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COMPRESSIBILITY EFFECTS ON TURBULENCE

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1. INTRODUCTION

Turbulence is a macroscopic state of flow in which the instantaneous flow variables exhibit a seemingly random variation in time and in the three spatial coordinates. The temporal and spatial disorder has finite decorrelation time and length scales. This article reviews those features of the turbulent state that depend on compressibility. Effects associated with the volume changes of fluid elements in response to changes in pressure are regarded as compressibility effects. These are contrasted with variable inertia effects associated with either variable composition or volume changes due to heat transfer. The behavior of incompressible turbulent flows has been frequently reviewed in this series and compressibility effects on turbulent shear layers have been reviewed by Bradshaw (1977). Morkovin (1961, 1992) has reviewed compressibility effects on wall-bounded and free-shear flows; Smits (1991) and Spina et al (1994) have recently reviewed the turbulent boundary layer structure in supersonic flow.

In recent years the renewed interest in high-speed civil transport aircraft and supersonic combustion ramjet engines for high altitude hypersonic propulsion has invigorated fundamental research on compressible turbulent flows. New experimental information on shear flows under compressible conditions has been gathered. Direct numerical simulations (DNS) of turbulence in compressible regimes have also been performed. Despite these research efforts the phenomenology of compressible turbulence is far from mature. This article is as much a review of what is not known about compressible turbulence vis-à-vis its incompressible counter-

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part, as it is a review of more commonly held views. Only macroscopic phenomena are reviewed and in most of the discussion the flow medium is an ideal gas. Chemical and thermodynamic nonequilibrium effects arising in hypersonic and rarefied flow regimes are not discussed; these flow regimes have been reviewed by Muntz (1989) and Stalker (1989). Issues related to supersonic combustion have been reviewed by Ferri (1973), Walthrup (1987), and Billig (1988). Compressible flows near the thermodynamic critical point have been reviewed by Kutateladze et al (1987); shock wave phenomena have been reviewed by Zel'dovich & Razier (1969) and Hornung (1986); and shock wave boundary layer interactions have been reviewed by Green (1970) and Adamson & Messiter (1980).

The organization of this article is as follows. Different facets of compressibility effects on turbulence and the dimensionless parameters characterizing them are summarized in Section 2. Homogeneous compressible flows are discussed in Section 3, followed by a discussion of simple inhomogeneous flows in Section 4. The review is thus limited to the simplest possible flows.

2. THE NATURE OF TURBULENCE IN A COMPRESSIBLE MEDIUM

2.1 *Modes of Fluctuations*

When a compressible medium is in turbulent motion the thermodynamic fluid properties such as density (mass per unit volume), specific entropy, as well as the thermophysical fluid properties such as viscosity coefficients and specific heats, undergo fluctuations. Measures of these fluctuations need to be included in a specification of the turbulent state. Yaglom (1948) and Moyal (1952) appear to be the first to formulate the equations governing the two-point correlations of these fluctuations. Kovasznay (1953) decomposed the turbulent fluctuations into vorticity, acoustic, and entropy modes of fluctuations. This decomposition is defined for (weak) turbulent fluctuations about a flow state of uniform mean velocity and spatially uniform mean thermodynamic properties. The three modes of fluctuations are dynamically decoupled from each other to first order in the fluctuation amplitude. At second order, mode couplings arise and a combination of any two modes (including self-interaction) generates other modes (Chu & Kovasznay 1958, Monin & Yaglom 1971).

If linearized perturbations about a nonuniform mean state are considered, mode couplings arise in the linearized equations. However, for linear unsteady disturbances about an arbitrary potential flow (over a body) the disturbance velocity can be decomposed into a "vortical" and "acoustic" part (Goldstein 1978a). The vortical disturbance is generally not solenoidal and can be calculated without a specific knowledge of the acoustic part. Analogous decomposition of unsteady disturbances about transversely sheared mean flows can also be obtained (Goldstein 1978b, 1979a). Such decompositions have been extended to flows with mean entropy gradient orthogonal to mean vorticity (Debieve 1978a, Dussauge et al 1988). Linear rapid-distortion theories have exploited such decompositions in studies as turbulence passing through a Prandtl-Meyer fan (Goldstein 1978a), turbulence generated by entropy fluctuations in a nonuniform flow (Goldstein 1979b), homogeneous turbulence subject to bulk compression (Durbin & Zeman 1992), and shock turbulence interaction (Ribner 1953, 1954, 1969, 1987; Moore 1953; Chang 1957; Kerrebrock 1956; McKenzie & Westphal 1968; Anyiwo & Bushnell 1982). Hot-wire measurements taken at a point in high-speed flow are commonly discussed in terms of fluctuation modes (Kovaznay 1950, Morkovin 1956, Smits & Dussauge 1988). In that context the "modes" are locally defined and associated with small deviations from a uniform state. Local fluctuations of mass flow and total temperature are obtained by exploiting the varying sensitivity of the hot-wire signal, as its operating conditions (such as overheat ratio) are varied and then decomposed into "modes."

2.2 *Evolution of Turbulent Kinetic Energy*

Nondimensional parameters characterizing the influence of compressibility on turbulence are contained in equations governing the turbulent flow. In this article the density-weighted form (Favre 1965a) is adopted, primarily for its compactness of notation. The density-weighted averaged velocity, internal energy per unit mass, and enthalpy per unit mass are denoted by $\tilde{u}_i$, $\tilde{e}$, and $\tilde{h}$, while the fluctuations from these are denoted by u_i'', e'', and h''. Ensemble- or Reynolds-averaged pressure and density are $\bar{p}$ and $\bar{\rho}$, and the fluctuation relative to the ensemble averages are p' or ρ' (denoted with a single prime). As there is no mass flux across the Favre-averaged streamlines (a property not shared by the conventional Reynolds-averaged streamlines) the equations for mean density, mean velocity, and mean enthalpy are more compact. Further discussion of the merits/limitations of Favre-averaging and its transformation properties can be found in Lele (1993).

As indicated in Figure 1 the evolution of turbulent kinetic energy (TKE) is coupled to the evolution of mean kinetic energy and the mean internal energy, and several pathways exist for these exchanges. Parameters measuring the influence of compressibility are defined by considering the importance of those pathways that exist only in a compressible flow relative to those occurring under incompressible conditions.

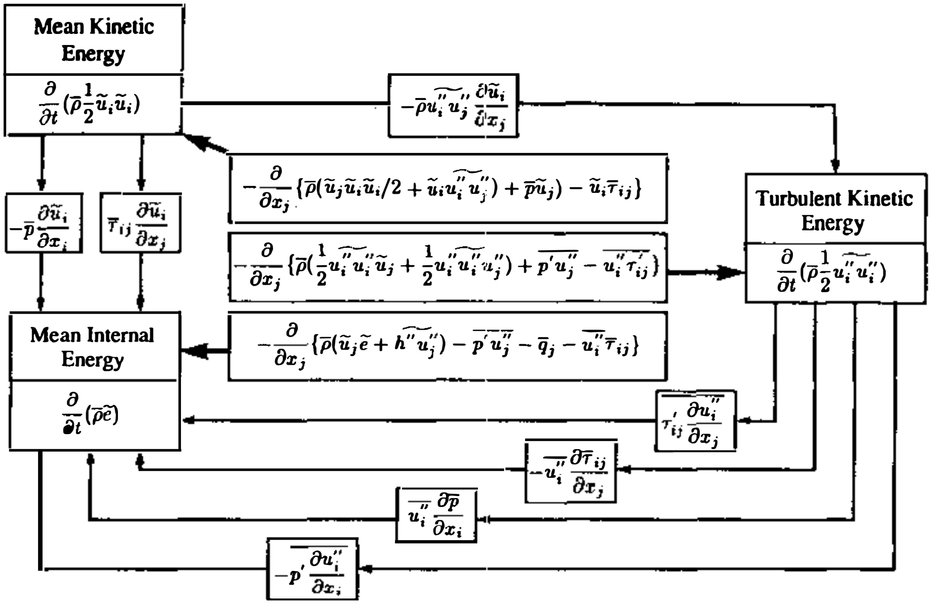

Figure 1 Balance of energy in a compressible turbulent flow. Exchange between the mean kinetic energy, mean internal energy, and turbulent kinetic energy occurs via the terms indicated by small arrows. All arrows pointing into the energy balance for some quantity Q are taken with a positive sign on the right-hand side; outward pointing arrows require multiplication by a negative sign.

Four terms couple TKE changes to the changes in $\bar{\rho}\tilde{e}$ (mean internal energy) and are shown in the bottom four paths in Figure 1. The viscous dissipation rate (per unit volume) of TKE, $\varepsilon = \overline{\tau_{ij}'u_{i,j}''}$ ($u_{i,j}''$ is shorthand for $\partial u_i''/\partial x_j$), is a significant contributor to the TKE budget in all turbulent flows (compressible or incompressible), with the possible exception of flows undergoing a rapid distortion (e.g. flow through a shock wave). Using the Stokes (Cauchy-Poisson) constitutive relation the viscous dissipation rate is

$$\Phi = \tau_{ij}S_{ij} = \mu_v\Theta^2+2\mu S_{ij}^{d}S_{ij}^{d},$$

where $S_{ij} = (u_{i,j}+u_{j,i})/2$ is the rate of strain, $\Theta = u_{i,i}$ is the dilatation, $S_{ij}^{d} = S_{ij}-\delta_{ij}\Theta/3$ is the deviatoric part of S_{ij}, μ_v is the bulk viscosity, and μ the (shear) viscosity. Thus if μ_v is not zero the volume changes of fluid elements lead to an additional dissipation. It is possible to rearrange the above as

$$\Phi = (\mu_v+\tfrac{4}{3}\mu)\Theta^2+\mu\omega_k\omega_k+\mu(u_iu_j)_{,ij}+\mu(u_ju_{i,j})_{,i}-3\mu(u_j\Theta)_{,j},$$

where ω_k is the vorticity. Under conditions of homogeneity (discussed in Section 3) the last three terms vanish for averaged quantities. The first of the surviving terms is called compressible or dilatational dissipation (Favre 1969, Moyal 1952, Zeman 1990, Sarkar et al 1991) and the second is regarded as incompressible or solenoidal dissipation. The compressible dissipation term is identically zero for a strictly incompressible flow but otherwise finite (even for $\mu_v = 0$).

The relative magnitude of ε compared to the rate of production of TKE (P) may depend on compressibility, but a priori estimates cannot be made with sufficient accuracy. For homogeneous flows ε may be approximated as

$$\varepsilon = \varepsilon_s + \varepsilon_c = \varepsilon_s(1 + \chi_\varepsilon), \quad \varepsilon_s = \bar{\mu}\overline{\omega_k'\omega_k'}, \quad \varepsilon_c = (\overline{\mu_v} + \tfrac{4}{3}\bar{\mu})\theta'^2,$$

$$\chi_\varepsilon = \left(\frac{4}{3} + \frac{\mu_v}{\bar{\mu}}\right)\overline{\theta'^2}/(\overline{\omega_k'\omega_k'}).$$

Terms arising from variations of μ and μ_v have been neglected; numerical simulations (Lee et al 1991, Blaisdell et al 1991, Erlebacher et al 1990) consistently show that the neglected terms are small. If ε_s is largely independent of the influence of compressibility, the ratio χ_ε may be regarded as a measure of the extra dissipation resulting from compressibility. Numerical simulations of decaying compressible isotropic turbulence (Sarkar et al 1991, Kida & Orszag 1992, Lee et al 1991, Blaisdell et al 1991) support this view. The simulations show that χ_ε depends on both the turbulence Mach number $M_t = q/C$ (q is a velocity scale characteristic of the turbulence and C is a representative speed of sound) and initial conditions such as initial density variance $\overline{\rho'^2}/\bar{\rho}^2$, and the initial partition ratio of the kinetic energy in the dilatational mode, χ_c (defined in Section 3). Simulations of compressible homogeneous shear flow (Blaisdell et al 1991, Sarkar et al 1993) also suggest that ε_s is largely independent of compressibility. Although the dissipation of TKE occurs at small scales, its rate ε is set by the energy-containing range of scales. It is unclear what attributes of the energy-containing scales, in a high Reynolds number flow, determine ε_s and ε_c. Whether ε_s/P is insensitive to compressibility in high Reynolds number flows, as found in the DNS at low Reynolds number, is an open question.

