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TRANSVERSE JETS FOR CONTROL OF MISSILE STEERING THRUSTORS

by

R. H. Nunn

10 April 1972

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LIST OF ILLUSTRATIONS

NOMENCLATURE

a	Speed of Sound
B	Mainstream duct height
C_D	Discharge coefficient
d_t	Width of jet nozzle throat
F	Thrust
F_x	Effective pressure force on "windward" side of jet; acting in the x-direction
h	Maximum height of sonic line above the wall
H	Jet slot width nondimensionalized by the mainstream duct height d_t/B
I_{sp}	Specific Impulse
$\dot{m}$	Mass Flow Rate
M	Mach number
m	See Eq. 5
P	Pressure
T	Temperature
V_e	Exit velocity of jet
x,y	Coordinates along, or normal to wall, respectively
ϵ	Angle of jet injection with respect to the vertical
	Ratio of specific heats
ρ	Density

SUBSCRIPTS

j	Jet conditions
m	Mainstream conditions
r	Reference conditions based on mainstream flow with no jet injection
PDM	Pulse-Duration-Modulation system
PROP	Proportional system

0 Upstream conditions for mainstream

1 Mainstream conditions at point of injection

2 Mainstream conditions at point of maximum jet penetration

SUPERSCRIPTS

o Stagnation conditions

* Sonic conditions

FUNCTION DEFINITIONS

A(M) Area ratio in locally isentropic flow

$$= \frac{1}{M}\left(\frac{1 + \frac{\gamma-1}{2} M^2}{\frac{\gamma+1}{2}}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

P(M) Pressure ratio in locally isentropic flow

$$= \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma}{1-\gamma}}$$

INTRODUCTION

A. BACKGROUND

This report describes the conduct and results of an investigation of the feasibility of aerodynamic throttling as a means of controlling transverse jets. The investigation is a part of a continuing study to identify, classify, and evaluate various fluidic and flueric concepts and devices that show promise for tactical missile control system applications.

At the present time the only well developed methods [Refs. 1,2] for employing fluid systems in missile control applications involve the utilization of pulse-duration-modulation (PDM) or flip-flop devices. A continuously recurring question concerning systems such as these (and, indeed, most fluidic systems) is the extent to which their performance is penalized by the loss of control fluid. In PDM systems this problem is especially critical since not only control fluid but power fluid is expended throughout the control mission and, in fact, the rate of thrustor mass flow is constant and independent of control requirements in PDM systems. Because of this apparent inefficiency in PDM systems, some activity has been directed toward the development and evaluation of proportional thrustor control methods that lend themselves to fluidic system technology.

One such method is that of injecting a high speed jet into the throat of a thrustor nozzle in an effort to control and throttle the total mass flow rate of the nozzle. This method is somewhat similar to that of a vortex valve in that a control jet is used to throttle the main flow, but the nature of the control jet/main flow interaction is significantly different in the two methods, as will be seen in the detailed analysis of section II of this report. Both methods are hampered by the fact that mass injected into the main flow, for the purpose of reducing the main flow, becomes a part of the total flow so that eventually a point is reached at which additional control fluid injection does not reduce the main flow to an extent sufficient to result in an overall reduction in the sum of the two flows.

The comparison of a proportional jet injection system and a PDM system is complicated considerably by the absence of a well described control mission. Some comparison can be made, however, on the basis of simple flow rate considerations. In order to accomplish this we consider the mass flow rates required of each system under the ground rules that both the PDM and proportional systems have the same thrustor characteristics in that for a given mass flow rate, each thrustor produces the same thrust. In other words, the thrustors for both systems have the same specific impulse (I_{sp}). We also require that each system be designed to deliver the same maximum control force (F_{max}).

For the PDM system a net control impulse is obtained by varying the duration of time over which opposing jets are actuated. For this system either one or the other (but not both) of the jets is always actuated so that the mass flow rate from the PDM system is simply

$$\dot{m}_{PDM} = \frac{F_{max}}{I_{sp}} \quad . \tag{1}$$

In the proportional system, the opposing jets may be throttled. If a given net thrust level, F, is required, it is obtained by throttling one jet the maximum amount (so that it's flow rate is $\dot{m}_{min}$) while at the same time providing sufficient mass flow ($\dot{m}$) in the opposite jet to give the desired net thrust. Thus, for the proportional jet

$$F = I_{sp} (\dot{m} - \dot{m}_{min}) \ . \tag{2}$$

If this system is also to be designed to have the capability of providing F_{max}, then

$$F_{max} = I_{sp} (\dot{m}_{max} - \dot{m}_{min}) \ . \tag{3}$$

The <u>total</u> mass flow rate required of the proportional system is the sum of that of both opposing thrustors so that

$$\dot{m}_{PROP} = \dot{m} + \dot{m}_{min} \ . \tag{4}$$

Equations (2) through (4) may be combined in the form

$$\dot{m}_{PROP} = \frac{F_{max}}{I_{sp}} \left(\frac{F}{F_{max}} + \frac{2m}{1-m} \right) \ , \tag{5}$$

$$\text{where } m = \frac{\dot{m}_{min}}{\dot{m}_{max}} \ .$$

The term m is a measure of the ability to throttle a thrustor jet - it is the ratio of the minimum attainable flow through a thrustor to the maximum (unthrottled) flow dictated by F_{max} (Eq. (3)). There are several interesting points to be made here: (a) If $m = 1$ (no throttling available) then $\dot{m}_{PROP} = \infty$ regardless of F/F_{max}. This is because two opposing jets can only produce a net thrust if the thrust of each is infinite. Thus as m approaches unity the jets must be made extremely large due to the small differences attainable between them. (b) If $m = 0$ (either or both of the jets can be completely shut off) then the mass flow required from the proportional system depends only upon the level of thrust desired. It is this latter feature, of course, that leads to the promise of the proportional thrust control system for missile steering applications.

In order to directly compare the PDM and proportional systems we need only combine Equations (1) and (5) to obtain

$$\frac{\dot{m}_{PROP}}{\dot{m}_{PDM}} = \frac{F}{F_{max}} + \frac{2m}{1-m} . \qquad (6)$$

This equation is illustrated in Fig. 1 for representative values of the parameters F/F_{max} and m.

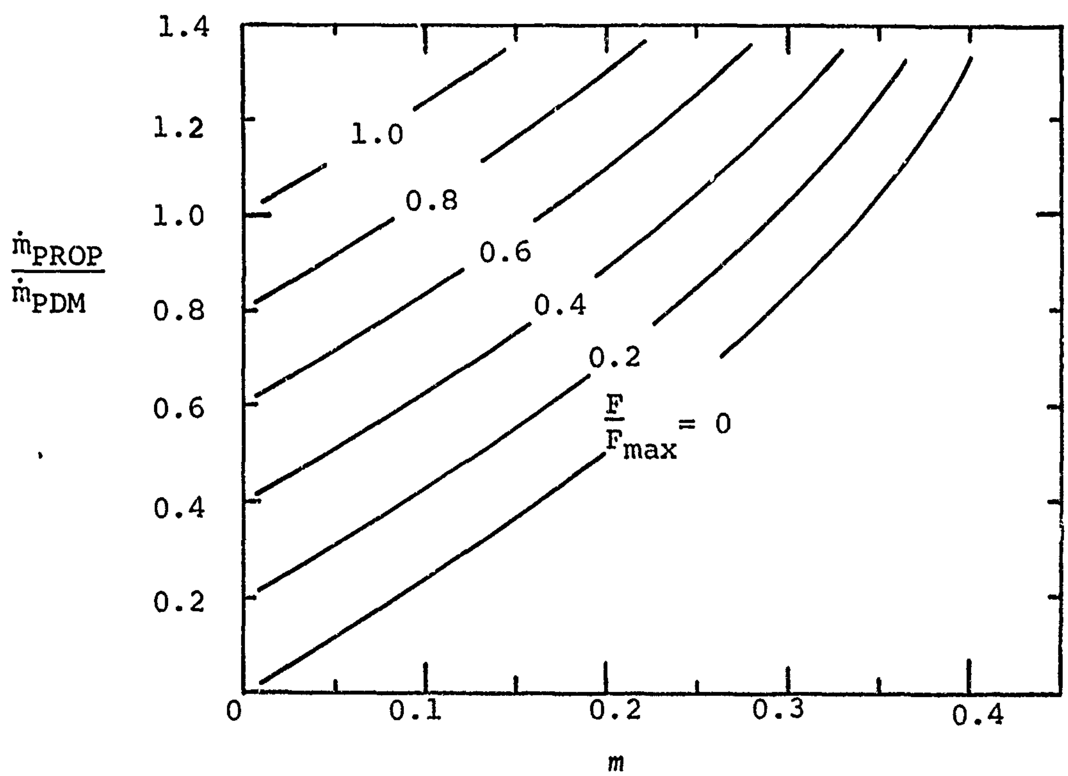


Figure 1. Comparison of PDM and Proportional Flow Requirements.

Since a general (but not critical goal) is to find those values of m that lead to less mass flow rate for the proportional system than for the PDM system, the region $\dot{m}_{PROP}/\dot{m}_{PDM} < 1$ is of general interest. It is seen that for $F/F_{max} = 0$ (no control force demanded) a value of $m \leq 1/3$ will yield improvements over the PDM system. For higher values of F/F_{max}, greater throttling capabilities are required and thus m must be smaller. For $m = 0$ the proportional system exceeds the PDM system in performance for all values of F/F_{max}.

There are, of course, many other factors affecting the comparison such as speed of response, autopilot logic, and continuity (smoothness) of control. In addition, the comparison of the PDM and proportional systems depends heavily upon the mission since missions in which little or no large control forces are required greatly favor the proportional system. It is clear, however, that if $m \leq 1/3$ is not attainable in a proportional system, little or no overall benefit can be expected from a stored mass point of view.

B. JET INTERACTION AT A SONIC THROAT

The problem under study seems to have been first investigated by Manoury, et al. [Ref. 3] who proposed an exit plane aerodynamic nozzle for gas turbine devices. This article introduced what has come to be known as "aerodynamic throttling." Aerodynamic throttling is the process by which the mass flow rate of a primary flow is controlled (throttled) by injecting a secondary (jet) flow into the mainstream. The jet/mainstream interaction creates what is referred to as an "aerodynamic throat" and has the general effect of reducing the mainstream mass flow rate. Since Manoury's article in 1955, the feasibility of aerodynamic throttling for some applications has been shown experimentally [Ref. 4-6] and predicted analytically [Ref. 7-9]. The knowledge gained from these studies has been used by designers of systems that involve the injection of one fluid into another such as variable-thoat nozzles, combustion chambers, and rocket nozzles cooled by injection.

As has been mentioned, this investigation was undertaken with the idea of applying the principle of aerodynamic throttling to a fluidic device which could be used in a missile control system. It was envisioned that a modified proportional amplifier, with throttling control, could be used as a moment producer for a missile steering system employing external jets. The advantage of such a device would lie in the fact that during no-control or low-level control phases of operation, the gas expended by the fluidic control system could be throttled thereby saving gas and allowing for either more control time, or an improvement in missile loading efficiency.

