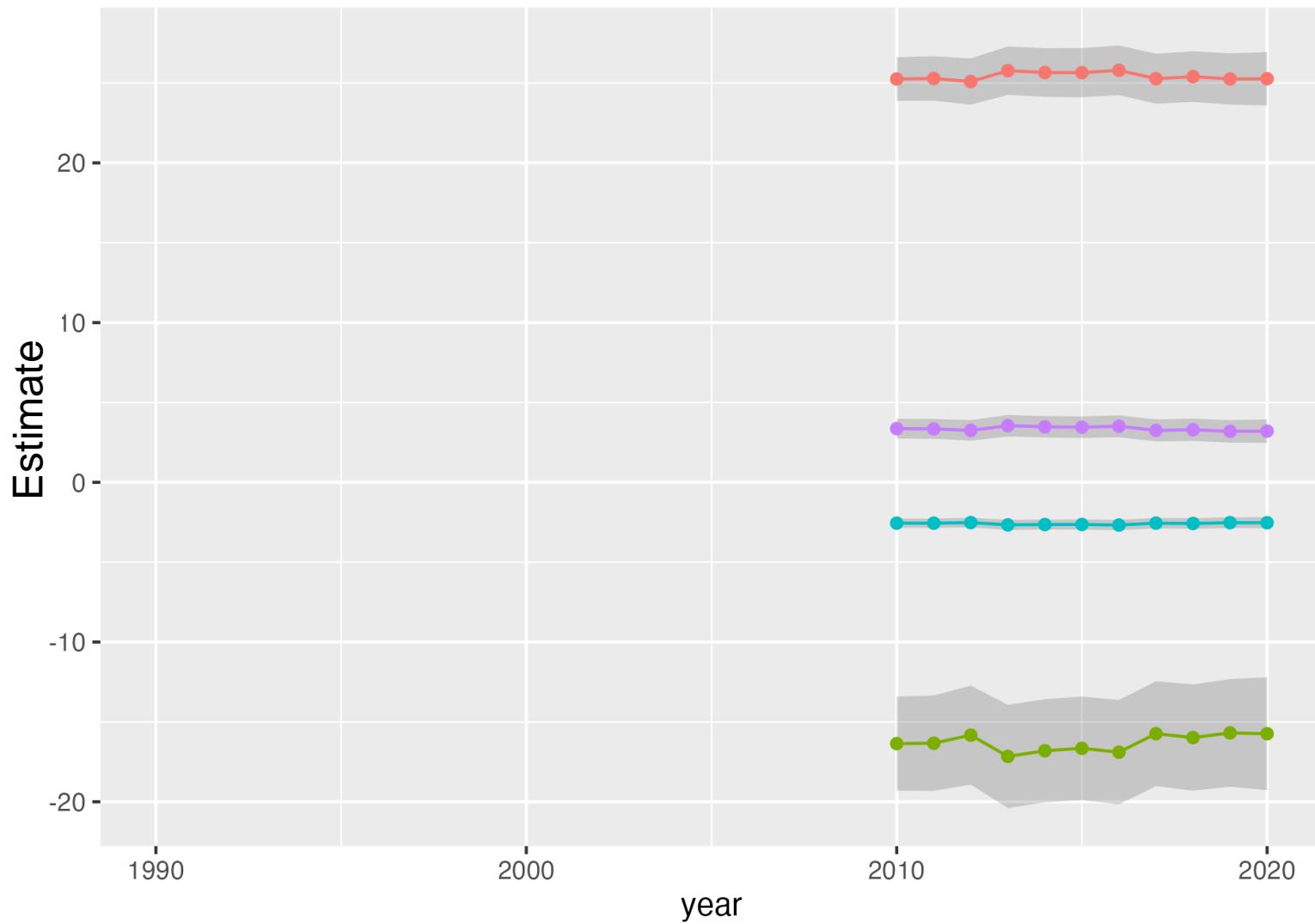
(both relatively & absolutely)