The second term $\overline{u_i''}\bar{\tau}_{ij,j}$ coupling TKE evolution to $\bar{\rho}\tilde{e}$, in Figure 1, is an artifact of Favre-averaging and, in principle, can have either sign. The third coupling term involving a Reynolds-averaged value of Favre velocity fluctuation is $\overline{u_i''}\bar{p}_{,i}$, which has been called enthalpic production of turbulence by exchange with enthalpic energy (Favre 1969) and may cause a gain or loss of TKE. It is natural to associate this exchange term with variable inertia in the flow rather than compressibility. When a body force or a mean pressure gradient acts on the fluid the regions with smaller inertia (per unit volume) can respond more quickly to the imposed change. Work done by the force is a gain in the kinetic energy of the flow, part of which contributes to TKE. Due to the acceleration reaction (Pope 1987, Batchelor 1967) the differential acceleration is $O(-\nabla\bar{p}/\bar{\rho})$, even when $\rho_{rms}/\bar{\rho}$ is not small, which is consistent with associating $\overline{u_i''}\bar{p}_{,i}$ with the rate of energy exchange to TKE. It appears that a formulation similar to Goldstein (1979b) may be useful in estimating this energy exchange (Hunt, cited in Fulachier et al 1989). The exact transport equation for $\overline{u_j''}$ (Taulbee & Van Osdol 1991) can also be used to estimate $\overline{u_j''}$. The leading order terms of this equation show that $\overline{u_j''}$ depends not only on $\bar{\rho}_{,k}$ as commonly assumed, but also on $\bar{p}_{,k}$ and $\tilde{u}_{k,k}$. An identity relating the energy fluxes

$$\bar{\rho}\widetilde{h''u_j''} = \bar{\rho}\widetilde{e''u_j''} + \overline{p'u_j''} + \bar{p}\overline{u_j''}$$

is also relevant to the models of $\overline{u_j''}$. For an ideal gas this identity implies that once models for $\widetilde{T''u_j''}$ and $\overline{p'u_j''}$ have been chosen the turbulent mass flux $\overline{u_j''}$ is already determined as $\overline{u_j''} = \widetilde{T''u_j''}/\tilde{T} - \overline{p'u_j''}/\bar{p}$. This consistent choice of $\overline{u_j''}$ is not always recognized in turbulence modeling. In recent work (Speziale & Sarkar 1991) models for $\widetilde{T''u_j''}$ and $\overline{u_j''}$ are chosen and $\overline{p'u_j''}$ is then consistently evaluated. However, the model forms are such that for inhomogeneous, uniform density, isothermal flows $\overline{p'u_j''}$ is always zero.

In external flows involving significant pressure gradients—such as shock-wave boundary layer interactions or streamwise acceleration/deceleration due to imposed pressure gradients or curved walls (Jayaram et al 1987, Fernando & Smits 1990, Dussauge et al 1988)—the turbulent mass flux $\overline{u_j''}$ is usually not associated with volume fluctuations of a fluid element as long as M_t is small. Hence effects associated with the enthalpic production of TKE, which commonly occur in compressible flows, are not compressibility effects but better regarded as effects of variable inertia. The importance of separating compressibility effects from effects of variable inertia cannot be overemphasized (Morkovin 1961, 1992). Enthalpic production of TKE is also important in Rayleigh-Taylor and Richtmeyer-Meshkov flows (Youngs 1989, Besnard et al 1989, Benjamin 1992, Meshkov 1992, Gauthier & Bonnet 1990). In premixed flames the differential acceleration is responsible for countergradient diffusion of species (Pope 1987). As M_t increases the distinction between variable inertia effects and compressibility effects gets blurred and the enthalpic production of TKE may introduce compressibility effects.

The fourth term coupling the evolutions of TKE and $\bar{\rho}\tilde{e}$, in Figure 1, is the net rate of work done by the pressure fluctuations due to the simultaneous fluctuations in dilatation, or the pressure dilatation (correlation), $\Pi_d = \overline{p'u''_{i,i}} = \overline{p'u'_{i,i}}$. In a compressible flow Π_d may take either sign and when negative represents a loss from TKE and gain of mean internal energy. This energy exchange does not change the mean entropy and thus can be regarded as reversible. In regions with negligible mean acceleration and negligible mean volume change, Π_d can also be interpreted as an exchange with acoustic potential energy (Section 2.5). The relative importance of Π_d compared to the Reynolds stress production of TKE is a natural measure of compressibility.

Estimates of Π_d can be obtained by considering the density ρ as a thermodynamic function of pressure, p, and specific entropy, s, (e.g. Thompson 1988) and by taking a material derivative to give (for an ideal gas)

$$\frac{\partial u_i}{\partial x_i} = -\frac{1}{\gamma p}\frac{Dp}{Dt} + \frac{1}{C_p}\frac{Ds}{Dt}. \tag{1}$$

Decomposing Equation 1 into its Reynolds-averaged mean and fluctuation gives (for high Reynolds number)

$$\overline{p'u'_{i,i}} \approx -\frac{\bar{p}}{2\gamma}\frac{\bar{D}}{\bar{D}t}\frac{\overline{p'^2}}{\bar{p}^2} - \frac{\bar{p}_{,i}}{\gamma\bar{p}}\overline{p'u'_i} - \frac{\gamma-1}{\gamma}\frac{\overline{p'^2}}{\bar{p}^2}(\varepsilon - \bar{q}_{j,j} + Q) + \ldots, \tag{2}$$

where only the leading terms are explicitly written. In Equation 2, $\bar{D}/\bar{D}t$ stands for the material derivative following the mean flow and Q is the external rate of energy addition per unit volume (e.g. due to chemical reactions). The first term of this equation can also be derived by considering linearized disturbance equations for homogeneous turbulence (Durbin & Zeman 1992). The approach adopted here (which is similar to that of Zeman & Blaisdell 1991) is more general. The expressions given here for an ideal gas are easily extended to an arbitrary equation of state.

The part of the third term in Equation 2 associated with the TKE dissipation rate ε always converts TKE into internal energy and is thus akin to *extra* dissipation due to compressibility, but does not cause an entropy increase. In a flow with mean deformation time scale $1/S$ and turbulence length scale l, the pressure fluctuation generated by the mean deformation p_r can be estimated as ρSlq, which leads to $\overline{p'^2}/\bar{p}^2 \sim \gamma^2\mathsf{M}^2M_t^2$, where $\mathsf{M} = Sl/C$. Thus the third term of Equation 2 can behave as extra dissipation which is proportional to M_t^2 when $\mathsf{M} \sim O(1)$. Terms representing compressible dissipation which depend on M_t have recently been proposed for use in turbulence models (Zeman 1990, Sarkar et al 1991). The first term of Equation 2 is $O(\mathsf{M}^2)$ relative to the TKE production (P) if $\bar{D}/\bar{D}t$ is estimated as $O(S)$, rough estimates for Π_d are consistent with the rapid distortion theory (RDT) predictions (Durbin & Zeman 1992, Sabel'nikov 1975) applicable in the limit of $S\bar{\rho}q^2/\varepsilon \gg 1$. The estimate of Π_d identifies M as an important compressibility parameter (in addition to M_t). This parameter can be regarded as the Mach number difference across an eddy (Durbin & Zeman 1992, who use the symbol Δm for it) or as a ratio of the acoustic propagation time across an eddy to the mean deformation time scale. With the latter interpretation if may be expected that when M is not small the loss of acoustic communication across an eddy may have a significant influence on the flow (Morkovin 1992, Papamoschou & Lele 1993). The importance of M has been underscored by Cambon et al (1992) who advocate a *pressure released* limit for $\mathsf{M} \gg 1$: Pressure fluctuations are ignored and the velocity fluctuations undergo a pure kinematic distortion due to the mean flow deformation.

The pressure fluctuation associated with the large eddies (in the absence of mean deformation), p_s, is expected to be ρq^2. The first term in (2) is then $\Pi_d/\mathsf{P} \sim M_t^2$ compared to $\Pi_d/\mathsf{P} \sim \mathsf{M}^2$ obtained earlier. Furthermore the ratio of p_r to p_s is $O(Sl/q)$ which is also the ratio of TKE production to its dissipation. In an equilibrium shear flow $Sl/q \sim O(1)$ or $\mathsf{M} \sim M_t$. Hence M and M_t can be regarded as independent parameters only in nonequilibrium flows, e.g. flows subject to rapid distortions.

The pressure fluctuation estimate ($p' \sim \rho Slq$) leading to $\Pi_d/\mathsf{P} \sim \mathsf{M}^2$ is valid only for the solenoidal fluctuations ($M_t \approx 0$) and should be revised when acoustic pressure fluctuations are dominant, e.g when M is large or when M_t is not small. If the characteristic acoustic particle velocity is $q_c = \chi_c^{1/2}q$, and the acoustic pressure $p_c \sim \rho C q_c$, it follows that $\Pi_d/\mathsf{P} \sim \chi_c$ (χ_c defined in Section 3). This suggests a decreasing importnce of Π_d in a nearly solenoidal flow subjected to a deformation with large M (Cambon et al 1992). A different approach using the Poisson equation for pressure (Sarkar 1992a) also yields similar estimates for Π_d.

The second term in Equation 2 is non-zero only for inhomogeneous flows. It may be significant in rapidly distorted flows, such as a boundary layer interacting with a shock wave. In hypersonic boundary layers, even over a flat surface, or in supersonic boundary layers on curved surfaces, the static pressure variation across the boundary layer is not negligible (Fernholz & Finley 1977). The second term may be important in these circumstances as well.

It should be stressed that while the parameter M_t naturally arises in nondimensional estimates of compressibility effects, it is generally not possible to set its value by design. The situation is analogous to not being able to set a value for the flux Richardson number in a turbulent flow involving buoyancy effects. Adjusting the speed of sound would provide the desired control if the turbulence was not affected by compressibility, but then M_t would also be a nonessential parameter. Just as the flux Richardson number can be indirectly varied by changing the imposed velocity difference (or temperature difference) the turbulence Mach number can be changed indirectly. Prescribing a turbulence Mach number M_t at a given turbulence Reynolds number is also equivalent to prescribing a Knudsen number: the ratio of microscopic to macroscopic length scales (see Lele 1993 for a discussion). Increasing M_t naturally increases the density fluctuations $\rho_{rms}/\bar{\rho}$, but this compressibility-associated increase is separate from the density fluctuations associated with temperature (entropy) fluctuations. The latter is dominant in turbulent flames, in turbulent boundary layers with $M_\infty \leq 5$ (Bradshaw 1977), and in low-speed buoyancy driven flows. The parameter M_t allows the compressibility effects to be separated from variable inertia effects and thus is preferred here. Furthermore it should be kept in mind that M_t is not the only parameter characterizing the compressibility effects. For nonequilibrium flows the deformation rate Mach number M is an independent parameter.

2.3 *Coupling between Momentum and Energy Exchanges*

The coupling between the momentum and energy exchanges in a compressible turbulent flow can be illustrated by considering the compressible mixing layer flow between two streams with free-stream speeds U_1 and U_2; where $\Delta U \equiv U_1 - U_2$, sound speeds are characterized by C, the width of the mixing layer at a station x is $\delta(x)$, the averaged flow speed $U_c = \frac{1}{2}(U_1 + U_2)$, the magnitude of turbulent velocity fluctuations is q, and magnitude of temperature fluctuations is T'. As the fluid particles that originate in the high speed stream exchange their excess momentum with the slower stream they are also compressed, which requires that work be done on them. Likewise the slower stream fluid gains momentum and loses work as it is entrained and mixed. The turbulent eddies affect this momentum and energy exchange between the two streams. The averaged rate of dilatation, $\bar{u}_{i,i}$, can be estimated using Equation (1) combined with the momentum balance and the entropy balance (Thompson 1988). This yields

$$\frac{\partial u_i}{\partial x_i} = -\frac{1}{c^2} u_i \frac{\partial(\frac{1}{2}u_k u_k)}{\partial x_j} + \frac{1}{c^2}\left[\frac{\partial(\frac{1}{2}u_k u_k)}{\partial t} - \frac{1}{\rho}\frac{\partial p}{\partial t}\right]$$

$$\frac{u_k f_k}{c^2} + \frac{\gamma - 1}{\gamma p}\left[\Phi + \frac{\partial q_k}{\partial x_k}\right], \quad (3a)$$

where u_i, p, c, f_k, q_k, and Φ, represent the instantaneous velocity, pressure, sound speed, external body force (per unit mass), heat flux vector, and viscous dissipation rate, respectively. Decomposing the variables into their Reynolds averages and fluctuations (for a stationary flow without external flow forces) gives

$$\bar{c}^{-2}\frac{\partial \bar{u}_i}{\partial x_i} = \frac{1}{2}\bar{u}_j\frac{\partial \bar{u}_k\bar{u}_k}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_k}\overline{u_j' u_k'} + \frac{1}{2}\bar{u}_j\frac{\partial \overline{u_k' u_k'}}{\partial x_j} + \bar{u}_k\overline{u_j'\frac{\partial u_k'}{\partial x_j}} + \overline{u_j' u_k'\frac{\partial u_k'}{\partial x_j}}$$
$$-\frac{1}{\bar{T}}\left(\bar{u}_k\frac{\partial \bar{u}_k}{\partial x_j}\overline{u_j' T'} + \bar{u}_j\frac{\partial \bar{u}_k}{\partial x_j}\overline{u_k' T'} + \bar{u}_j\bar{u}_k\overline{u_k'\frac{\partial T'}{\partial x_j}}\right) + \frac{\gamma-1}{\bar{\rho}}\left(\bar{\Phi} - \frac{\partial \bar{q}_j}{\partial x_j}\right) + \dots, \tag{3b}$$

where T denotes the absolute temperature, $\bar{c}^2 = \gamma R\bar{T}$, and a Taylor series expansion of c^{-2} about $\bar{c}$ is used. Individual terms of Equation (3b) have been discussed in Lele (1993).