An important aspect of the aerodynamic throttling process is the extent to which the jet to mainstream total pressure ratio affects the penetration of the jet into the mainstream. This study attempts to improve the analytical prediction of the penetration height of the jet and also to extend the information available about the throttling process by experimentally determining the effect of high total pressure ratios. Previous investigators have not gone above jet to mainstream total pressure ratios of 5:1.

II. ANALYSIS

A. GENERAL CONSIDERATIONS

The goal of this study was to examine the jet penetration and throttling characteristics of a transverse jet injected into a sonic throat. The jet, shown schematically in Fig. 2, is sonic, underexpanded, two-dimensional and inclined at an angle α into the mainstream.

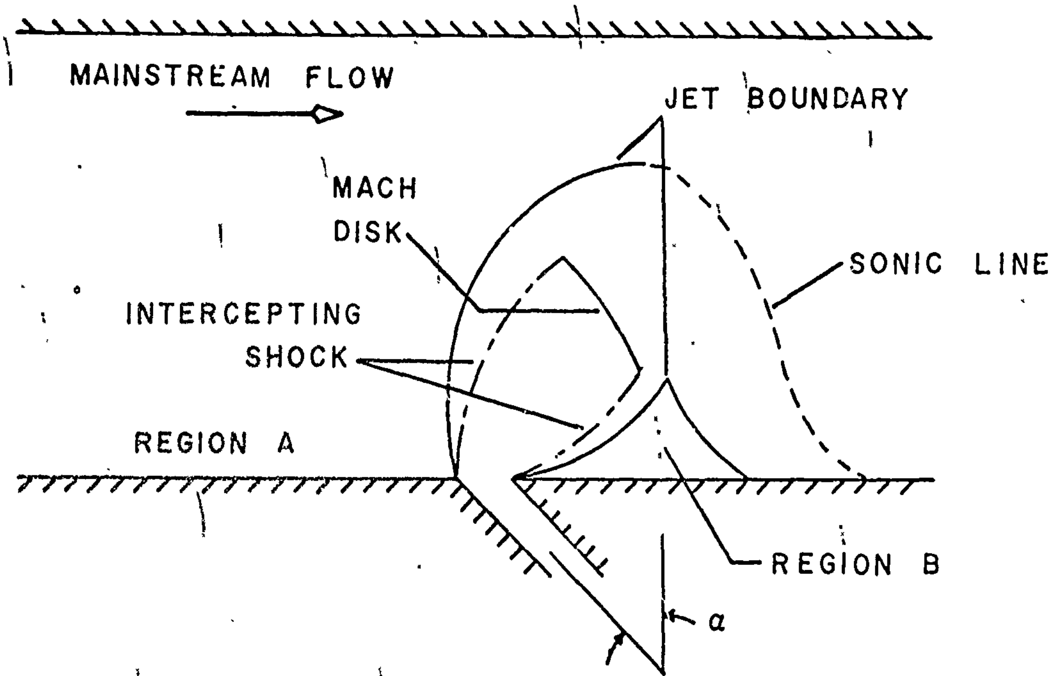


Figure 2. Region of Interest

The basic structure of the jet in the vicinity of the nozzle exit is similar to that of a free jet expanding into a quiescent atmosphere except that the pressure in region A (windward side) is higher than that in region B (bubble side). This pressure imbalance results in a jet plume that is skewed relative to the nozzle axis. For a given injector geometry and specific gas properties, the jet total pressure and the exterior pressure in regions A and B determine the location of the jet boundaries. The jet flow in the core of the plume quickly becomes overexpanded and must pass through a strong jet shock while the flow along the boundary of the jet passes through a series of oblique shocks and remains supersonic. The strong jet shock, known as the Mach disk, is nearly normal to the jet flow direction. The jet flow immediately downstream of the Mach disk is subsonic and at a pressure that is higher than the mainstream pressure. Beyond this point the jet reaccelerates and expands in an attempt to achieve pressure equilibrium with the mainstream. Mixing between the jet and the mainstream is considered to be negligible in the region prior to the Mach disk.

B. JET PENETRATION

In the jet penetration studies conducted by Schetz and Billig [Ref. 10] and Orth and Funk [Ref. 11] the characteristic dimension of the interaction region was taken to be the height to the center of the Mach disk. This dimension was considered to be a measure of the jet penetration into the mainstream. Barnes, et al. [Ref. 12] as well as Spaid and Zukoski [Ref. 13] chose as the characteristic dimension the height of the sonic line occurring in the jet at a short distance beyond the Mach disk. The actual penetration of the jet can only be defined in terms of its effect upon the crossing flow. The characteristic height, whether that of the Mach disk or the sonic line, is useful mainly as a scale for the penetration. Both of these heights have been found to be valid parameters for the correlation of jet interaction data. In this analysis the sonic line is used as a more convenient characteristic height and, in addition, it is felt that this measure is more nearly the actual or "effective" depth of penetration as discussed below. To the extent that the calculated penetration is the actual penetration, the throttling characteristics of the system can be calculated by a direct application of the principle of mass flow continuity.

As the jet is turned by the bounded mainstream, the Mach disk becomes more nearly perpendicular to the wall. In this configuration, a measure of jet penetration based on the distance to the center of the Mach disk would only account for the penetration of a portion of the jet. Simply adding half the width of the Mach disk to this height would still not account for the full penetration. Vick, et al. [Ref. 14] has shown that for three-dimensional free jets at low total pressure ratios, the Mach disk occurs close to the point at which the jet plume achieves its maximum width and that the Mach disk diameter is no more than sixty per cent of the maximum jet diameter. It appears reasonable then to assume that for a two-dimensional jet the maximum penetration into a bounded mainstream would be some distance greater than the height to the center of the Mach disk.

Although the flow immediately behind the Mach disk is subsonic and the mainstream would be expected to strongly affect the flow in this region, there is jet flow along the windward boundary of the jet that has not passed through the Mach disk and remains supersonic for some distance downstream of the Mach disk. Vaughan [Ref. 15] and Maurer [Ref. 16] have observed, using shadowgraphs and oil films respectively, that the jet achieves a sonic velocity within a very short distance after the Mach disk and that negligible mixing occurs between the jet and mainstream prior to the sonic line. Thus, it is hypothesized that the supersonic flow along the boundary shields the subsonic core of the jet from massive mixing with the mainstream until the region of the sonic line is reached. Beyond this point it is assumed that the jet loses its identity, mixes with the mainstream, and ceases to affect the mainstream in the sense of penetration or throttling.

C. METHODS OF ANALYSIS

1. Previous Work

The problem of a secondary jet issuing into a bounded mainstream for the purpose of throttling the primary flow has been analyzed by numerous investigators in the past. In a recent analysis, Nunn [Ref. 17] makes use of a momentum balance across a control volume which includes both the mainstream and the jet. This momentum balance is then combined with continuity expressions for the mainstream and the jet along with a statement of pressure equilibrium between the jet and mainstream at the point of maximum penetration. To account for jet total pressure losses, it is noted that the strongest shock that can occur in the jet is at a jet Mach number which corresponds to an isentropic jet expansion to the critical mainstream static pressure. The solution of the system of equations presented in Nunn's work is based on the assumption that the strongest shock (i.e., the Mach disk) does occur in the jet and that it is located at the point of maximum penetration. It is felt that this assumption places an unrealistic constraint upon the flow field when a wide range of total pressure ratios is considered.

Nunn's analysis determines the shock strength purely on the basis of the ratio of total pressures, P_j^o/P_{mo}^o. This is equivalent to assuming that the jet is exhausting into a quiescent atmosphere ($P_{mo}^o = P_\infty$), and does not take into consideration the crossing flow and the effect of the turning of the jet. Due to the curvature of the jet plume, the flow within the jet experiences an additional compression that will cause the shock to occur at a lower Mach number than would be predicted for a free jet.

It has been shown by Adamson and Nicholls [Ref. 18] as well as Vick, et al. [Ref. 14] that for free jets, the distance to the Mach disk follows an approximately linear variation with P_j^o/P_∞ when plotted on log coordinates. Additionally, Vick has shown that the ambient pressure (P_∞) has a definite effect on the Mach disk location. For a given pressure ratio, P_j^o/P_∞, the lower the ambient pressure, the less the distance from the nozzle exit to the Mach disk. Based on the above findings, it would seem unrealistic to assume that the Mach disk, for all pressure ratios, would position itself at the point of maximum penetration.

It is advantageous to seek an analytical model whereby the jet boundaries as well as the Mach disk are more accurately predicted for jets which experience not only a different pressure on each side, but also encounter a varying pressure on the windward face. A computer program was written employing the method of characteristics to determine the shape of the jet plume along with the Mach number distribution on the centerline. This program was successfully completed for jets which experience a constant pressure on each face, but attempts to modify the program to account for a varying pressure were discontinued for two main reasons. First, it is not known *a priori* just what the pressure distributions are on the faces of the jet, and secondly the complexity of the computer program that would be necessary to handle this problem by means of the method of characteristics was deemed to be prohibitive. Therefore, it was decided that an alternate approach to the problem was needed. A brief description of the method of characteristics calculations, together with a computer listing, may be found in Ref. 19. A primary consideration of this alternate approach was to avoid the assumption that the Mach disk occurs at any particular location in the jet.

2. Present Analysis

As indicated previously, the jet is assumed to be sonic at injection, underexpanded, two-dimensional and inclined at an angle α into the mainstream. The jet undergoes an initial isentropic expansion and is then recompressed by a strong normal shock, the Mach disk. The jet fluid leaving the Mach disk again expands to sonic velocities a short distance downstream. The flow is assumed to reattach itself to the wall and continue to flow parallel to it.

