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EXPERIMENTAL INVESTIGATION OF FLAME-HOLDING CAPABILITY OF HYDROGEN TRANSVERSE JET IN SUPERSONIC CROSS-FLOW

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This paper describes an experimental effort to characterize the flame-holding process of a hydrogen jet injected into a high total enthalpy supersonic cross flow. An expansion tube is used to provide a correct simulation of true flight combustion chemistry, including ignition delay and reaction times. This approach permitted a number of unique experiments involving acceleration of radical-free air to high total enthalpies.

The experiments were designed to map the near-field flow characteristics and autoignition process of an underexpanded transverse hydrogen jet injected into flight-Mach number 10 and 13 total enthalpy flow conditions. Flow visualization techniques included planar laser-induced fluorescence (PLIF) of OH and schlieren imaging applied simultaneously. Schlieren images show the shock structure around the jet and the periodically formed coherent structures in the jet–free-stream interface. Overlaid OH-PLIF and schlieren images allow characterization of the autoignition of a hydrogen jet in air cross flow for different jet-to-free-stream momentum flux ratios at both flow conditions. Transverse jet autoignition and flame-holding characteristics observed in both side view and top view images by OH-PLIF reveal differences with previous results in the literature. In the present experiments, the first OH signals are obtained in the recirculation region upstream of the jet exit and in the bow shock region, while in past experiments with similar geometry but lower total enthalpy conditions, no strong OH signal was observed within the first 10 jet diameters. The OH-PLIF results for Mach 10 conditions also show that the OH signal level decreases significantly as the mixture expands around the jet flow field, indicating a partial quenching of the ignition. This indicates that combustion of hydrogen and air in these high total enthalpy conditions is a mixing-limited process. It is evident from the results that improved injection schemes will be required for practical applications in scramjet engines.

Introduction

The success of future hypersonic air-breathing propulsion systems will be largely dependent on efficient injection, mixing, and combustion processes inside a supersonic or hypersonic combustion chamber. To promote efficient performance of this type of very high speed system, the air must be compressed in the diffuser to minimum velocities in order to increase the flow residence time and therefore to allow a combustor of reasonable length. However, the minimum burner entry velocity (Mach number) is restricted by the maximum allowable compression temperature T_3 (in the range of 1440–1670 K as noted in Ref. [1]) to prevent excessive dissociation in the exhaust flow. Under these constraints, typical values of burner entry Mach number, M_3, can be calculated as a function of flight Mach number, M_0 (see Fig. 1.) As shown in this figure, for hypersonic flight (beyond $M_0 > 6$), a supersonic combustion ramjet (scramjet) where the flow remains supersonic and hypersonic throughout the engine should be considered.

Most supersonic combustion research in the open literature has focused on flight speeds of Mach 8 and below [2–7], and there are relatively few works that have been performed for higher flight Mach numbers [8–11]. The experimental simulation in ground facilities frequently utilizes combustion heating and oxygen replenishment (vitiation heating) as a means of generating a supersonic flow test environment. These facilities have long test times (~10 s); however their application is limited to low-speed flight conditions (at flight Mach 8, the air kinetic energy and potential combustion heat release are roughly equal). Therefore, due to the large enthalpies associated with high flight Mach numbers (beyond Mach 8), only impulse facilities are capable of providing the required total temperature and Mach number to replicate a combustor environment. Of the possible choices for an impulse facility, an expansion tube is uniquely attractive due to its potential to provide a wide range of total enthalpies without exposing the flow to high temperatures and pressures. With this approach, a correct simulation of the true flight combustion chemistry can be achieved to study autoignition and flame-holding characteristics of different