# Decadal estimates by county since 1990

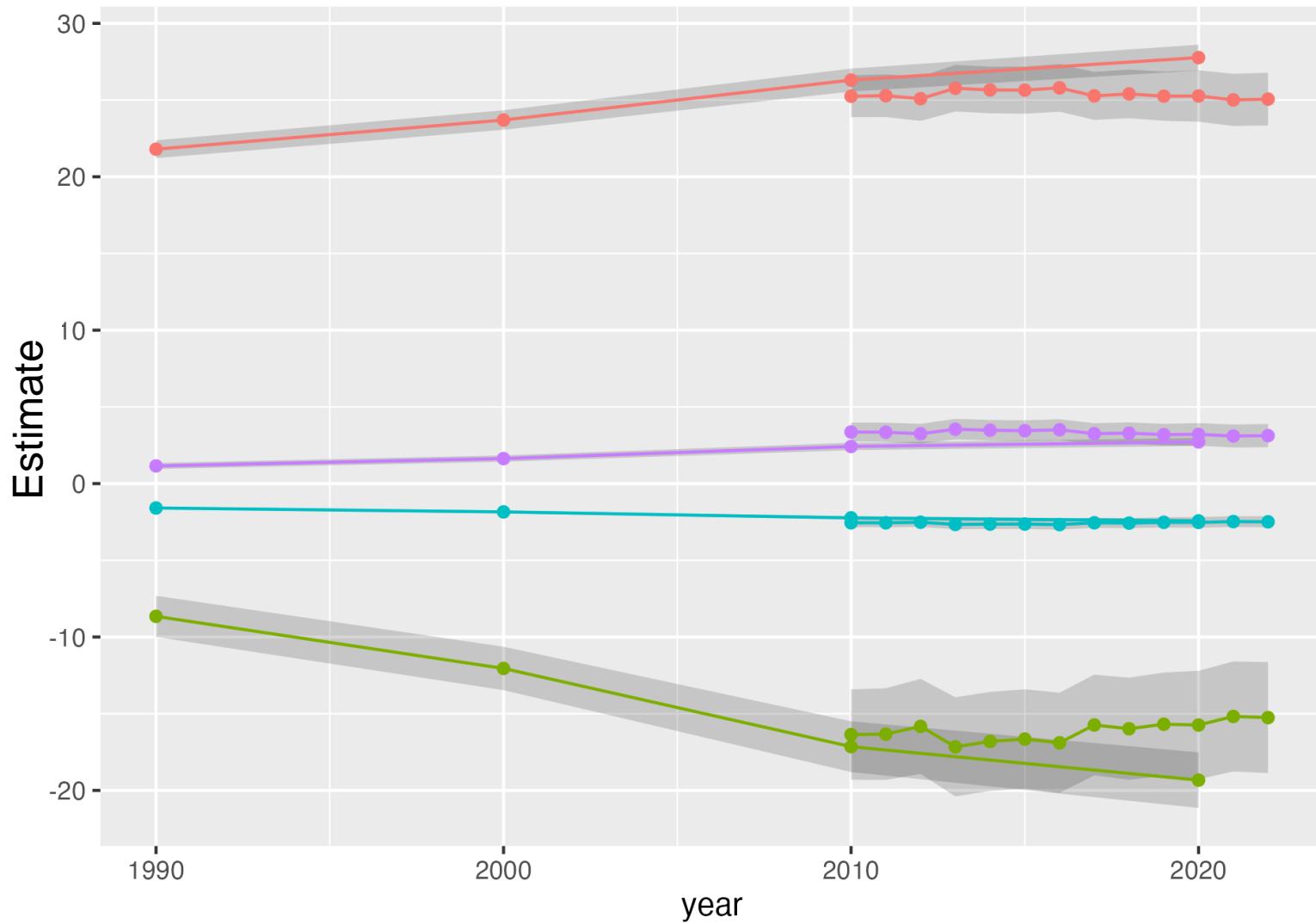


$$\log(\text{pop}) \sim 1 + \log(\text{rank}) + \text{gini} + \log(\text{rank}) * \text{gini}$$



**Yearly  
estimates  
by city  
since 2010**

$$\log(\text{pop}) \sim 1 + \log(\text{rank}) + \text{gini} + \log(\text{rank}) * \text{gini}$$



