A Study of the Interaction of Gaseous Jets from Transverse Slots with Supersonic External Flows

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The paper describes a study concerning the sonic injection of a gaseous jet through a transverse slot nozzle in a wall into an external flow which is uniform outside of a turbulent boundary layer. An analytic model of the flowfield has been constructed in which conservation of momentum is applied to a control volume at the jet nozzle exit. A series of flat-plate experiments was conducted with normal, sonic jets at external flow Mach numbers of 2.61, 3.50, and 4.54. Pressure data near separation and the plateau were in agreement with existing correlations. Comparisons of the trends predicted by the analysis with two-dimensional force data from these experiments and from other sources showed good agreement. Values of amplification factor, the upstream interaction force plus the jet thrust divided by the vacuum thrust of a sonic jet, of 2.9 to 3.2 were measured. The amplification factor is relatively insensitive to variations in external flow Mach number and variations in injectant gas properties. A correlation of data obtained from experiments with finite-span slots demonstrates that the effective jet penetration height and the slot span are the important characteristic dimensions of such flowfields.

Nomenclature

a_j^* = jet gas velocity where $M = 1$
b = slot nozzle span
c = nozzle discharge coefficient
d = slot nozzle width
F_i = interaction force, upstream of jet nozzle
h = two-dimensional jet penetration height (calculated), or step height
K = amplification factor = $(F_i + T)/T_{sv}$
K_3 = K for a finite-span slot with F_i evaluated as the total interaction force upstream of the y axis (see Fig. 3)
K_3' = K for a finite-span slot in which F_i is evaluated by integrating the pressure distribution upstream along the x axis, and values of T and T_{sv} per unit span are used
L = distance from slot to plate leading edge
M = Mach number
$\dot{m}$ = mass flow rate
P = pressure
$\bar{P}_b$ = average pressure on downstream face of control volume, Fig. 1b
$\bar{P}_f$ = average pressure on forward face of control volume, Fig. 1b
P_p = separation plateau pressure
Re = Reynolds number
T = thrust of injectant; temperature
T_{sv} = sonic vacuum thrust of the injectant
U_2 = velocity at outer edge of shear layer
V_j = average velocity of jet material in x direction leaving the control volume
x = coordinate parallel to solid surface, aligned with the external flow (see Fig. 3)
X_s = value of x at separation point
ΔX_s = x measured from separation point
y = coordinate transverse to flow direction (spanwise direction)
α = parameter used in analysis, see Eq. (4)
β' = parameter used in analysis, see Eq. (2)
β = function of β', γ, M_2; see Eq. (3)
γ = specific heat ratio; without subscript, it refers to external flow
δ = undisturbed boundary-layer thickness just upstream of separation point
ρ = density

Subscripts

0 = stagnation conditions
1 = region just upstream of separation outside of the boundary layer
2 = region downstream of separation shock, at edge of shear layer
= refers to the jet

Received May 9, 1967; revision received October 2, 1967. This work was supported by the Missile and Space Systems Division of the Douglas Aircraft Company under the sponsorship of the Douglas Independent Research and Development Program, by the Daniel and Florence Guggenheim Jet Propulsion Center of the California Institute of Technology, and by NASA, through the Solid Propellant Engineering Section of the Jet Propulsion Laboratory, California Institute of Technology.

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Introduction

THE description of the flowfield which results from the interaction of a gaseous jet with a supersonic primary flow, sometimes called the jet-interaction flowfield, is a problem of current engineering interest. This type of flow is associated with thrust vector control of rocket motors by gas injection, and occurs when gaseous fuel is injected into a supersonic burner. Jet interaction also is a possible technique for the control of high-speed flight vehicles, particularly for applications in which aerodynamic heating is severe. The complex interaction between the two fluid streams is accompanied by a high-pressure region on the wall near the jet nozzle exit, which is the source of an interaction force that augments the thrust of the control jet.

The situation chosen for study is the sonic injection of a gaseous jet through a transverse slot in a wall into an external flow that is uniform and rectilinear outside of a turbu-

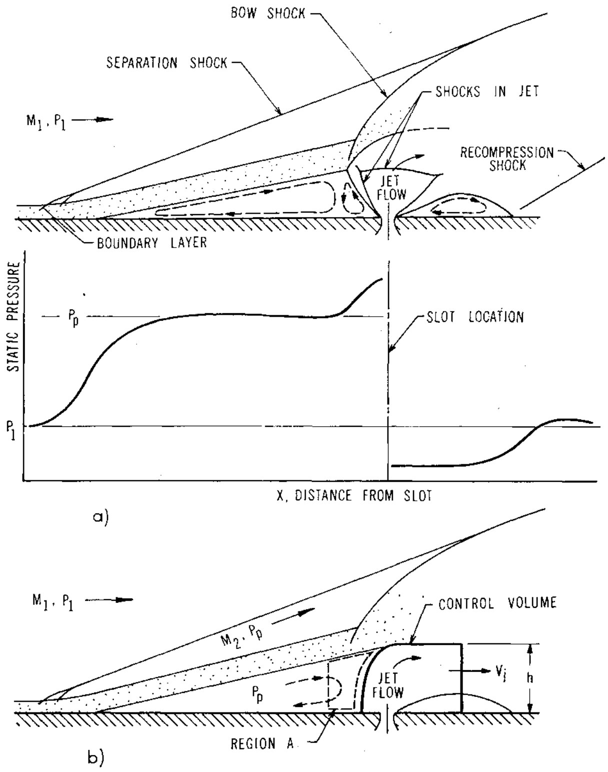

Fig. 1 Sketches of a) flowfield and pressure distribution and b) simplified flowfield used in calculation.

lent boundary layer. The analysis is two-dimensional; the experiments have been made as nearly two-dimensional as possible.