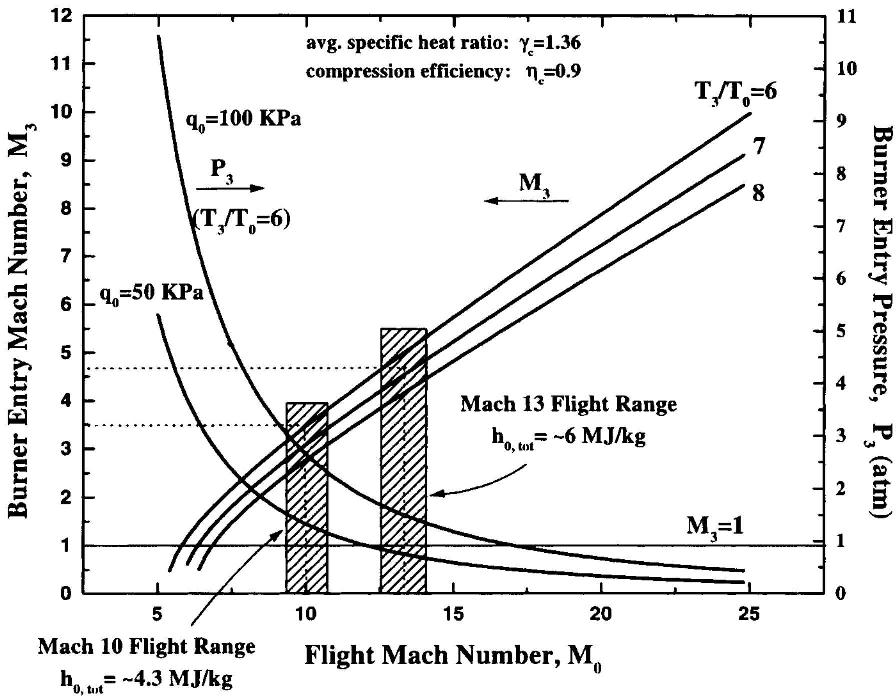

FIG. 1. Typical scramjet burner entry conditions. The burner entry Mach number, M_3, is calculated for different temperature ratios, T_3/T_0, assuming adiabatic compression. The burner entry pressure, P_3, is evaluated assuming fixed dynamic pressure, $q_0 = 1/2\ \rho_0 v_0^2$ (50 and 100 Kpa) and compression efficiency of 0.9 (Ref. [1]). A detailed explanation of this calculation can be found in Ref. [14].

fuel-injection schemes, though with limited test times (~0.3–0.5 ms). In the present study, the Stanford expansion tube facility is used to generate total enthalpy conditions in the Mach 10 and Mach 13 flight range (see Fig. 1).

Flame-holding and mixing are critical issues in the design of supersonic/hypersonic combustors, and these must be considered as coupled phenomena. In general, mixing and flame stabilization can be achieved with the following techniques: (1) by organization of an upstream recirculation area, (2) by formation of coherent structures containing unmixed fuel and air, where a diffusion flame occurs as the gases are convected downstream, and (3) by interaction with a shock wave.

Transverse injection, a commonly used flame stabilization scheme in supersonic combustors, applies two of the three foregoing techniques. A schematic of the underexpanded transverse injection flow field appears in Fig. 2. This illustration describes the qualitative features of the injection flow field where the supersonic cross-flow is displaced by the fuel jet as if a bluff body was inserted into the flow. As a result, a three-dimensional bow shock upstream of the injector exit is formed causing the upstream wall boundary layer to separate. In the separation region, the boundary layer and jet fluids mix subsonically upstream of the jet exit. This region is important in transverse injection because of its flame-holding capability in combusting situations.

Transverse injection schemes have two main points where the ignition is likely to occur. The region behind the jet bow shock where high temperatures and pressures are obtained, and the recirculation regions ahead of and behind the base of the jet providing long residence time and high temperatures. The residence time of the hydrogen-air mixture in the bow shock region will be short, since the mixture expands around the jet flowfield immediately after compression in the bow shock. For the higher Mach number flows, however, ignition may still be initiated in this region owing to the relatively high static temperature, though a more likely place for ignition to occur is in the recirculation region upstream the jet exit.