The dotted line in Fig. 3 is designated as the sonic line and is the transition from sonic to supersonic velocities. The actual geometry of the sonic line is unknown. The geometry has, however, no effect on the results

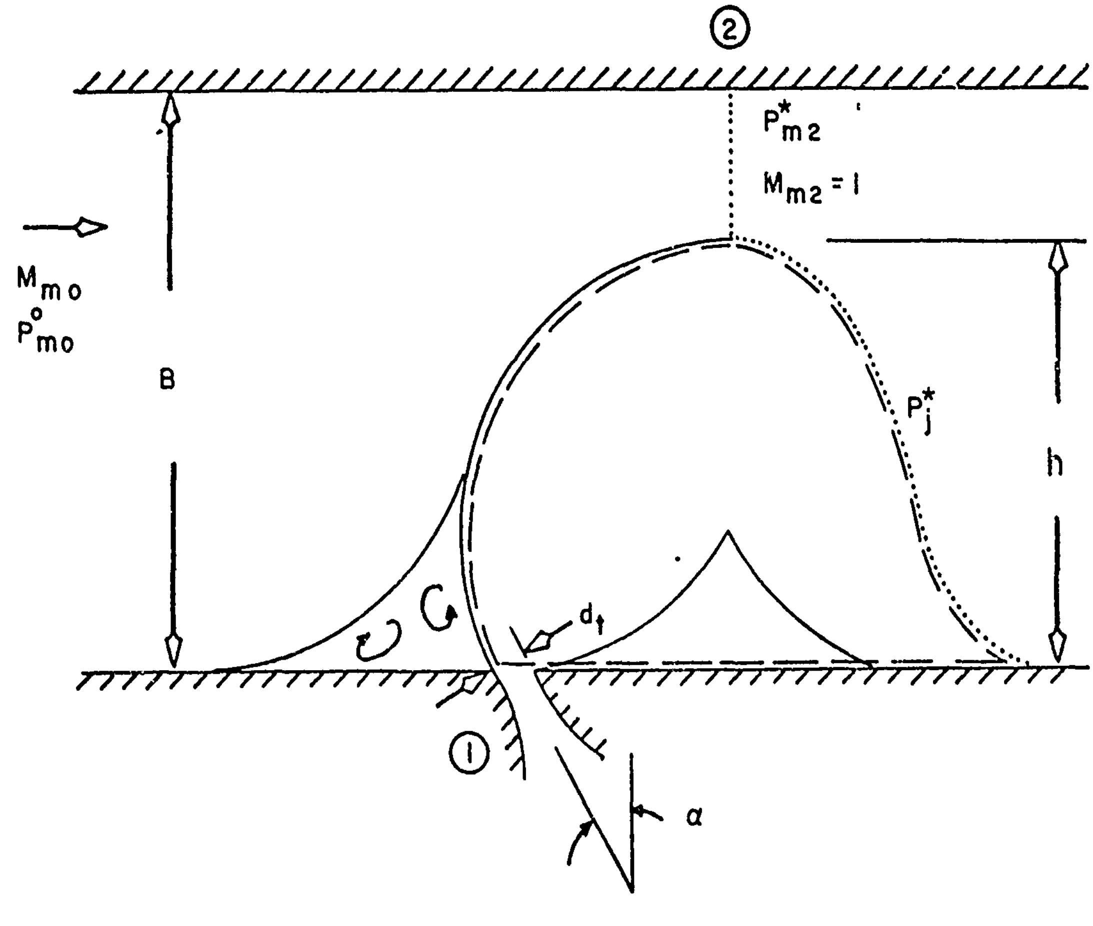


Figure 3. Schematic Diagram of Throttling Region Showing Control Volume.

of the analysis if it is assumed that the turning of the jet occurs prior to the sonic line so that the average momentum of the jet fluid at the sonic line is parallel to the wall. This assumption is the same as that made by Barnes, et al. [Ref. 12] and Cassel, et al. [Ref. 20] in their studies of jets in an unbounded mainstream.

A control volume (dashed line in Fig. 3) is defined which is bounded by the windward face of the jet plume from the nozzle to the jet sonic line (this is also the point of minimum mainstream flow area), the jet sonic line to the wall, and the plane of the wall.

Conservation of jet momentum requires that the change in the x-component of momentum be equal to the forces in the x-direction acting upon the control volume. Recalling that the momentum vector of the jet fluid at the sonic line is assumed to be parallel to the wall, the conservation of momentum can be expressed as follows:

$$F_x - P_j^* h = \dot{m}_j [a^* + V_e \sin \alpha] \tag{7}$$

In this expression F_x is the average pressure force acting in the x-direction on the windward face of the jet and will be evaluated in subsequent paragraphs. It has been further assumed here that the shear stresses acting along the jet/mainstream interface do not contribute significantly to the x-component of the net force on the plume. This assumption is characteristic of earlier jet interaction analyses (including those previously mentioned) and is based upon the hypothesis that the drag due to shear will only play a significant role at downstream regions where turbulent mixing is the dominant exchange mechanism for mainstream and jet momentum. This assumption presupposes an initial transverse jet momentum sufficient to lead to a large penetration in a short distance. As the distance to jet maximum penetration increases (as, for instance the upstream angle of injection increases) the neglect of shear stresses becomes less credible.

Equating the mass flow rate at the nozzle throat to that at the sonic line yields:

$$\dot{m}_j = \rho_t\, d_t\, a_t = \rho^* h\, a^* = \gamma \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \frac{P_j^o\, d_t}{a_t} \tag{8}$$

where the subscript (t) refers to conditions at the injection sonic throat. The latter relationship in Equation (8) restricts the analysis to conditions in which the jet behaves as an ideal gas and the process is adiabatic. Thus, for situations in which the jet total temperature varies widely from that of the mainstream, it may become necessary to consider the transfer of heat across the jet mainstream boundary. For the adiabatic process, $a^* = a_t$ and Equation (8) produces the following relationships:

$$\frac{\rho^*}{\rho_t} = \frac{d_t}{h} = \frac{P_j^*}{P_t} = \frac{P^{o*}}{P_j^o} \tag{9}$$

It should be noted that Equations (8) and (9) are the same as those obtained by Barnes, et al. [Ref. 12] in their study of transverse jets interacting with an unbounded free stream. Substituting Equations (8) and (9) into the momentum equation yields an expression for the maximum height of the sonic line above the wall:

$$\frac{h}{d_t} = \frac{P_j^o}{\frac{F_x}{h}} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \left[\gamma \left(1 + \frac{V_e}{a_t} \sin\alpha\right) + 1\right] . \tag{10}$$

Since the experiments that were conducted for this study treated only jets which were sonic at injection, $V_e = a_t$, Equation (10) takes the final form of:

$$\frac{h}{d_t} = \frac{P_j^o}{\frac{F_x}{h}} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \left[\gamma \left(1 + \sin \alpha\right) + 1\right] . \tag{11}$$

The height of the sonic line, h, as given by Equation (11), is a characteristic dimension of the flow field which according to the previous arguments is an estimate of the depth of jet penetration into the mainstream.

If Equation (11) could be solved directly for h, then it would be a simple matter to determine the effective throttling of the mainstream. The problem, however, lies in determining the effective average pressure, F_x/h, acting on the windward boundary. Since the mainstream is bounded on all sides, a sonic throat will be formed in the mainstream at the point of maximum jet penetration. The pressure acting on the windward face of the jet is assumed to vary from the mainstream static pressure at station 1 [Ref. 17] down to P_m^* at station 2. See Fig. 3.

The effective average pressure acting on the windward boundary can be expressed in integral form as:

$$\frac{F_x}{h} = \frac{1}{h} \int_{y_1}^{y_2} P \, dy \tag{12}$$

Expressing this integral in terms of Mach number gives:

$$\frac{F_x}{h} = \frac{P_{mo}^o}{h} \int_{M_{mo}}^{1} P(M) \frac{dy}{dM} \, dM \tag{13}$$

Relating y to M

$$\frac{B-y}{B} = \frac{A}{A_1} = \frac{A(M)}{A(M_{mo})}$$

and differentiating y with respect to M gives:

$$\frac{dy}{dM} = -\frac{B}{A(M_{mo})}\frac{d}{dM}[A(M)] \tag{14}$$

Upon evaluating $\frac{d}{dM}$ [A(M)] Equation (14) becomes:

$$\frac{dy}{dM} = \frac{B\,A(M)}{A(M_{mo})}\left(\frac{1}{M} - \frac{\frac{\gamma+1}{2}M}{1+\frac{\gamma-1}{2}M^2}\right)$$

and substitution of this expression into Equation (13) gives the following for the effective pressure:

$$\frac{F_x}{h} = -\frac{B\,p^o_{mo}}{h}\int_{M_{mo}}^{1} P(M)\,\frac{A(M)}{A(M_{mo})}\left(\frac{1}{M} - \frac{\frac{\gamma+1}{2}M}{1+\frac{\gamma-1}{2}M^2}\right) dM\,. \tag{15}$$

Finally, if Equation (15) is substituted into Equation (11) an equation in terms of the mainstream Mach number, M_{mo}, appears as follows:

$$M_{mo}\left(\frac{1+\frac{\gamma-1}{2}M^2_{mo}}{\frac{\gamma+1}{2}}\right)^{\frac{\gamma+1}{2(1-\gamma)}}\int_{M_{mo}}^{1}\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}\left[\frac{1}{M^2}\left(\frac{2}{\gamma+1}\right)\left(1+\frac{\gamma-1}{2}M^2\right)-1\right]dM$$

$$= \frac{d_t}{B}\frac{p^o_j}{p^o_{mo}}\left[\gamma\,(1+\sin\alpha)+1\right]\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \tag{16}$$

For a given set of parameters $\frac{d_t}{B}$, α, and $\frac{p^o_j}{p^o_{mo}}$, Equation (16) can be solved for the mainstream Mach number M_{mo}.

From the isentropic area relationship:

$$\frac{A}{A^*} = \frac{B}{B - h} = A(M_{mo})$$

an expression for the maximum jet penetration, h, can be obtained:

$$\frac{h}{B} = \frac{A(M_{mo}) - 1}{A(M_{mo})} \tag{17}$$

D. THROTTLING CHARACTERISTICS

A reference mass flow rate is defined as that which would flow isentropically in the mainstream with no jet injection;

$$\dot{m}_r = K\, P^o_{mo}\, B$$

where

$$K = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \left(\frac{\gamma}{RT^o_m}\right)^{\frac{1}{2}}$$

Using this reference mass flow rate, the throttled mainstream and jet mass flow rates may be expressed in nondimensional terms as:

$$\frac{\dot{m}_m}{\dot{m}_r} = \frac{1}{A(M_{mo})} \tag{18}$$

$$\frac{\dot{m}_j}{\dot{m}_r} = \frac{d_t P^o_j}{B\, P^o_{mo}} \tag{19}$$

The theoretical curves for flow throttling are shown in Figure 4.