Although a considerable body of data and several studies related to this type of flowfield have been published (see Refs. 1 and 2 and their reference lists), considerable disagreement exists among proposed theories, particularly concerning the influence of the external flow Mach number. A review of this situation has been presented by Hawk and Amick,[3] together with an attempt to correlate the available data. In the opinion of the present authors, the magnitude of three-dimensional flowfield effects, which are present in data obtained from experiments with finite-span slots, have seldom been evaluated properly. As a result, no correlations or comparisons of data with theory have yet been presented in which effects of parameters, such as external-flow Mach number, can be distinguished clearly and independently of three-dimensional influence. Maurer[4] and Romeo[5] have studied the problem of three-dimensional flows associated with finite-span slots. In each case, however, the data that were presented covered a limited range of experimental variables, e.g., Mach number, so that the resulting conclusions were necessarily limited in scope.

The objectives of the present study were to determine the influence on the two-dimensional flowfield of parameters such as Mach number, pressure ratio, and injectant gas composition when the influences of three-dimensional flow were clearly absent, and to determine the magnitudes of three-dimensional influences for finite-span jets.

Description of the Flowfield

Data obtained from this study and others[1-9] have made it possible to describe the flowfield, and the more prominent features are well-known. Some of these are shown in Fig. 1a, which is a schematic diagram of a typical flowfield with the associated static pressure distribution on the wall. In this example, the jet is underexpanded, and the effective obstruction to the external flow that is produced by the jet is significantly larger than the undisturbed boundary-layer thickness.

Boundary-layer separation is an important aspect of the jet interaction flowfield. Therefore, some of the results pertaining to turbulent separation which are of special interest to this investigation must be reviewed. Data presented by Sterrett and Holloway[10] indicate that two-dimensional turbulent boundary-layer separation produced by the flow over a forward-facing step is the same as the separation flowfield produced by a jet, if the region in the immediate vicinity of the step or jet is excluded, and if the step height and jet penetration distance are adjusted so that the lengths of the separated regions are approximately the same. As a result, it is possible to utilize results which were obtained from studies of flow over forward-facing steps to describe jet-induced separation. A review of turbulent boundary-layer separation in front of a forward-facing step has been presented by Zukoski,[11] and some of the conclusions of that paper will be summarized here.

At the beginning of the interaction (see Fig. 1a), the static pressure on the wall rises to the separation pressure in about two boundary-layer thicknesses and then rises more slowly to a first maximum or plateau value. The separation point lies about four step heights upstream of the step, and the plateau pressure is reached at a point about two step heights from the step. The angle formed by the wall and a line drawn from the separation point to the top of the step is approximately 13°, and the pressure rise in the plateau region corresponds to that which occurs when an inviscid flow is turned through this angle. The pressure rise to the actual separation point is about 75% of that to the plateau pressure.

These results were found to be almost independent of Reynolds number, Mach number, and ratio of step height to boundary-layer thickness for the conditions that boundary layer was turbulent, that the Mach number was between 2 and 6, and that the boundary-layer thickness was less than the step height. In addition, the plateau pressure P_p was found to depend only on the undisturbed static pressure P_1 and Mach number M_1, and is roughly correlated by

$$P_p/P_1 = (1 + M_1/2) \quad (1)$$

Analytic Model

The jet-interaction flowfield is exceedingly complex, and contains several features that individually are subjects of current research. Because of this complexity, a highly simplified analytic model of the flowfield has been constructed, which includes only those features believed to have a first-order influence on the structure of the interaction. The analysis is primarily intended to provide correlating parameters and scaling laws for such a flowfield.

The general approach is similar to that used for the three-dimensional work.[6,12,13] A simple body shape is selected to represent the dividing streamline between injectant and external flow. A momentum balance is used to calculate a characteristic dimension of the equivalent body, and the interaction force is then estimated. The calculations are partially based upon experimental information obtained in studies of flows over forward-facing steps.

Consider first the calculation of the characteristic dimension of the equivalent body. A simplified sketch of the flowfield upon which the analysis is based is given in Fig. 1b. The control volume selected consists of the upstream interface between the jet and the external flow, the exit plane of the nozzle and a portion of the wall downstream, and a plane which is normal to the wall and to the primary flow direction and which passes through the downstream separated region. It is assumed that no mixing occurs between the jet and the primary flows along the boundaries of the control volume, and that all shear stress on the control volume can be neglected. The former assumption implies that the structure of the flowfield is determined by condition in the immediate vicinity of the slot. The x momentum balance reduces to equating the net force or drag acting on the boundaries of the control volume to the x component of the momentum flux of injectant as it leaves the volume, $\dot{m}_j V_j$. These quantities must now be evaluated.

When the jet is underexpanded, most of the jet flow passes through a normal shock before much of the turning has occurred.[1,7] For low jet-to-freestream pressure ratios, the jet may be near-sonic or subsonic as it is turned. Apparently, the jet flow will be either subsonic or near-sonic relatively early in the turning process over a wide range of pressure ratios. Because the recompression shock, shown in Fig. 1a, is observed to extend much closer to the wall than a characteristic dimension of the jet, it is evident that the majority of the jet flow passes through it, and that the jet flow must be supersonic near reattachment. Hence it is reasonable to assume that the jet flow will be approximately sonic as it leaves the control volume. If the jet flow is also adiabatic then V_j can be replaced by a_j^*, where a_j^* can be evaluated from the jet reservoir conditions and the properties of the jet gas.