For scramjet combustors, where relatively cold hydrogen is injected into hot air, there will be a significant variation of temperature with equivalence ratio (ϕ) through the mixing layer around the jet. Because the temperature of the mixture will be higher at low equivalence ratios, and because ignition time is a

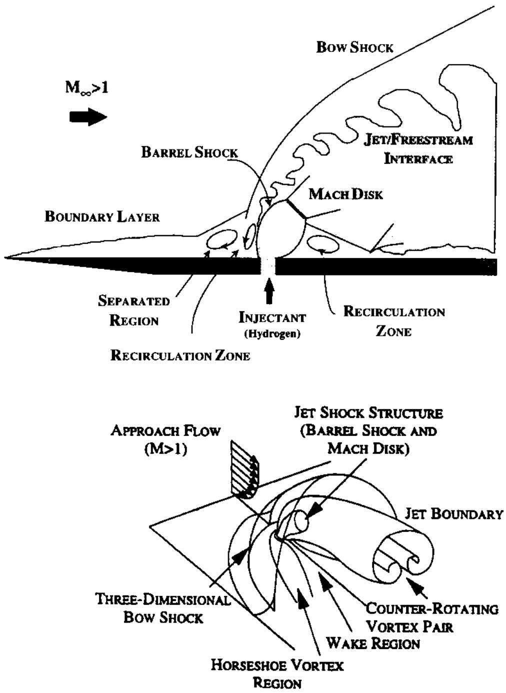

FIG. 2. Schematic of underexpanded transverse injection into supersonic flowfield: (a) side view at the centerline axis of the jet and (b) 3-D perspective (Gruber et al. [5]).

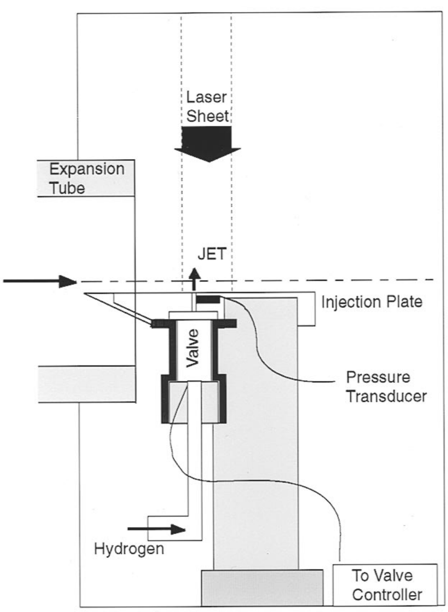

FIG. 3. Injection system schematic.

strong function of the mixture temperature, it is expected that the self-ignition point will be on the lean side of the mixing layer around the jet ($\phi \cong 0.2$ as noted in Ref. [12]).

In order for self-ignition (and therefore combustion) to be accomplished in a flowing combustible mixture, it is necessary that four quantities have suitable values: static temperature, static pressure, fuel–air mixture, and the residence time at these conditions. In a reacting system, ignition is considered accomplished when sufficient free radicals are formed to initiate the reaction system, even though no appreciable heat has yet been released. When the conditions of spontaneous ignition exist, the distance l_i at which it occurs in a medium flowing at a velocity u is:

$$l_i = u\tau_i$$

Because the ignition delay time, τ_i, varies inversely with pressure, because of the two body reactions involved in the ignition chemistry of hydrogen and air, the product $\tau_i p$ is reasonably constant for a given temperature and fuel–air equivalence ratio. This allows use of the binary scaling law for the ignition of hydrogen–air mixtures in the following form:

$$\frac{pl_i}{u} \approx \text{constant}$$

which means that for a given flow velocity and temperature, the ignition characteristics of hydrogen–air are the same for different scaled geometries. Therefore, the correct simulation of temperature and velocity is essential and generally sufficient to study the ignition capability of a given geometry.

