人工智能基础习题课

第五、六和十一次作业

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第五次作业

7.12 本习题将考察子句和蕴涵语句之间的关系。

a. 证明子句($\neg P_1 \vee \ldots \vee \neg P_m \vee Q$) 逻辑等价于蕴涵语句($P_1 \wedge \ldots \wedge P_m$) $\Rightarrow Q$。

b. 证明每个子句（不管正文字的数量）都可以写成($P_1 \wedge \ldots \wedge P_m$) $\Rightarrow$ ($Q_1 \vee \ldots \vee Q_n$)的形式，其中 P 和 Q 都是命题符号。由这类语句构成的知识库是表示为**蕴涵范式或称科瓦尔斯基（Kowalski）范式**的。

c. 写出蕴涵范式语句的完整归结规则。

a. $(P_1 \wedge \cdots \wedge P_m) \Rightarrow Q$ 等价于 $\neg(P_1 \wedge \cdots \wedge P_m) \vee Q$（蕴含消去）

$\neg(P_1 \wedge \cdots \wedge P_m)$ 等价于 $(\neg P_1 \vee \cdots \vee \neg P_m)$ （摩根律）

因此，$(P_1 \wedge \cdots \wedge P_m) \Rightarrow Q$ 等价于 $(\neg P_1 \vee \cdots \vee \neg P_m \vee Q)$

b. 一个子句会有一些正文字和负文字，先将它们排列成$(\neg P_1 \vee \cdots \vee \neg P_m \vee Q_1 \vee \cdots \vee Q_n)$，然后设$Q$ 为 $Q_1 \vee \cdots \vee Q_n$，则同a可以得到，$(P_1 \wedge \cdots \wedge P_m) \Rightarrow (Q_1 \vee \cdots \vee Q_n)$。

c. 对于原子语句p_i, q_i, r_i, s_i，其中$p_j = q_k$，

$$\frac{\begin{array}{c} p_1 \wedge \cdots \wedge p_j \wedge \cdots \wedge \mathrm{p}_{n_1} \Rightarrow r_1 \vee \cdots \vee r_{n_2} \\ s_1 \wedge \cdots \wedge s_{n_3} \Rightarrow q_1 \vee \cdots \vee q_k \vee \cdots \vee q_{n_4} \end{array}}{(p_1 \wedge \cdots \wedge p_{j-1} \wedge p_{j+1} \wedge \cdots \wedge \mathrm{p}_{n_1} \wedge s_1 \wedge \cdots \wedge s_{n_3} \Rightarrow r_1 \vee \cdots \vee r_{n_2} \vee q_1 \vee \cdots \vee q_{k-1} \vee q_{k+1} \vee \cdots \vee q_{n_4})}$$

第五次作业

$\equiv$ ✓ $\Leftrightarrow$ ✗

a. $(P_1 \wedge \cdots \wedge P_m) \boxed{\Rightarrow} Q$ $\boxed{\text{等价于}}$ $\neg(P_1 \wedge \cdots \wedge P_m) \vee Q$ （蕴含消去）

$\neg(P_1 \wedge \cdots \wedge P_m)$ 等价于 $(\neg P_1 \vee \cdots \vee \neg P_m)$ （摩根律）

因此，$(P_1 \wedge \cdots \wedge P_m) \Rightarrow Q$ 等价于 $(\neg P_1 \vee \cdots \vee \neg P_m \vee Q)$

b. 一个子句会有一些正文字和负文字，先将它们排列成$(\neg P_1 \vee \cdots \vee \neg P_m \vee Q_1 \vee \cdots \vee Q_n)$，然后设$Q$ 为 $Q_1 \vee \cdots \vee Q_n$，则同a可以得到，$(P_1 \wedge \cdots \wedge P_m) \Rightarrow (Q_1 \vee \cdots \vee Q_n)$。