The drag may be represented by $(\bar{P}_f - \bar{P}_b)(h)$, where $\bar{P}_f$ and $\bar{P}_b$ are the average pressures acting on the upstream and downstream faces of the control volume, respectively, and h is the jet penetration height. The flow in the region forward of the injector must be analyzed to make a crude estimate of $\bar{P}_f$. Investigation of the flow over a forward-facing step[11] shows that the separated flow region may be considered as a constant-pressure region with pressure P_p, which is fixed by the turning angle θ. The pressure on the face of the step is fixed in part by the general pressure rise in the separated region, $P_p - P_1$, and in part by the pressure rise required to turn the recirculating flow near the face of the step, $\bar{P}_f - P_p$. An estimate for $\bar{P}_f - P_p$ can be made from a momentum balance for region A of Fig. 1b. This leads to

$$(\bar{P}_f - P_p)h \approx \int_0^h \rho U^2 dy \approx \beta' \rho\{P_p, T_0\} U_2^2 h \qquad (2)$$

where the integral is evaluated along the upstream (left) face of region A, and the quantity β' is a proportionality constant. The value of β' has been selected as 0.062 in order to give agreement with jet interaction force data. This order of magnitude is consistent with measurements of maximum reverse flow speeds in separated regions reported by Vas and Bogdonoff,[14] and mean pressure data on forward faces of steps indicate that β' is a function of h/δ, when $h/\delta < 1$.[11] Manipulation of Eq. (2) leads to the result that

$$\bar{P}_f \approx \left[1 + \beta' \frac{\gamma M_2^2}{1 + [(\gamma - 1)/2]M_2^2}\right] P_p \equiv [1 + \beta] P_p \qquad (3)$$

where β is a function of M_1 through its dependence on M_2.

The average base pressure can be formally written as αP_1, where α must be evaluated. The minimum wall pressure level downstream of the injection point is roughly $(0.4 \pm 0.1)P_1$ for a wide range of conditions. Estimates of pressure in the injection flow suggest that the static pressure is greater than or equal to P_1. Hence, $1.2 \geq \alpha \geq 0.4$ probably covers the reasonable range for α. If the x momentum balance is combined with the expressions for $\bar{P}_f$ and $\bar{P}_b$,

$$h = \dot{m}_j a_j^* / [(1 + \beta)(P_p - P_1) + (1 + \beta - \alpha)P_1] \qquad (4)$$

The interaction F_i is approximately given by $(P_p - P_1)(X_s/h)h$ because in the simple model used here, the pressure in the separated region is P_p. This definition of F_i, combined with Eq. (4), can be used to derive an expression for

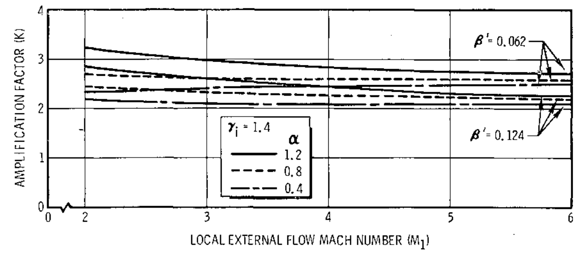

Fig. 2 Dependence of calculated values of amplification factor on Mach number for various values of parameters α and β'.

amplification factor K defined as follows:

$$K \equiv \frac{F_i + T}{T_{sv}} = \frac{[\gamma_j/(\gamma_j + 1)](X_s/h)}{(1 + \beta) + (1 + \beta - \alpha)[P_1/(P_p - P_1)]} + \frac{T}{T_{sv}} \qquad (5)$$

where T is the jet thrust, T_{sv} is the vacuum thrust of a sonic jet, and their ratio is approximately 1 when $P_{0j}/P_1 > 20$.

Continuation of the calculation requires introduction of empirical data obtained from step tests and restricted to the separation of a turbulent boundary layer. It has been found[11] that $(P_p - P_1) \approx (0.5 M_1 P_1)$, $X_s/h \approx 4.2$, and $M_2 \approx \frac{3}{4} M_1$ are good approximations for the turbulent case when $2 \leq M_1 \leq 6$.

Equation (5) was evaluated with $\gamma_j = 1.4$ and $P_{0j}/P_1 > 20$ for the approximations given previously and for several values of α and β'. The results, shown in Fig. 2, indicate that K is only a weak function of α, β', and Mach number. The dependence on injectant properties comes in through the $\gamma_j/(\gamma_j + 1)$ terms and indicates that K should increase very slightly as γ_j increases. The injectant flow rate, pressure ratio, or slot width do not enter at all, and thus K is independent of these parameters. These conclusions are not necessarily valid for very small flow rates because the approximations made in evaluating Eq. (5) are not valid when $h/\delta < 1$. Because h decreases as $\dot{m}_j a_j^*$ approaches zero, one would expect K to vary as well because of this h/δ effect. In general, the drag of a step decreases[11] as h/δ decreases below 1. Thus, β will also decrease, and, consequently, K will increase as h/δ approaches zero.

The calculation performed here for the turbulent case justifies and rationalizes to some extent the experimental results discussed later, and obviously is not intended as an a priori calculation. However, because the dependence of K on α and β' is so weak, one would expect the calculation to at least predict the correct trends.

When transition occurs in the shear layer between the separation point and the jet, approximately the same value of F_i would be expected as in the fully turbulent case for the following reasons. Under these conditions, wall static pressures immediately upstream of the jet are observed to be approximately the same as in the fully turbulent case, and this implies that the local flow structure and h would also be the same. Because the increase in wall static pressure in the separated region is approximately proportional to the local turning angle of the external inviscid flow, it can be shown that the total force upstream of the jet is roughly independent of the distribution of external flow turning angle or wall static pressure for a fixed jet penetration distance.