Terms 4, 5, 8, and 10 on the right-hand side of (3b) are expected to be small in high Reynolds number flows (Tennekes & Lumley 1972). The magnitude of the remaining terms (1–3, 6, 7, and 9) of (3b) may be estimated for the mixing layer example as

$$\frac{\partial \bar{u}_i}{\partial x_i}\bigg/\frac{\partial \bar{u}}{\partial y} \sim O\left(\frac{d\delta}{dx}\frac{U_c^2}{C^2}\right) + O\left(\frac{q^2}{C^2}\right) + O\left(\frac{d\delta}{dx}\frac{q^2}{C^2}\right)$$
$$+ O\left(\frac{U_c}{C}\frac{q}{C}\frac{T'}{\bar{T}}\right) + O\left(\frac{d\delta}{dx}\frac{\Delta U}{C}\frac{q}{C}\frac{T'}{\bar{T}}\right) + \left(\frac{q}{\Delta U}\frac{q^2}{C^2}\right).$$

Evidently, the mean dilatation relative to mean shear $\partial\bar{u}/\partial y$ is the largest of $O(d\delta/dx\, U_c^2/C^2)$, $O(q^2/C^2)$, and $O[(U_c/C)(q/C)(T'/\bar{T})]$. From Equation (2) it follows that $d\delta/dx \sim O[q^2/(U_c\Delta U)]$
this, $d\delta/dx\, U_c^2/C^2 \sim O[q^2/C^2(U_1+U_2)/(U_1-U_2)]$ which makes the first term in Equation (3b) at least as large as the second term and the seventh term smaller than the sixth. Under the assumption of the Strong Reynolds Analogy (SRA) which is valid if $T_{01} \approx T_{02}$ (discussed in Section 4.3) the temperature fluctuations are estimated as $T'/\bar{T} \sim -(\gamma-1)(\bar{u}^2/C^2)(u'/\bar{u})$ so that $(U_c/C)(q/C)(T'/\bar{T})$
mates of the mean flow divergence is the parameter $M_t = q/C$. The mean divergence relative to mean shear is proportional to M_t^2 in a canonical mixing layer. Since *extra strain rates* are known to produce effects on turbulence that may be substantially larger than the measures of the extra strain rates (Bradshaw 1974), it may be anticipated that compressibility effects may become significant when M_t exceeds 0.2–0.3.

2.4 *Entropy Changes*

From a thermodynamic point of view the entropy changes associated with the different exchange processes are fundamental. The equation for the rate of change of specific entropy, s, can be obtained from the primitive (nonaveraged) mass, momentum, and energy balances. Its Favre-averaged form is (Favre 1969)

$$\frac{\partial}{\partial t}\bar{\rho}\tilde{s}+\frac{\partial}{\partial x_j}(\bar{\rho}\tilde{s}\tilde{u}_j+\bar{\rho}\widetilde{s''u_j''})-\frac{\partial}{\partial x_j}(\overline{kT_{,j}/T})=\Upsilon=\overline{\Phi/T}+(\overline{k|\nabla T/T|^2}),$$

where the Fourier law of heat conduction $q_j=-kT_{,j}$ with thermal conductivity k has been used. The irreversible increase of mean entropy Υ may be approximated by $\bar{\Phi}/\bar{T}+k(\bar{T})|\nabla\bar{T}/\bar{T}|^2$, if $T_{\mathrm{rms}}/\bar{T}$ is small. The mean rate of viscous dissipation, $\bar{\Phi}$, is the sum of dissipation due to the mean flow $\tilde{u}_i$ and the mean dissipation of TKE, ε. The dissipation rate due to the mean flow (deformation) is typically R^{-1} relative to ε, where R is a Reynolds number based on the energy-containing eddies, and thus relatively small except in the immediate vicinity of a no-slip surface or in the interior of a shock wave. It may seem that the entropy flux due to turbulent motions, $\bar{\rho}\widetilde{s''u_j''}$,

entropy analysis. Under certain assumptions, given in Section 2.7, useful estimates can be made. It is shown there that $\widetilde{s''u_j''}/C_v\approx\overline{p'u_j''}/\bar{p}-\gamma\overline{\rho'u_j'}/\bar{\rho}$. In a steady flow an integration of the mean entropy equation along the Favre-averaged streamlines gives

$$\Delta\tilde{s}/C_v=\int_{\xi_a}^{\xi_b}(\bar{\rho}\tilde{u}_s C_v)^{-1}\left\{\Upsilon-\frac{\partial}{\partial x_j}[\bar{\rho}\widetilde{s''u_j''}-(\overline{kT_{,j}/T})]\right\}d\xi,$$

where ξ and $\tilde{u}_s$ denote the position and mean velocity along the mean streamline, respectively. The contribution of the energy dissipation due to turbulence, $\varepsilon\approx\bar{\rho}q^3/\delta$, towards an entropy increase (over a streamwise distance L) may be estimated as $\gamma(\gamma-1)(q/U_s)(L/\delta)M_t^2$. In a mixing layer flow $(q/U_s)(L/\delta)\sim(U_1-U_2)/q$ making the entropy rise $\Delta\tilde{s}/C_v\sim M_t^2(U_1-U_2)/q$. At moderate and high $M_c=(U_1-U_2)/(C_1+C_2)$, this entropy rise and the associated total pressure loss, $\Delta p_0/p_0=\exp[-(\Delta\tilde{s}/C_v)/(\gamma-1)]$, may be significant. A similar conclusion about total pressure losses in a compressible mixing layer has been obtained using a different method of analysis (Papamoschou 1993).

2.5 *Role of Pressure Fluctuations*

The energy equation expressed in terms of mean pressure $\bar{p}$ serves to emphasize the important role of the pressure transport term in compressible flows. From the ideal gas relation, $p = \rho h(\gamma-1)/\gamma$, the equation governing enthalpy can be rearranged to givc

$$\frac{\bar{D}}{\bar{D}t}\bar{p} = -\gamma\bar{p}(\bar{u}_{j,j})-(\gamma-1)\overline{p'u'_{j,j}}-\frac{\partial}{\partial x_j}(\overline{p'u'_j})+(\gamma-1)\overline{u_{i,j}\tau_{ij}}+(\gamma-1)\bar{q}_{j,j}, \qquad (4)$$

$$\frac{\bar{D}}{\bar{D}t}\overline{p'^2}/2 = -\frac{\partial\bar{p}}{\partial x_j}\overline{p'u'_j}-\gamma\overline{p'^2}\bar{u}_{j,j}-\gamma\bar{p}\overline{p'u'_{j,j}}-(2\gamma-1)/2\overline{p'^2u'_{j,j}}$$
$$-\frac{\partial}{\partial x_j}(\overline{p'^2u'_j})/2+(\gamma-1)\overline{p'u_{i,j}\tau_{ij}}+(\gamma-1)\overline{p'q'_{j,j}}, \qquad (5)$$

whcrc $\bar{D}/\bar{D}t = \partial/\partial t+\overline{u_j}\,\partial/\partial x_j$. An approximate boundary layer form of Equation (4) was used by Brown & Roshko (1974) and the terms causing deviations from incompressible behavior (i.e. $\bar{u}_{j,j} \approx 0$) were estimated. From Equation (4) it follows that $\bar{u}_{j,j}/S \sim O(M_t^2 q/\Delta U)+O(M_t^2 M^2)$, where S is mean deformation rate. The first estimate is based upon the pressure transport and/or dissipation term and the second is based on the pressure dilatation term. In a thin shear flow the static pressure variation across the flow is $O(\bar{\rho}\overline{v'^2})$ so that the contribution of $\bar{D}\bar{p}/\bar{D}t$ to $\bar{u}_{j,j}/S$ becomes $O(M_t^2 d\delta/dx)$ and is small relative to the right-hand side of Equation (4). To correctly predict the compressible mean flow it is neccssary to corrcctly predict the $O(M_t^2)$ relative variation of static pressure in the direction normal to the shear layer.

Equation (5) governing the pressure variance is exact (Sarkar 1992a) and is related to the approximate Equation (2) discussed earlier. The appearance of the pressure dilatation term in this equation, also anticipated by classical acoustic theory, motivates associating $\overline{p'^2}/(2\gamma\bar{p})$ with the *potential* energy (PE) of the compressible fluctuations (Zeman 1991, Sarkar et al 1993). This identification is rigorous for small-amplitude acoustic waves when $\nabla\bar{p} = 0$ and $\bar{u}_{j,j} = 0$. In this situation (for example, the turbulence evolution downstream of a shock wave discussed in Section 4.1) the sum of TKE and PE change along the mean streamlines primarily due to a net imbalance between the production of TKE and its dissipation, with pressure dilatation Π_d representing an exchange between TKE and PE. Terms in Equation 5 involving $\nabla\bar{p}$ or $\bar{u}_{j,j}$ do not represent an exchange with kinetic energy. Generalized definitions of acoustic wave energy density and acoustic energy flux for arbitrary time-independent mean flows are available (Myers 1991, Pierce 1981, Morfey 1971, Cantrell & Hart 1964). These definitions have not, so far, been systematically exploited in studies of turbulent flows.

2.6 *Acoustic Energy Loss*

In a compressible flow an additional mechanism of energy loss due to acoustic radiation to the far field becomes active. Estimates of such losses can be made following the pioneering work of Lighthill (1952). In a turbulent jet the total acoustic power lost to the far field increases as M^8, where M is the Mach number of the jet. The ratio of the radiated acoustic power to the rate at which the turbulence extracts energy from the mean flow is a measure of acoustic efficiency, which increases as M^5 for small M, but because the coefficient in front is $O(10^{-4})$ only a small fraction of TKE production is lost to acoustic waves. The acoustic radiation remains an insignificant contributor to the TKE budget in compressible mixing layers at least up to convective Mach number $M_c = (U_1 - U_2)/(C_1 + C_2) \approx 0.6$ (Lele & Ho 1993). For M near unity the acoustic efficiency rises with M more gradually than M^5. Experimental data on supersonic jets summarized by Goldstein (1976) show that for $M \geq 1$ the acoustic efficiency becomes independent of M. It has been suggested (Liepmann 1979) that the energy loss due to acoustic radiation may be significant for hypersonic turbulent boundary layers but conclusive evidence for it is not available. Laufer (1962) has estimated that in a boundary layer at $M_\infty = 5$ the acoustic energy loss is of the order of 1% of the net work ($\tau_w U_\infty$) done by the wall shear stress τ_w.

Measurements of the acoustic radiation from supersonic boundary layers were made by Laufer (1962, 1964) who emphasized the Mach wave radiation as its primary mechanism. Mach wave radiation requires that the acoustic sources move at a supersonic convection velocity relative to the free stream. In the far field the distance r between the observation point and the acoustic source associated with a radiating eddy is large compared to both the eddy size and the distance traveled by the eddy over its radiation life. As a result the Mach waves in the far field take the form of spherical wave fronts and the acoustic intensity due to Mach wave radiation from a volume of size l^3 varies as $\overline{\rho'^2}/\bar{\rho}^2 \sim M^3 l^2/r^2$ (Ffowcs Williams 1966). In the near field, however, the Mach waves appear as straight wave fronts lying on Mach cones generated by the relative supersonic motion of the radiating eddies. Mach wave radiation has also been identified as a primary radiation mechanism in supersonic jets (Tam & Morris 1980, Tam & Burton 1984) and is consistent with acoustic efficiency becoming independent of the Mach number. Theoretical estimates of the energy loss have also been made (Ffowcs Williams & Maidanik 1965). For Mach wave radiation an acoustic efficiency of 1% is also reported for supersonic hot jets (Seiner 1992).