c. 对于原子语句p_i, q_i, r_i, s_i，其中$p_j = q_k$，

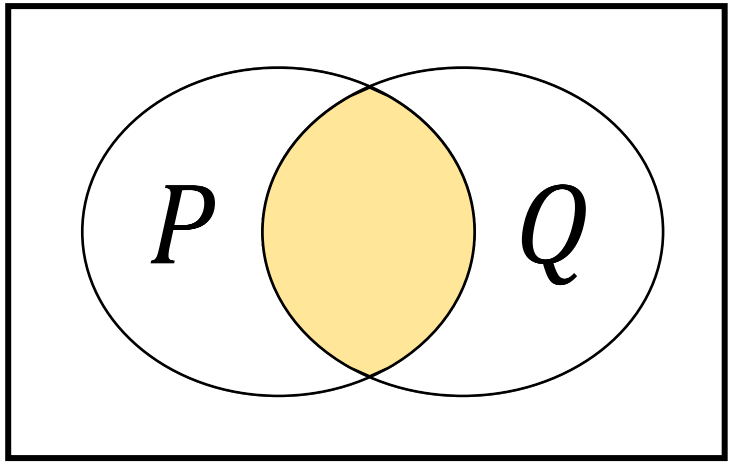
$$p_1 \wedge \cdots \wedge p_j \wedge \cdots \wedge \mathrm{p}_{n_1} \Rightarrow r_1 \vee \cdots \vee r_{n_2}$$

$$s_1 \wedge \cdots \wedge s_{n_3} \Rightarrow q_1 \vee \cdots \vee q_k \vee \cdots \vee q_{n_4}$$

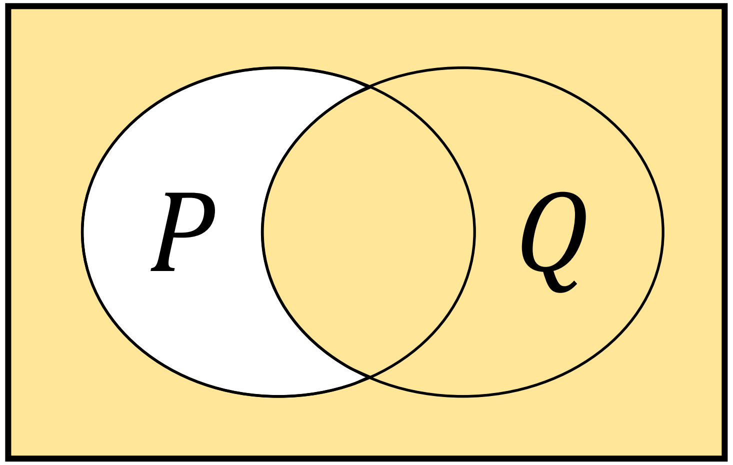
$$(p_1 \wedge \cdots \wedge p_{j-1} \wedge p_{j+1} \wedge \cdots \wedge \mathrm{p}_{n_1} \wedge s_1 \wedge \cdots \wedge s_{n_3} \Rightarrow r_1 \vee \cdots \vee r_{n_2} \vee q_1 \vee \cdots \vee q_{k-1} \vee q_{k+1} \vee \cdots \vee q_{n_4})$$

第五次作业

$$P \wedge Q \longrightarrow$$



$$P \Rightarrow Q \longrightarrow$$



第五次作业

- 证明前向链算法的完备性

很容易看出，前向链接是**可靠**的：每个推理本质上是分离规则的一个应用。前向连接也是**完备**的：每个被蕴涵的原子语句都将得以生成。验证这一点的最简单方法是考察 *inferred* 表的最终状态（在算法到达**不动点**以后，不可能再出现新的推理）。该表对于在推理过程中参与推理的每个符号都包含 *true*，而所有其它的符号为 *false*。我们可以把该推理表看作一个逻辑模型；而且，*原始 KB 中的每个确定子句在该模型中都为真*。为了看到这一点，假定相反情况成立，也就是说某个子句 $a_1 \wedge \ldots \wedge a_k \Rightarrow b$ 在模型中为假。那么 $a_1 \wedge \ldots \wedge a_k$ 在模型中必须为真，b 在模型中必须为假。但这和我们的假设，即算法已经到达一个不动点相矛盾！因而，我们可以得出结论，在不动点推理的原子语句集定义了原始 *KB* 的一个模型。此外，被 *KB* 蕴涵的任一原子语句 q 在它的所有模型中必须为真，尤其是这个模型。因此，每个被蕴涵的语句 q 必定会被算法推断出来。