Description of Experiments and Data Presentation

A series of experiments was conducted in the 20-in. supersonic wind tunnel of the Jet Propulsion Laboratory, California Institute of Technology. The model configuration,

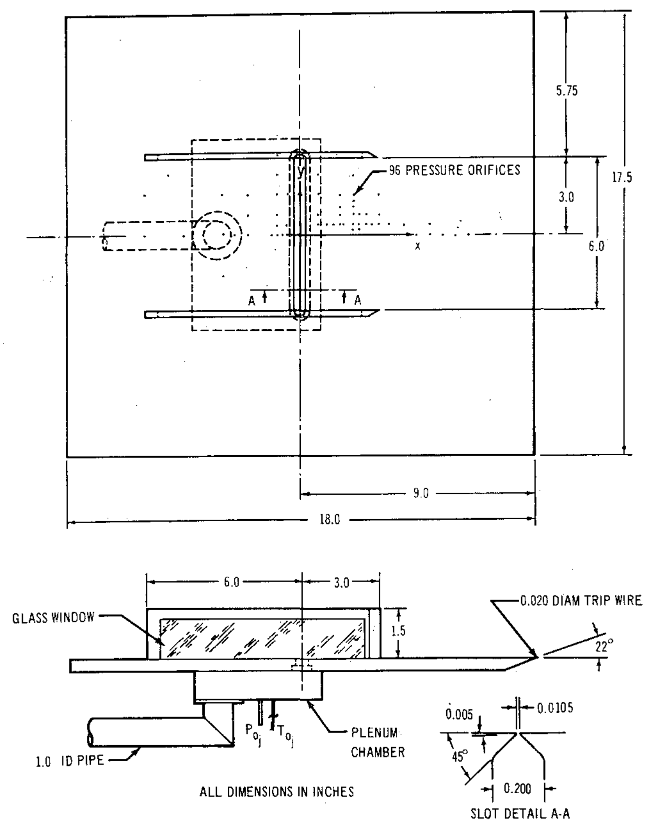

Fig. 3 Sketches of experimental model.

dimensions, and the coordinate system for data presentations are given in Fig. 3. During the experiments, a sonic jet of gas was injected through a transverse slot in the surface of the model, normal to the external flow. End plates having glass inserts usually were mounted at either end of the slot to create a flowfield that would be as nearly two-dimensional as possible. The experiments were conducted at test-section Mach numbers of 2.61, 3.50, and 4.54. Data are presented at a fixed value of test-section unit Reynolds number at each Mach number. At Mach numbers of 2.61 and 3.50, examination of shadowgraph photographs indicated that boundary-layer transition occurred 2 to 3 in. downstream of the plate leading edge.

Measured quantities consisted of test-section flow parameters, jet-reservoir flow parameters, and static-pressure distributions on the surface of the plate near the jet. The discharge coefficients for this slot were taken from the calibration of a very similar nozzle.[6] A set of tabulated data from this experiment is given in Table 1, and static–pressure distributions are presented in Figs. 4–7. Solid symbols have been used in Figs. 4–7 to denote tests in which the upstream separated region extends forward of the end plates.

Figure 4 shows data obtained with nitrogen and helium as injectant at $M_1 = 2.61$ for various values of P_{0j}/P_1. The upstream plateau is well-defined for separation distances of approximately 2 in. or more; the pressure immediately upstream of the slot did not reach a limiting value. The minimum pressure observed downstream of the slot was more nearly constant, except for the highest jet total pressure or mass flow rate. The static pressure in the downstream region with nitrogen as injectant did not exceed P_1 near reattachment. The downstream static pressure levels obtained with helium as injectant were considerably higher than those obtained with nitrogen. All but one of the sets of data shown in Fig. 4 were obtained with the end plates installed, but one set, which was obtained with end plates removed, is included for comparison.

Data obtained with nitrogen as injectant at a test-section Mach number of 3.50 are presented in Fig. 5. Upstream of the jet, the data show essentially the same general features as the data of Fig. 4. Each of the pressure distributions in the downstream region shows a distinct overshoot of P_1, in contrast to the data obtained at Mach 2.61. Similar data obtained at Mach 4.54 with nitrogen and helium as injectant are presented in Fig. 6. The character of the pressure distributions upstream of the jet, and evidence from the shadowgraph photographs, indicates that these data correspond to transitional separation. Downstream of the jet, the static–

