Experimental Study of the Interaction of Multiple Jets with a Cross Flow

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This paper discusses the results from an experimental study of twin jets injected normal to a cross flow at a jet:cross-flow velocity ratio of 2. Turbulence data from hot-wire anemometry are presented and comparison is made with single-jet data of the present and earlier studies. The profiles of mean velocities and Reynolds stresses for the single and tandem jets show very similar behavior, indicating that the turbulent nature of the jet/s is caused by the same mechanism/s for both configurations.

Nomenclature

D	= jet exit diameter, m
U	= axial component of velocity, m/s
R	= velocity ratio, V_j/U_∞
u', v', w'	= instantanous values of the velocity fluctuations, m/s
u, v, w	= rms values of the velocity fluctuations, m/s
$\widetilde{uv}$	= $\overline{u'v'}$
$\widetilde{uw}$	= $\overline{u'w'}$
X, Y, Z	= Cartesian coordinates as shown in Fig. 2

Subscripts

j	= jet flow conditions at injection point
$(\bar{\ })$	= time-averaged values
∞	= freestream (tunnel flow) conditions

Introduction

FLUID mechanics research workers often encounter problems in which one or more jets enter a cross flow at large angles to the mainstream. The lift jets of V/STOL aircraft is one example of jets in a cross flow. Other examples of gaseous jets in a cross flow are the gas injection into combustion chambers, cooling jets on turbine blades, and secondary injection in rocket nozzles for thrust vector control. In most of these cases, the jets enter at right angles to the mainstream. A different, but equally important, area in which jets in cross flow play a major role concerns environmental pollution. A key factor preventing the use of coal to alleviate the energy shortage is the hazard from air pollution. Cooling tower plumes and pollutant discharges into rivers constitute other examples in this regard. In many such situations, one encounters more than one jet in proximity to another. A detailed knowledge of the jet/s' behavior in these examples is quite important from an engineering standpoint. In the case of coolant jets in combustion chambers, their effectiveness is dependent on how fast the jets and the combustion gases mix. The aerodynamic characteristic of V/STOL aircraft is greatly influenced by the behavior of the deflected lift jets and the thrust vector control effectiveness is dictated by the modification of the pressure distribution around the nozzle walls. The location of chimney stacks, the rate of discharge of pollutant, etc., can be decided with greater confidence if one is able to determine the characteristics of pollutant dispersal.

There have been several analytical and experimental studies of a single jet exiting normal to a cross flow.[1-14] There have also been a few studies of multiple jets in a cross flow.[15-22] A review of some of these studies is given here. The early researchers in this field were concerned with velocity and pressure measurements on the experimental side and a momentum integral approach on the analytical side. Abramovich[1] has given a detailed description of the processes involved in the deflection of a turbulent jet in a cross flow. He observed that the jet cross section deformed into a horseshoe shape that could be approximated as an ellipse. For a jet directed normal to a cross wind, Keffer and Baines[2] established a functional behavior between the axial jet velocity and the velocity ratio by considering the jet to originate from a virtual source. They then deduced the similarities in the lateral velocity profiles when scaled by suitable length scales. The studies by Ramsey and Goldstein[3] dealt with a heated jet with deflecting flow for normal and inclined injections. Temperature, velocity, and turbulence intensity data were obtained by these investigators. They varied the mass flux parameter in the range of 0.1-2. Other studies[4-12] dealt with different aspects of the jet flow problem. It is only recently that detailed measurements of turbulence parameters of a jet in cross flow have been made.[13,14]

The literature on multiple jets in a cross flow is still very limited. Kamotani and Greber[15] reported axial and transverse distributions of the velocity and temperature for single and multiple circular jets for jet: cross-flow momentum ratios of 8-72. Penetration and mixing of multiple cold-air jets into a ducted subsonic heated mainstream flow have been investigated by Walker and Kors[16] for momentum flux ratios of 6-60. Mixing was evaluated by an energy exchange parameter they developed. Their data also include some information on the temperature distribution within the flowfield. Schwendemann[17] has given trajectory data for tandem jets in a cross flow for normal and inclined injections and for double jets arranged side-by-side for normal injection. Ziegler and Wooler[18] compared Schwendemann's data with their analysis of multiple jets in a cross flow. They assumed that the leading

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jet is not influenced by the rear jet and that the rear jet is modified by the reduced dynamic pressure behind the leading jet. Isaac and Schetz[19] performed a momentum integral analysis of double jets and obtained good agreement with Schwendemann's data. Makihata and Miyai[20] performed experimental and theoretical studies on twin jets in a cross flow. Their results do not identify individual trajectories for the front and rear jets. Surface pressures induced on a body of revolution and a flat plate with various dual-jet configurations were reported recently in Ref. 21.