2.7 *Thermodynamic Relations*

It is useful to know the extent to which the thermodynamic relations applying to instantaneous state variables may be applied to the mean or averaged values of the state variables and if the fluctuations in state variables may be related in some fashion. In the following discussion the equation of state for an ideal gas will be commonly used. Relations holding for fluids with an arbitrary equation of state may similarly be constructed. Taking the instantaneous density ρ and entropy s as two independent state variables, the pressure p and temperature T may be expressed by thermodynamic relations $p = \vartheta(\rho, s)$ and $T = \mathsf{T}(\rho, s)$, which for an ideal gas become $\vartheta(\rho, s) = p_r e^{s/C_v}(\rho/\rho_r)^{\gamma}$ and $\mathsf{T}(\rho, s) = \vartheta(\rho, s)/(R\rho)$, where p_r and ρ_r are reference state values. Since the functions ϑ and T are generally nonlinear, $\bar{p} = \overline{\vartheta(\rho, s)} \neq \vartheta(\bar{\rho}, \bar{s})$, etc. However, if the fluctuations in ρ and s are small, a Taylor series expansion of $\vartheta(\rho, s)$ about the state $(\bar{\rho}, \bar{s})$ yields

$$\bar{p} = \vartheta(\bar{\rho}, \bar{s})\,[1 + \tfrac{1}{2}\gamma(\gamma - 1)\overline{\rho'^2}/\bar{\rho}^2 + \tfrac{1}{2}\overline{s'^2}/C_v^2 + \gamma\overline{\rho' s'}/(\bar{\rho} C_v) + \ldots].$$

Treating entropy as a dependent variable gives

$$\bar{s}/C_v = S_{\rho p}(\bar{\rho}, \bar{p}) - \frac{1}{2}\overline{p'^2}/\bar{p}^2 + \frac{\gamma}{2}\overline{\rho'^2}/\bar{\rho}^2 + \ldots,$$

where $S_{\rho p}$ is a thermodynamic entropy function with the indicated arguments for an ideal gas. Evidently, the leading-order error in applying the thermodynamic relations to the mean state variables are quadratic in the fluctuation. Relations applying to thermodynamic fluctuations can be similarly obtained. Such relations are useful in eliminating a chosen variable in terms of other variables. The entropy flux associated with turbulence may be evaluated using these as $\overline{s' u_j'}/C_v \approx \overline{p' u_j'}/\bar{p} - \gamma\overline{\rho' u_j'}/\bar{\rho}$ and using $\widetilde{s'' u_j''} \approx \overline{s' u_j'}$. Similarly it follows that $\overline{p' u_j'}/\bar{p} \approx \overline{T' u_j'}/\bar{T} + \overline{\rho' u_j'}/\bar{\rho}$.

Relations analogous to the Gibbs relations, Maxwell relations, etc. which hold for the averaged thermodynamic state variables can also be obtained by a Taylor series expansion.

3. COMPRESSIBLE HOMOGENEOUS TURBULENCE

A turbulent flow is considered homogeneous if the statistics of turbulent fluctuations are independent of position (Batchelor 1953, Monin & Yaglom 1971). For a compressible flow the restrictions on the mean fields necessary to maintain spatial homogeneity (Blaisdell et al 1991, Cambon et al 1992, Feiereisen et al 1981, Delorme 1985, Dang & Morchoisne 1987) are more stringent than the restrictions for an incompressible flow. Volume-preserving mean deformations, except for a
are excluded. Pure rotation is also excluded. Suitable combinations of time-dependent rotation and dilatational straining flows are permitted. Pure shearing is the only time-independent deformation allowed. A flow not strictly satisfying the homogeneity conditions, but with small devi-

ations from them (over an integral scale), is termed quasi-homogeneous (Goldstein & Durbin 1980, Hunt & Carruthers 1990, Durbin & Zeman 1992).

The set of equations referred to as the *low Mach number equations* or *anelastic approximation* (Spiegel 1971, Rehm & Baum 1978, Majda & Sethian 1985) is intermediate to the incompressible and compressible equations. The low Mach number equations describe the effects of variable fluid inertia but do not contain acoustic wave propagation. The restrictions for maintaining homogeneity for velocity and density fluctuations (but inhomogeneous temperature fluctuations) within this approximation are similar to the strict incompressible case. Studies of variable density (near zero Mach number) homogeneous turbulence are relevant to low-speed combustion flows. Many compressible flows also have accompanying variable inertia effects which are distinct from compressibility effects. Studies of variable density homogeneous turbulence are presently not available. Discussion is, hereafter, limited to the fully compressible case.

For homogeneous turbulence there is no distinction between the Reynolds-averaged and Favre-averaged velocity (Blaisdell et al 1991) making the velocity fluctuations relative to these averages also identical. The velocity fluctuations, u_i, can be uniquely decomposed (Moyal 1952) via Helmholtz decomposition into a solenoidal part, u_i^s (whose Fourier-Stieltjes transform, $\hat{u}_i^s(\mathbf{k})$, is orthogonal to the wavenumber vector $\mathbf{k}$), and a dilatational part, u_i^c [with $\hat{u}_i^c(\mathbf{k})$, parallel to $\mathbf{k}$]. Since the solenoidal and dilatational projections of u_i are orthogonal with respect to the L_2 norm, the variance u_iu_i (not the kinetic energy $\overline{\rho u_iu_i}/2$) can be split into a sum of solenoidal and dilatational parts. The dilatational fraction of the variance $\chi_c = \overline{u_i^c u_i^c}/\overline{u_iu_i}$ is a diagnostic of compressibility in the energy-containing range (Blaisdell et al 1991, Kida & Orszag 1990a, Erlebacher et al 1990). Splitting the Reynolds stress $\overline{\rho u_iu_j}$ on the basis of u_i^s and u_i^c leads to solenoidal, dilatational, and cross Reynolds stresses (Blaisdell et al 1991). A related split, which accounts for the density variations, is developed (Kida & Orszag 1990a,b, 1992) by decomposing $w_i = \sqrt{\rho}u_i$ into its solenoidal, dilatational, and mean parts. If the flow is nearly incompressible the fluctuations in thermodynamic quantities can also be split by a systematic appeal to a low Mach number expansion. Given the nonuniqueness of the low Mach number expansions (Zank & Matthaeus 1991, Bayly et al 1992) the choice must be based on physical information about the flow regime. Separating the pressure fluctuations into vortical and compressible parts—a decomposition distinct from the solenoidal/dilatational split discussed above—has been helpful in developing turbulence models for terms containing pressure fluctuations (Sarkar et al 1993, Durbin & Zeman 1992, Blaisdell et al 1991).

Theoretical prediction of compressibility corrections to Kolmogorov-Obukhov inertial range spectra has been surveyed elsewhere (Passot & Pouquet 1987, Passot et al 1988). The estimated correction to the inertial range slope is significant only for $M_t \sim O(1)$, conditions which may be impossible to realize in laboratory experiments. To our knowledge these predictions have remained unconfirmed. The equation governing the two-point correlations of density fluctuations resembles a wave equation (Chandrasekhar 1951). For high Reynolds number nearly-incompressible flows, density fluctuations are predicted to possess a $-\frac{5}{3}$ inertial subrange (Bayly et al 1992, Montgomery et al 1987) and are offered as a possible explanation of the Kolmogorov spectra observed in the interstellar media (Armstrong et al 1981, Goldstein & Siscoe 1972).

3.1 *Isotropic Turbulence*

In isotropic turbulence the statistical correlations are invariant to an arbitrary rotation of the coordinates (Hinze 1975, Batchelor 1953). Additional reflectional symmetry which makes the turbulence free of helicity (Lesieur 1990) is also commonly imposed. Isotropic turbulence (in absence of any mean flow deformation) is devoid of any mechanism to produce TKE and must decay in the absence of external forcing. Viscous dissipation of TKE is accompanied by a decrease of the turbulence Mach number M_t while the total energy $\bar{\rho}(C_v\tilde{T}+\widetilde{u_k''u_k''}/2)$ is conserved. The evolution of density (and pressure) fluctuations depends strongly on the initial conditions, which in turn influence the kinetic energy evolution. Initial conditions include specifying the partition of the velocity field into its solenoidal and dilatational parts; specifying the density, pressure fluctuations; and specifying the values of M_t, Pr (the Prandtl number). In the limit of small M_t the qualitative behavior of the system depends sensitively on χ_c: When χ_c is $O(M_t)$ the vortical and compressible parts (defined as the deviation from the incompressible limit) of the flow evolve independently at leading order with the compressible part obeying linearized acoustic equations; for larger χ_c nonlinearity (wave steepening) cannot be ignored in the evolution of u_i^c and shocks form when $\chi_c \sim O(1)$ (Erlebacher et al 1990, Ghosh & Matthaues 1992). Numerical simulations of forced isotropic turbulence (Kida & Orszag 1990a) demonstrate that the coupling between the vortical and compressible parts of the flow is weak.

The sensitivity of u_i^c to the initial conditions is maintained even when M_t is not small (Blaisdell et al 1991, Sarkar et al 1991). In the numerical simulations the decay rate of TKE slightly increases with M_t. The contribution of dilatation to TKE decay, defined by χ_ε, increases with M_t and depends sensitively on the initial values of χ_c and $\rho_{rms}/\bar{\rho}$ (Blaisdell et al 1991, Sarkar et al 1991) which makes its parameterization difficult. The pressure dilatation, $\Pi_d = \overline{p'u_{i,i}}$ (which is always an exchange term between TKE and $\bar{\rho}\tilde{e}$), is also an exchange term between the kinetic energy, $\bar{\rho}u_i^c u_i^c/2$, and the potential energy, $\overline{p_c^2}/(2\gamma\bar{p})$, in a small-amplitude acoustic field (Morse & Ingard 1968, Lighthill 1978). The numerical simulations suffer from a poor statistical ensemble of the largest scales in the flow. This is particularly severe for the acoustic fluctuations generated by the vortical motions at small M_t (Blaisdell et al 1991). As a result, when the contribution of these scales to the statistics being studied becomes important, temporal oscillations with a time scale commensurate with the acoustic propagation time over these spatial scales become evident. The oscillations provide a mechanism of exchange between the kinetic and potential energy of the acoustic field with the total acoustic energy staying constant, and are also observed in TKE when χ_c is appreciable. When entropy fluctuations are small, i.e. $s_{rms}/C_v \ll \gamma\rho_{rms}/\bar{\rho}$, then $\Pi_d \approx \overline{\rho' u_{i,i}}R\bar{T}$ so that for temporally decaying turbulence Π_d is expected to be positive (Sarkar et al 1991) due to the appearance of $-\overline{\rho' u_{i,i}}$ in the exact equation for density variance. Numerical simulations consistently show that for isotropic-decaying turbulence, pressure dilatation is positive on average (Blaisdell et al 1991, Sarkar et al 1991) and its contribution to TKE decay is small. However, in shear flows and straining flows it can be a significant contributor. These flows are discussed later in Sections 3.2, 3.3, and 4.

Models proposed for the compressible dissipation ε_c (Zeman 1990, Sarkar et al 1991) have been shown to give better predictions of the spreading rate of a compressible mixing layer (Zeman 1990, 1992b; Sarkar & Lakshmannan 1991; Viegas & Rubesin 1991). Calibration of ε_c from numerical simulations of decaying isotropic turbulence is not reliable (Blaisdell et al 1991, Zeman 1992a) due to the sensitivity of χ_ε to initial conditions and the low values of the microscale Reynolds number R_λ. However, the suggested model form of Sarkar et al (1991) is consistent with numerical simulations having initial $\chi_c = 0$ (Sarkar et al 1991, Blaisdell et al 1991). Figure 2 reproduces the data from Lee et al (1991). Even though χ_ε varies over almost two decades (highest in case A and lowest in case E) this variation collapses to within a factor of two when plotted as χ_ε/M_t^2. Similar trends are reported by Blaisdell et al (1991). The DNS data, as in Figure 2, is limited to low values of R_λ and does not allow a test of specific Reynolds number dependence of χ_ε/M_t^2 based on spectral-similarity arguments (Zeman 1992a). The collapse in Figure 2 suggests that χ_ε might be modeled as αM_t^2 with α in the range 0.1–0.2; however, in homogeneous shear flow α is in the range 0.5–1. How much of this variation reflects a dependence on the flow type is presently unclear.