第五次作业

Proof of completeness（完备性）

FC可推出每个被KB蕴涵的原子语句

1. FC到达不动点以后，不可能再出现新的推理。
2. 考察inferred表的最终状态，参与推理过程的每个符号为true，其它为false。把该推理表看做一个逻辑模型*m*
3. *原始KB中的每个确定子句在该模型m中都为真*
   证明：假设某个子句 $a_1 \wedge \ldots \wedge a_k \Rightarrow b$ 在*m*中为false
   那么 $a_1 \wedge \ldots \wedge a_k$ 在*m*中为true，b 在*m*中为false
   与算法已经到达一个不动点相矛盾
4. m是*KB*的一个模型
5. 如果 $KB \models q$，q在*KB*的所有模型中必须为真，包括*m*
6. q在*m*中为真→在inferred表中为真→被FC算法推断出来

http://staff.ustc.edu.cn/~linlixu/ai2022spring/ai2022spring.html

$Takes(x, c, s)$: student x takes course c in semester s;
$Passes(x, c, s)$: student x passes course c in semester s;
$Score(x, c, s)$: the score obtained by student x in course c in semester s;
$x > y$: x is greater than y;
F and G: specific French and Greek courses (one could also interpret these sentences as referring to *any* such course, in which case one could use a predicate $Subject(c, f)$ meaning that the subject of course c is field f;

a. Some students took French in spring 2001.
$$\exists\, x \;\; Student(x) \wedge Takes(x, F, Spring2001).$$
b. Every student who takes French passes it.
$$\boxed{\forall\, x, s} \; Student(x) \wedge Takes(x, F, s) \; \boxed{\Rightarrow} \; Passes(x, F, s).$$
c. Only one student took Greek in spring 2001.
$$\exists\, x \;\; Student(x) \wedge Takes(x, G, Spring2001) \wedge \forall\, y \;\; y \neq x \Rightarrow \neg Takes(y, G, Spring2001).$$
d. The best score in Greek is always higher than the best score in French.
$$\forall\, s \;\; \exists\, x \;\; \forall\, y \;\; Score(x, G, s) > Score(y, F, s).$$
e. Every person who buys a policy is smart.
$$\forall\, x \;\; Person(x) \wedge (\exists\, y, z \;\; Policy(y) \wedge Buys(x, y, z)) \;\Rightarrow\; Smart(x).$$
f. No person buys an expensive policy.
$$\forall\, x, y, z \;\; Person(x) \wedge Policy(y) \wedge Expensive(y) \;\Rightarrow\; \neg Buys(x, y, z).$$
g. There is an agent who sells policies only to people who are not insured.
$$\exists\, x \;\; Agent(x) \wedge \forall\, y, z \;\; Policy(y) \wedge Sells(x, y, z) \;\Rightarrow\; (Person(z) \wedge \neg Insured(z)).$$
h. There is a barber who shaves all men in town who do not shave themselves.
$$\exists\, x \;\; Barber(x) \wedge \forall\, y \;\; Man(y) \wedge \neg Shaves(y, y) \;\Rightarrow\; Shaves(x, y).$$

第六次作业

$$\forall x \;\; \mathrm{At}(x, \mathrm{USTC}) \;\wedge\; \mathrm{Smart}(x)$$ 🤔

$$\forall x \;\; \mathrm{At}(x, \mathrm{USTC}) \Rightarrow \mathrm{Smart}(x)$$ 😊

$$\exists x \;\; \mathrm{At}(x, \mathrm{USTC}) \;\wedge\; \mathrm{Smart}(x)$$ 😊

$$\exists x \;\; \mathrm{At}(x, \mathrm{USTC}) \Rightarrow \mathrm{Smart}(x)$$ 🤔

$Takes(x, c, s)$: student x takes course c in semester s;
$Passes(x, c, s)$: student x passes course c in semester s;
$Score(x, c, s)$: the score obtained by student x in course c in semester s;
$x > y$: x is greater than y;
F and G: specific French and Greek courses (one could also interpret these sentences as referring to *any* such course, in which case one could use a predicate $Subject(c, f)$ meaning that the subject of course c is field f;

a. Some students took French in spring 2001.
$\exists x \;\; Student(x) \land Takes(x, F, Spring2001).$

b. Every student who takes French passes it.
$\forall x, s \;\; Student(x) \land Takes(x, F, s) \;\Rightarrow\; Passes(x, F, s).$

c. Only one student took Greek in spring 2001.
$\exists x \;\; Student(x) \land Takes(x, G, Spring2001) \land \forall y \;\; y \neq x \Rightarrow \neg Takes(y, G, Spring2001).$