The main objective of this study was to investigate the combustion characteristics of a hydrogen transverse jet in order to obtain a complete picture of its autoignition and flame-holding capability. Simultaneous schlieren and OH-PLIF imaging, performed in the near field of the jet, provides an insight on the location of shock waves, the jet penetration, and the region of combustion.

Experimental Approach

The experiments reported here were conducted in the Stanford expansion tube facility [13], with its dedicated lasers and optical equipment which allows application of simultaneous schlieren and planar laser-induced fluorescence of OH radicals. The flow facility is 12 m in length (including the dump tank) with an inner diameter of 89 mm. The injection system, as described in Fig. 3, is mounted downstream of the exit of the expansion tube, inside a square viewing chamber of 27 × 27 cm cross section, equipped with an opposed pair of square quartz windows for observation.

The schlieren system used in these experiments is set up in a standard Z-arrangement. The light source constitutes of a Xenon flash lamp (Strobotac, type 1539-A). Two *f*/10, 20-cm-diameter concave mirrors are used to collimate the light through the test section and then refocus it onto a knife-edge. The test object is imaged with an *f*/6, 76-mm lens onto the 576 × 384 pixel array of an intensified CCD camera.

The laser sheet for PLIF imaging is formed from the frequency-doubled output of a dye laser pumped by a pulsed Nd:YAG laser. Rhodamine 590 dye is used for OH-PLIF transitions near 283 nm, with pulse energies of approximately 8 mJ. The sheet is roughly 0.5 mm thick × 3 cm wide at the viewing section. The fluorescence signal is collected through the same exit window as that of the schlieren system. To separate both light signals, a 5-cm-diameter dichroic mirror is mounted at 45° to the optical axis perpendicular to the exit window. The dichroic, designed for larger than 99% reflectivity between 300 and 320 nm, reflects the OH fluorescence but is transparent to the schlieren beam. The reflected fluorescence is collected onto the 578 × 364 pixel array of a second ICCD camera.

The injection system (see Fig. 3) was designed to inject gaseous fuel normal to the cross-flow. The system consists of a flat plate with an attached high-speed solenoid valve (General Valve Series 9, Iota One controller) that allowed for near-constant injection flow rates during the expansion tube test time period. An underexpanded jet of hydrogen was injected from a circular port, 2 mm in diameter, located at a distance 30 mm downstream of the tube exit.

Flow characterization efforts, described previously by Ben-Yakar et al. [14], allowed flow quantities to be determined using the combined data of wall pressure, pitot pressure, and IR emission. Table 1 summarizes test conditions used in the experiments.

OH-PLIF and Its Interpretation

PLIF imaging of reactive flows relied on OH, a naturally occurring combustion radical, as the fluorescent tracer. The relationship between fluorescence signal and number density or mole fraction have been described by Hanson et al. [15]. Briefly, the fluorescence is proportional to the laser energy, the fluorescence yield, the species number density, and the Boltzmann population fraction for the absorbing transition. Effective combustion visualization with OH PLIF requires that the variation in OH mole fraction in the region of interest affect the fluorescence signal more effectively than changes in other parameters described earlier. The most critical issue, typically, is the temperature dependence of the Boltzmann fraction of the absorbing state. At the combustion pressures obtained in this work, the fluorescence signal can be modeled as [15]

$$S_f \propto \chi_{\mathrm{OH}}\ [f_{J''}/T^{1/2}]$$

where χ_{OH} is the OH mole fraction, and $f_{J''}$ is the Boltzmann fraction of OH molecules in the absorbing state. For the absorption transition considered here—the $Q_1(7)$ transition of the $A^2\Sigma^+ \leftarrow X^2\Pi\,(1,0)$ band of OH, located at 283.31 nm—such effects play a relatively minor role in interpreting the signal in the regions observed to contain OH, and the fluorescence intensity can be qualitatively linked to OH mole fraction.