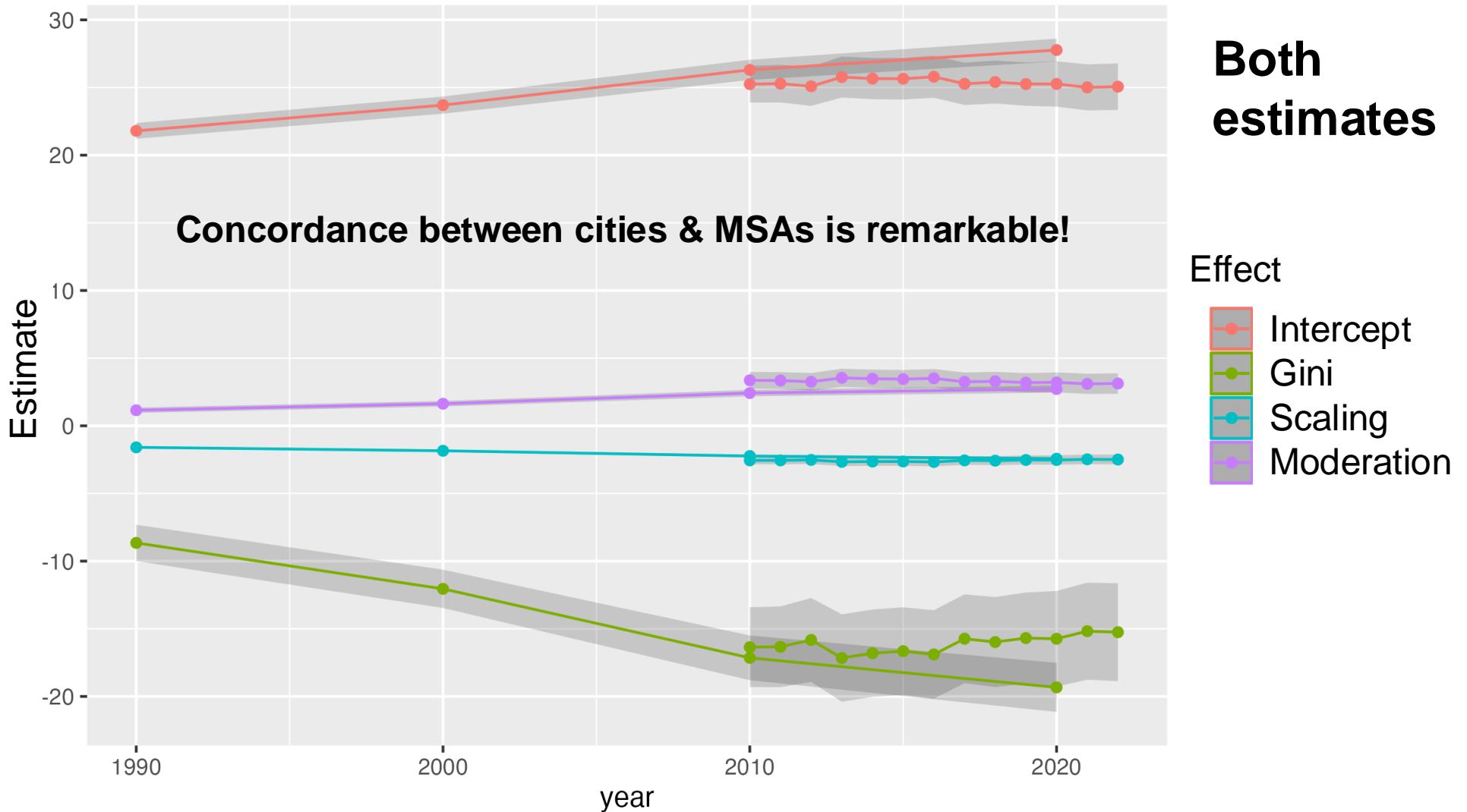
$$\log(\text{pop}) \sim 1 + \log(\text{rank}) + \text{gini} + \log(\text{rank}) * \text{gini}$$