Table 1 Test data summary

	M_1	P_{0_1}, psia[a]	T_{0_1}, °R	Re, 1/ft $\times 10^{-6}$	Injectant gas	P_{0j}, psia	T_{0j}, °R	$\frac{P_{0j}}{P_1}$	c	F_i, lb/in.	$\frac{F_i}{\dot{m}_j a_j^*}$	K or K_3'	
1	2.61	19.3	572	3.5	...	...	...	...	...	...	...	...	No injection scan
2	2.61	19.4	572	3.5	N_2	23.1	536	23.5	0.929	0.640	3.84	3.21	
3	2.61	19.15	572	3.5	N_2	43.5	530	44.5	0.942	1.118	3.50	3.08	
4	2.61	19.30	572	3.5	N_2	83.1	524	82.9	0.954	1.990	3.23	2.88	
5	2.61	19.4	573	3.5	N_2	151.5	519	154.0	0.967	...	...	...	Separation forward of end plates
6	2.61	19.4	563	3.5	He	45.0	535	45.6	0.923	1.295	3.68	3.28	
7	2.61	19.4	564	3.5	He	85.2	540	86.5	0.935	2.43	3.58	3.23	
8	2.61	19.0	567	3.5	N_2	42.4	521	44.0	0.941	(1.01)	(3.24)	(2.87)	End plates removed
9	2.61	19.8	570	3.5	N_2	115.1	519	116.5	0.961	(2.24)	(2.60)	(2.49)	End plates removed
10	3.50	34.8	556	4.0	N_2	7.59	525	16.55	0.907	0.200	3.74	3.13	
11	3.50	34.9	560	4.0	N_2	14.85	527	32.4	0.920	0.408	3.84	3.22	
12	3.50	34.9	564	4.0	N_2	28.1	527	81.0	0.933	0.675	3.31	2.92	
13	3.50	34.9	566	4.0	N_2	55.1	525	120.2	0.947	1.394	3.43	3.00	
14	3.50	34.9	567	4.0	N_2	97.3	521	212	0.958	...	...	...	Separation forward of end plates
15	4.54	54.2	565	3.9	...	...	...	...	...	...	...	...	Separation forward of end plates
16	4.54	54.1	575	3.9	N_2	10.12	533	59.1	0.912	0.258	3.59	3.09	
17	4.54	54.01	579	3.9	N_2	18.06	529	101.9	0.924	0.465	3.59	3.09	
18	4.54	53.9	581	3.9	N_2	36.2	525	212	0.938	...	...	...	Separation forward of end plates
19	4.54	54.1	587	3.9	He	38.3	527	224	0.920	...	...	...	Separation forward of end plates

[a] Nominal values.

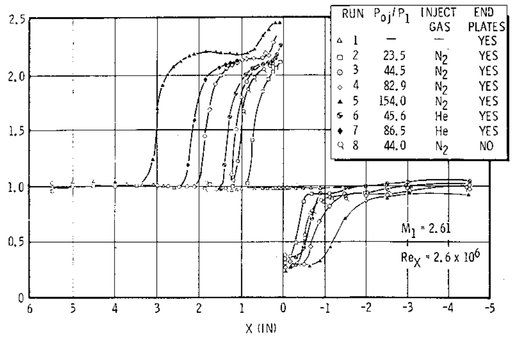

Fig. 4 Static pressure distributions, $M_1 = 2.61$.

pressure overshoot near reattachment is considerably greater than that shown by the data obtained at Mach 3.50. In addition, the data obtained with helium as injectant show much higher static-pressure levels in the downstream region than do the nitrogen data.

Static-pressure data obtained with and without end plates are presented in Fig. 7, at Mach 2.61. Data for two sets of test conditions are presented, including data obtained at various values of y.

Separation Pressure Profile and Downstream Region

In an examination of pressure profiles produced by forward-facing steps, Zukoski[11] showed that profiles from a wide range of test conditions were well-correlated by a relationship of the form $(P - P_1)/(P_p - P_1) = f\{\Delta X_s/\delta\}$. Data obtained in the present investigation and that reported by Romeo and Sterrett[1] agree well with the previous correlations. Hence, as one would expect for a free interaction region, the separation process for solid spoilers and jets appears to be identical. In addition, the location of the separation point ahead of the jet [taken as the point at which $(P_s - P_1) = \frac{3}{4}(P_p - P_1)$] was found to be roughly $4h$ when h was calculated from Eq. (4) for data from the present report, and for the data with end plates from Refs. 15 and 18. Because of this dual dependence of scale length, δ upstream and h downstream of separation, it is clear that the complete pressure field cannot be scaled with a single parameter. Consequently, the induced force may depend on both δ and h.

The static-pressure distributions obtained downstream of the jet showed a region of relatively constant static pressure, less than P_1, immediately downstream of the nozzle exit. The value of this pressure was affected only by the external-flow Mach number and the injectant-gas molecular weight. In the region downstream from reattachment, the pressure was observed to exceed P_1, except at Mach 2.61, and the

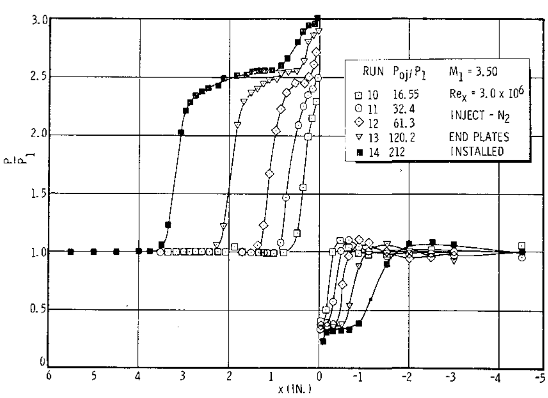

Fig. 5 Static pressure distributions, $M_1 = 3.50$.

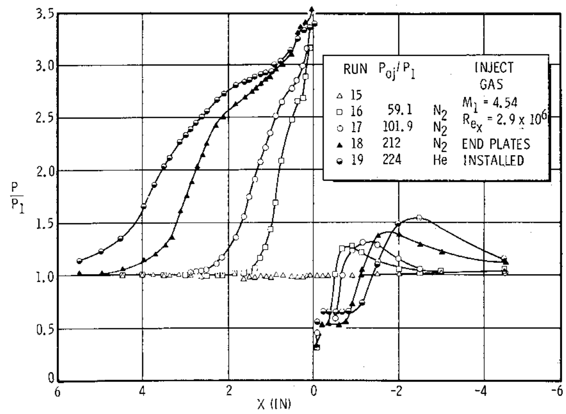

Fig. 6 Static pressure distributions, $M_1 = 4.54$.

magnitude of this overshoot was observed to increase with increasing M_1. The data obtained at Mach 2.61 and 3.50 showed significant unfavorable interaction forces downstream of the jet; however, at Mach 4.54, the net downstream interaction force was found to be almost exactly zero with nitrogen as injectant, when the pressure integration was continued sufficiently far downstream. A similar static-pressure overshoot in the downstream region has been observed in the case of injection through a circular hole,[13] but the magnitude of the overshoot was observed to decrease with increasing M_1. The experiments with circular hole injection[13] also show higher static pressures in the downstream region with helium as injectant than with nitrogen.