It is evident from the foregoing discussion that wide gaps exist in the understanding of jets in a cross flow, especially in the case of multiple jets. The present study was meant to close some of these gaps. The single jet was first investigated; the data from this study were compared with published data from Refs. 13 and 14. This afforded a qualitative check on the present experiments. The double-jet characteristics were then examined. It was felt that a thorough study of a few transverse planes would be more useful than a cursory examination of several cases.

The experimental investigation reported in this paper deals with single and double jets exhausting from a flat plate perpendicular to a cross flow as shown in the schematic diagram of Fig. 1. The double jets were arranged in tandem and side-by-side configurations with a spacing of 4 jet diameters. The jet:cross-flow velocity ratio was 2 for all the experiments. These values of the jet spacing and jet:cross-flow velocity ratio were thought to be representative of many engineering situations involving jets in a cross flow. Hot-wire anemometry was use to measure the axial mean velocities, three components of the normal stresses, and two components of the shear stresses. A flow visualization study using tufts was performed to estimate the flow angles and the spread of the jets. One-dimensional energy spectra were obtained for several stations to examine their dependence on the parameters of the problem. In the present study, no attempt was made to model any one particular application. Thus, for instance, the effects of the boundary layer were not accounted for. Also, the thermal and chemical species diffusion characteristics, which are significant in the cases of cross flow with a chemical reaction, were not considered. Instead, emphasis was focused on the aerodynamic interaction of two equal and relatively closely spaced jets, arranged in a tandem configuration and exposed to a uniform cross flow.

Experimental Details

The experiments were conducted in the VPI&SU 1.83×1.83 m subsonic stability wind tunnel. The experimental setup (Fig. 1) included two jet nozzles securely mounted in a flat-plate arrangement. The jet streams were supplied by dc blowers whose speeds and the resulting flow rates were carefully controlled. The jet exit diameters were 5.08 cm each. The jet and cross-flow (tunnel) velocities were 30.48 and 15.24 m/s, respectively, which gave a jet:cross-flow velocity ratio R of 2. The Reynolds number based on the jet diameter and the tunnel speed was 0.56×10^6. The flat-plate boundary layer was approximately 1.27 cm thick at the jet exit. The jets were tested independently to determine the velocity profile at the jet exit. The jet exit velocity profile was found to be of a top-hat shape and identical profiles were obtained for different azimuthal angles. The 30 deg elbow in the rear jet circuit was not found to alter the jet exit velocity profile.

With the model in position, but without the jets blowing, the tunnel flow had a maximum deviation in mean velocity of 0.6% and a maximum turbulence intensity of 0.04%. The mean flow inclination in the same configuration was slightly less than 0.5 deg above the horizontal. All the data were taken at a freestream velocity of 15.24 m/s. The maximum difference in temperature between the plenum and the freestream was 11°C. Recent studies by Wark[12] have found that a 11°C overheat in the jet plenum implies a maximum overheat of 3°C at $X/D = 3$. Therefore, it is safe to assume that the present data are not influenced by the temperature differences between the jet and the freestream. A hot-wire rake consisting of one normal single-wire probe and two end-flow X probes was used to measure the flow properties. The rake was mounted on a traverse system allowing for remotely controlled three-dimensional motion. Data were taken for single, tandem, and side-by-side plume configurations. Four downstream stations ($X/D = 3$, 7, 10, and 14) were selected for the single jet and two downstream stations ($X/D = 3$ and 10) for the double jets. While the single jet was being tested, the second jet was closed with very thin and smooth adhesive tape.