**Both  
estimates**

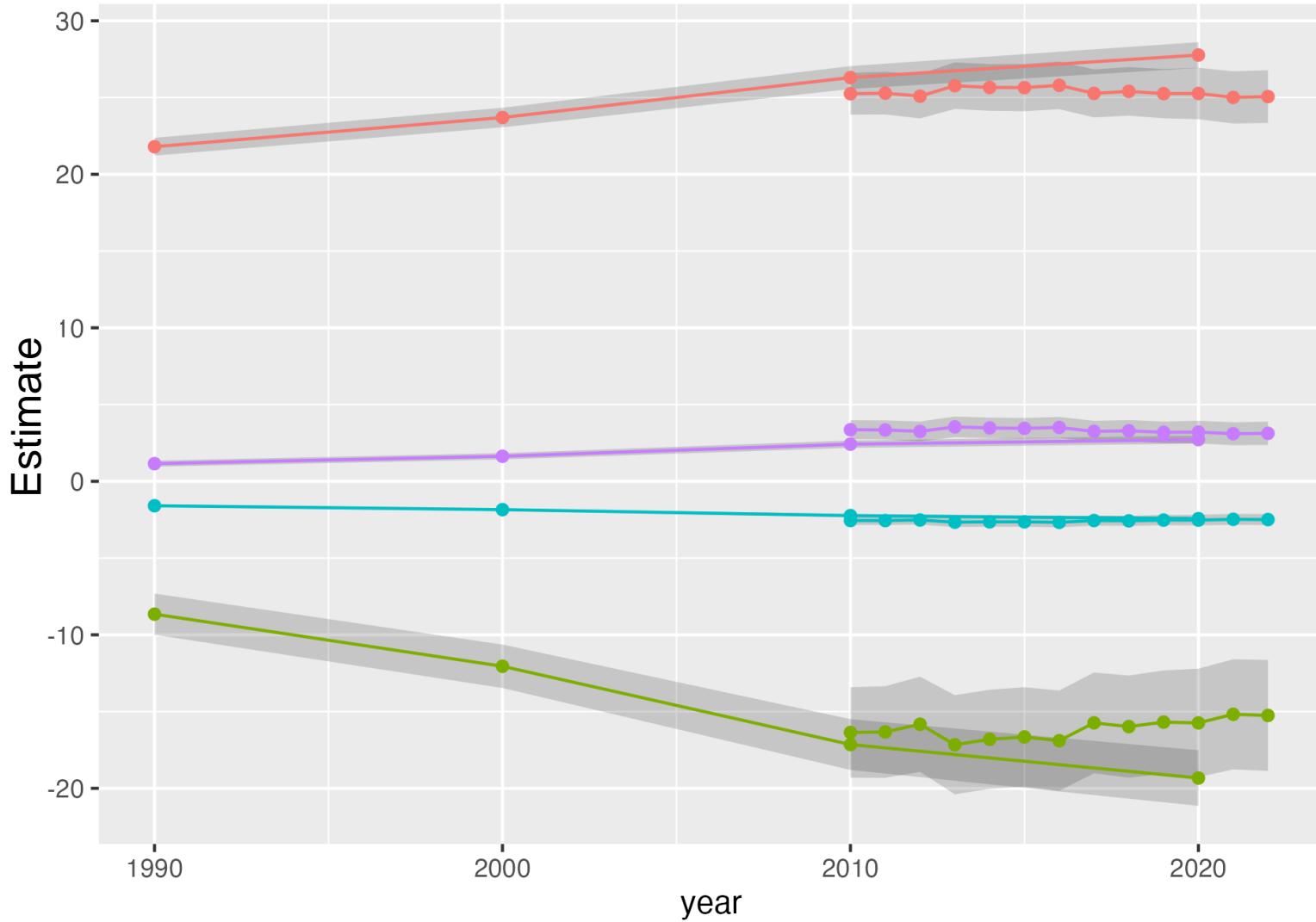
Effect

- Intercept
- Gini
- Scaling
- Moderation

**Both  
estimates**



$$\log(\text{pop}) \sim 1 + \log(\text{rank}) + \text{gini} + \log(\text{rank}) * \text{gini}$$



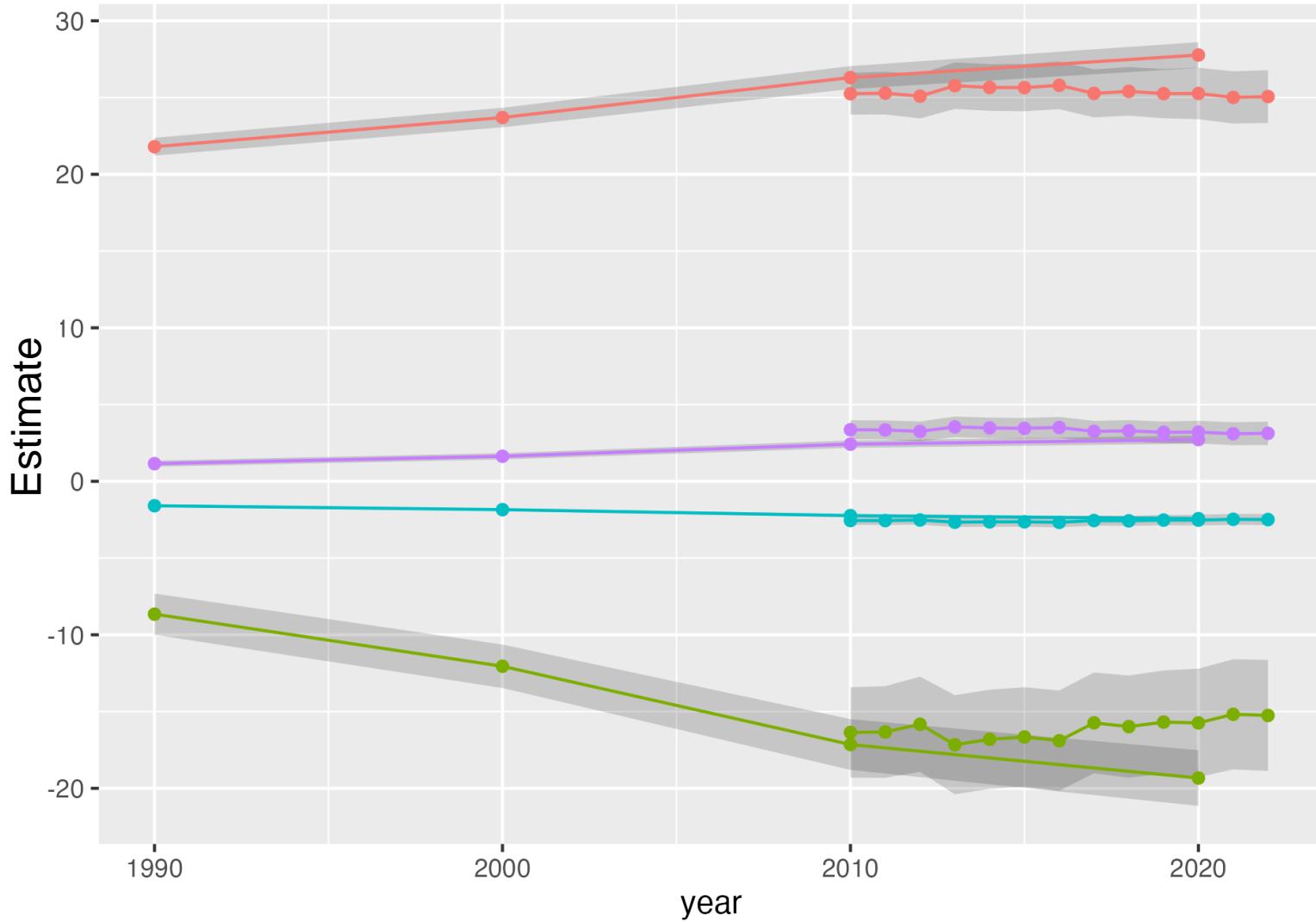
**Both  
estimates**

Effect

- Intercept
- Gini
- Scaling
- Moderation

**Increasing city  
rank decreases  
city population,  
holding city  
inequality fixed**

$$\log(\text{pop}) \sim 1 + \log(\text{rank}) + \text{gini} + \log(\text{rank}) * \text{gini}$$