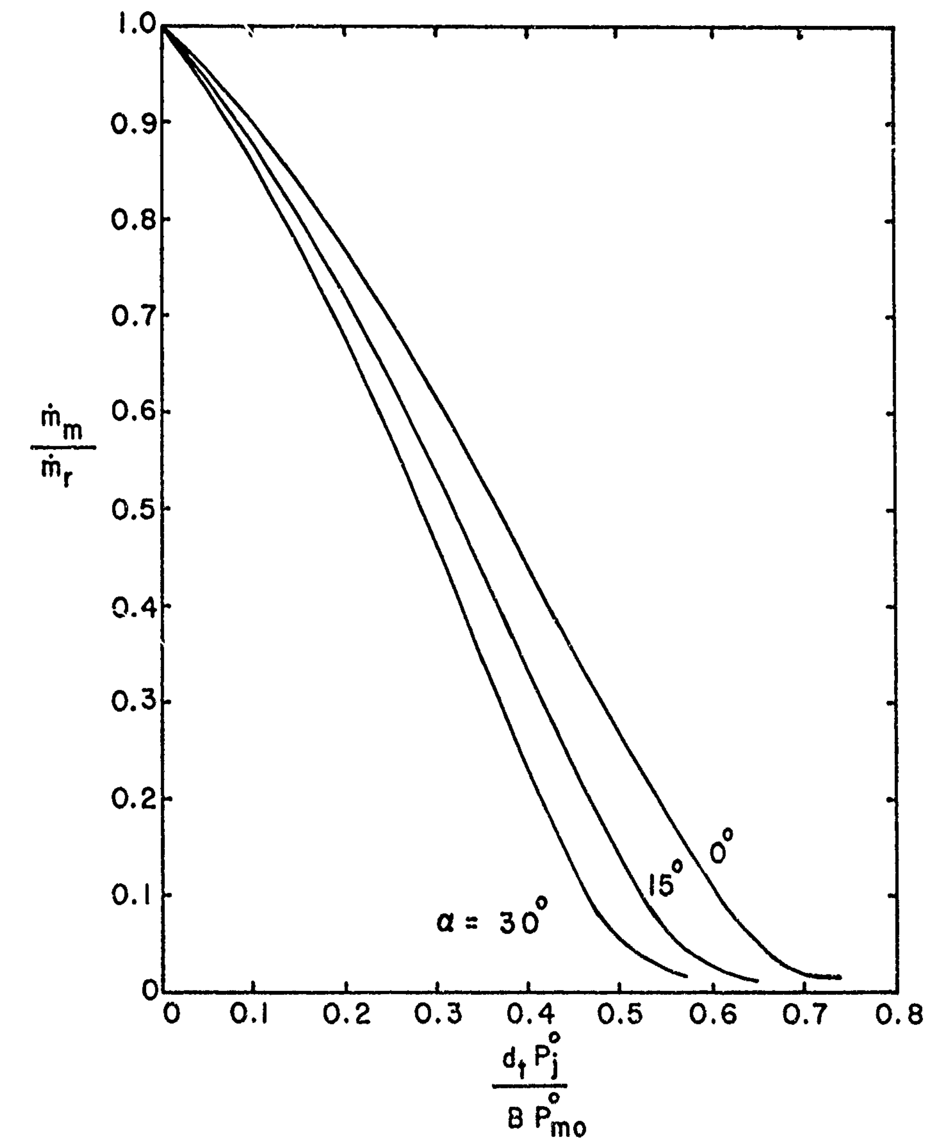


Figure 4. Throttling for Various Injection Angles (Unmodified Theory).

III. APPARATUS AND EXPERIMENTAL METHODS

The goals of the experimental portion of this study have been to evaluate the analytical model and examine the throttling effects of a transverse jet with various slot widths, d_t, and angles of injection, α, when injected into a sonic throat. To achieve these goals, it was desired to experimentally determine the mass flow rates of the mainstream and jet during the throttling process.

The experimental apparatus consisted of four distinct functional units: the first being the primary air flow system; the second being the test section itself; the third is the jet or secondary flow system; and, the fourth is the instrumentation for gathering flow information.

The primary system, shown in Fig. 5, consisted of a 200 psig air compressor and two 117 cu. ft. tanks. With both tanks pressurized to maximum capacity, it was possible to conduct blow-down tests of approximately one minute duration with a reference mass flow rate of 1.19 lbm/sec. The air left the tanks and passed through a pressure regulating valve just downstream of each tank. The pressure regulators were remotely controlled by a compressed nitrogen control system and were capable of regulating the mainstream pressure from 0 psig to 150 psig. After the regulators, the flow from both tanks was combined in a single 3-inch pipe in which was installed a remote controlled on-off valve. Downstream of the on-off valve, the 3-inch pipe was fitted with a standard ASME flat plate orifice which was used to obtain mass flow measurements for the primary system. The air then flowed into a 5-inch pipe and finally to a transition section that changed the circular cross-section to a rectangular cross-section measuring 2.625" x 3.0".

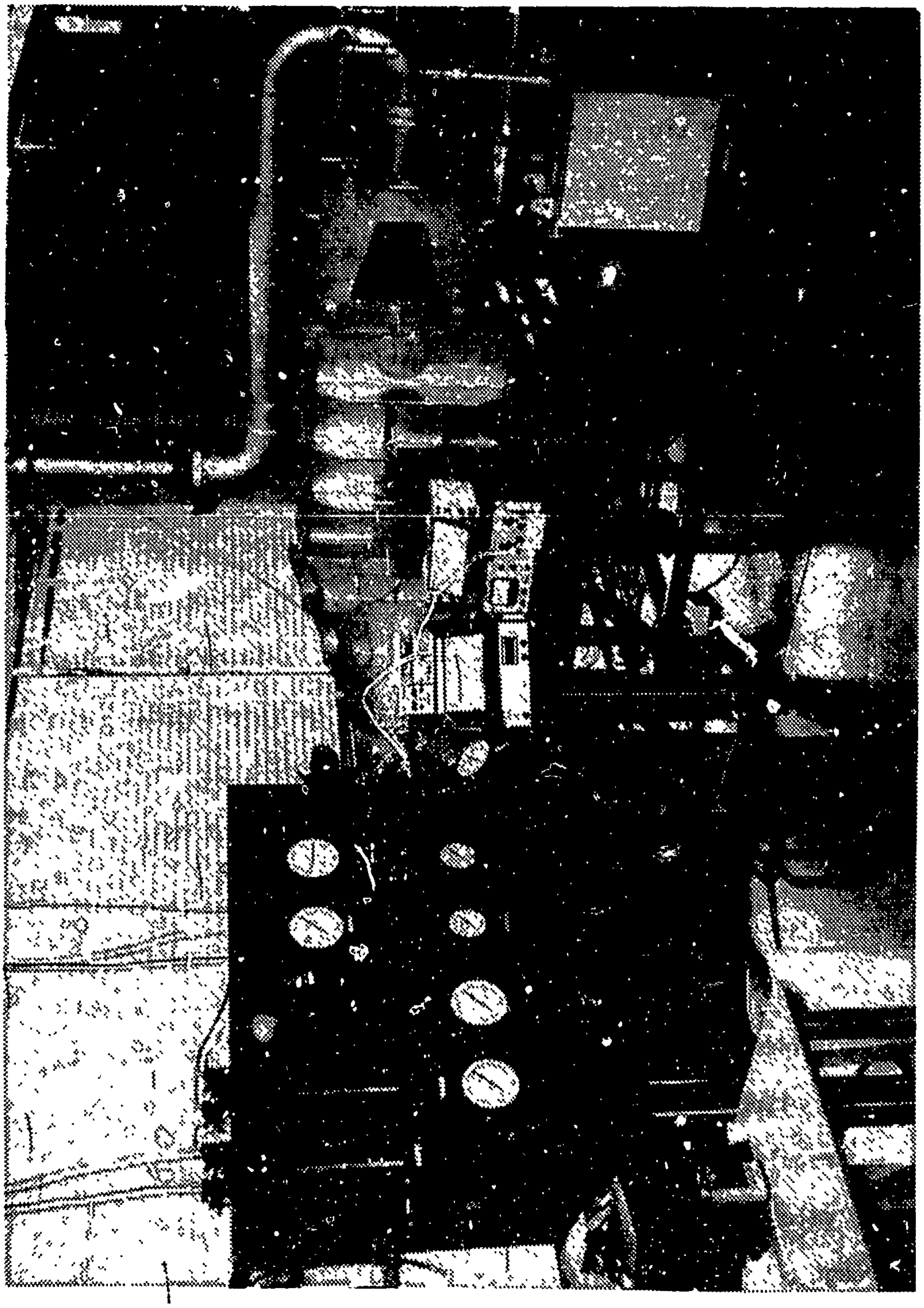


Figure 5. Primary Flow System with Control Panel.

The 5-inch pipe and subsequent transition section were part of a small wind tunnel which had been obtained as surplus government equipment. Since the available compressed air tanks could not provide sufficient blow-down times through the existing cross-section, the test section that was installed in the tunnel had plexiglass inserts on either side which reduced the width of the tunnel from 2.625" to 1.4". In this way the rectangular cross-section just prior to the test section had the dimensions of 1.4" x 3.0".

The test section, shown in Fig. 6, consisted of a contoured upper block and a flat lower block which contained the jet slot. The contoured upper block choked the flow at a 1.4" x 0.737" throat. The air then exited through a nozzle which was divergent on one side only, the upper block; the straight wall of the lower block served as a plane of symmetry for simulating symmetric injection. The lower block was fitted with variable inserts to allow for a change in both the slot width, d_t, and the angle of injection α (see Fig. 7). Care was taken in the design of the inserts to insure that the jet occupied the entire jet slot (from side wall to side wall) upon entry into the main-stream flow.

The secondary flow followed a path similar to the primary system in reaching the test section. Eight bottles of compressed nitrogen, manifolded together, provided the secondary flow. (See Fig. 8). After leaving the manifold, the nitrogen passed through a remote controlled pressure regulating valve. The regulator would reduce the 1800 psig bottle pressure of the nitrogen to values ranging from 0 psig to 1500 psig. The nitrogen then flowed through a remote controlled on-off valve and on through a section of 1" stainless steel tubing which was fitted with a standard ASME flat plate orifice to a plenum chamber located in the lower block of the test section.

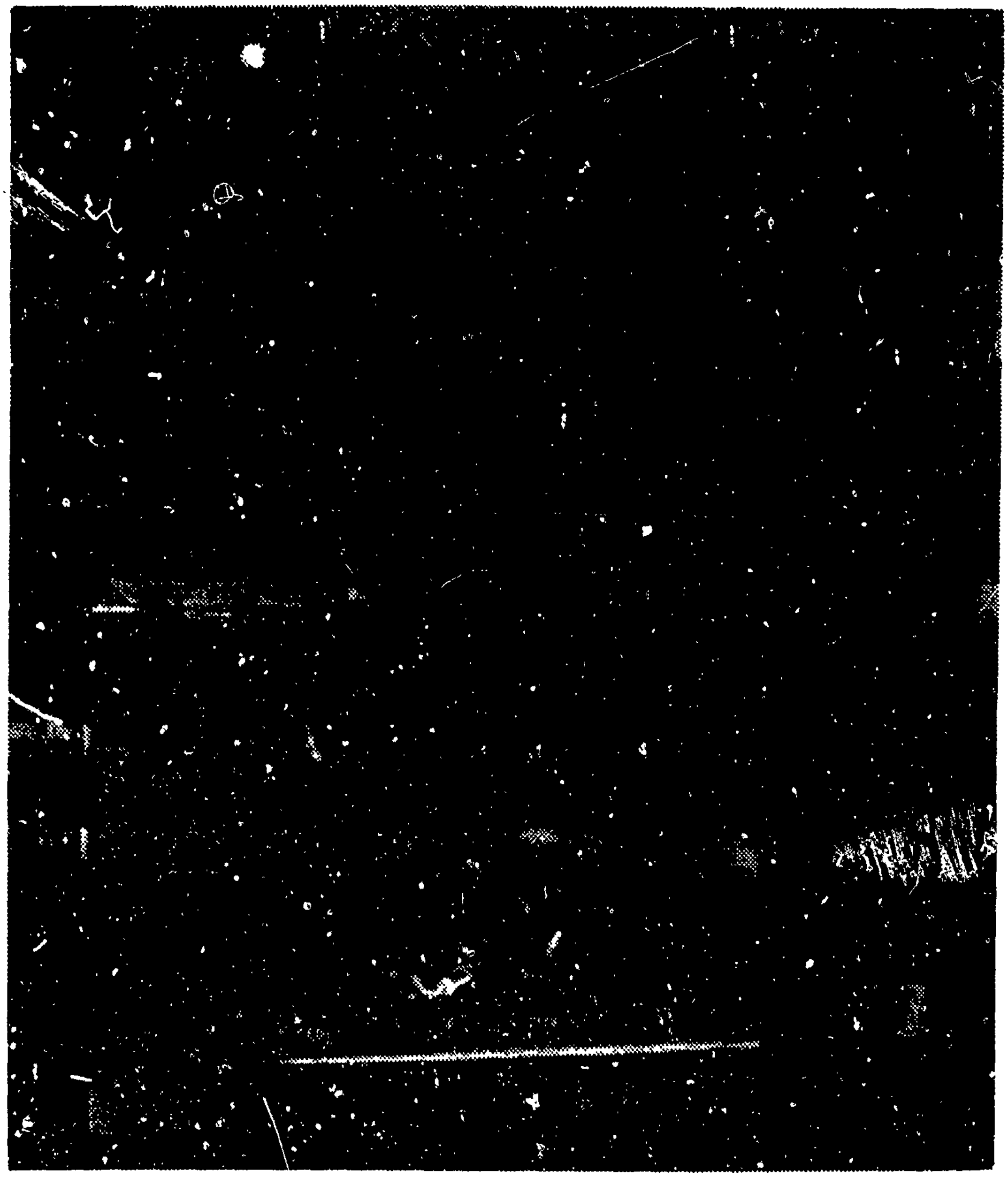


Figure 6. Test Section.