Two types of hot-wire probes were used in the present study, normal single-wire (TSI model 1210) and end-flow X-wire probes (TSI model 1241). The probe sensors were 5.8 μm in diameter and 1.27 mm in length. The single-wire probe was used to measure the mean flow and normal stress in the axial direction. The two sensors of the X-wire probe, oriented at 90

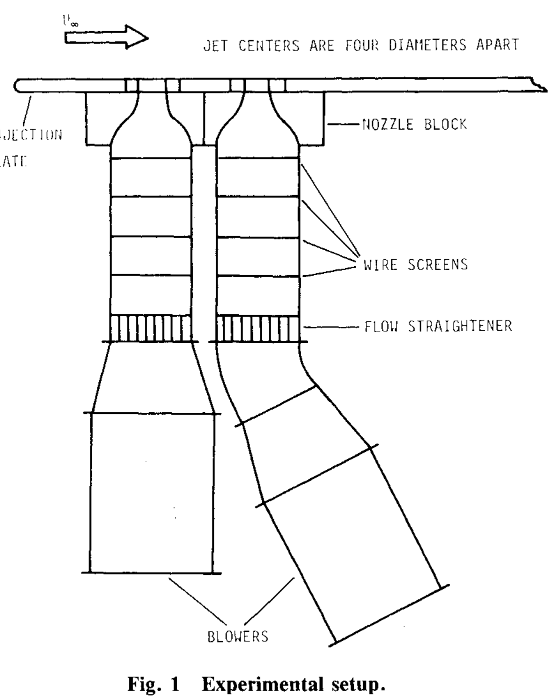

Fig. 1 Experimental setup.

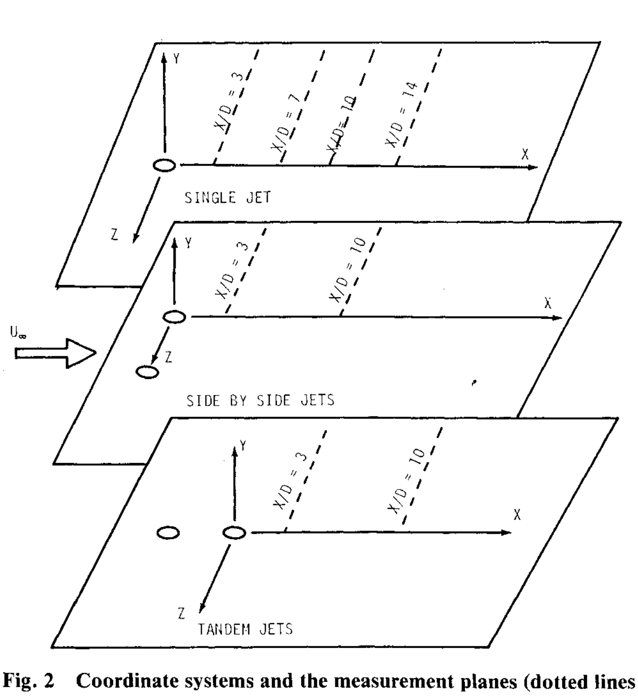

Fig. 2 Coordinate systems and the measurement planes (dotted lines show the planes where measurements were made).

deg to each other, could detect mean flow and turbulent fluctuations in directions perpendicular to the two sensors. One of the X-wire probes had its sensors in the horizontal (XZ) plane and the other in the vertical (XY) plane, enabling measurements of the shear stress components $-\widetilde{uv}$ and $-\widetilde{uw}$. The anemometers used were TSI models 1050 and 1755 and DISA type 55D01. These anemometers had very low noise (less than 0.007% equivalent turbulence intensity). Results from the two TSI anemometers were compared and found to agree very well. A TSI 1052 linearizer was used to process the output from the TSI anemometers and a DISA type 55D10 linearizer that from the DISA anemometers. The fourth-degree polynomial curve fit by the TSI 1052 and the exponential curve fit by the DISA 55D10 were both found to give good results, with a deviation from linearity of less than 1% in the range of velocities encountered in the measurements. Summing and differencing of the instantaneous analog outputs from the linearizers for the two sensors of the X-wire probe were performed by the TSI 1015C correlator and DISA type 52B25 turbulence processor. The 1.83×1.83 m stability tunnel data acquisition system was used to perform the on-line calibration of the apparatus, as well as to record the dc signals from the anemometers on magnetic tape. The data acquisition system consisted of a Hewlett-Packard (HP) 3495A channel selector, HP 3455A digital voltmeter, HP 9825A computer, and an HP 9872A digital plotter. The voltmeter had a sampling rate of 20 samples/s. Each dc voltage from the anemometers was sampled for 3 s and the average, maximum, and minimum values recorded. A DISA type 55D35 rms voltmeter was used to measure the rms values of the fluctuating voltage.