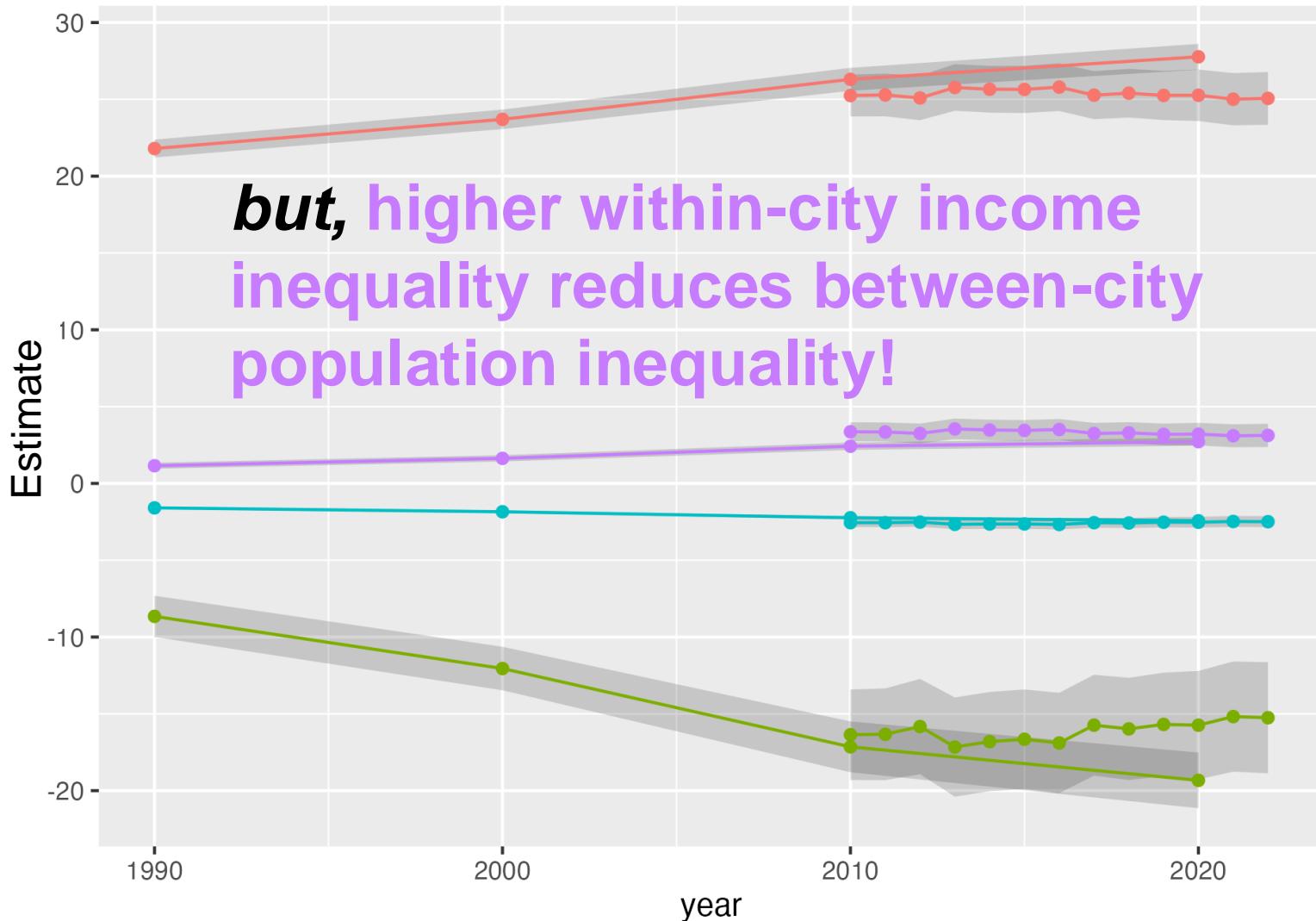
**Both  
estimates**

Effect

- Intercept
- Gini
- Scaling
- Moderation

At a fixed rank in  
the city system,  
unequal cities  
will be smaller...

$$\log(\text{pop}) \sim 1 + \log(\text{rank}) + \text{gini} + \log(\text{rank}) * \text{gini}$$