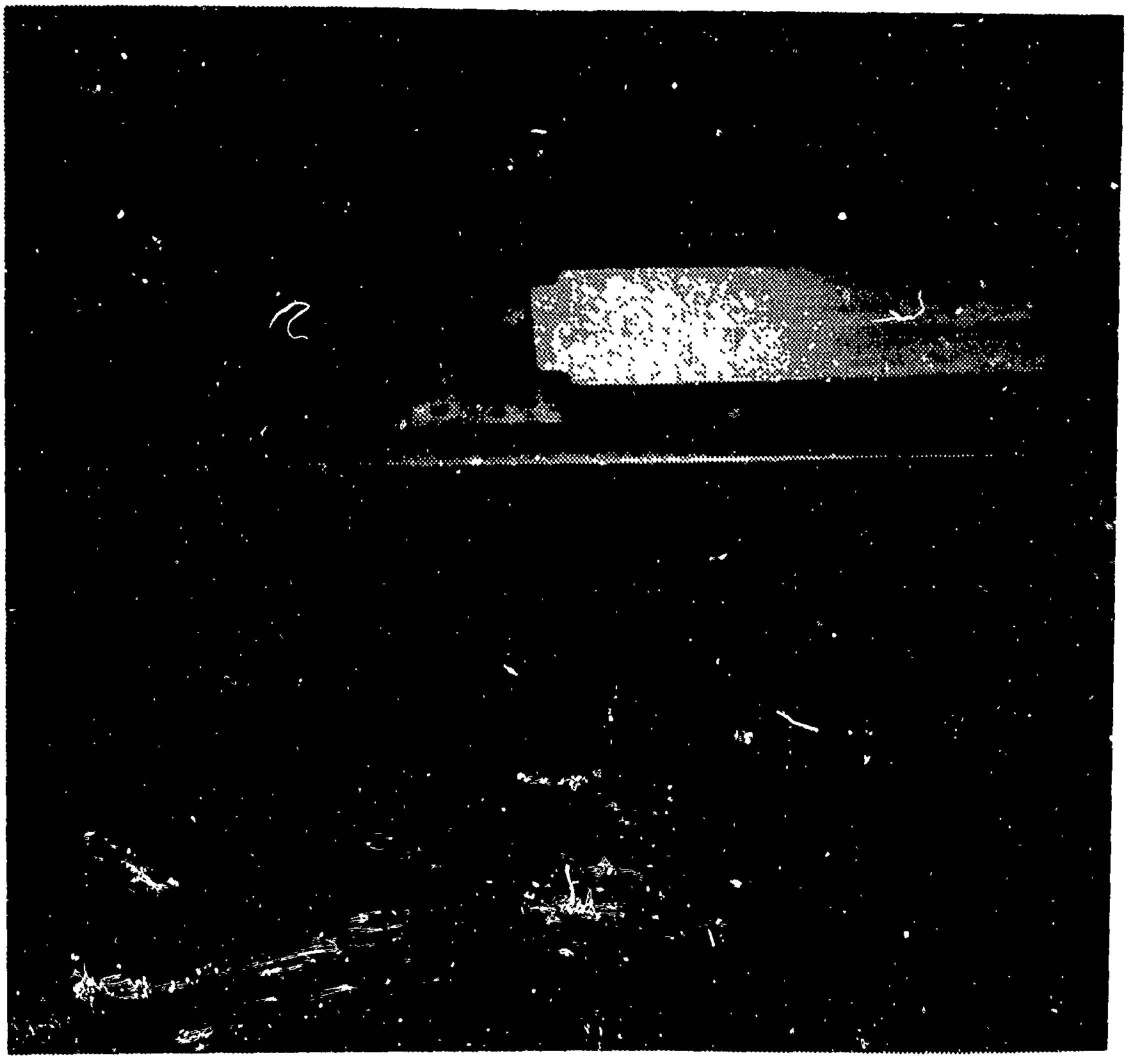


Figure 7. Lower Block with Removable Insert.

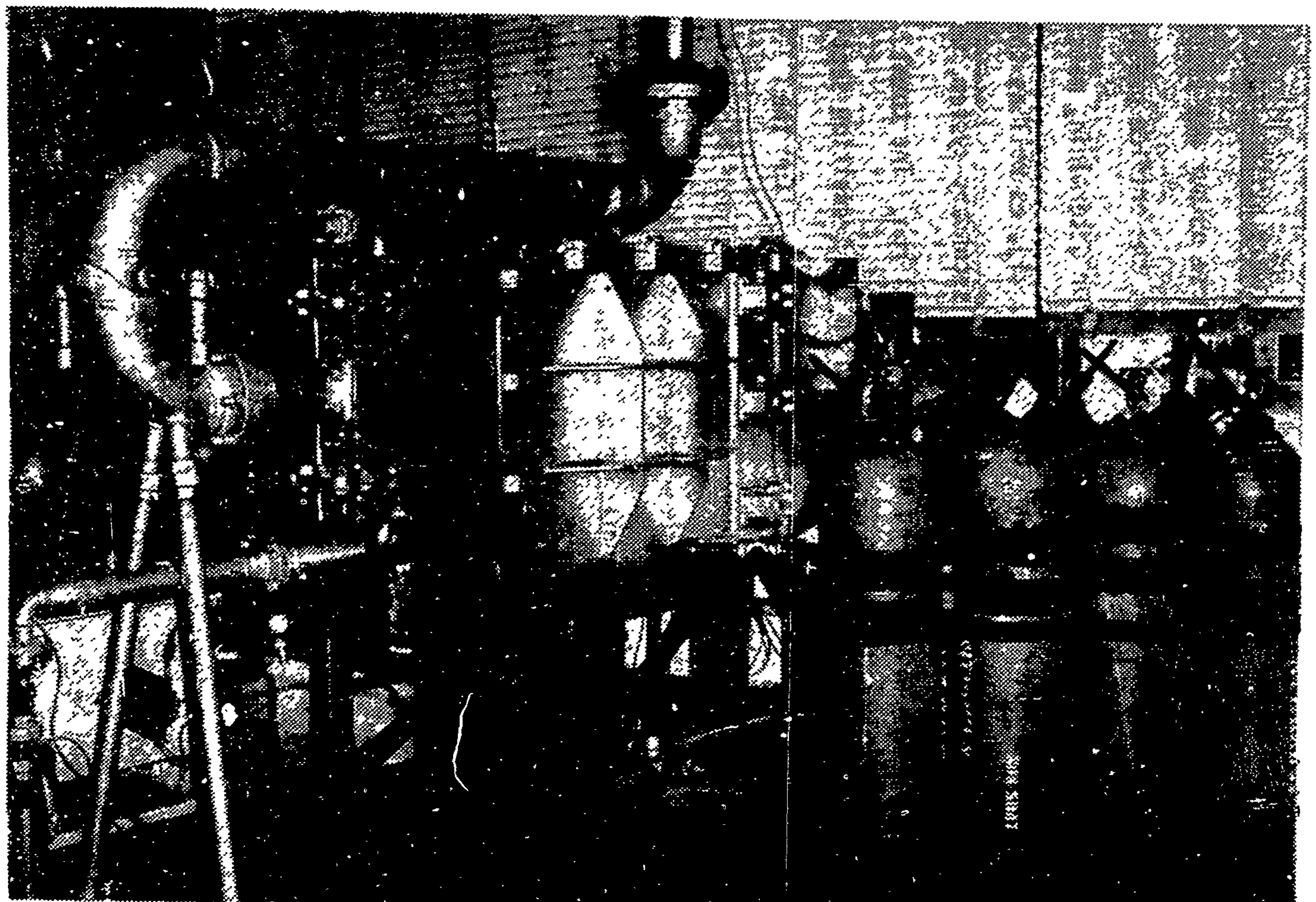


Figure 8. Secondary Flow System.

From the plenum chamber the secondary flow was exhausted through a slot of variable width, and into the primary flow at the sonic throat.

The instrumentation for both the primary and secondary systems consisted of variable reluctance pressure transducers whose output was continuously recorded on strip chart recorders. In this way, the orifice pressure drops and corresponding upstream pressures required for determining mass flow rates by the ASME Power Test Code methods, [Ref. 21], as well as the total pressure measurements for both the primary and secondary system, were obtained as a continuous record during any experimental run. The total pressure probe for the primary system was mounted in the transition section just prior to the test section, and the total pressure probe for the secondary system was mounted in the plenum chamber in the lower block of the test section. Along with the output from the pressure transducers, bourden tube gages installed in a control panel, Fig. 5, registered the total pressure of both systems, reservoir pressure of the primary systems, and control pressure for the primary and secondary pressure regulating valves. The remote controls for the reducing valves as well as the on-off valves were also located on this control panel for convenience of operation.

The mainstream supply pressure, set at P^o_{mo} = 35.0 psig, was chosen for convenience and to provide sufficient run times. The flow of the unthrottled nozzle was measured for mainstream total pressures of 30.0 psig to 50.0 psig with an average discharge coefficient in the range of 1.05 ± .02. The discharge coefficient used here is defined to be the actual flow divided by the theoretically predicted flow. Considerable effort was expended to reduce the system leakages to a minimum value so that all of the measured primary flow actually passed through the throat. The value of the discharge coefficient given above indicates that this effort was not completely successful but further reductions would have entailed massive modifications to the equipment on hand. Further comments concerning the difficulties with the experimental apparatus are given in the section on recommendations for future work. At the supply pressure of 35.0 psig and air temperature of 530°R the resulting mass flow rate of the mainstream air was 1.19 lbm/sec. This value was used for $\dot{m}_r$ in the reduction of data after correcting for slight variations in $\dot{P}^o_{mo}$.