d. The best score in Greek is always higher than the best score in French.
$\boxed{\forall s \;\; \exists x \;\; \forall y} \; Score(x, G, s) > Score(y, F, s).$

e. Every person who buys a policy is smart.
$\forall x \;\; Person(x) \land (\exists y, z \;\; Policy(y) \land Buys(x, y, z)) \;\Rightarrow\; Smart(x).$

f. No person buys an expensive policy.
$\forall x, y, z \;\; Person(x) \land Policy(y) \land Expensive(y) \;\Rightarrow\; \neg Buys(x, y, z).$

g. There is an agent who sells policies only to people who are not insured.
$\exists x \;\; Agent(x) \land \forall y, z \;\; Policy(y) \land Sells(x, y, z) \;\Rightarrow\; (Person(z) \land \neg Insured(z)).$

h. There is a barber who shaves all men in town who do not shave themselves.
$\exists x \;\; Barber(x) \land \forall y \;\; Man(y) \land \neg Shaves(y, y) \;\Rightarrow\; Shaves(x, y).$

第六次作业

$$\forall x \ \ \exists y \ \ x < y$$
$$\exists y \ \ \forall x \ \ x < y$$

$Buys(x, y, z)$: x buys y from z (using a binary predicate with unspecified seller is OK but less felicitous);
$Sells(x, y, z)$: x sells y to z;
$Shaves(x, y)$: person x shaves person y
$Born(x, c)$: person x is born in country c;

e. Every person who buys a policy is smart.
$\forall x \;\; Person(x) \land (\exists y, z \;\; Policy(y) \land Buys(x, y, z)) \;\Rightarrow\; Smart(x).$

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$\exists x \;\; Barber(x) \land \forall y \;\; Man(y) \land \neg Shaves(y, y) \;\Rightarrow\; Shaves(x, y).$

i. A person born in the UK, each of whose parents is a UK citizen or a UK resident, is a UK citizen by birth.

$$\forall x\ \boxed{Person(x)} \wedge \boxed{Born(x, UK)} \wedge \boxed{(\forall y\ \ Parent(y, x) \Rightarrow ((\exists r\ \ Citizen(y, UK, r)) \vee Resident(y, UK)))} \Rightarrow Citizen(x, UK, Birth).$$

j. A person born outside the UK, one of whose parents is a UK citizen by birth, is a UK citizen by descent.

$$\forall x\ \ \boxed{Person(x)} \wedge \boxed{\neg Born(x, UK)} \wedge \boxed{(\exists y\ \ Parent(y, x) \wedge Citizen(y, UK, Birth))}$$
$$\Rightarrow\ Citizen(x, UK, Descent).$$

k. Politicians can fool some of the people all of the time, and they can fool all of the people some of the time, but they can't fool all of the people all of the time.

$$\begin{aligned}
&\forall x\ \ Politician(x)\ \Rightarrow \\
&\qquad (\exists y\ \ \forall t\ \ Person(y) \wedge Fools(x, y, t)) \wedge \\
&\qquad (\exists t\ \ \forall y\ \ Person(y)\ \Rightarrow\ Fools(x, y, t)) \wedge \\
&\qquad \neg(\forall t\ \ \forall y\ \ Person(y)\ \Rightarrow\ Fools(x, y, t))
\end{aligned}$$

$Born(x, c)$: person x is born in country c;
$Parent(x, y)$: x is a parent of y;
$Citizen(x, c, r)$: x is a citizen of country c for reason r;
$Resident(x, c)$: x is a resident of country c;
$Fools(x, y, t)$: person x fools person y at time t;

第六次作业

8.15 解释下面给出的 wumpus 世界中相邻方格的定义存在什么问题：

$$\forall x, y \quad Adjacent([x, y], [x+1, y]) \land Adjacent([x, y], [x, y+1])$$