**Both  
estimates**

Effect

- Intercept
- Gini
- Scaling
- Moderation

At a fixed rank in  
the city system,  
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$$\log(\text{pop}) \sim 1 + \log(\text{rank}) + \text{gini} + \log(\text{rank}) * \text{gini}$$

[nature](#) > [scientific data](#) > [data descriptors](#) > [article](#)

Data Descriptor | [Open access](#) | [Published: 28 February 2024](#)

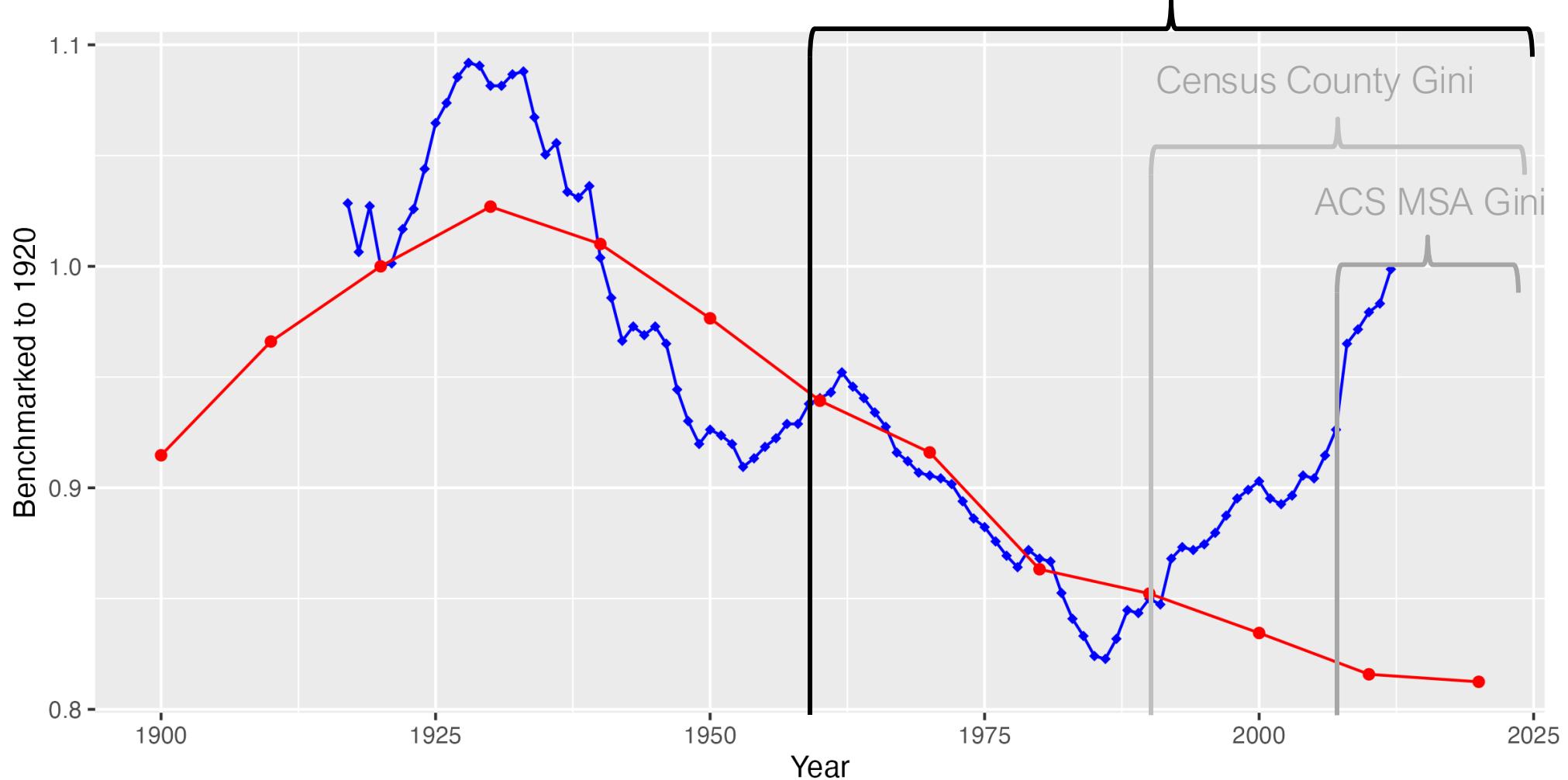
## GEOWEALTH-US: Spatial wealth inequality data for the United States, 1960–2020

[Joel Suss](#), [Tom Kemeny](#)  & [Dylan S. Connor](#)