Two-Dimensional Force Data

An important aspect of the force data presentation is the selection of correlating parameters. If we consider the external flow and the jet to be the same gas at equal stagnation temperatures, and if effects of heat transfer to the wall can be neglected, then requirements of fluid-dynamic similarity will be satisfied by fixing the following set of parameters: M_1, P_{0j}/P_1, d/L, Re_L. Results obtained from experiments with forward-facing steps[11] show that h/δ and M_1 are the important parameters when δ is evaluated at the separation point in the absence of interaction, and that the Reynolds number is important only through its effect on δ. The parameter h/δ can be related to more easily measured variables by use of Eq. (4). Thus, if injectant fluid properties and M_1 are fixed,

$$h \propto \frac{(P_{0j}/P_1)d}{(1 + \beta)(P_p/P_1 - 1) + (1 + \beta - \alpha)} \tag{6}$$

The denominator of Eq. (6) is most probably a function of h/δ because the pressure rise, β, and probably α, depend upon this ratio. Consequently, this result can be written formally as:

$$h/\delta = g\{P_{0j}d/P_1\delta\} \tag{7}$$

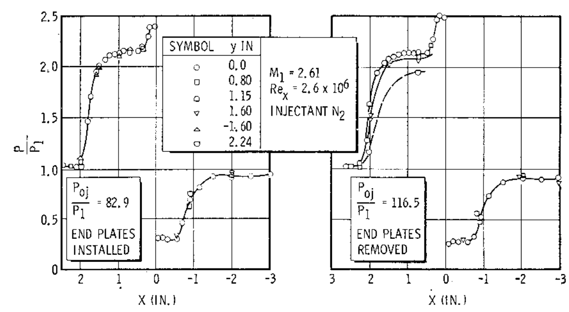

Fig. 7 Static pressure distributions showing effects of end plates.

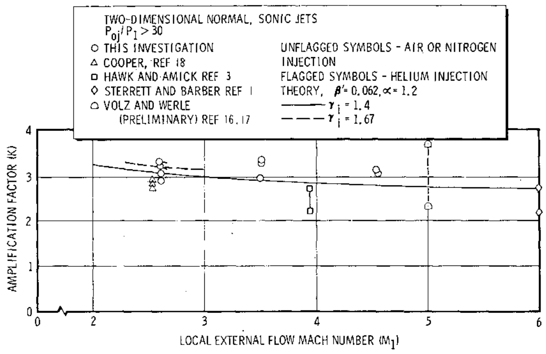

Fig. 8 Dependence of experimental values of amplification factor on Mach number.

The majority of the experimental evidences[1,2,15–17] show that $(P_{0j}d)/P_1$ correlates data obtained at a fixed M_1 for various values of d; therefore, d and P_{0j}/P_1 need not be taken as independent parameters. However, the data of Sterrett et al.[11] and the data of Volz and Werle[16,17] do show a small but systematic variation with slot width. Therefore, if boundary-layer separation to the plate leading edge is excluded, the majority of the preceding experimental evidence, supported by the analysis, indicates the choice of M_1 and h/δ or $(P_{0j}d)/(P_1\delta)$ as the appropriate correlating parameters. Explicit dependence on Reynolds number has been omitted, based upon the results of data obtained with forward-facing steps.[11]

Other investigators[1,2] have used the parameter $(P_{0j}d)/(P_1L)$, which is comparable to use of h/L. If this group and M_1 are held fixed for a particular piece of equipment and Re_L is varied, then h/δ will also change, and fluid-dynamic similarity will not be maintained. In comparing different experiments, h/L and h/δ also may give different correlations because of the influence on δ of boundary-layer trips and transition position. Hence we feel that h/δ is preferable to h/L even though data are not available at present which would distinguish clearly between the two. However, when d and P_{0j}/P_1 are varied for fixed external flow conditions, both $(P_{0j}d)/(P_1L)$ and $(P_{0j}d)/(P_1\delta)$ should correlate the data equally well, since under these conditions one is a unique function of the other.

The data that have been chosen for comparison with the present results are limited to those obtained in tests in which end plates enclosed the separated flow regions in order to obtain as nearly two-dimensional a flow as possible. Interaction force data presented from this investigation and from Cooper,[18] Volz and Werle,[16,17] and Sterrett et al.[15] were obtained by integrating pressure distributions upstream of the nozzle exit. In the case of Ref. 15, only the data obtained with end plates were used, and these are suspect since the end plates did not completely enclose the jet. Data obtained by Hawk and Amick[3] were obtained by direct measurement of forces.

Amplification factor data from the present investigation (see Table 1) and from Refs. 16 and 17 show a decreasing trend with increasing $(P_{0j}d)/(P_1\delta)$ at small values of this parameter, followed by a region of constant amplification factor. The present data were not obtained over a wide range of $(P_{0j}d)/(P_j\delta)$, and the region of constant K with increasing $(P_{0j}d)/(P_1\delta)$ is not as clearly established as in the latter set. The turbulent boundary-layer data of Hawk and Amick,[3] which include a wide range of both P_{0j}/P_1 and d, show no dependence on $(P_{0j}d)/P_1$. The data of Sterrett et al.[15] show constant K for a fixed d and variation in P_{0j}/P_1, but show a variation with d. From all these data, it may be concluded that the two-dimensional value of K is independent of $(P_{0j}d)/(P_1\delta)$ if the value of this parameter is sufficiently large, a result which is consistent with the analysis.