The error estimates of the hot-wire data were largely based on previous experience; they were estimated to be within 2-3% for the mean flow and 3-6% for the low and medium turbulence intensities (u/U less than 0.2). However, the accuracy of hot-wire data deteriorated rapidly for high levels of turbulence. Therefore, it is felt that the numerical values of the data corresponding to $X/D = 3$ and 7 (not shown due to space limitations) should be interpreted with caution.

Results and Discussion

A great deal of data has been acquired during the present study; however, space limitations restrict us to present only a few significant results in this paper. Readers interested in more detailed results are directed to Ref. 22. The coordinate systems used for presentation of the data and discussion of the results are shown in Fig. 2. Figures 3 and 4 show the vertical profiles of the flow properties at different transverse locations in the YZ plane at $X/D = 10$ for the single and tandem-jet cases, respectively. Single-jet data at $X/D = 10$ and 14 and tandem-jet data at $X/D = 10$ (in the coordinate system shown in Fig. 2) are compared in Fig. 5. Published data from Ref. 13 are also included in Fig. 5 for comparison. All of the data shown in Fig. 5 are for the plane of symmetry ($Z/D = 0$).

It may be observed from Fig. 3 that the low axial mean velocity (U) and high turbulence intensities (u/U, v/U, and w/U) occur in the same region above the injection surface. The profiles of vertical turbulence intensity (v/U) in Fig. 3 are similar in shape to the axial ones; however, their maximum values are higher. The transverse turbulence intensity (w/U) profiles in Fig. 3 show marked differences from their axial and vertical counterparts. These differences are pronounced near the plane of symmetry ($Z/D = 0$) in the region close to the injection surface. The w/U distribution in this region starts at high values at the stations closest to the wall (where the measurements were made); these high values of w/U persist for quite some distance and then decay gradually to zero. From these considerations, it was felt that examination of the individual components of the Reynolds stress tensor, rather than quantities such as the turbulent kinetic energy, might be a more useful approach to understanding the very complex nature of the flowfield under consideration. The reason for the large differences in the behavior of the w/U profiles compared to that of u/U or v/U may lie in the fact that the gra-

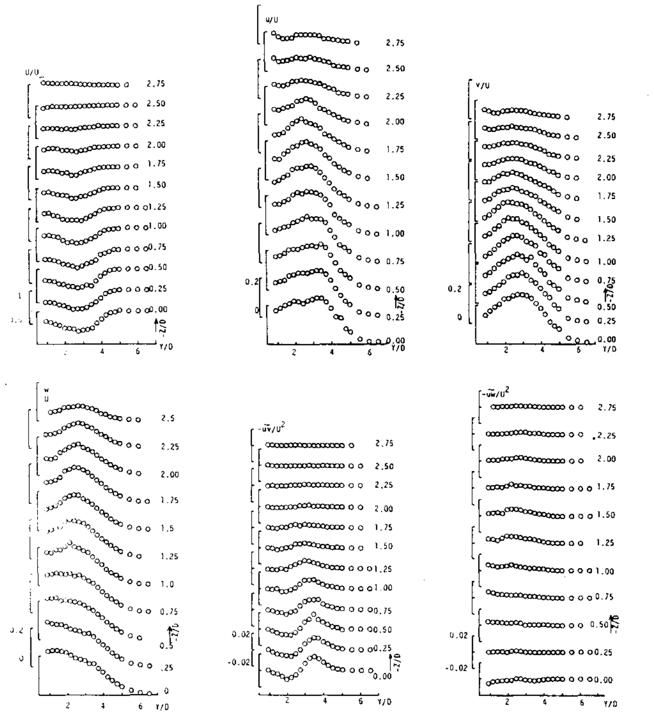

Fig. 3 Profiles of flow parameters (mean velocity U/U_∞, axial u/U, vertical v/U, and horizontal w/U turbulence intensities and shear stresses $-\widetilde{uv}/U^2$ and $-\widetilde{uw}/U^2$ are shown for a single jet at $X/D = 10$).