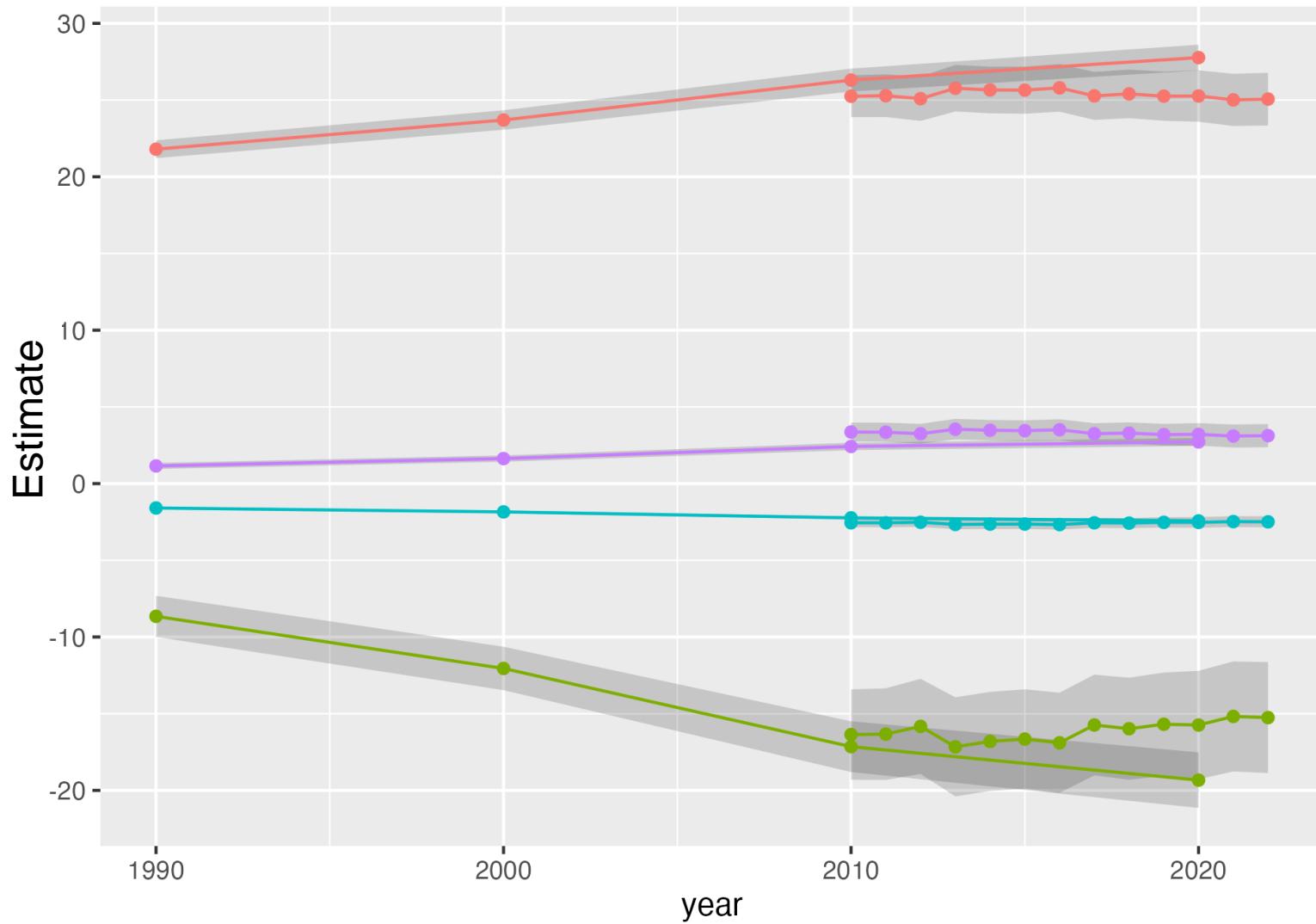
[Scientific Data](#) 11, Article number: 253 (2024) | [Cite this article](#)

481 Accesses | [Metrics](#)

# GEOWEALTH County Gini



Between-city population inequality moved with macro-level income inequality until the 21<sup>st</sup> Century...



$$\log(\text{pop}) \sim 1 + \log(\text{rank}) + \text{gini} + \log(\text{rank}) * \text{gini}$$