The force data have been plotted vs the remaining parameter M_1 in Fig. 8. Data are presented for normal, sonic jets of air, nitrogen, and helium, exhausting through slot-type nozzles, and for conditions of both fully turbulent or transitional flow in the upstream separated region. A curve determined from Eq. (5) is included for comparison. A wide range of $(P_{0j}d)/(P_1\delta)$ has been included, although the comparison has been limited to $P_{0j}/P_1 > 30$ to avoid variations caused by changes in jet thrust coefficient with pressure ratio. The lower data points are probably the most relevant for comparison, since they correspond to the larger values of $(P_{0j}d)/(P_1\delta)$, or $h/\delta > 1$.

The data from the present investigation are the only set obtained with relatively large end plates which include measurements made at several values of external-flow Mach number in the same facility and with the same model and instrumentation. These data indicate that the influence of M_1 on K is very small, a result which is in good agreement with the analysis. The fact that both transitional and turbulent data correlated well suggests that the normalized interaction force may be independent of Reynolds number. The data obtained with helium as injectant show an increase in K of from 6 to 7%, as compared with nitrogen, and this effect is also in good agreement with the prediction of Eq. (5). Comparison of the present data with that of other experiments shows an appreciable scatter, but the data, taken as a whole, show little systematic Mach-number dependence.

Three-Dimensional Effects

The preceding conclusions depend strongly upon the criteria used in choosing the data points for comparison, in particular, the limitation to test conditions in which end plates fully enclosed the separated flow regions. The purpose of the present section is to justify the restriction and to show the importance of three-dimensional effects in reducing interaction forces from values corresponding to two-dimensional flow.

The pressure distributions of Fig. 7 for flows with and without end plates show that with end plates the pressure field is relatively independent of y position on the plate surface. When end plates are used, no strong flow in the y direction is expected; some oil flow studies, such as those of Heyser and Maurer,[9] show that none exists. However, when the end plates are removed, the pressure distribution becomes a function of y, e.g. in Fig. 7, the length of the separated region decreases, and strong lateral flows from the upstream separated region are indicated by oil flow studies.[9] Finally, the plateau pressure and hence the initial separation angle near the plate centerline are not affected by the removal of end plates.[9,15,18] However, when transverse flows exist, the streamline dividing the flow passing over the jet from that entering the recirculation zone must now shift outward to include a part of the boundary-layer flow upstream of the separation point. This shift has the effect of moving the separation point toward the disturbance. In addition, the new dividing streamline is now located in a region of higher average velocity in the shear layer than that for the two-dimensional flow, and this produces an increase in the drag of the effective obstacle produced by injection, a decrease in penetration height, and an additional decrease in the length of the separated region. If the ratio of penetration height X_s to slot span b is sufficiently large, these transverse flows will become negligible; they can be prevented by the use of end plates.

Although end plates will introduce spurious effects themselves, the work of Lewis, Kubota, and Lees[19] shows that end plates can be used to achieve an excellent approximation to a two-dimensional flow, for the case of laminar separation induced by a wedge.

The principal approximation made concerning data obtained without end plates is that the centerline pressure distribution closely approximates the two-dimensional value even when X_s/b is as large as 1.2.[15] Examination of the present and other published data show this approximation to be grossly incorrect. For example, data from the present investigation, Refs. 15 and 9, show that the integral of the centerline pressure distribution increases when end plates are installed, and that this change always increases as $(P_{0j}d)/P_1$ is made larger. This change was 22% when $X_s/b = 0.35$ in the present experiments, 13% when $X_s/b = 0.44$ for data of Ref. 15, and over 40% when $X_s/b = 0.6$ for data of Ref. 9. Interpretation of data of Ref. 9 is complicated because the end plates were not large enough to give a good approximation to two-dimensional flow except at very small values of P_{0j}/P_1.[20]

Thus, it may be concluded without ambiguity that without end plates, the pressure distribution upstream of the jet will be a function of y, and that the centerline pressure distribution will be significantly affected unless X_s/b is very small. Hence, in examining the dimensionless scaling parameters for the three-dimensional case, one must add the span b to the groups discussed for the two-dimensional case. Because neither h/δ nor M influenced K strongly for the two-dimensional case, it appears reasonable that a single group such as h/b should correlate the three-dimensional values of the amplification factor, even though it is clear that h, calculated from Eq. (4), is not directly proportional to the penetration of a jet for a finite-span slot.

Examination of the three-dimensional results[4,5,9,15,21] shows that h/b is a satisfactory correlating parameter. For example, values of an amplification factor for finite-span slots, K_3', based on the integral of the upstream centerline pressure distribution, are presented in Fig. 9 as a function of h/b with h calculated from Eq. (4). Note that the data include a wide range of experimental conditions and that $(P_{0j}d)/(P_1L)$ would not have given a correlation. The data of Romeo[5] are of particular interest since b, d, and P_{0j}/P_1 are varied separately in three series of tests.

Except at low values of h/b where data of Maurer[4] are badly scattered, the data show a consistent variation of K_3' with h/b, and hence they indicate that h/b satisfactorily correlates the three-dimensional effects for the complete range of parameters. The good agreement of the data of Ref. 15 with other more clearly three-dimensional results shows that the decrease in K_3' with increasing $(P_{0j}d)/(P_1L)$ reported by these authors is actually a result of three-dimensional and *not* two-dimensional effects.