Results and Discussion

This investigation employed both schlieren and PLIF measurements to create a complete picture of combustion phenomena in a reacting supersonic flow field around a fuel jet injected normal to the flow. The schlieren technique, a direct indicator of density gradients in the flow, is appropriate for defining the position of shocks in the flow and the location of the hydrogen jet penetration. On the other hand, PLIF imaging of the flow relied on OH as a fluorescence tracer that defines the structure and evolution of the ignition process. As we combined these two diagnostic techniques simultaneously and overlaid the images, the exact location of the combustion process relative to the jet and its characteristics were defined. Furthermore, the camera gate widths of 200 ns for the schlieren image and less than 150 ns for OH are short enough to effectively freeze the flow, providing temporally resolved images of the phenomena.

The data acquired as part of this investigation include two basic flow conditions representing total enthalpies of flight Mach numbers 10 and 13, as described in Table 1. The images chosen to be presented in this paper are the experimental results performed at flow condition 1. (Images from condition 2 can be found in a previous publication [14].)

TABLE 1
Test gas flow properties of the current experiments. The corresponding flight Mach numbers were obtained from Fig. 1

Condition	1	2
Flight simulation	Mach 10	Mach 13
Total enthalpy, MJ/kg	4.0	6.0
Mach number	3.46	4.70
Static temperature, K	1300	1300
Static pressure, atm	0.32	0.05
Velocity, m/s	2420	3300
Test time, μs	260	400

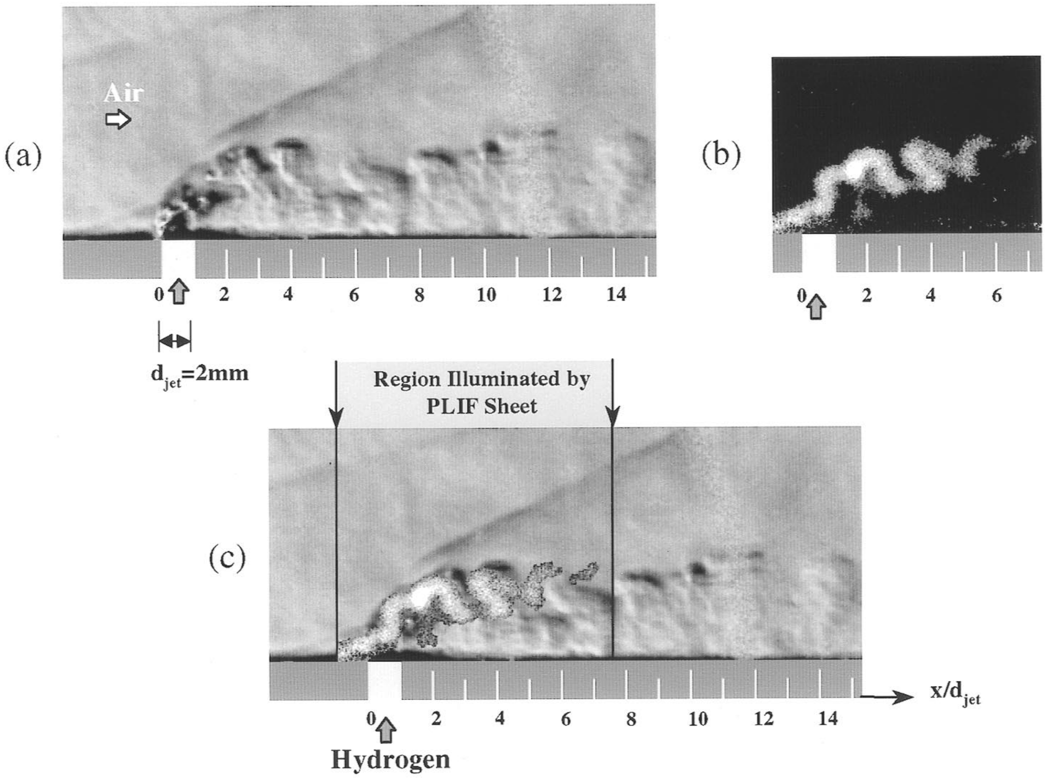

FIG. 4. Simultaneous OH-PLIF and schlieren images visualizing hydrogen injection into supersonic cross-flow. Free-stream conditions are $M = 3.46$, $T = 1300$ K, $P = 0.32$ atm, $V = 2420$ m/s. The jet-to-free-stream momentum flux ratio is $J = 1.4$. (a) Schlieren image, (b) OH-PLIF image demonstrating the ignition and combustion regions of jet-in-cross-flow at high enthalpy condition, and (c) overlaid OH-PLIF and schlieren images.