The M_t^2 dependence of χ_ε (Sarkar et al 1991) rests upon the expectation that χ_ε is proportional to χ_c, which is in turn estimated by invoking the

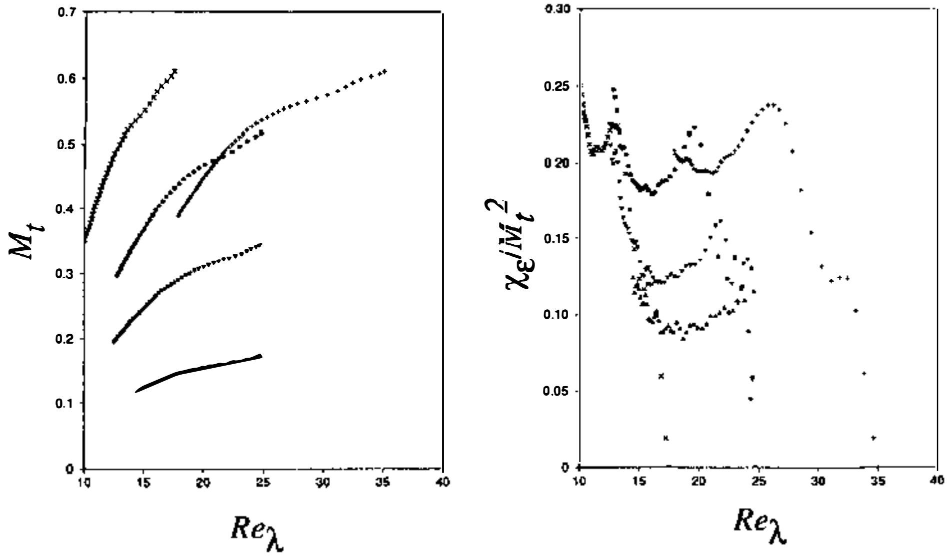

Figure 2 Evolution of isotropic compressible turbulence. The data of Lee et al (1991) are plotted showing the turbulence Mach number M_t against the Reynolds number Re_λ for cases A (+), B (×), C (●), D (▼), and E (▲). The corresponding values of χ_c/M_t^2 are also plotted against Re_λ.

equipartition of kinetic and potential energy for the acoustic mode (Morse & Ingard 1968, Lighthill 1978) and assuming that pressure fluctuations due to vortical and acoustic modes are comparable. The proportionality between χ_ε and χ_c is observed in numerical simulations only when the acoustic mode is not prescribed independently from the vortical mode (Blaisdell et al 1991). In the simulations of Lee et al (1991) the condition of no initial pressure fluctuations ensures that p^s and p^c are comparable and the observed M_t^2 dependence is thus not unexpected. Lee et al also report the occurrence of eddy shocklets for cases with high M_t. Parameterization of the dissipation due to the eddy shocklets is the centerpiece of the Zeman (1990) model. The detailed shocklet modeling of Zeman has not been confirmed by numerical simulations since the shocklets observed in the simulations contribute only a small fraction of the observed compressible dissipation; their contribution appears to increase with R_λ and M_t. Most of the compressible dissipation in numerical simulations occurs at large scales (Sarkar et al 1991, Blaisdell & Zeman 1992) and the proposed models are in reasonable agreement with it. Whether the same would hold in high Reynolds number flows

it may be noted that the spectral equations governing the dilatational velocity field $u_i^c(k)$ contain terms analogous to those describing passive scalar advection by the solenoidal velocity. This may cascade the acoustic energy to smaller scales. The associated spectral behavior does not seem to have been studied.

Measurements of decaying compressible isotropic turbulence are nonexistent due to the practical difficulties in generating a quasi-homogenous turbulent flow with significant levels of M_t. Space-time correlations (Favre et al 1965b; Wills 1964, 1971) are commonly used to provide statistical measures of the spatial scales in the flow and their temporal persistence. They also provide a statistical definition of the convection velocity of various scales (Wills 1964, 1971), a notion well suited for the vortical mode. The acoustic mode is, by definition, associated with wave propagation rather than convection which makes its space-time correlation rather distinct from the vortical mode (Lee et al 1992). In a regime of significant coupling between the acoustic and vortical modes (e.g. with significant shock-generated vorticity) the space-time correlations of the vortical mode may also show some change. The phenomenology of such regimes are presently unknown. In flows with high M_t the influence of compressibility on spectral energy transfer and influence on local isotropy (e.g. due to the additional large scale–small scale coupling due to eddy shocklets) have not been examined. A complete theory of the final period of decay of compressible isotropic turbulence is not available. The degeneracy of the nearly incompressible flows will need to be resolved in such a theory. Unlike its incompressible counterpart the constraints governing the slopes of the low wavenumber energy spectra are also not available.

3.2 *Homogeneous Sheared Turbulence*

Spatially-uniform, time-independent shearing is the only nontrivial volume-preserving mean deformation that strictly satisfies the homogeneity constraints for a compressible flow. Mean flow shear is common to most turbulent flows and the homogeneous idealization allows a study of some effects of compressibility that are shared by many turbulent flows. The idealization is unable to retain other aspects critical to the inhomogeneous shear flow and boundary layer turbulence: Energy loss by acoustic radiation to the far field, which may be important under hypersonic free-stream conditions, and the strong mean density stratification near a no-slip surface in supersonic and hypersonic turbulent boundary layers are two such effects. Measurements realizing quasi-homogeneous compressible uniformly sheared turbulence are not available, unlike its incompressible counterpart (Tavoularis & Corrsin 1981, Tavoularis & Karnik 1989), and the following discussion is largely based on numerical simulations.

Feiereisen et al (1981) appear to have conducted the first numerical simulations of homogeneous sheared compressible turbulence. Due to their particular choice of initial conditions no significant effects of compressibility were reported. Delorme (1985) and Dang & Morchiosne (1987) also performed DNS and LES (Large Eddy Simulations) of this flow. Recently Blaisdell et al (1991) and Sarkar et al (1993) have conducted extensive studies of this flow for values of M_t exceeding the range covered in previous work. The simulations impose a spatially uniform shear, $S = dU/dy$, on an isotropic field of fluctuations with prescribed energy spectra. Under the influence of mean shear the fluctuations develop toward a "realistic" state of turbulence. Typically this adjustment takes a time $St \approx 4$–6 depending upon the initial conditions (developed isotropic turbulence requires a smaller adjustment time). After this the fluctuation quantities display an approximately exponential growth.

The incompressible homogeneous shear flow contains two time scales, i.e. $\bar{\rho}q^2/\varepsilon$ and $1/S$. The ratio of these, $S_* = S\bar{\rho}q^2/\varepsilon$, is also proportional to production $\mathsf{P} = -\bar{\rho}S\overline{uv}$ divided by the dissipation ε, if the structure parameter $a_1 = -\overline{uv}/q^2$ is a constant. In the exponential growth phase a_1 and S_* are approximately constant and an almost constant fraction, σ, of P goes towards increasing TKE. With increasing compressibility a larger fraction of P is "dissipated," lowering the normalized growth rate, σ/S. Analysis of the TKE budget indicates (Blaisdell et al 1991, Sarkar et al 1993) that the increased "dissipation" is a result of the loss due to Π_d and dissipation due to ε_c. The energy supply rate P also depends on turbulence (Rogers et al 1986 have shown the influence of initial conditions) and appears to decrease with increasing compressibility for large St (Blaisdell et al 1991, Sarkar et al 1993).

Numerical simulations (Blaisdell et al 1991, Sarkar et al 1993) show that the compressible dissipation fraction χ_ε becomes independent of the initial values of χ_c and $\rho_{rms}/\bar{\rho}$ in an evolution time $St \approx 8$–10; χ_c, $\rho_{rms}/\bar{\rho}$ also show a similar trend. In this evolved flow χ_ε, χ_c, $\rho_{rms}/\bar{\rho}$, and $p^c_{rms}/\bar{p}$ all exhibit an M_t^2 dependence (Sarkar et al 1993). There is some uncertainty in the value of χ_ε/M_t^2; Sarkar et al suggest a value of 0.5, revising their previous estimate of 1.0 which was chosen to model the decay of isotropic compressible turbulence. Blaisdell et al (1991) find the latter to be consistent with their simulations, although no best fit is attempted. DNS of other homogeneous flows (e.g. compressed turbulence, discussed in Section 3.3) suggest that an algebraic closure of χ_ε in terms of M_t alone is too restrictive.

Structural parameters such as the Reynolds stress anisotropy, $b_{ij} = \overline{\rho u_i'' u_j''}/\overline{\rho u_k'' u_k''} - \delta_{ij}/3$, structure tensor y_{ij} (Reynolds 1989), and ratios of integral length scales, which characterize the sheared turbulence, are similar in value to their incompressible analogs (Blaisdell et al 1991). Small-scale features, such as the preferred alignment between the intense vorticity and the intermediate principal strain rate direction, and most probable principal strain rate ratios in intense dissipating regions are also largely similar to the incompressible case (Blaisdell et al 1991, Sarkar et al 1993, Erlebacher & Sarkar 1992). Splitting the velocity field, however, reveals that the dilatational part of the velocity field has a structure distinct from the solenoidal part. Unlike u_i^s for which $\overline{u_s^2} > \overline{w_s^2} > \overline{v_s^2}$ is generally observed in shear flows, u_i^c shows that $\overline{v_c^2} > \overline{u_c^2} \approx \overline{w_c^2}$ (Blaisdell et al 1991). The contribution of $\overline{v_c^2}$ to $\overline{v^2}$ becomes significant with increasing M_t even when the contribution of u_i^c to other covariances is still small. Since u_i^c contributes little to scalar mixing (Blaisdell et al 1991) the mixing across the imposed shear is preferentially decreased as M_t increases (Blaisdell et al 1991, Sarkar et al 1993). This distinct anisotropy of u_i^c is also reflected in the anisotropy of the compressible dissipation which scales better with b_{ij}^c than with b_{ij} (Blaisdell et al 1991). Both ε_c and Π_d, whose importance is an indicator of compressibility, preferentially oppose the growth of transverse fluctuations. This in turn reduces the growth of Reynolds shear stress, and finally the energy supply rate P.

The turbulence Mach number M_t grows with time in the simulations and it is natural to ask if the late time evolution of homogeneous shear turbulence is always influenced by compressibility even when the initial M_t is quite small. The energy balance equations can be rewritten to yield an equation for M_t which for homogeneous shear flow at high Reynolds number simplifies to

$$\frac{\partial M_t^2}{\partial t} = M_t^2 S(\sigma_* - \hat{\gamma} M_t^2/\hat{S}),$$

where $\hat{\gamma} = \gamma(\gamma - 1)$, $\sigma_* = \sigma/S = (\partial q^2/\partial t)/(Sq^2)$, and $\hat{S} = S\bar{\rho}q^2/(\varepsilon - \Pi_d)$. Simulations have shown that σ_* is a decreasing function of M_t, while the dependence of $\hat{S}$ on M_t is not certain but it seems probable that $M_t^2/\hat{S}$ increases with M_t: Thus a fixed point of M_t (such that $\partial M_t/\partial t = 0$) may be approached in the long term evolution of the flow, although with initially small M_t the time for this approach may be very long. An equilibrium value of M_t has been noted by Zeman & Blaisdell (1991) and Zeman (1992a) but the DNS data, at present, do not show a limiting behavior (the long time behavior of DNS may suffer from inadequate sampling of the large scales). Asymptotic values of M_t have also been reported in turbulence model based calculations of compressible mixing layers (Zeman 1990, 1992b; Sarkar & Lakshmanan 1991).

To date the simulations have been limited to parameter ranges $S_* = S\bar{\rho}q^2/\varepsilon \approx 5$–15 and $M_t \approx 0$–0.65. Estimates of S_* in typical shear flows (e.g. in the logarithmic and outer regions of a boundary layer and channel flow) range between 5 and 10, a range spanned by the simulations. Self-preserving incompressible mixing layers have $S_* \approx 8$ (Rogers & Moser 1993). In the immediate vicinity of a no-slip wall, however, S_* can be as large as 30 (Lee et al 1990). Reliable estimates for the near-wall region of compressible boundary layers are currently not available. Incompressible homogeneous shear turbulence with large S_* has been successful in explaining several structural features of near-wall turbulence (Lee et al 1990). In the compressible case large S_* may also imply large M, since $\mathsf{M} \sim S_* M_t$, and as noted in Section 2 additional effects of compressibility associated with loss of acoustic communication may arise for large M.[1] Such strongly sheared compressible turbulence has as yet not been explored. The importance of the shearing rate Mach number M has also been recently stressed by Sarkar (1992b).

3.3 *Homogeneous Compressed Turbulence*

Irrotational, time-dependent mean-deformation can still satisfy the compressible homogeneity constraints when the mean dilatation is not zero. The principal strain rates can be used to classify the dilatational straining mode as either one-dimensional, planar, axisymmetric, spherically-symmetric, or three-dimensional. One-dimensional compression arises in shock-turbulence interaction and in the compression stroke of a piston engine; planar compressions and expansions (generally inhomogeneous) are common in planar mean flows; axisymmetric strains arise in axisymmetric nozzle and diffusor flows or in attached axisymmetric external flows; spherically-symmetric strain is an idealization of implosions; and three-dimensional strains arise in complex flow fields. Bulk expansions generally dampen the turbulence and if sufficiently strong may even lead to relaminarization (Morkovin 1955, Narasimha & Sreenivasan 1979, Dussauge & Gaviglio 1987). Bulk expansions are not discussed in detail in this review; reference can be made to Bradshaw (1974, 1977), Dussauge & Gaviglio (1987), Jayaram et al (1989), and Smith & Smits (1991) for further details.

