Exam

Algorithms on Sequences

Definition. For a natural number t, a word u is called a non-trivial t-power if there exists a non-empty word v such that $u = v^t$.

A. Given a word w and a number t, compute the number of distinct factors $w[i..j]$]of w such that $w[i..j]$ is a non-trivial ℓ-power, for some $\ell \geq 2$.

Complexity: $O(n^2 \log\log n)$, but everything strictly faster than $O(n^3)$ is fine.

Example: For $w = a^{10} = aaaaaaaaaa$ and $t = 3$, the **distinct** factors of w which are non-trivial 3-powers are $a^3 = a \cdot a \cdot a$, $a^6 = aa \cdot aa \cdot aa$ and $a^9 = aaa \cdot aaa \cdot aaa$. So, there are 3 distinct 3-powers. Note that, each of these non-trivial 3-powers occurs multiple times in w (for instance, a^3 occurs on positions $1, 2, \ldots, 8$ of w), but each of them is counted exactly once in the answer. One can count similarly the distinct 2-powers, 4-powers, 5-powers, etc, but one must pay attention to not count a factor twice: for instance, a^6 is a 6-power but also both a 2-power and a 3-power.