Transverse Jet Characteristics

The underexpanded jet penetration into a supersonic flow is known to be strongly correlated with the jet-to-free-stream momentum flux ratio (J) defined as

$$J = \frac{(\rho u^2)_j}{(\rho u^2)_\infty} = \frac{(\gamma p M^2)_j}{(\gamma p M^2)_\infty}$$

where the subscript j corresponds to the jet exit conditions and ∞ corresponds to free-stream conditions ahead of a bow shock. In the experimental results presented here, the jet-to-free-stream momentum flux ratio is $J = 1.4$.

A schlieren image of a jet in reacting supersonic cross-flow revealing the instantaneous vortex structure of the flowfield is given in Fig. 4a. Free-stream fluid flows from left to right, and the jet fluid enters from the bottom centered at $x/d_{\text{eff}} = 0$. Large-scale jet-shear layer vortices generated by the jet–free-stream interaction can clearly be observed along the jet–free-stream interface, starting in the region near the injector exit. These periodically formed eddies tend to enlarge and engulf free-stream fluid as they travel downstream with the flow. The jet penetrates to an average height of 7 mm and fully aligns with free-stream flow six jet diameters downstream. The bow shock around the jet curves sharply upstream and appears to be stationary with no large fluctuations in position from shot to shot. The schlieren images also show a separation shock wave upstream of the jet, demarcating the separation of the coming boundary layer. The jet exit is located at a distance 55 mm downstream of the flat plate leading edge. At this location, the expected boundary layer thickness to jet diameter ratio is approximately $\delta/d_{\text{jet}} \approx 1/2$.

Autoigniton and Flame Holding

Figure 4 demonstrates an example of simultaneous schlieren and a side view OH-PLIF image overlaid in a single image (Fig. 4c). These images are obtained at the flow conditions simulating flight Mach 10. Apparent in the images are the regions containing OH molecules, indicating the location of

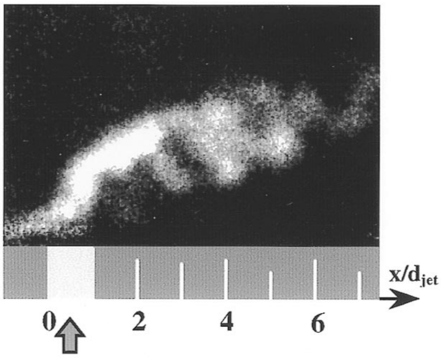

FIG. 5. An average of four side-view OH-PLIF images. Free-stream conditions are $M = 3.46$, $T = 1300$ K, $P = 0.32$ atm, $V = 2420$ m/s. The jet-to-free-stream momentum flux ratio is $J = 1.4$.

the reaction zone. The structural evolution of reaction zone is in good agreement with the jet position determined by the schlieren imaging, although there is a small shift between their position. This shift (about 1 mm) compares well with the fact that schlieren image was taken 2 μs after the PLIF image. A significant and fairly uniform level of OH in the recirculation area confined by the separation wave upstream of the injector is visible. This is followed by a thin filament on the order of the jet diameter (2 mm) along the outer edge of the plume attached to the recirculation zone ignition region. However, farther downstream, a significant decrease of OH fluorescence is obtained as the mixture expands around the jet flow field, indicating quenching of the ignition around the jet. As explained earlier, the fluorescence signal in this case is essentially proportional to OH mole fraction. Therefore, the significant decrease in signal level observed in the sheet-corrected PLIF images is a direct indication of decrease in OH mole fraction. The same behavior is consistent from shot to shot as can be seen in Fig. 5, where an average of four side-view PLIF images is shown. In contrast, continuous and intense OH signals around the jet were observed at condition 2 (flight Mach 13 range), thereby indicating a better flame-holding capability of this transverse jet. However, more detailed study at this condition is required in order to fully characterize the flame holding at increased Mach numbers.