The influence of compressibility on compressed turbulence depends on the turbulence Mach number M_t, the deformation (rate) Mach number M, and on the mode of compression. The rapidity of compression characterized by $S_* = S\bar{\rho}q^2/\varepsilon$ relates the two characteristic Mach numbers: $\mathsf{M} \sim S_* M_t$. Thus nearly solenoidal turbulence ($M_t \approx 0$) when compressed sufficiently rapidly [so that $\mathsf{M} \sim O(1)$] can exhibit significant com-

[1] In boundary layers the effects associated with strong mean density stratification are large and compressibility effects are difficult to isolate.

pressibility effects (Zeman & Coleman 1993). When the fluctuations are themselves compressible ($M_t \neq 0$), even moderate values of S_* are associated with $\mathsf{M} \geq O(1)$. When M is large an approximation reflecting the lack of acoustic communication should become applicable. In the context of homogeneous turbulence a *pressure released* RDT (Cambon et al 1992) has been proposed. Response of incompressible turbulence to bulk compression has been studied theoretically in the limit of rapid compression (Batchelor & Proundman 1952, Ribner & Tucker 1953, Batchelor 1955, Hunt 1978, Lee 1989, Coleman & Mansour 1993). Numerical simulations of solenoidal compressed turbulence (Wu et al 1985, Coleman & Mansour 1991) and compressible compressed turbulence (Coleman & Mansour 1991, Zeman & Coleman 1993, Cambon et al 1992) have considered both rapid and slow compressions for spherically-symmetric and one-dimensional compression modes. Theoretical analysis of compressible rapid compressions (Durbin & Zeman 1992, Cambon et al 1992, Sabelnikov 1975) has been used to aid the modeling of compressed turbulence.

The evolution of TKE for isotropic turbulence subjected to a rapid one-dimensional compression with a fixed initial compression rate S_* presented by Zeman & Coleman (1993) shows the interesting result that the TKE increase for a given value of mean density change is smaller when the initial M_t is small. The DNS results (Cambon et al 1992) show that in compressions with $S_* \gg 1$ the rapidity of compression (S_*) continues to influence the TKE change unless $\mathsf{M} \gg 1$ in which case the pressure released limit is approached. Cambon et al (1992) use $\Delta m = M_t S/\omega$, where ω is the rms vorticity, as the important compressibility parameter. The Mach number Δm defined using the acoustic time based on a Taylor microscale is related to M by $\Delta m \sim \mathsf{M}R^{-1/2}$, where $R = \rho q^4/(\nu\varepsilon)$. Compressibility parameters such as M or Δm are better suited than M_t in characterizing the compressibility influence. This is illustrated by Figure 3 (courtesy of G. Coleman) which shows the TKE change as a function of the net compression for different S_* (with initial $S_* > 47$ and $S/\omega > 2$) and M_t. Evidently the TKE change for a given compression depends monotonically on Δm but the dependence on M_t is not monotonic.

For small M but $S_* \gg 1$ the incompressible RDT limit is obtained, and for large M with large S_* the pressure released limit is achieved. As shown in Figure 3 the TKE growth for a given volume compression increases with compressibility (measured by Δm). This is opposite to the trend observed in homogeneous shear flow (Section 3.2). Cambon et al (1992) have emphasized the two different roles played by the pressure fluctuations, i.e. through Π_d in overall TKE balance and through the pressure strain rate correlation. For small but nonzero M the pressure dilatation Π_d is the dominating influence and as a result the TKE growth approaches the

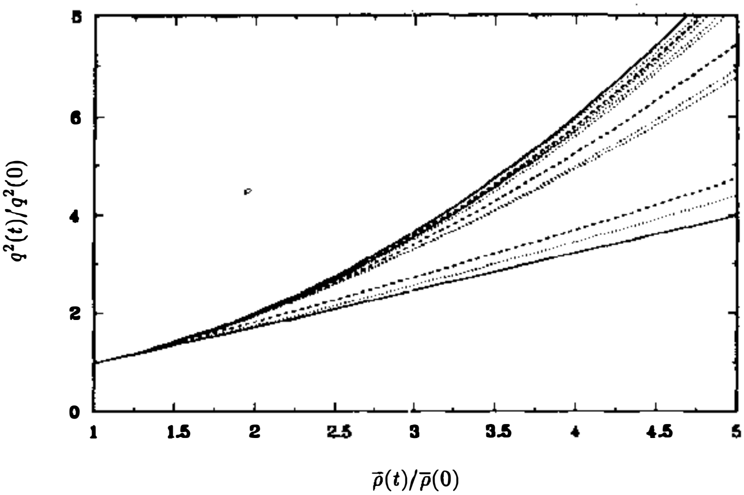

Figure 3 Turbulent kinetic energy histories for one-dimensional compression. Data taken from Cambon et al (1992). The lower solid line is the solenoidal RDT limit and the upper solid line is the *pressure released* RDT limit. DNS data lie between these limits varying monotonically with initial Δm; dotted curves are DNS with initial Δm ranging from 0.3 (lower) to 8.0 (upper), dashed curves are DNS cases: lower $(M_t, \Delta m) = (0.03, 0.3)$, upper $(0.1, 7.0)$, middle $(0.3, 1.0)$.

corresponding solenoidal case with the same S_* (toward the bottom curve in Figure 3). The RDT analysis (Durbin & Zeman 1992) also shows the dependence of Π_d on the mode of compression: The relative importance of Π_d decreases as the compression mode is changed from one dimensional to planar to spherically symmetric. At larger M the pressure strain rate term dominates; the DNS results show that the component energy redistribution is less effective as M increases resulting in a larger TKE increase (Cambon et al 1992). A limitation of the pressure-released RDT arises from the fact that flows that acquire large values of deformation rate Mach number may also be expected to show some effects of chemical/thermodynamic nonequilibrium. Such influences are not discussed in this article.

One-dimensional rapid compressions lead to negative Π_d (loss of TKE) regardless of the initial conditions, but for slow compressions Π_d displays significant positive and negative oscillations (G. Coleman 1993, private communication), reminiscent of the isotropic decay case (Section 3.1). For rapid spherically-symmetric compressions the sign of Π_d depends on the initial conditions (G. A. Blaisdell 1993, private communication). The inability of irrotational mean straining flows to erase the memory of the initial conditions rules out simple parametrizations of the compressible dissipation parameter χ_ε (Coleman & Mansour 1993). Rapid deformations also influence the overall TKE dissipation rate ε. For compressed flows, the need to account for the variations of kinematic viscosity in the transport equation for ε has been stressed by Coleman & Mansour (1991, 1993). Removing an imposed one-dimensional compression (G. Coleman 1993, private communication) shows a relaxation on an acoustic time scale but the return towards isotropy remains incomplete while the turbulence continues to decay.

3.4 *Homogeneous Complex Shear Flow Turbulence*

Homogeneous turbulence subjected to simultaneous mean dilatation and shear may be viewed as an idealized flow pertinent to distorted boundary layer turbulence. The passage of sheared turbulence through a shock suggests the idealization of homogeneous turbulence subjected to simultaneous shear and one-dimensional compression. Analysis using incompressible RDT (Mahesh et al 1993) has shown that the resulting amplification of TKE is influenced by the initial Reynolds stress anisotropy and the ratio of shearing rate to compression rate. The dependence arises through the pressure strain rate terms. For sufficiently strong normal compressions the Reynolds shear stress changes its sign. It is presently not known how M_t may influence these results. Other combinations of bulk compression and shear—e.g. shear with plane compressive strain, shear with lateral compression/divergence—are also relevant to distorted boundary layer applications. In light of the reported nonlinear behavior when multiple extra strain rates such as bulk compression, lateral divergence, and streamline curvature are combined with canonical boundary layers (Fernando & Smits 1990, Smits & Wood 1985, Smits et al 1979), the homogeneous idealizations (incompressible or compressible) of complex shear flows may prove useful for conceptual understanding, even though they do not contain the inhomogeneous transport effects of the large eddies.

4. SIMPLE INHOMOGENEOUS FLOWS

4.1 *Idealized Shock-Turbulence Interactions*

The interaction of turbulence with shocks is a common element of external aerodynamic flows. In experiments it is common to study the interaction of an oblique shock (created by a wedge in a supersonic flow) with a turbulent boundary layer or the supersonic flow in a compression ramp (Settles et al 1979, Dolling & Or 1985, Andreopoulos & Muck 1987, Smits & Muck 1987, Kuntz et al 1987, Selig et al 1989). Although these flows are an idealization of shock-turbulence interactions they still constitute a complex turbulent flow (Green 1970). To help unravel the basic processes occurring in such flows a further idealization may be undertaken: The simplest is to consider the interaction of a normal shock with turbulence that is homogeneous in directions transverse to the mean shock front. Theoretical analyses linearize the upstream turbulence into vorticity, and acoustic and entropic modes and impose linearized Rankine-Hugoniot jump conditions across the shock. Predictions are then made of the downstream statistics in terms of the upstream statistics. Experiments attempting to study the idealized interaction are difficult (Jacquin et al 1991, Honkan & Andreopoulos 1992, Keller & Merzkirch 1990, Debieve & Lacharme 1986) and the currently available data are limited. Recent numerical simulations (Lee et al 1993, Rotman 1991) have confirmed some of the predictions of the linear analyses and identified nonlinear aspects of the problem. Linear analysis has also been used to estimate the turbulent corrections to the classical shock jump relations and shock speed formulae (Lele 1992).

When the turbulent pressure fluctuations are small compared to the pressure rise across the shock $[M_t^2 < 0.1(M_t^2 - 1)]$ the shock front is weakly distorted and the linearization of the Rankine-Hugoniot conditions used in theoretical analyses is justified. For stronger turbulence the shock structure is strongly modified and the instantaneous pressure rise along the mean streamlines does not remain monotonic (Lee et al 1993). A similar strong interaction is reported in experiments (Hesselink & Sturtevant 1988) on propagation of weak shocks in a medium with strong density fluctuations. In passing across the shock the vorticity fluctuations are strongly amplified and consequently the dissipation rate is enhanced (Lee et al 1993, Jacquin et al 1991). Linear analysis predicts the vorticity amplification accurately. Components of potential vorticity (vorticity divided by density) lying in the plane of the shock remain unaltered and the potential vorticity normal to the shock is diminished by the bulk compression. The net result is that the vorticity components in the plane of the shock increase in proportion to the density ratio across the shock and the normal component of vorticity remains unchanged. The linear analysis also predicts an amplification of the TKE across the shock (see Jacquin & Cambon 1992 for a comparison of different linear analyses).

The DNS results to data have been limited to weak shocks. Simulations using two-dimensional Euler equations (Rotman 1991) cover a wider range of shock strengths. The reported TKE amplification for small M_t is in reasonable agreement with a two-dimensional version of Ribner's analysis (S. Lee 1992, private communication). The TKE amplification across a shock of a prescribed strength is reported to decrease as M_t (i.e. turbulence intensity) is increased. This trend is shared by DNS (Lee et al 1993) and experiments (Hokan & Andreopoulos 1992). However, the DNS shows that the TKE increases not simply across the intermittent zone occupied by the shock but in a region downstream of it. Numerical simulations for stronger shocks using a shock-capturing ENO scheme in the region surrounding the shock (Lee 1992) also show a clear TKE rise but, as in DNS of weak-shock cases (Lee et al 1993), the increase occurs in a region downstream of the oscillating shock.

This downstream increase of TKE is due to an energy exchange between the potential energy in the pressure fluctuations just downstream of the shock and the kinetic energy of the acoustic mode. Pressure fluctuations are amplified across the shock and decay downstream. The acoustic energy density (KE plus PE) remains approximately constant downstream of the shock, causing a downstream rise of TKE. In the TKE balance of Figure 1 the potential energy of acoustic modes is not explicitly accounted for, and the PE to KE exchange mechanism is manifested via the pressure dilatation and pressure transport terms.

Experiments of Jacquin et al (1991) report no significant TKE rise (for $M_1 = 1.4$) for isotropic upstream turbulence but a clear rise when (anisotropic) jet turbulence interacts with the shock cells of an over-expanded jet. The latter flow is significantly inhomogeneous and comparisons with homogenous rapid distortion theory can at best be qualitative. Other experiments (Honkan & Andreopoulos 1992, Debieve & Lacharme 1986) report clear TKE amplification. To help clarify these trends, RDT calculations have been performed (Mahesh et al 1993) for the interaction of anisotropic turbulence with normal and oblique shocks. The anisotropy is observed to make a substantial difference in the TKE amplification as anticipated by Jacquin et al (1991). The dependence of TKE amplification on the initial anisotropy decreases with the obliquity of the shock.