The dotted line in Fig. 9 is the mean value of Romeo's[5] data (with $d = 0.1$, 0.05, and 0.02 in.) for an amplification factor K_3 based on the total interaction force for the region upstream of a line drawn through the slot (the y axis). These data are also correlated (±10%) by use of the parameters h/b and again indicate that slot span is an important parameter. Also note that the values of K_3' and K_3 are quite different. This emphasizes the result that K_3' is a poor measure of interaction force when a plate large enough to contain the entire separated region is considered.

In view of the two- and three-dimensional data presented previously, it is evident to the authors that the reduction in amplification factors [with increasing values of $(P_{0j}d)/(P_1L)$] reported in the literature, e.g., Fig. 20 of Ref. 15 and Fig. 8 of Ref. 2, are caused by three-dimensional effects. Judging from Fig. 9, three-dimensional effects will certainly influence the centerline pressure distribution when $h/b > 0.05$ or when X_s/b is as small as 0.2.

Conclusions

Consider first conclusions pertinent to two-dimensional flows with fully developed turbulent boundary layers upstream of separation. Widely varied predictions of the influence of Mach number and injectant flow parameters on forces associated with jet interaction have been made in the

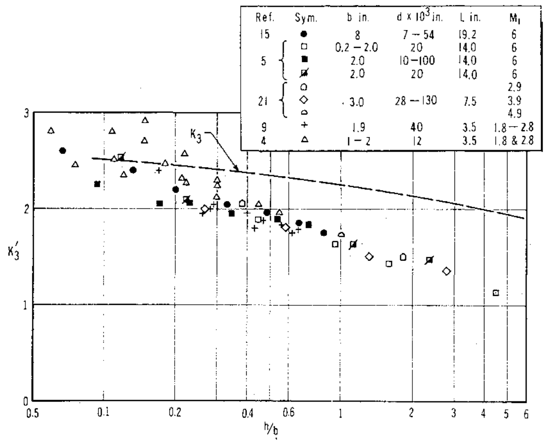

Fig. 9 Dependence of K_3 and K_3' on ratio of penetration height to slot span.

past. The principal result of this study is that both Mach number and injectant flow parameters have little influence on the normalized force variable, K. For example, in the present experiments which covered the Mach number range 2.6 to 4.5 with a single piece of experimental equipment, K varied between 3.2 and 2.9. Comparison of the present and previous results showed some differences between the experiments, e.g., $2.2 \leq K \leq 3.2$, but no systematic variations with Mach number. The data also suggest that the jet-to-freestream pressure ratio and injectant molecular weight have no influence on K for a fixed value of $(P_{0j}/P_1)(d/\delta)$, and that K varies quite slowly with this parameter when $h/\delta < 1$ and is a constant when $h/\delta > 1$. Slightly higher values of K are obtained for higher values of γ_j.

A highly simplified model of the flowfield has been developed which is based on data obtained from forward-facing step experiments. The model predicts trends which are in good agreement with the experimental results.

End plates must be used in flat-plate experiments to achieve a reasonable approximation of two-dimensional flow, except when the separation distance is very small compared to slot span. The end plates must extend forward of the interaction region. Examination of data obtained from experiments with finite-span slots without end plates shows that the induced force (in the region upstream of the slot) decreases with increasing values of the ratio of jet penetration height to jet span, and that the decrease is well-correlated by this parameter alone for a wide range of injectant and external stream parameters. It is clear that, for finite-span nozzles, the slot span and not the more commonly used plate length is the important characteristic dimension of the nozzle.

Beside the force data, information has been obtained concerning the separation and reattachment processes. Wall static-pressure distributions in the neighborhood of the separation point form a universal curve in the co-ordinates $(P - P_1)/(P_p - P_1)$ vs $\Delta X_s/\delta$. The static pressure in the separated region immediately downstream of the jet nozzle exit is usually lower than P_1, but, near reattachment, pressures exceeding P_1 have been observed. The pressure distribution in this region depends upon M_1, m_j, and the injectant fluid properties.

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FEBRUARY 1968 AIAA JOURNAL VOL. 6, NO. 2

Experimental Studies on the Lip Shock

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The lip shock, which emanates from the separation edge of a 6° half-angle wedge, has been investigated by means of optical observations as well as surface- and pitot-pressure measurements in the Mach number range 2.0 $\sim$ 4.5 and in the Reynolds number range 0.2 $\times$ 10^6 $\sim$ 2.0 $\times$ 10^6. First, the lip-shock strength is found to be quite substantial, contrary to the prevailing belief. The flow around the separation edge actually overexpands first and is then recompressed to the base pressure through the lip shock. The relative orientation between the lip shock and the wake shock appreciably varies depending upon the Mach number and the Reynolds number. Anomalous behavior in the static-pressure recovery distribution along the wake centerline, such as peak and hump, can be attributed to the interaction between the two shocks. The interaction further results in a distorted near-wake velocity profile, particularly in the turbulent case. Finally, the essential cause of the lip shock appears to be the viscous separation effect, similar to the separation shock emanating from a circular cylinder, rather than the inviscid rotational-field phenomena.

I. Introduction

THE supersonic near-wake problem is the subject of current intensive investigations. The flow behavior around a separation edge, however, had been rather poorly understood. Only quite recently, a more rigorous theoretical consideration on the separating boundary layer was put forward.[1]

During the course of our experimental investigations of the base-pressure and near-wake phenomena, the so-called lip shock, which emanates from the sharp separation edge, was not only clearly identified but also was observed to interact

Presented as Paper 67-29 at the AIAA 5th Aerospace Sciences Meeting, New York, January 23–26, 1967; submitted December 22, 1966; revision received August 30, 1967. The author is greatly indebted to A. Roshko, L. Lees, and T. Kubota of the California Institute of Technology for their many valuable suggestions. This paper presents the results of one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology, under Contract NAS7-100, sponsored by NASA.

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