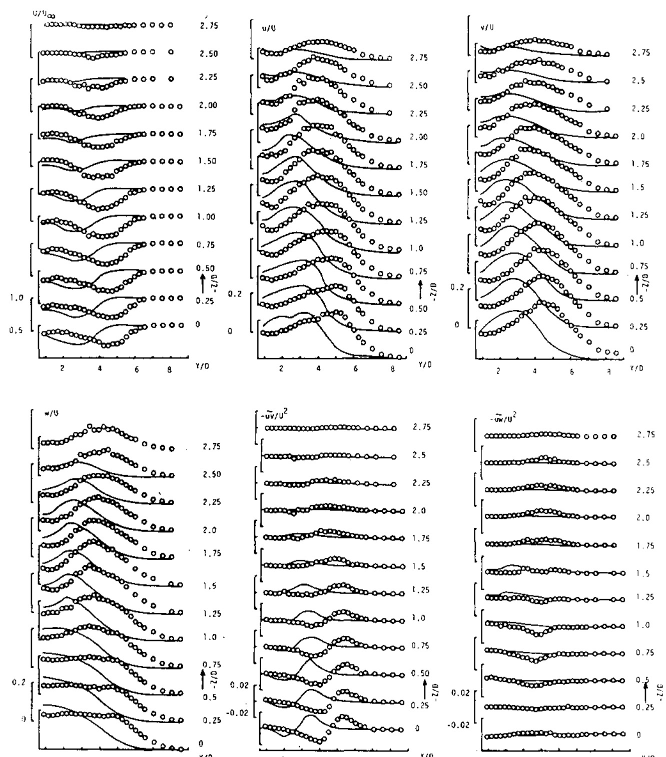

Fig. 4 Comparison of single and tandem jets (solid curves represent single-jet data and circles tandem-jet data for the corresponding stations).