1. 仅考虑上相邻格和右相邻格：Adjacent([1,2], [1,1])作为相邻格不符合定义
2. 忽略边界情况
3. 必须写为有“⇔”的形式

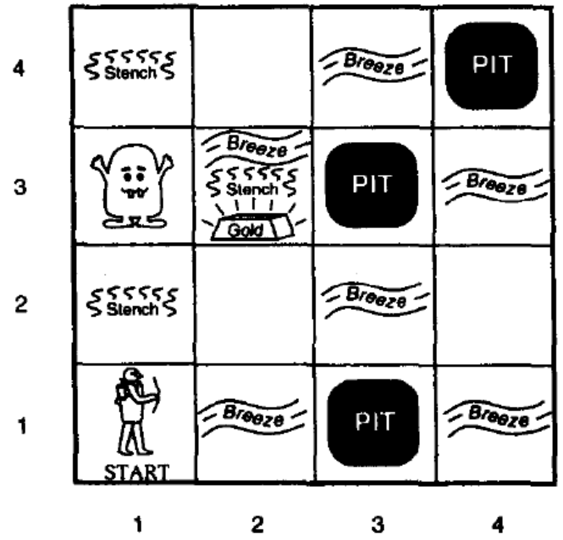

图 7.2 一个典型的 wumpus 世界。智能体位于左下角

第十一次作业

已知正例点 $x_1=(1,2)^{\mathrm{T}}, x_2=(2,3)^{\mathrm{T}}, x_3=(3,3)^{\mathrm{T}}$，负例点 $x_4=(2,1)^{\mathrm{T}}, x_5=(3,2)^{\mathrm{T}}$，试求Hard Margin SVM的最大间隔分离超平面和分类决策函数，并在图上画出分离超平面、间隔边界及支持向量。

第十一次作业

假设超平面 $w^T x + b = 0$

$$\begin{cases} \min \frac{1}{2}\|w\|^2 \\ s.t.\ \ y_i(w^T x_i + b) \geq 1 \end{cases}$$

$x_1 = \begin{pmatrix}1\\2\end{pmatrix}$ $x_2 = \begin{pmatrix}2\\3\end{pmatrix}$ $x_3 = \begin{pmatrix}3\\3\end{pmatrix}$

$x_4 = \begin{pmatrix}2\\1\end{pmatrix}$ $x_5 = \begin{pmatrix}3\\2\end{pmatrix}$ $y = (1,1,1,-1,-1)$

拉格朗日对偶

$$\max_{\alpha} \sum_{i=1}^{n} \alpha_i - \frac{1}{2} \sum_{i,j=1}^{n} \alpha_i \alpha_j y_i y_j ({x_i}^T x_j)$$

subject to $\alpha_i \geq 0$, $\sum_{i=1}^{n} \alpha_i y_i = 0$

$\alpha_1 + \alpha_2 + \alpha_3 - \alpha_4 - \alpha_5 = 0$ 代入目标函数，有

$$\max\ \ 2(\alpha_1 + \alpha_2 + \alpha_3) - \frac{1}{2}(4{\alpha_1}^2 + 2\,{\alpha_2}^2 + {\alpha_3}^2 + 2{\alpha_4}^2 + 4\alpha_1\alpha_2 + 6\alpha_1\alpha_4 + 2\alpha_2\alpha_3 + 2\alpha_3\alpha_4)$$

第十一次作业

首先，求解上式各偏导/梯度 = 0

$$\alpha_1 = -0.4<0, \ \alpha_2 = 1.2, \ \alpha_3 = \alpha_4 = 0$$

不满足约束条件，故最大值一定在可行域的边界处取到；又从上述目标函数可以直观地看出最大值处 α_4 必为0 （因为含有α_4的项为 $-\alpha_4{}^2 - 3\alpha_1\alpha_4$- $\alpha_3\alpha_4$ ）
因此上式简化为

$$\max \ \ 2(\alpha_1+\alpha_2+\alpha_3)-\frac{1}{2}(4\alpha_1{}^2+2\ \alpha_2{}^2+\alpha_3{}^2+4\alpha_1\alpha_2+2\alpha_2\alpha_3)$$

求解上式各偏导/梯度=0，无解；故α_1 , α_2 α_3至少有一个为0，即边界值

α_1 =0时， 求解偏导为0，可得 α_2 =0 ， α_3 =2，f=2

α_2=0时， 求解偏导为0，可得 α_1 =0.5 ， α_3 =2，f=2.5

α_3 =0时， 求解偏导为0，可得 α_1 =0 ， α_2 = 1，f=1

第十一次作业

$$\alpha_1 = \frac{1}{2} \quad \alpha_2 = 0 \quad \alpha_3 = 2 \quad \alpha_4 = 0 \quad \alpha_5 = \frac{5}{2}$$

$$w = \sum \alpha_i y_i x_i = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad b = -2$$

决策函数：f(x)=sign(-$\boldsymbol{x_1 + 2x_2 - 2}$）