In order to achieve a more complete picture of the combustion, we also obtained top PLIF views of the jet. A set of four instantaneous top view images collected at different heights above the jet exit is shown in Fig. 6 (the white dots in the images indicate the center of the jet exit). The results show OH around the jet, while the center of the plume has no OH formation. The bottom image at 1 jet diameter above the plate shows two main features: (1) the jet spreads very quickly in the lateral direction (up to about 8 jet diameters) assuming that the flame is around the jet–air interface, and (2) the OH concentration downstream of the jet remains nearly constant. By contrast, the other three images obtained at 2, 2.5, and 3 jet diameters height demonstrate the same tendency of the side views, namely, that the

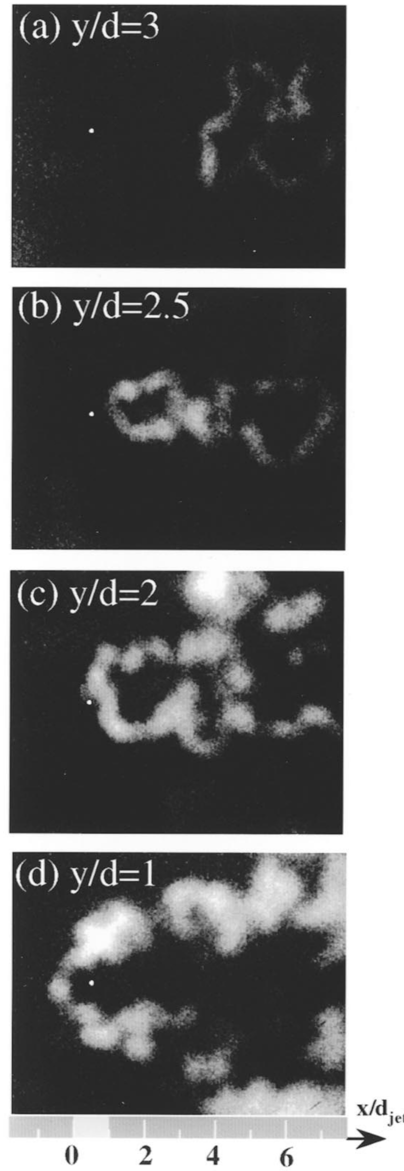

FIG. 6. Instantaneous top-view OH-PLIF images obtained at different height above the injection plate. Free-stream conditions are $M = 3.46$, $T = 1300$ K, $P = 0.32$ atm, $V = 2420$ m/s. The jet-to-free-stream momentum flux ratio is $J = 1.4$. (a) $y/d_{jet} = 3$, (b) $y/d_{jet} = 2.5$, (c) $y/d_{jet} = 2$, and (d) $y/d_{jet} = 1$ above the injection plate.

OH concentration decreases significantly as the jet moves downstream.

Additionally, side-view OH-PLIF images were obtained off the centerline axis of the jet, in order to observe ignition occurring in the boundary layer just downstream of the jet exit (as is consistent with Fig. 6d). A comparison between on- and off-centerline axis (~3 mm off the axis) performed in Mach 13 flight conditions for the case of $J = 2$ can be found in our previous publication [14]. Even though both cases exhibit a flame front visible along the free-stream–jet interface, only the PLIF image acquired along the off-centerline axis shows the presence of OH in the boundary layer downstream of the jet. The difference between the on- and off-centerline axis OH data is apparently a result of the horseshoe vortex that wraps around the jet column, as shown in the 3-D jet flow-field schematic of Fig. 2b. The OH radicals produced during autoignition of the recirculation zone are convected downstream with the horseshoe vortex. Continuous and intense OH signals around the jet were observed at condition 2 (flight Mach 13 range), thereby indicating a better flame-holding capability of this transverse jet. However, more detailed study at this condition is required in order to fully characterize the flame-holding at increased Mach numbers.