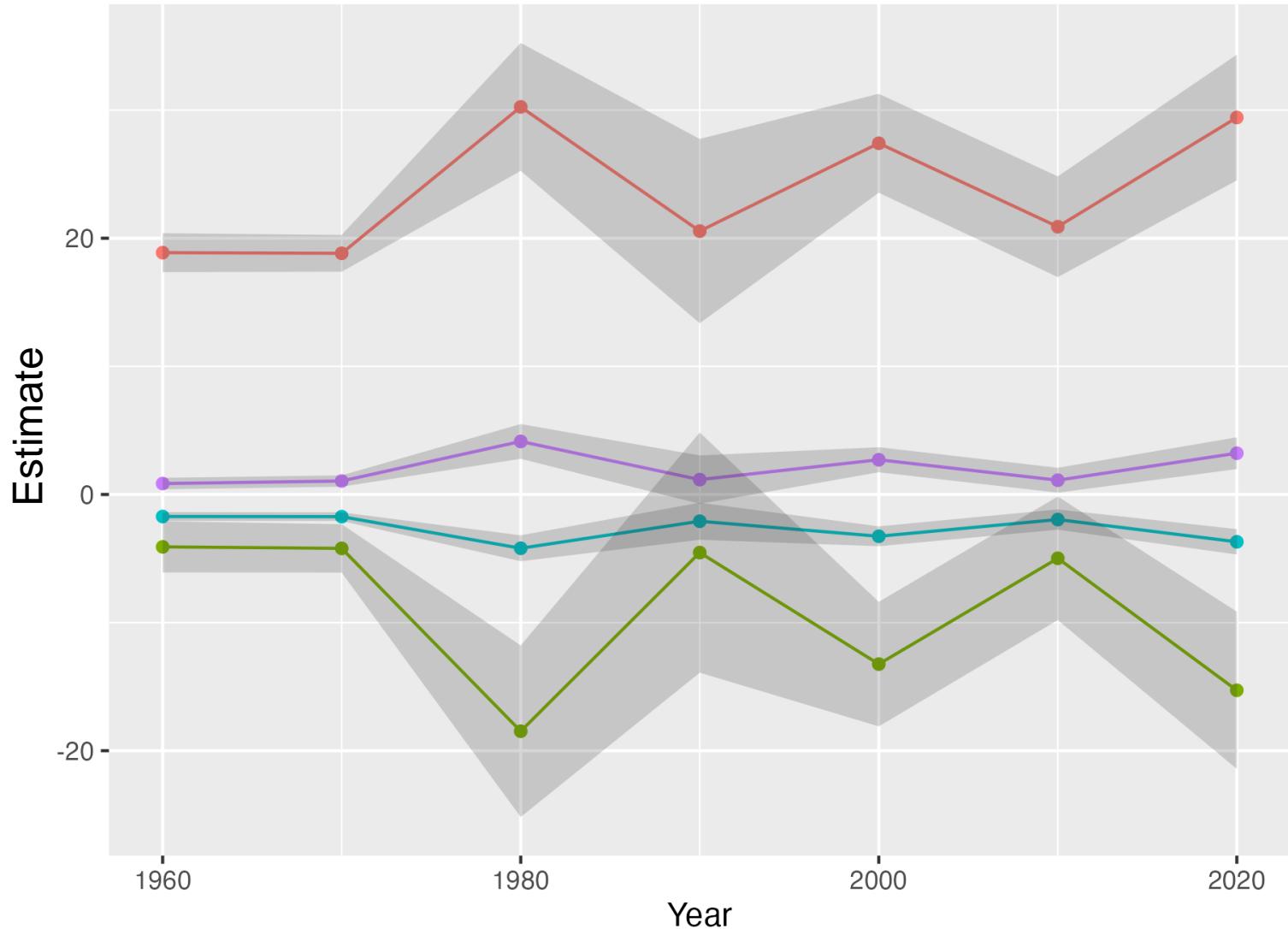
# Both estimates

Effect

- Intercept
- Gini
- Scaling
- Moderation

Holding city rank constant, large income inequality is associated with smaller city population

# Suss et al. estimates



Effect

- Intercept
- Gini
- Scaling
- Moderation

Holding city rank  
constant, large  
income  
inequality is  
associated with  
smaller city  
population

$$\log(\text{pop}) \sim 1 + \log(\text{rank}) + \text{gini} + \log(\text{rank}) * \text{gini}$$

## Suss et al. estimates



Effect

- Intercept
- Gini
- Scaling
- Moderation

Holding city rank constant, large income inequality is associated with smaller city population

$$\log(\text{pop}) \sim 1 + \log(\text{rank}) + \text{gini} + \log(\text{rank}) * \text{gini}$$

## Urban scaling laws arise from within-city inequalities

Arvidsson et al. (2023)

### City size-dependent cumulative advantage

- (1) Cities benefit their inhabitants
- (2) Benefits are unevenly distributed
- (3) In big cities, the top “gets” more from more people
- (4) Disparity compounds over time due to generational transfer and migration
- (5) This top causes scaling to emerge between cities

**Within-city inequality doesn't go hand-in-hand with pop growth**

**Economic inequality**  
no longer associates w/  
**economic opportunity!**

*generative value*

*regularity value*

*How often is this true?*

*contextual value*

*Where/when is this true?*

*generative value*

*How is this true?*

City size distributions often  
follow Zipf's Law...

... in “coherent” urban  
systems ...

... because of within-city  
inequality?

*regularity value*

*How often is this true?*

*contextual value*

*Where/when is this true?*

*generative value*

*How is this true?*

**Lots of data about the present**

**Lots of data about the present, disaggregated by geography or time**

**Lots of data about, disaggregated by geography and time**

# **Causal Values in City Science**

What do we mean by it?

## **Regularity and Contextuality**

City scaling gives an example

## **Moving to generative value**

Winner-take-all city systems?

*Talk outline*

# **CITY SYSTEMS AND THE CAUSE OF INEQUALITY**

**A CITY SCIENCE APPROACH**

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**University of Bristol**

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