The test section was provided with windows so that visual observations of the jet/mainstream interaction could be made. Visual flow patterns were obtained by coating the inside surfaces of the plexiglass inserts with a mixture of titanium dioxide (TiO_2) and 200 Dow Corning Silicone Oil with a viscosity of 350 centistokes. The oil and titanium dioxide were mixed in the proportions of 7 cc of oil with 1 cc of TiO_2. The mixture was applied with a brush with little regard to the coating thickness since the viscosity is the parameter which dictates the speed with which the coating is blown off. Photographs were taken of the resulting flow patterns. Although it is difficult to obtain any quantitative results from these pictures, they were useful in qualitatively examining the jet/mainstream interaction region.

A simplified linear error analysis was performed on the experimental data in an attempt to arrive at bounds on the uncertainty of the mass flow ratios of $\dot{m}_m/\dot{m}_r$ and $\dot{m}_j/\dot{m}_r$. The sources of error in the experimentally determined mass flow rates fall into three main categories: the first being the error associated with the various parameters which enter into the equation for determining the mass flow rates from the ASME Power Test Code; the second is the error in the pressure transducers and associated electronics; and, the third is the error in reading the pressure traces on the strip chart recorders.

The error associated with the various constants appearing in the ASME mass flow rate equation was found to be insignificant. Furthermore, when examining the error in the ratio of mass flow rates, i.e., $\dot{m}_m/\dot{m}_r$, it was found that any error in the previously mentioned quantities would have no effect on the error of the ratio since the same error appears in $\dot{m}_m$ and $\dot{m}_r$ and would therefore cancel out. The total error in the ratio was found to be a function of the error in measuring the pressure drop across the orifice plate, as well as in measuring the upstream static pressure. Furthermore, the error in both of these pressure measurements is dependent upon transducer error and reading error.

Each individual pressure transducer along with the associated electronics and strip chart recorder was calibrated as a total unit. In this way, much of the error which might have been inherent in single components was calibrated out of the system. It is felt that the transducers and electronics used for determining the upstream static pressure and the total pressure for both the primary and secondary systems, were capable of determining the pressure to within ± 1.0 psi. Combining the transducer error with the error associated in reading the trace on the strip chart recorders, provided a measure of the total error in determing the upstream static pressure and total pressure of the flow. It was found that the static pressure and the total pressure of the primary and secondary systems could be determined to within ± 1.25 psi and ± 3.0 psi respectively. The sensitivity of the differential transducers used for measuring the pressure drop across the orifice plates was such that the differential pressures could be determined to within ± 0.1 psi. This value includes the error in reading the strip chart recorder. By using the previously determined errors for the individual pressure measurements, upper and lower bounds were determined for the uncertainty in the nondimensionalized ratios of $\dot{m}_m/\dot{m}_r$ and $\dot{m}_j/\dot{m}_r$. For $\dot{m}_m/\dot{m}_r = 0.9$ the per cent uncertainty is ± 0.3 per cent, and for $\dot{m}_m/\dot{m}_r = 0.2$, the per cent uncertainty is ± 24.0 per cent. The high error for the latter case (high throttling) comes from the measurement of the primary orifice pressure drop. As previously mentioned, the total uncertainty in measuring the pressure drop across the orifice plate was ± 0.1 psi. For low values of $\dot{m}_m/\dot{m}_r$ (high throttling) the differential pressure is less than 0.5 psi, therefore the per cent uncertainty in this measurement alone is on the order of ± 20.0 per cent. This problem could be corrected if the flow measuring device would give significantly larger pressure drops at the lower values of $\dot{m}_m/\dot{m}_r$. In the case of an orifice meter, this would entail using orifice plates with smaller diameters as the throttling was increased. The experimental setup for this study did not lend itself to changing the orifice plates without considerable effort, so all experimental runs were conducted with the same size orifice plate. It should be noted however, that the consistency of the data seems to indicate that the data falls well within the bounds of uncertainty. A similar analysis was performed for the ratio $\dot{m}_j/\dot{m}_r$ and it was found that the per cent uncertainty in this ratio was on the order of ± 1.5 per cent over its entire range.

Whereas the uncertainty is given for the ratio of mass flow rates, it must be remembered that the uncertainty for the primary or secondary mass flow rate alone, may differ significantly from the uncertainty for the nondimensionalized ratio. For example, for $\dot{m}_m/\dot{m}_r = 0.484$ the per cent uncertainty in this ratio was ± 3.8 per cent. However, the per cent uncertainty in $\dot{m}_m$ alone was ± 5.5 per cent.

IV. DISCUSSION OF RESULTS

Throttling measurements were obtained for values of H of 0.02, 0.04 and 0.08 for perpendicular injection ($\alpha = 0°$). Measurements were also taken for $\alpha = 15°$ with $H = 0.023$, and $\alpha = 30°$ with $H = 0.027$. For each of these five settings the jet supply pressure was varied so that data were obtained for $\dot{m}_j/\dot{m}_r$ over a range of 0 to 0.6. Figures 9 and 10 show the results of the throttling measurements.

Examination of the data in Fig. 9 shows that although the data is consistent and repeatable, there is a general departure of the theory from the data. An improvement on the analytical model will be discussed in detail below.

In Fig. 10, it is evident that the effect of angle of injection as predicted by the theory is qualitatively verified. However, it appears that as the angle of injection exceeds approximately 30° the effect on the throttling becomes negligible. This phenomenon has also been observed by Nunn [Ref. 8]. At high angles of injection it is probable that the primary air flow partially fills the jet slot and the effective angle of injection becomes something less than the designed-for value. (See Fig. 11) The departure of the theory from the data at high angles of injection can be partially attributed to the fact that the analytical model fails to account for such phenomena.

The general departure of the theoretical curves from the data led to a re-evaluation of the analytical model. It is felt that the weakest part of the analysis is in the determination of the pressure acting on the windward face of the jet by means of the assumption that the pressure varies

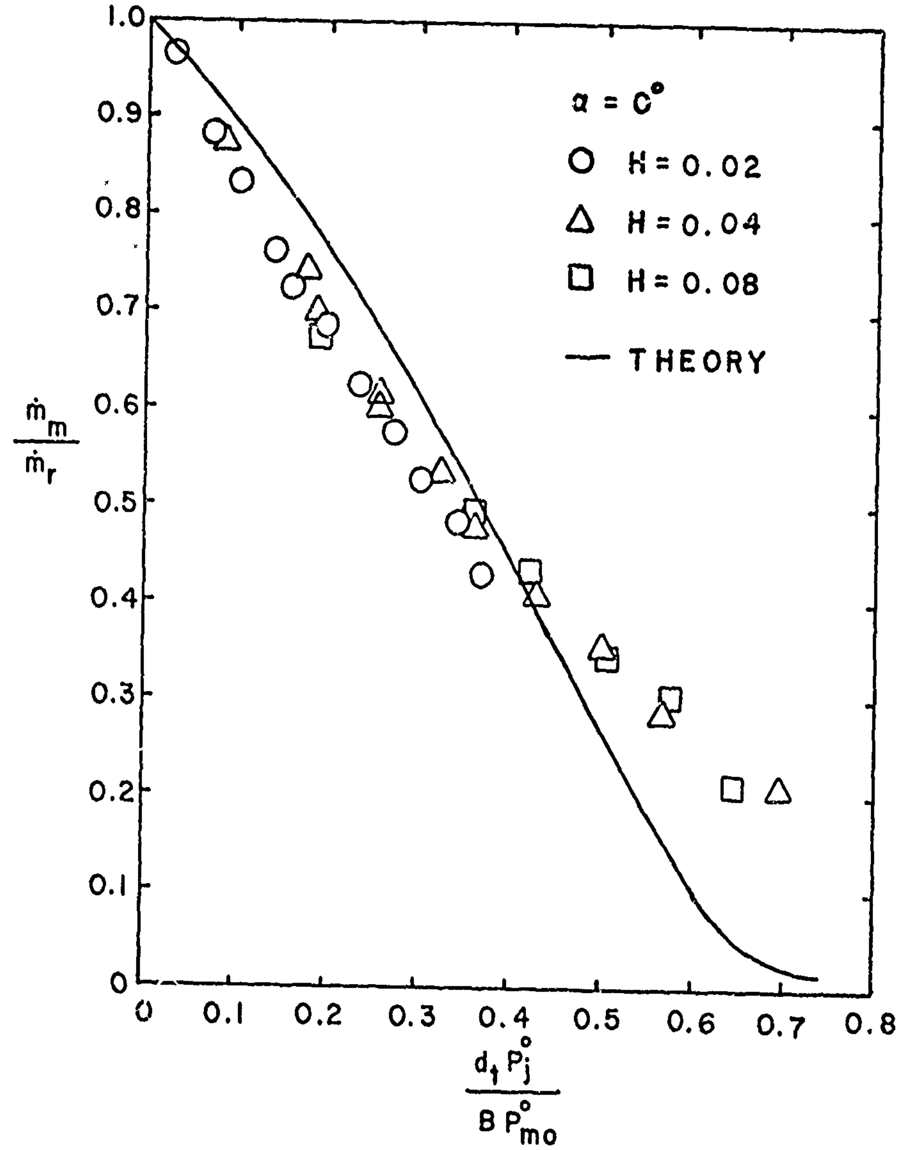


Figure 9. Throttling for Normal Injection (Unmodified Theory).

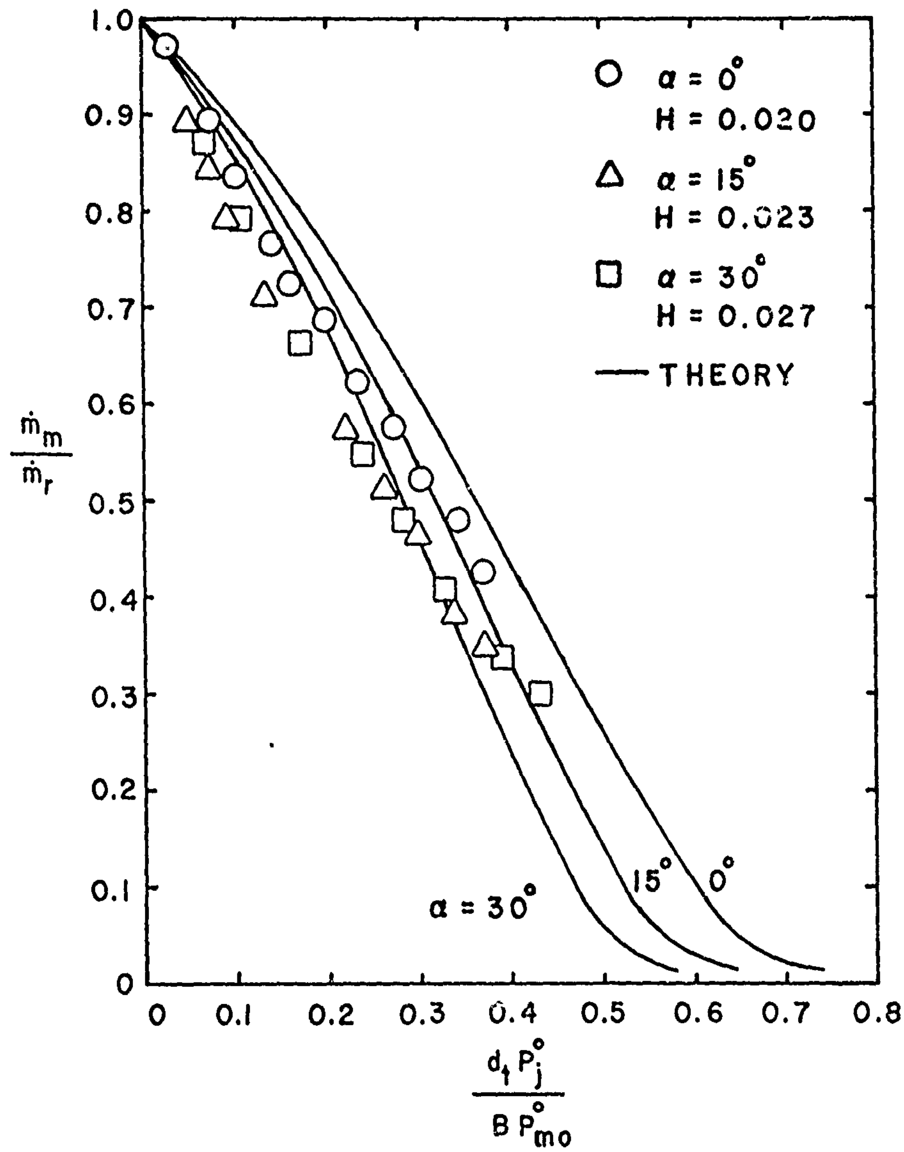


Figure 10. Throttling For Various Angles of Injection (Unmodified Theory).