Conflicting claims about the changes in the length scales of turbulence when passing across a shock have appeared in the literature. Honkan & Andreopoulos (1992) in their experiment ($M_t = 1.24$) suggest that $L_{\varepsilon 1} = \bar{\rho}(\overline{u^2})^{3/2}/\varepsilon_{11}$ increases across the shock; this is in conflict with DNS. Taylor microscales for velocity fluctuations decrease across the shock in DNS which is in agreement with Ribner's (1953, 1987) theory. An apparent conflict with earlier experiments (Debicve & Lacharme 1986) has bcen attributed to a false microscale increase due to shock front intermittency (Lee et al 1993). Figure 4 (courtesy of S. Lee) shows the evolution of different physical length scales across the shock from a shock-capturing simulation. For the case shown $M_1 = 2.0$, $M_t = 0.08$, and $R_\lambda = 18$ just upstream of the shock. The region blanked out from the plot corresponds

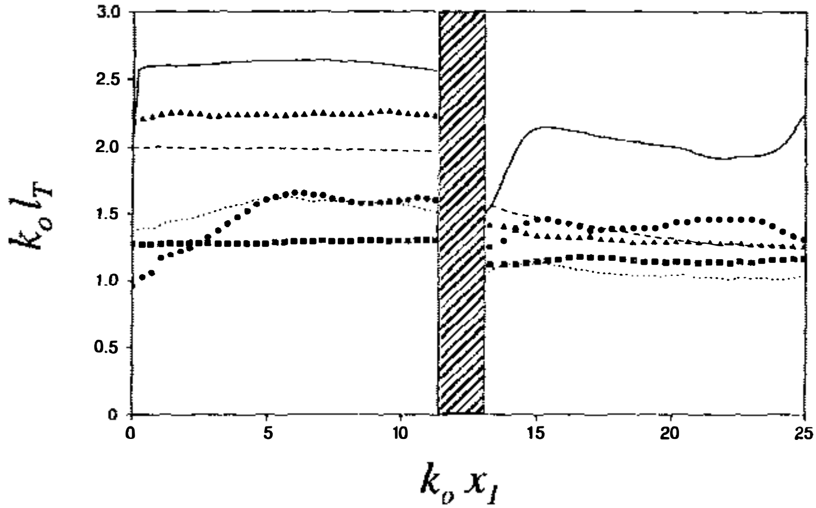

Figure 4 Evolution of several physical length scales in a shock-turbulence interaction. Figure taken from Lee et al (1994). Isotropic turbulence with $M_t = 0.08$, $R_\lambda = 18$ just upstream of the shock interacts with a $M_1 = 2.0$ normal shock; dissipation length $\bar{\rho}q^3/\varepsilon$ (*solid line*), longitudinal velocity microscale $\lambda_2(u_2)$ (*dashed line*), density microscale λ_2 (*dotted line*), longitudinal velocity integral scale $\Lambda_2(u_2)$ (▲), lateral velocity integral scale $\Lambda_2(u_1)$ (■), and integral scale for density fluctuations Λ_2 (●).

to the intermittent region occupied by the shock whose extent is defined by $\bar{u}_{i,i} = 0$ (Lee et al 1993). The simulation shows, in agreement with the DNS of weak-shock cases, that characteristic length scales—including longitudinal and lateral velocity integral scales, longitudinal velocity microscale, dissipation length scale $\bar{\rho}q^3/\varepsilon$, as well as the integral scale and microscale of density fluctuations—decrease across the shock. The result for density microscale is in conflict with the experimental observation of Keller & Merzkirch (1990) who claim that the Taylor microscale of density fluctuations increase across the shock. At present this conflict remains unresolved. It is unclear how closely the experiments approximate the homogeneous idealization of numerical simulations, and there may very well be practical limitations in improving them. In the future, as better diagnostics improve the experimental data base, numerical simulations that retain more of the physical effects occurring in laboratory flows are likely to further our understanding of shock-turbulence interaction phenomena (i.e. DNS of idealized shock-turbulence interactions which include the effects of mean shear and mean density gradient, LES of shock wave/turbulent boundary layer interaction, LES of compression corner flows, etc).

4.2 *Compressible Mixing Layers*

The turbulent mixing between two streams of different velocity has served as the most vivid experimental demonstration of the effect of compressibility on turbulence (Birch & Eggers 1972, Brown & Roshko 1974). The notion of convective Mach number, M_c (Papamoschou & Roshko 1988, Bogdanoff 1983), has been moderately successful in collapsing the normalized growth rate data onto a single curve. The collapse represented in Figure 5 (assembled by D. Papamoschou) still has significant scatter, perhaps serving as a reminder of the sensitivity of the mixing layer flow to background disturbance environment. Some of the scatter may also be due to the variable degree to which a self-preserving flow is realized in different experiments (Clemens & Mungal 1992) and to uncertainty in the normalizing incompressible mixing layer growth rate used in Figure 5.

It has also been suggested that a single compressibility parameter M_c may not be sufficient (Dimotakis 1991, Hall 1991) particularly when data from mixing layers with one subsonic stream are included (Viegas & Rubesin 1991). It is possible that the anomalous behavior of the small ρ_2/ρ_1 data (e.g. Hall 1991) in Figure 5 may be due to an inaccurate

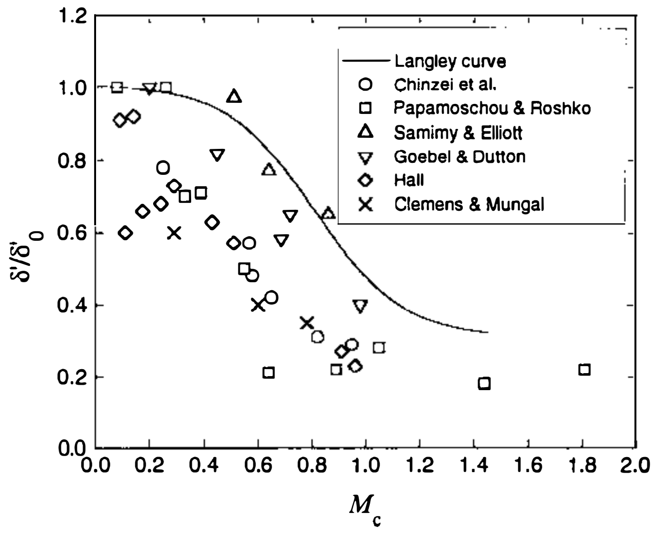

Figure 5 Normalized compressible mixing layer growth rates plotted against the convective Mach number M_c. Data assembled by D. Papamoschou. Different experiments employ different definitions for the mixing layer thickness and to compare them estimates of the factors relating one definition to another are needed. The factors recommended by the original authors were used when available.

extrapolation of the incompressible mixing layer growth rate data to extreme values of ρ_2/ρ_1. If it is assumed that the incompressible mixing layer growth rate is proportional to the maximum spatial amplification rate of a linearized disturbance, an alternate extrapolation of mixing layer growth rates to small ρ_2/ρ_1 can be plotted. With the new normalization the anomalous behavior is no longer evident (Lu & Lele 1993). Experimental data on low speed mixing layers with small ρ_2/ρ_1 are needed to satisfactorily resolve this issue. Data for $M_c \gg 1$ are also very limited; it is unclear if the limiting behavior suggested by Figure 5 is influenced by walls or possible pressure wave systems in the confining channel (Dimotakis 1991).

Turbulence statistics in a compressible mixing layer have been measured with hot wires (Barre et al 1992) and laser Doppler anenometers (LDA) (Elliott & Samimy 1990, Samimy & Elliott 1990, Goebel & Dutton 1991, Debisschop & Bonnet 1993). They exhibit a decrease of turbulent fluctuations (normalized with ΔU) with increasing M_c. Limited measurements of the third moments $\overline{v'(u'^2+v'^2)}/(\Delta U)^3$ and $\overline{v'^3}/(\Delta U)^3$ also show a decrease with M_c. The correlation coefficient $r_{uv} = \overline{u'v'}/u'_{\rm rms}v'_{\rm rms}$ appears to be unchanged from its incompressible value. The data on anisotropy $v'_{\rm rms}/u'_{\rm rms}$ have conflicting trends in different experiments. Elliott & Samimy (1990) and Debisschop & Bonnet (1993) claim that both $u'_{\rm rms}/\Delta U$ and $v'_{\rm rms}/\Delta U$ decrease with M_c, while Goebel & Dutton (1991) claim that only the v fluctuations diminish with M_c leading to increasing anisotropy at higher M_c. The peak values of $k\, [= \frac{1}{2}(\overline{u'^2}+\overline{v'^2}+\overline{w'^2})]$ in the three cases ($M_c = 0.51$, 0.64, 0.86) studied by M. Samimy (private communication, 1992) correspond to M_t values of 0.27, 0.32 to 0.43 (defined as $\sqrt{2k}/C_1$); the values relative to the low speed stream sound speed C_2 would be 0.22, 0.24, and 0.26. The deformation rate Mach number $\mathsf{M} = Sl/C = S_* M_t$ may be estimated to be in the range 2–3 for these flows where S_* is taken as 8. Measurements of density fluctuations in the range $0.3 \leq M_c \leq 0.9$ where a significant growth rate reduction occurs are not available. Measurements of scalar mixing are also limited. Clemens et al (1991) report a decrease in normalized scalar variance (unmixedness parameter) with increasing M_c but express caution in interpretation due to the limited signal-to-noise ratio and spatial resolution available in the experiment. Hall (1991) finds the opposite trend of a decrease in the fraction of mixed fluid with M_c in the compressible reacting mixing layers. Further experiments using "cold-chemistry" as well as a passive scalar to measure the molecular mixing (Clemens & Paul 1993) find that compressibility has only a small effect on mixing efficiency even though the turbulence structure visualized in the images change significantly with M_c.

The decrease of mixing layer growth rate with M_c is widely agreed upon;

but its cause is widely debated. Linear stability growth rates exhibit a similar reduction with M_c (Gropengeisser 1970, Ragab & Wu 1989, Jackson & Grosch 1989, Sandham & Reynolds 1990) which persists into the nonlinear phase (Sandham & Reynolds 1990, 1991; Lele 1989). Although the connection of the linear instability properties to the turbulent flow evolution (Morkovin 1992) is not mathematically rigorous, models based upon it (Planche & Reynolds 1992, Morris et al 1990, Tam & Hu 1990) do indeed explain some of the observations. Figure 6 (from Lu & Lele 1993) shows the correlation between the maximum spatial amplification rate and the mixing layer growth rates measured in different experiments. As in Figure 5, suitable scaling factors K are necessary to compare different experiments that use different definitions of mixing layer thickness. Acoustic radiation properties of supersonic jets have also been predicted using instability wave based models (Tam & Morris 1980, Seiner 1992). Recent observations of supersonic reacting mixing layers (Hall 1991, Miller et al 1993) have also been interpreted using linear stability analysis and nonlinear simulations (Planche & Reynolds 1992).

These approaches study the compressibility effects by examining inviscid energy exchanges. One alternative point of view centers on the dissipative

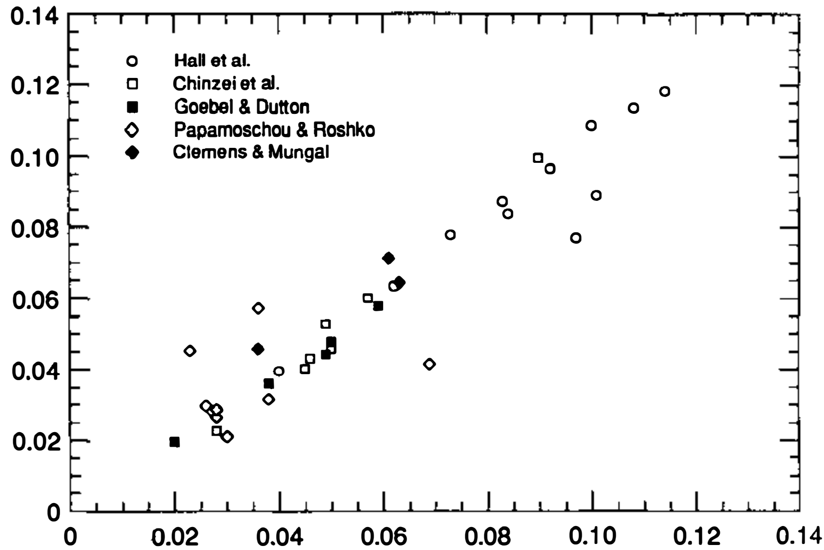

Figure 6 Comparison of the maximum amplification rate $|\alpha_i|_{max}/K$ with the growth rate δ'_{exp} measured by Hall et al (1991) ($K = 0.31$); Chinzei et al and Goebel & Dutton (1991) ($K = 0.51$); Papamoschou & Roshko (1988) and Clemens & Mungal (1992) ($K = 0.38$).

process (Zeman 1990, Sarkar et al 1991) and invokes additional compressible dissipation of TKE and pressure-dilatation. Another alternative is an interpretation in terms of the reduced communication between different (streamwise) zones of the flow (Papamoschou 1991b), a process necessary to establish the instability eigenfunctions or vortex interactions (Morkovin 1992). Model problems studied by Papamoschou & Lele (1993) have further supported this view. In the context of a turbulent flow, communication effects enter through terms involving pressure perturbations. A further observation is that numerical simulation of homogeneous shear flow (Blaisdell et al 1991) shows a preferential suppression of v'_{rms}, which in turn decreases $-\overline{u'v'}$, TKE and, for inhomogenous flow, turbulent momentum transport.