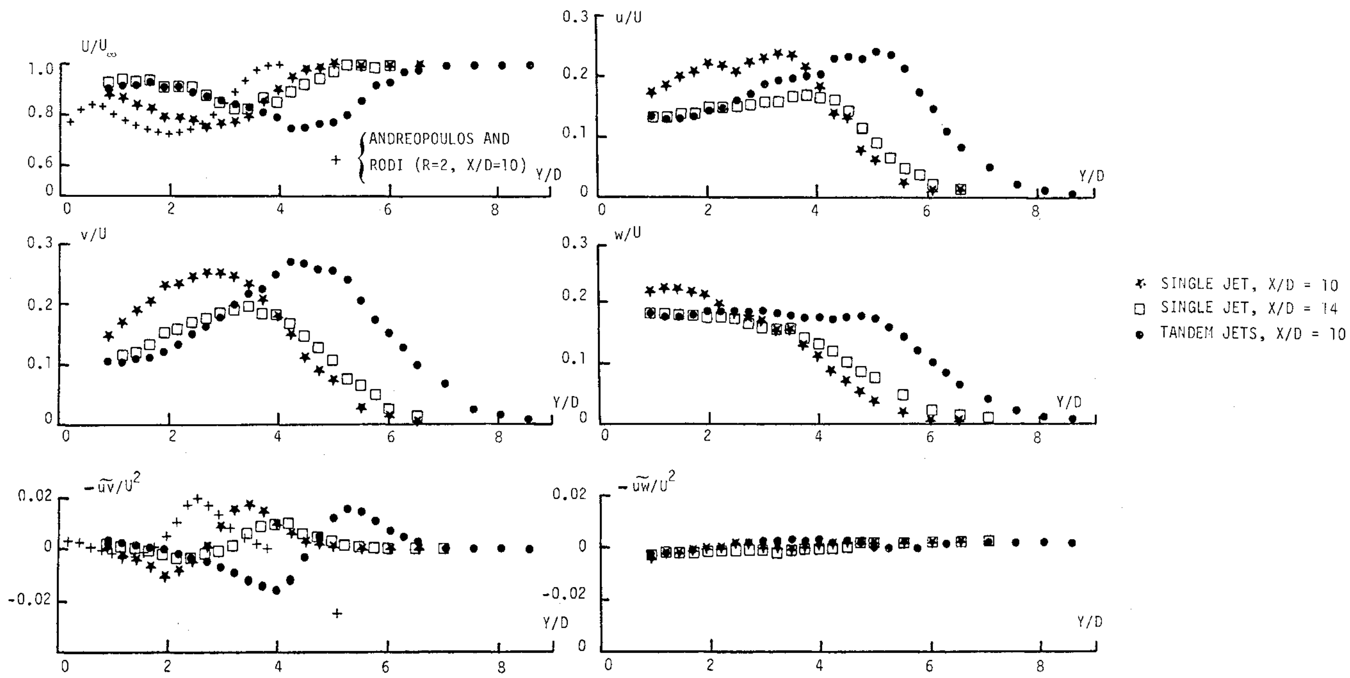

Fig. 5 Flow properties in the plane of symmetry for single jet ($X/D = 10$ and 14) and tandem jets ($X/D = 10$). Andreopoulos and Rodi13 data for a single jet for $R = 2$ at $X/D = 10$ are shown by the symbol +.

dients of the transverse mean velocity W show large values near the plane of symmetry.

The data of Ref. 13 show that the mean velocity W does possess large gradients $\partial W/\partial z$ near the plane of symmetry. The high values of w near the $Z = 0$ plane can also be explained physically, by realizing that the two counter-rotating vortices which develop in the jet wake eventually meet at the aft midplane, causing large values of the normal stress w^2 in the Z direction. The v/U profiles do not show the same trend, probably because $\partial U/\partial x$ and $\partial W/\partial z$ may have opposite signs in the region under discussion. Therefore, the effect of large $\partial W/\partial z$ may be offset by the $\partial U/\partial x$ term having the opposite sign.

The Reynolds shear stress $-\widetilde{uv}/U^2$ profiles in Fig. 3 show a minimum and a maximum and the variation can be seen to be rather sharp with fairly rapid changes in slope; its values also decay rather rapidly with increasing distance from the plane of symmetry. The maximum value of the shear stress $-\widetilde{uv}/U^2$ (≈ 0.0175) is seen to be less than half of the maximum of the normal stresses u^2/U^2, v^2/U^2, and w^2/U^2 (≈ 0.06). As noted in Ref. 13, the downstream flow is wake-like in character with low mean velocity gradients and, therefore, it does not have large values for the shear stresses typical of shear-layer flows. A direct comparison of the shear:normal stress ratios in the present study with those of Ref. 13 is not possible because the normal stresses are lumped into a single parameter (turbulent kinetic energy) in Ref. 13. The shear stress ($-\widetilde{uw}/U^2$) profiles in Fig. 3 show very small values in comparison to the $-\widetilde{uv}/U^2$ values as well as the normal stresses. It is well known from previous studies that the jet cross section deforms into a kidney shape. This deformation of the jet cross section causes the axial mean velocity gradients to be much larger in the vertical Y direction than those in the transverse Z direction. If the shear stress $-\widetilde{uw}/U^2$ is assumed to be the result of the mean velocity gradients $\partial U/\partial z$ and $\partial W/\partial x$ (an assumption that cannot be fully supported by the experimental evidence), then it can be seen that the shear stress $-\widetilde{uw}/U^2$ must be much smaller than the shear stress $-\widetilde{uv}/U^2$ due to the relatively large spread of the jet in the transverse direction and the resultant small mean velocity gradients $\partial U/\partial z$, as well as the small variation of W in the X direction. Even though experimental evidence indicates the presence of recirculation zones (making the flowfield elliptic in nature), it is reasonable to assume that the gradients in the axial X direction are smaller than those in the vertical Y direction and, to a lesser extent, in the transverse Z direction.

The six variables plotted in Fig. 3 for the single jet are also plotted for tandem jets in Fig. 4. In order to provide an easy comparison between the single- and tandem-jet cases, the single-jet data are also shown by means of solid lines in Fig. 4. The first observation that one makes when comparing these curves is the striking similarity between the general shape of the two sets of data. It is as though the two jets have lost their separate identities and have become a single jet. It should be remembered that in the double-jet case the front and the rear jets originate at 14D and 10D, respectively, ahead of the measurement plane (see Fig. 2). This and the fact that the tandem jets have twice the momentum of the single jet at the jet exit make simple comparisons between the two of limited value. Nevertheless, the present results do have their merits and show that further studies may allow more general conclusions to be drawn regarding jets in a cross flow.