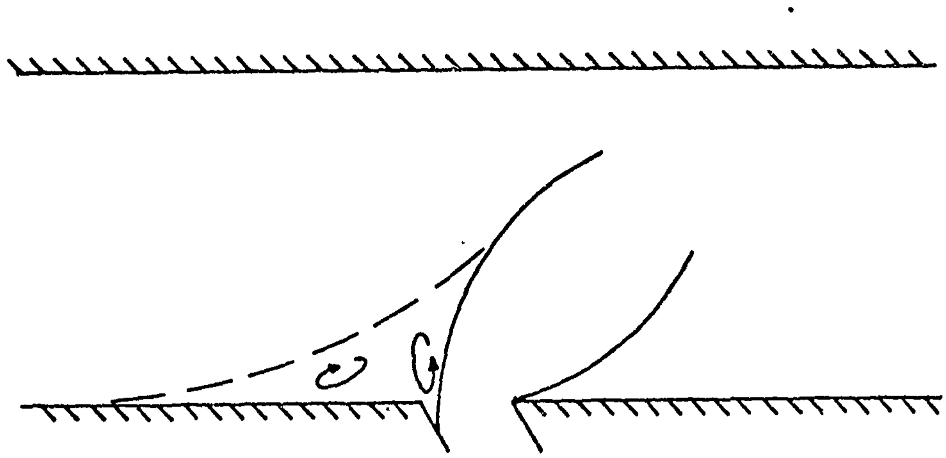


Figure 11. Effect of Mainstream on Jet at High Angles of Injection.

from the static pressure at station 1 down to P_m^* at the point of maximum penetration. Although P_m^* is fixed since the point of maximum jet penetration is the point of minimum flow area for the primary fluid, it can be argued that the pressure in the recirculatory region at the base of the windward face of the jet may be greater than the mainstream static pressure. A pressure weighting factor, β, was sought which would allow for the mainstream total pressure to act of the windward face of the jet at station 1. The following form of β:

$$\beta = \frac{P^o_{mo}}{P} - \left(\frac{P^o_{mo}}{P} - 1\right)\left(\frac{y}{h}\right)^s , \tag{20}$$

when substituted into Equation (12) satisfies the boundary conditions:

$$\beta P = P^o_{mo} \quad \text{at} \quad y = 0$$

$$\beta P = P \quad \text{at} \quad y = h$$

The effect of β is thus to weight the pressure distribution in such a way as to yield higher pressures at the base of the windward face of the jet while maintaining the condition $P = P^*_m$ at $y = h$. The effective average pressure acting on the windward boundary of the jet is now expressed with βP replacing P:

$$\frac{F_x}{h} = \frac{1}{h} \int_{y_1}^{y_2} \beta P \, dy \tag{12a}$$

If, following previous developments, Equation (12a) is expressed in terms of Mach number and substituted into Equation (11), an equation in terms of the mainstream Mach number, M_{mo}, appears as follows:

$$\frac{d_t P^o_j}{\beta P^o_{mo}} = \frac{[A(M_{mo})]^{s-1}}{C[A(M_{mo})-1]^s} \int_{M_{mo}}^{1} \left[1 - \frac{A(M)}{A(M_{mo})}\right]^s \left(\frac{1}{M} - \frac{\frac{\gamma+1}{2} M}{1 + \frac{\gamma-1}{2} M^2}\right) \left[P(M) - 1\right] dM$$

$$+ \frac{1}{C[A(M_{mo})]} \int_{M_{mo}}^{1} A(M) \left(\frac{1}{M} - \frac{\frac{\gamma+1}{2} M}{1 + \frac{\gamma-1}{2} M^2}\right) dM \tag{16a}$$

where:

$$C = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \left[\gamma \left(1 + \sin \alpha\right) + 1\right]$$

Before Equation (16a) can be solved, a suitable value of s must be estimated. Figure (12) shows the dependency of β on s. It seems most probable that if a stagnation region exists at station 1, the mainstream stagnation pressure would act on the windward face of the jet over a short height (y very small) and decrease rapidly to the values calculated in the unmodified analysis. From this reasoning, a value of $s<<1$ would be the logical choice based on Fig. 12. The modified theoretical curve for $\alpha = 0°$ and $s = 0.25$ is shown in Fig. 13. Also shown in this figure is the unmodified theoretical curve and the theoretical curve from Nunn's analysis [Ref. 12]. The agreement between the data and the modified theory is extremely good up to $d_t P_j^o / BP_{mo}^o \approx 0.5$. For higher values of this parameter, the theory predicts greater throttling of the mainstream than was found experimentally.

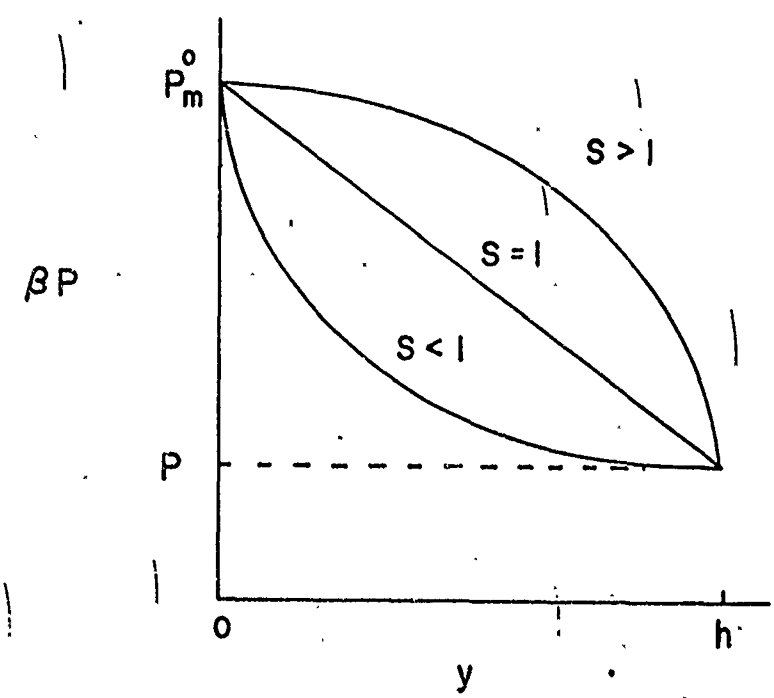


Figure 12. Effect of Exponent s on Pressure Weighting Factor.