Laser sheet images (Clemens et al 1991, Clemens & Mungal 1992, Elliott et al 1992) show the instantaneous structures to be highly three dimensional in high M_c flows, with some tendency towards quasi two-dimensional structures when M_c is small. The lack of any noticeable two-dimensional structures for $M_c > 0.5$ is consistent with oblique wave linear instabilities being the most amplified waves at these M_c (Sandham & Reynolds 1990, 1991). No dominant oblique structures have been observed. Some difference between the high- and low-speed edges of the mixing layers have also been noted as M_c increases (Clemens & Paul 1993, Bonnet & Debisschop 1993).

The increased three-dimensionality is also evident in the two-point correlation measurements of static pressure: Samimy et al (1992) report a decrease of the spanwise coherence with M_c. The convection speed of large structures measured by double exposure photography (Papamoschou 1991a, Fourguette et al 1990, McIntyre & Settles 1991) by pressure records from transducers in tunnel walls (Hall et al 1991) have consistently differed from an isentropic theoretical estimate (Papamoschou & Roshko 1989) when $M_c > 0.5$. Two alternative views have been offered to reconcile these differences: an interpretation using linear stability theory (Jackson & Grosch 1989, Sandham & Reynolds 1990, Tam & Hu 1991) relates the discrepancy to fast and slow supersonic instability modes and possible obliquity of these waves; Papamoschou (1991a) and Dimotakis (1991) offer a different interpretation, proposing that total pressure losses due to eddy shocklets alter the convection velocity. Mach waves generated by supersonically moving disturbances have been widely reported but shocklets associated with large structures have not been observed. Numerical simulations of two-dimensional mixing layers (Lele 1989, Sandham & Reynolds 1990, Soetrisno et al 1989) display eddy shocklets for $M_c \geq 0.8$, but in three-dimensional simulations (Sandham & Reynolds 1991, Chen 1993) shocklets are not observed up to $M_c = 1$. This difference between two- and three-dimensional cases is not unexpected, as the critical M_c for shocklet formation in three-dimensional flows is expected to be higher than the critical M_c for two-dimensional flows. Whether shocklets occur in the highly three-dimensional flows at higher M_c is presently unknown.

A representative convection velocity is needed to determine the entrainment ratio in the mixing layer (Dimotakis 1991). Statistical estimates of the convection velocity U_c can be obtained from space-time correlations. Such techniques have been widely used in compressible boundary layer studies (Spina et al 1991) but space-time correlation data on compressible mixing layers are very limited. The convection velocity of the strong local extrema of pressure have been presented in the form of histograms (Samimy et al 1992); they show a significant spread in the U_c values and a dependence on the lateral coordinate. Near the free-stream edges the average U_c is closer to the free-stream speed, but in the middle of the shearing zone it is well approximated by the isentropic formula (for $\gamma_1 = \gamma_2$) $U_{c(\text{isentropic})} = (C_2U_1 + C_1U_2)/(C_1 + C_2)$. Measurements of the convection velocity of the large structures (Papamoschou 1991a and others, summarized by Dimotakis 1991) have differed consistently from the isentropic formula and have motivated suggestions of eddy structures with embedded shocks. Sufficient measurements are presently unavailable to judge if the overall entrainment ratio and chemical product formation are significantly influenced by the deviation of the convection speed from $U_{c(\text{isentropic})}$.

The influence of chemical heat release on compressible mixing layers and jets has not been well explored but is a subject of current studies. In low-speed studies (or incompressible flows) the primary effect of heat release is the displacement effect resulting from thermal expansion. The spreading rate of the mixing layer shows only a mild change and decreases with increasing heat release (Hermanson & Dimotakis 1989). The maximum shear stress is also observed to decrease with heat release with most of the decrease attributed to the lowered mean density. This is analogous to the density stratification effect common to most compressible boundary layers (Morkovin 1961). The entrainment ratio also changes with heat release. Dimotakis (1991) summarizes the effects of chemical kinetics (finite reaction rates), Reynolds number, and streamwise pressure gradient. Under compressible conditions the entrainment, stirring, and the subsequent heat release are intimately linked. The instability characteristics are qualitatively different from the nonreacting case: The heat release establishes local maxima in $\bar{\rho}(d\bar{U}/dy)$ which occur close to the free-stream edges (Planche & Reynolds 1992, Shin & Ferziger 1992). The most unstable disturbances are less oblique, dispersive (i.e. not subject to pairing/subharmonic instability), and appear to limit their growth by baroclinic vorticity generation and acoustic radiation (Planche & Reynolds 1992). The mixing in such a flow has been described in terms of two co-layers (Planche & Reynolds 1992). Current experimental evidence (Miller et al

1993, Hall 1991) is consistent with such an interpretation, but is too limited to draw any firm conclusions.

4.3 *Compressible Boundary Layers*

Bradshaw (1977), Morkovin (1961, 1992), and most recently Spina et al (1994) have reviewed the information on compressible turbulent boundary layers. A density gradient caused by the dissipative heating near the no-slip wall is the primary effect of increasing the mean flow Mach number. The importance of the dissipative heating is appreciated by noting that for boundary layers on adiabatic walls the ratio of wall temperature to the free-stream temperature rises from $T_w/T_\infty = 1.9$ at $M_\infty = 2.2$ to $T_w/T_\infty = 4.7$ at $M_1 = 4.5$ and is nearly 20 at $M_\infty = 10$. The variable mean density is largely responsible for a decreased skin-friction coefficient, smaller turbulence intensity, viscous effects, and for modifications to the incompressible law of the wall. [Due to the large variation in fluid properties a single Reynolds number is not sufficient to characterize the flow (Smits 1991).]

Accounting for the passive effects of the mean density variation via transformations of the low-speed analyses (Van Driest 1951, Coles 1964, Rotta 1960, Morkovin 1961) [summarized in Bradshaw (1977) and Fernholz & Finley (1977, 1980)] allows engineering prediction of the canonical adiabatic flat-plate boundary layers. For hypersonic, cooled boundary layers an additional modification is introduced. The variable density transformations are similar to the Howarth-Dordonitsyn transformation for compressible laminar boundary layers. Due to the closure problem their application to turbulent flow requires assumptions that are not obtained by first principles. The experimental data summarized by Fernholz & Finley (1980) show that such transformations are effective. The measurements of fluctuations are limited and accurate estimates of peak M_t or M for different M_∞ are currently not available. Spina et al (1994) provide a recent perspective on the available experimental data and Zeman (1993) addresses the modeling of compressibility effects inherent in them. One way to identify the variable inertia effects imbedded in compressible boundary layers is to contrast the high Mach number boundary layers with strongly heated, low-speed boundary layers (Cheng & Ng 1982, 1985; Thunker 1991). With a heated wall at 1100 K the primary effect, in the low-speed case, is the reduction of the Reynolds shear stress $-\overline{\rho u v}$ due to the change in $\bar{\rho}$ across the boundary layer (Cheng & Ng 1985) while the kinematic Reynolds stress $-\overline{uv}$ is close to the isothermal case. It is possible that the mean velocity profiles in these flows can be predicted using mean density-weighted transformations. Such comparisons should help in isolating the high-speed effects in compressible boundary layers.

An approximation called the Strong Reynolds Analogy (SRA) is introduced in the context of adiabatic flat-plate boundary layers (Gaviglio 1987, Bradshaw 1977, Morkovin 1961). SRA has been applied to other configurations (Barre et al 1992) and is used for turbulence modeling purposes (Rubesin 1990). The assumptions needed to derive SRA can only be validated by the experimental measurements of the flow under consideration. Gaviglio (1987) has examined these assumptions and proposed an extension. In Morkovin's proposal for SRA two assumptions are required: 1. negligible total temperature fluctuation and 2. negligible pressure fluctuation.

The following analysis is helpful in assessing the conditions under which these assumptions may apply. Consider small-disturbance inviscid equations for perturbations to a sheared mean flow, $[\bar{U}(y), 0, 0]$, with mean entropy variation due to TKE dissipation or heat transfer across the mean flow streamlines. If the disturbance velocity is nearly solenoidal the density and entropy fluctuations are expressible as $\rho' \sim -\eta(\partial\bar{\rho}/\partial y)$ and $s' \sim -\eta(\partial\bar{s}/\partial y)$, where η is a Lagrangian displacement. The thermodynamic relations (Section 2.7) connect the mean density and entropy gradients as

$$\frac{\partial \bar{s}}{\partial y}\Big/ C_{\rm v} \approx \frac{-\gamma}{\bar{\rho}}\frac{\partial\bar{\rho}}{\partial y}[1 + O(\rho'^2/\bar{\rho}^2) + O(M_t^2)]$$

which leads to

$$s'/C_{\rm v} = -\gamma\rho'/\bar{\rho} + O(\overline{\rho'^2}/\bar{\rho}^2) + O(M_t^2).$$

Since the equation of state provides

$$p'/\bar{p} = \gamma\rho'/\bar{\rho} + s'/C_{\rm v} + \dots$$

it follows that $p'/\bar{p} \ll \rho'/\bar{\rho}$. Thus in flows with small M_t, small mean streamline curvature, and small $\rho'_{\rm rms}/\bar{\rho}$, but with a significant mean entropy variation across the flow, a large negative correlation coefficient between s' and ρ', or between T' and ρ', is expected. If such a flow is subjected to some further distortion (extra strain rate) the pressure fluctuations in the distorted flow may no longer be negligible (as noted in Sections 2.2 and 3.3, bulk compression can generate significant pressure fluctuations).

The justification for neglecting the total temperature fluctuation in SRA is less clear. From the definition of total temperature as $C_{\rm p}T_0 = C_{\rm p}T + u_k u_k/2$ it follows that

$$T'_0/\bar{T} = T'/\bar{T} + \bar{U}u'_1/(C_{\rm p}\bar{T}) + [u'_k u'_k - \overline{u'_k u'_k}]/(2C_{\rm p}\bar{T}).$$

Estimating the streamwise velocity and temperature fluctuation (Gaviglio

1987) as $u'_1 \sim -\eta(\partial\bar{U}/\partial y)$ and $T' \sim -\eta(\partial\bar{T}/\partial y)$ (which ignore the fluctuations of the x-direction pressure gradient), it follows that $T'_0/\bar{T} \ll T'/\bar{T}$ if the "mean" total temperature, $\bar{T}+\bar{U}^2/(2C_p)$, is constant across the flow. Thus under the strict SRA assumptions $T'/\bar{T} \approx -\bar{U}u'_1/(C_p\bar{T})$; or, T' and u'_1 have a correlation coefficient R_{uT} close to -1. For a known variation of $\bar{T}_0$ across the flow this formulation predicts a value of R_{uT} (Gaviglio 1987) close to but not equal to -1. Compressible boundary layer measurements over adiabatic walls commonly show R_{uT} between -0.8 and -1, and the measured $\bar{T}_0$ variation is consistent with the values of R_{uT} or the extended SRA relation between T' and u'_1 (Gaviglio 1987). Conditions under which the variation of $\tilde{T}_0$ across the flow is small can be obtained from the equation governing $\tilde{T}_0$ and associated boundary conditions (see Lele 1993 for a discussion).

The similarity between the x-momentum transport and the transport of heat (passive scalar) added at the wall is almost perfect in an incompressible flow near a no-slip wall (Guezennec et al 1990, Kim & Moin 1989) and progressively becomes imperfect in regions away from the wall. Extending such studies to compressible flows may shed further light on the SRA relations. In flows subjected to sudden distortions or that have curved mean streamlines the acceleration of fluid elements cannot be ignored and neglecting total temperature fluctuation is inaccurate. Likewise in flows with high M_t or high density fluctuations SRA may break down (Bradshaw 1977).

5. CLOSURE

Some miscellaneous topics of fundamental importance that are not discussed in this article include: (*a*) combustion instability phenomena arising from the coupling between the unsteady compressible flow and chemical reactions, (*b*) the behavior of acoustic turbulence, e.g. the turbulent field dominated by a random distribution of shock waves (Porter et al 1992), (*c*) the spectral or two-point closure theories applied to compressible turbulence, and (*d*) the related topic of subgrid-scale models for compressible turbulent flows. Recent formulations of dynamic subgrid-scale models (Moin et al 1991) have suggested an approach for applying the customary parametrizations (Yoshizawa 1991, Erlebacher et al 1992, Zang et al 1992) to a wide variety of physical problems. If the subgrid models prove to be robust they will lead the way for future large eddy simulations of compressible turbulent flows.

At present measurements that allow the compressibility effects on turbulence to be isolated are very limited. The development of new measurement techniques combined with numerical simulations of compressible turbulent flows will continue to test our perspective. Faced with this data explosion, the efforts aimed at physical modeling of the compressible flow phenomena will be even more critical to advancing our knowledge.

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