The following differences between the two cases (Figs. 3 and 4) may be noted:

1) The tandem jets show a slight upward shift. The Y coordinates corresponding to the upper edge of the jets (roughly defined by the location at which the turbulence intensities go to zero) are seen to be higher for the tandem jets.

2) The general shape of the mean velocity plots is seen to be similar for both the single and tandem jets. In particular, the minimum values of the mean velocity are nearly the same for the two cases represented by Figs. 3 and 4.

Observations similar to these can also be made regarding the general shapes and the maxima and minima of the Reynolds stress profiles. This suggests that, perhaps, there exist similarity parameters that, when used for scaling the parameters of the problem, may enable the data to be correlated in a simple manner. It is known that self-preservation exists in other classes (axial jets, wakes, shear layers) of turbulent flows; therefore, it may not be too surprising that such properties exist for jets in cross flow as well. (However, the task of identifying the appropriate similarity variables remains.)

A direct comparison of the data for the plane of symmetry for the three cases (single jet at $X/D = 10$ and 14 and tandem jets at $X/D = 10$) can be made from the plots of Fig. 5. Many of the observations made by comparing the profiles of Figs. 3 and 4 are made clearer in this figure. The axial mean velocity U/U_∞ and the shear stress $-\overline{uv}/U^2$ from Ref. 13 for $R = 2$ and $X/D = 10$ for a single jet are also shown in Fig. 5. These values were obtained by cross plotting from Figs. 7 and 24 in Ref. 13. The trajectory of Ref. 13 is lower than the single-jet trajectory of the present study. This difference in the jet trajectory is probably caused by the differences in the experimental conditions; the boundary layer in Ref. 13 was tripped ahead of the jet exit, making it fully turbulent in the interaction region. The jet flow also was tripped to make it a turbulent pipe flow. In the present study, the flat-plate boundary layer was not tripped and the jet exit flow was nearly irrotational. Therefore, it is natural to expect the upward momentum in the present experiments to persist longer, giving more penetration of the jet fluid into the freestream. Thus, one can expect the potential core, which may exist outside the jet exit of the present experiments, to make the flowfield in the initial region different from that of Ref. 13. A turbulent flow simply facilitates more efficient mixing than a laminar or potential flow.

The initial flatness of the w/U profiles is more pronounced for the tandem jets when compared to that of the single-jet w/U profiles. Also, at $X/D = 10$ for the single-jet case, the maximum w/U is higher than that of the tandem jets. This apparent anomaly may be explained on the grounds that the tandem jets interact more extensively with each other and with the freestream. Therefore, the tandem jets become wake-like in nature earlier than the single jet does, with a more uniform distribution of turbulence intensity across the cross section. The maximum of the shear stress $-\overline{uv}/U^2$ is larger for the single jet than its minimum (in absolute value); the data of Ref. 13 also show the same behavior, although the difference between the maximum and minimum values in this case is more pronounced. The absolute values of the maximum and minimum values of $-\overline{uv}/U^2$ for the tandem jets are seen to be nearly equal. This may again be due to the jets losing some of the influence of the complex system of vortices[13,24] developed upstream and the jets starting to behave like a wake in the vicinity of the turbulent wall boundary layer. The $-\overline{uw}/U^2$ profiles shown in Fig. 5 confirm that $\overline{uw}$ is zero in the plane of symmetry. The small nonzero values of $-\overline{uw}/U^2$ in the figure may be attributed to the probable imperfections in geometry as well as experimental uncertainties.

Conclusions

The present investigation of multiple jets in a cross flow has been helpful in obtaining a deeper insight into the dynamics involved in the complex problem of jet cross-flow interaction. The data from this study, in conjunction with those from other studies, show that the initial conditions do have a bearing on the development of the jet trajectory, even though the details of the turbulent parameters themselves are largely unaffected by the initial conditions. The transverse turbulence intensity profiles showed significantly different behavior when compared with the axial and vertical turbulence intensity profiles. Perhaps, the most significant finding of the present study is the striking similarity in the details of the flow within the cross sections (in terms of the mean velocity and the turbulence parameters) of the single and tandem jets in the region downstream of the near-wake, at $X/D > 10$. The presently available data are, however, not sufficient to reach a more broad-based conclusion regarding such problems as the existence of similarity variables. A planned continuation of this study will address some of these important questions.

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