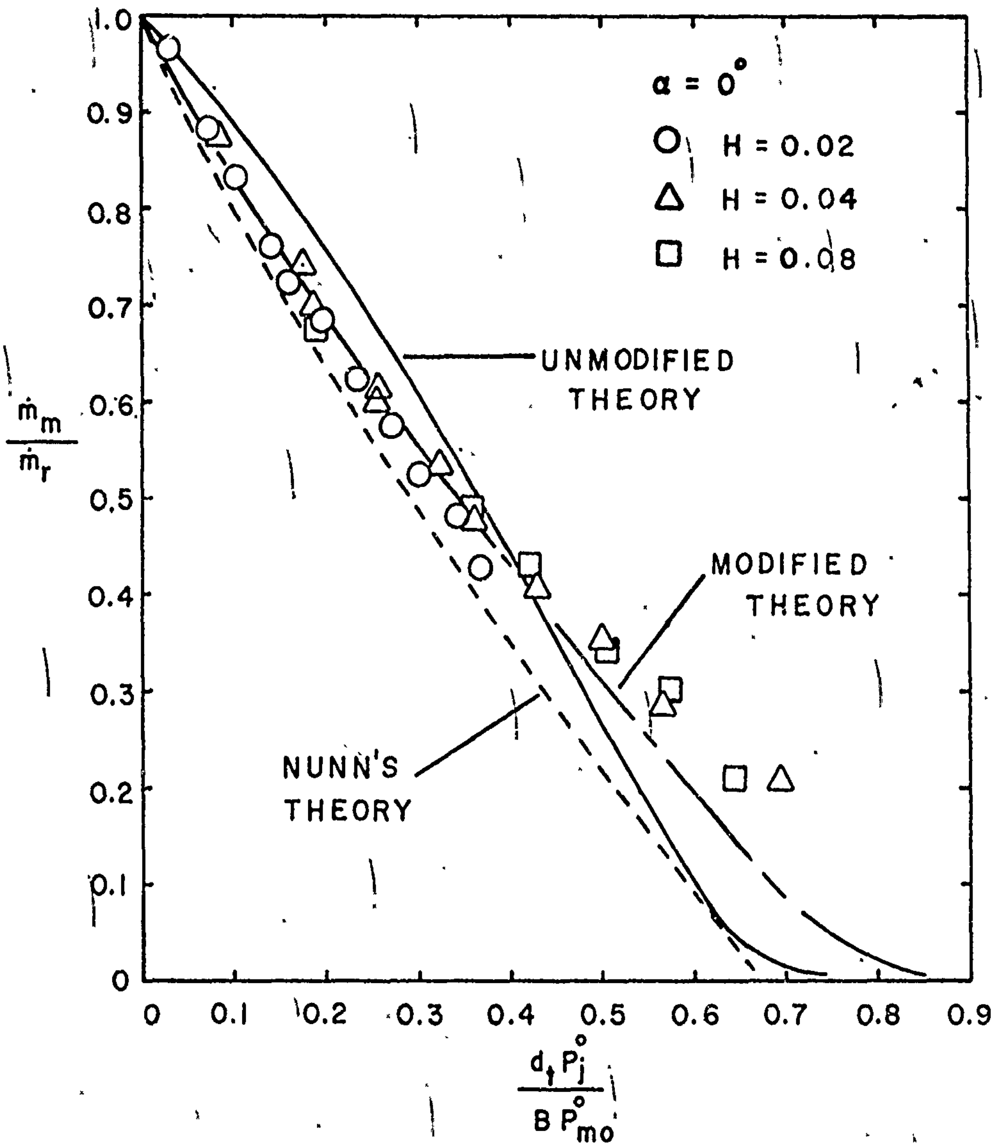


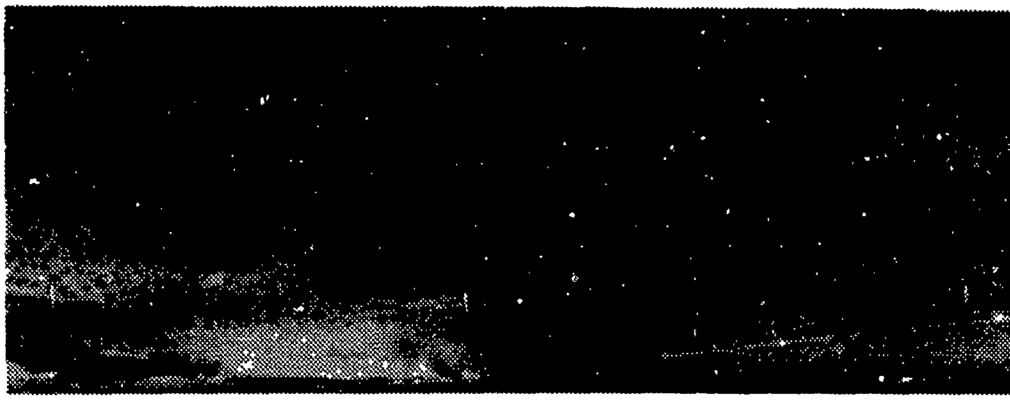
Figure 13. Comparison of Theoretical Curves for Normal Injection.

It should be noted that Nunn's analysis predicts, for a given secondary flow, an improvement in the throttling of the primary flow as the slot width is reduced. The theory presented in this report does not predict such a dependence on slot width. Within the scatter of the data presented in Fig. 13, it is felt that a dependency upon slot width, if any, is not clearly discernable. Further, it should be pointed out that Nunn's data included slot widths that were three times greater than the maximum slot width examined in this study.

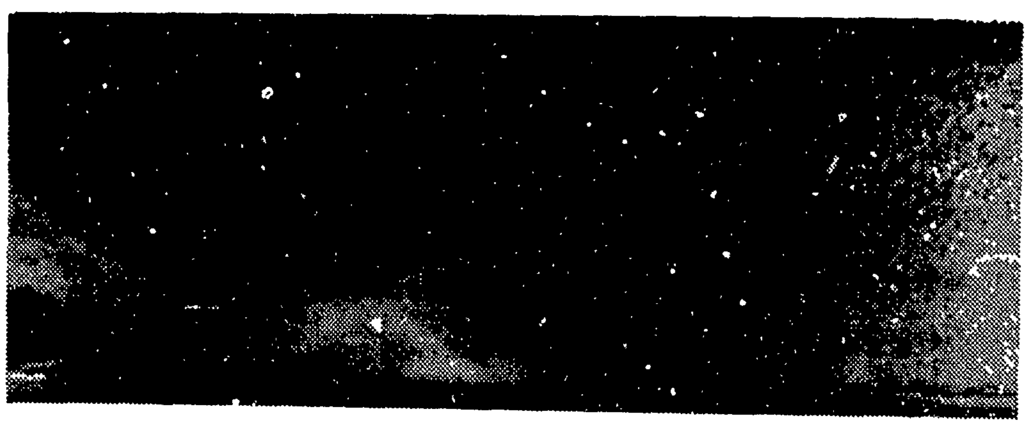
Figure 14 shows a sequence of oil flow photographs taken for various values of P_j^o/P_{mo}^o with $d_t = 0.059$ and $\alpha = 0°$. All three photographs exhibit the "Mach-bottle" characteristic of underexpanded jets as is expected. The photographs show very good definition of the windward boundary of the jet up to the point of maximum jet penetration. This lends a considerable amount of confidence to the assumption that there is a negligible amount of mixing between the jet and mainstream prior to the point of maximum penetration.

The shock pattern within the jet plume is not readily discernable from the photographs. It can probably be stated, however, that the Mach disk occurs somewhere close to the point of maximum penetration. It is clearly evident from the photographs that the jet reattaches itself to the wall after the point of maximum penetration. At the reattachment point the mark of an oblique shock can be seen coming off from the lower surface. This shock serves to curve the jet fluid back so that it again flows parallel to the lower block of the test section.

Examination of the oil streaks on the surface of the lower block of the test section indicated that a small amount of the mainstream fluid flowed



(a) $\frac{d_t P_j^o}{BP_{mo}^o} = 0.504$



(b) $\frac{d_t P_j^o}{BP_{mo}^o} = 0.65$

Figure 14. Oil Flow Photographs, $d_t = 0.059$, $\alpha = 0°$.

around the sides of the jet sheet thereby creating a three-dimensional problem rather than the hoped for two-dimensional situation.

The oil streak which appears in the interior of the jet plume is thought to be oil which is introduced into the jet at the sides by the recirculatory flow in the bubble region.

V. CONCLUSIONS

1. The important parameters in the process of throttling a sonic flow by the injection of a transverse jet, are the jet slot width (d_t), the angle of injection (α), and the ratio of jet to mainstream total pressures.
2. The sum $\dot{m}_m + \dot{m}_j$ is the controlling factor in determining the effectiveness of the throttling process and not $\dot{m}_m$ alone, since the secondary fluid, introduced for the purpose of throttling, contributes to the overall flow through the nozzle. A practical limit to the throttling process is around 60 per cent reduction of the mainstream flow. Reduction of the mainstream beyond this level requires an increase in the secondary flow which is greater than the additional throttling achieved in the mainstream. For example, for normal injection, it requires a jet mass flow rate of approximately 40 per cent of the mainstream reference flow to throttle the mainstream by 60 per cent. The best overall throttling of a nozzle flow that can be attained is therefore approximately a 20 per cent reduction when the sum of $\dot{m}_m$ and $\dot{m}_j$ is considered. Figure 14 shows the net throttling attainable ($\dot{m}_m + \dot{m}_j$) based upon the modified analytical model.
3. Within the range of parameters investigated here, the analytical model which includes the pressure weighting factor, β, with a value of $\beta = 0.25$, is adequate to predict the mainstream throttling up to the 60 per cent level.
4. To achieve the maximum reduction in mainstream flow for a given level of secondary flow, the angle of injection should be a maximum. However,

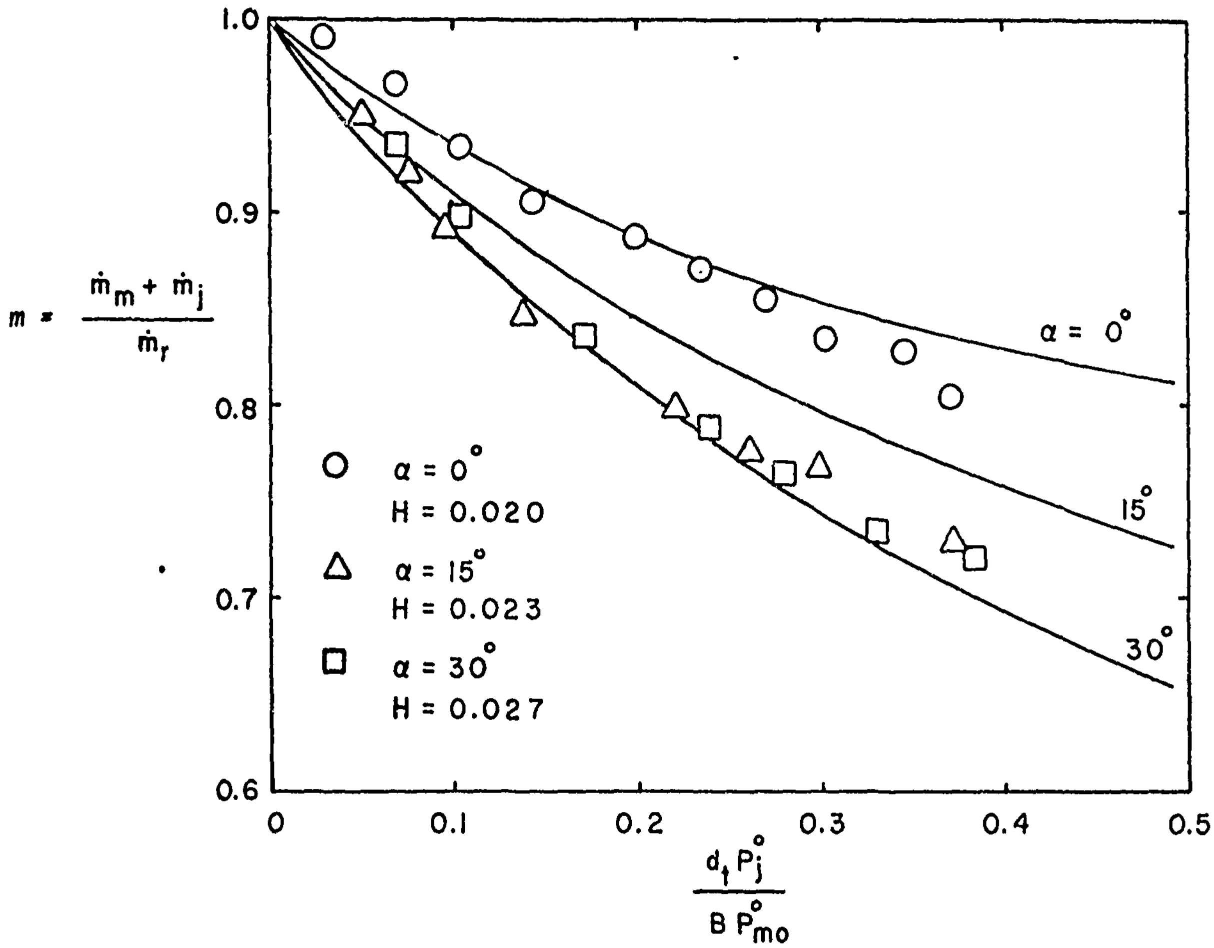


Figure 15. Net Throttling Available for Various Angles of Injection.

an increase in the angle of injection beyond a value of approximately 30° does not seem to hold promise in terms of throttling effectiveness. This conclusion is tempered by the rather uncertain nature of the injection conditions at high angles of injection, and the apparent inability to analytically predict the throttling under these conditions. There is some hope that practical throttling values ($m \leq 1/3$) can be obtained by opposing (90°) injection. The uncertainty of payoff associated with this area of investigation has, however, led to a rather low priority for additional tests.

5. It is clear that the values of m attainable by direct injection at angles up to 30° are not sufficiently small to warrant serious consideration of aerodynamic throttling for missile control jets. In this investigation the lowest value obtained was $m = 0.7$ which is clearly inferior to the comparable PDM system (see Fig. 1). As noted above, there is some promise in proceeding with 90° injection but this task holds less promise than the consideration of other systems such as hybridizations of PDM and vortex or aerodynamic methods.

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