It is worth noting that these results demonstrate substantial differences with previous results published by McMillin et al. [4]. They also studied transverse hydrogen injection into supersonic free-stream conditions ($M = 1.4$, $p = 0.4$ atm, $T = 2200$ K) but with lower total enthalpies ($H_{\text{tot}} = 1.9$ MJ/kg). In their experiments, no strong OH signal was observed within the first 10 jet diameters. Beyond this point, pockets of OH were found first within the lower plume region near the wall and, at slightly delayed distances, in filaments along the jet–free-stream shear layer. The lack of significant OH formation in these measurements is likely due to the longer ignition delay times associated with their conditions, where the flow temperature is as high as 2200 K but the free-stream velocity is lower (1190 m/s).

Ignition in the current experiments is likely due to the hot and radical-rich separation region upstream of the jet exit where the boundary layer and jet fluids mix subsonically. Although these recirculation kernels in general are small in volume, in high enthalpy flows, the temperature of these zones can be as high as the stagnation temperature of the bulk flow allowing autoignition of hydrogen air mixture. However, the ignition is quenched as the gases expand around the jet and the local mixture temperature falls, resulting finally in unsustained combustion.

It is evident from these results that improved injection schemes with sustained combustion would be required for practical applications in scramjet engines. Oblique shock-wave impingement into the jet is one method known to enhance the molecular mixing between supersonic air and gaseous fuel. The vorticity generated when a shock wave interacts with a shear layer has immediate significance to the mixing enhancement in supersonic flows resulting in enhanced combustion efficiency [17]. Another possible scheme is staging of the injectors to provide improved mixing and therefore additional flame-holding, especially in the recirculation region between the injectors. An injection system consisting of two ports was in fact previously studied in reflected shock tunnel experiments by Parker et al. [3]. Their results showed a continuous level of OH concentration along the fuel–air interface beginning in the boundary layer of the first jet. Although these results indicated flame-holding capability, one should remember that free-stream flows generated in shock tunnels may consist of significant levels of radicals. The boundary layer developed along the nozzle of the shock tunnel, in addition to radicals present in the free stream, might thus have promoted the ignition and flame-holding characteristics of the flow field. Such results underscore the importance of testing at stagnation temperatures representative of those that will be experienced in the hypersonic flight envelope but while retaining correct chemical composition of the free-stream air.

Conclusions

The problem of hydrogen transverse injection and its flame-holding capability was studied at high-total-enthalpy flow conditions generated in an expansion tube. The experiments applied simultaneous OH-PLIF and schlieren imaging to map the regions where combustion occurs relative to the jet position. The main results are summarized as follows:

Overlaid OH-PLIF and schlieren images indicated the autoignition of a hydrogen jet in air cross flow simulating flight Mach 10 and 13 conditions. OH fluorescence appears first in the recirculation region upstream of the jet and extends along the outer edge of the jet plume adjacent to the wall. However, at Mach 10 condition, a decrease in OH signal level was observed in the PLIF side view, beginning about 6 jet diameters downstream of the jet. This implies quenching of the ignition that is initiated in the recirculation region. The tendency of the OH signal level to decrease was consistent in top-view images too, except for planes imaged close to the wall.

Although quenching of the combustion was observed at the downstream jet–free-stream interface, the results still show potential for flame-holding. The high stagnation temperatures that exist at hypersonic flight conditions appear to provide instantaneous ignition in the case of jet in cross-flow. However, in order to achieve sustained combustion, improved injection schemes, possibly involving staged jets, will be required for practical applications in scramjet engines.

Acknowledgments

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