A New Reynolds Stress Damping Function for Hybrid RANS/LES with an Evolved Functional Form

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A new methodology for finding hybrid RANS/LES damping functions is proposed. The Flow Simulation Methodology framework is modified from Speziale's original formulation [AIAA 36,173 1998] to allow for fewer grid points away from solid boundaries. The proposed functional form is regressed using an evolutionary algorithm. The solution is trained by considering DNS data of a separated flow. This sophisticated approach to building the damping function allows for the development of a hybrid methodology adaptable to any level of required modelling. The regressed model is compared to highly resolved reference LES on the classical two dimensional periodic hills case, for which the agreement is very good.

Keywords: Hybrid RANS/LES; FSM; Evolutionary Optimisation; Symbolic Regression.

1. Introduction

The cost of Large Eddy Simulation (LES) is too great for industrially relevant flows, whereas Reynolds-Averaging the Navier-Stokes (RANS) equations is too inaccurate in many cases. Instead Hybrid RANS/LES methods have appeared that aim to be cheaper turbulence resolving approaches by leaning on a RANS model in some way.

The Reynolds-averaging and low-pass spatial filtering of the (incompressible) Navier-Stokes equations form structurally similar transport equations,

$$\partial_t u_i + u_j \partial_{x_j} u_i = -\partial_{x_i} p + \nu \partial^2_{x_j x_j} u_i + \partial_{x_j} \tau_{ij}^{MOD} \tag{1}$$

where u_i is the resolved velocity. This similarity makes it possible to construct closures τ_{ij}^{MOD} that are a blend of both τ_{ij}^{RANS} and τ_{ij}^{LES}.

Flow Simulation Methodology (FSM) is an adaptable hybrid approach for which

$$\tau_{ij}^{MOD} = F(\Delta, ...) \cdot \tau_{ij}^{RANS}. \quad (2)$$

τ_{ij}^{RANS} models the turbulence of the flow field and the function F automatically damps this to ensure that the overall τ_{ij}^{MODEL} is representative of the local sub-grid scales (SGS). F is defined between 0 and 1, the DNS and RANS limits for total resolution or complete turbulence modelling. A non-linear stress-strain relation is often used for SGS anisotropy on coarse meshes. The damping function depends locally on the grid spacing Δ and also on some turbulent length scales in order to estimate local grid resolution. The success of a given hybrid closure crucially depends on the formulation of F, however it is not trivial to derive a functional form. Instead, authors have generally defined F on an *ad-hoc* basis, by either making assumptions or considering a certain philosophy. In the latter, we mean a hybrid approach has been formulated by *implicitly* defining F. In this case, no explicit thought is necessarily given to the RANS and DNS limiting regimes.

For example, Speziale[1] defined F as a function of the Kolmogorov microscale ℓ_k,

$$F = \left(1 - e^{-\beta \frac{\Delta}{\ell_k}}\right)^n, \quad (3)$$

where $\beta = 0.001$ and $n = 1$. These coefficients are set so F behaves well in the DNS limit. However as $\Delta/\ell_k >> 1$, it is unclear whether this ratio is a suitable parameter for the required level of modelling and worse the coefficients dictate such a slow response to lower mesh resolutions that the RANS limit, or even coarse LES regimes are unrecoverable. Rather FSM acts as a poorly calibrated implicit LES which lacks the required model dissipation to be numerically stable.

Alternatively, Spalart *et al.*[2] formulated a hybrid approach known as Detached-Eddy Simulation (DES) — which has become extremely popular and has met great success at predicting flows where the geometry determines the unsteadiness. The concept, which has recently been adopted by the developers of FSM,[3] is to damp the length scale ℓ_t in the RANS dissipation term,

$$D_{hyb} = \frac{k^{3/2}}{\ell_{hyb}}, \quad \ell_{hyb} = F(\Delta, ...) \cdot \ell_t. \quad (4)$$

By considering the Boussinesq approximation, Eq. 2 and Eq. 4 are almost identical in method and so damping functions are interchangeable between the two approaches. For DES, F is defined as,

$$F = \min(\frac{d}{\ell_t}, C_{DES}\frac{\Delta}{\ell_t}), \tag{5}$$

where the d is the wall distance and $C_{DES} = 0.65$. Away from solid boundaries, the value of C_{DES} makes the recovery of the unsteady RANS regime impossible. Consider that for this limit, $F \approx 1$ which implies $\Delta > \ell_t$. Therefore the model will provide too little dissipation. This is again another issue with constructing the damping function on an *ad-hoc* basis.

This paper outlines a more rigorous construction of a new damping function F. A data driven approach is taken to formulate F applicable to a wider range of flows. Whilst the data set used in this paper is not yet big enough, the methodology is introduced and a new damping function F is proposed, more as a proof of concept. Subsequent studies will expand on the data employed here, in order to produce a truly adaptable damping function.

2. Construction of the Damping Function

2.1. *Process*

The essential concept behind the new formulation methodology is to use a regression analysis, with large datasets representing idealised forms of F. To use a regression analysis, training data is required to fit potential solutions. Here DNS data is used to calculate the idealised damping function, then a functional form can be found via symbolic regression. To evaluate the idealised form, the DNS flow field must be made to represent a hybrid RANS/LES flow field. Hybrid RANS/LES flow fields are under-resolved due to the effective filter created by large grid scales Δ and time steps Δ_t. To mimic this, filters are applied to DNS snapshots removing the small scales. The large scales are those that would still be resolved by a hybrid model and the small scales removed by the filter are those that must be picked up by τ_{ij}^{MOD}.

This decomposition creates a hybrid flow field without having to perform an under resolved DNS. This is undesirable not just for time constraints but also for non-physical dissipation levels which would impact on the quality of the data. Filtered DNS (FDNS) also allows for the creation of a pseudo hybrid flow field without having to run any hybrid model. This is beneficial for several reasons. Firstly, FDNS is free from any model assumptions, reducing bias. Secondly, FDNS allows testing without having to repeatedly run hybrid simulations to assess potential candidates.

The filtering for the FDNS is performed locally, such that the strength of the filter depends on the grid size of the DNS. The number of points used in the filter is kept constant, so that the filter width varies. In other words the pseudo hybrid grid is still fine near solid boundaries and in coarser regions the filter is stronger. This is desired because it means that the size of the filter mimics a grid used in a hybrid simulation. The transfer function T is inspected for a given filter and the hybrid grid width is defined by the point where $T = 0.5$.

The ideal form of the damping function describes the amount of resolved over total stress, derived[4] from Eq. 2,

$$F = 1 - \frac{\tau_{ij}^{RANS} T_{ij}}{\tau_{mn}^{RANS} \tau_{mn}^{RANS}}, \tag{6}$$

where $T_{ij} = \langle u_i u_j \rangle - \langle u_i \rangle \langle u_j \rangle$ is the time averaged resolved stress. Due to this time average Eq. 6 is not suitable for actual CFD because of the increase in computational time. However, Eq. 6 can be evaluated for a range of FDNS datasets, using a series of snapshots to build a picture of the damping function for different flow fields.

Equation 6 is free from any explicit length scale dependence, allowing freedom in the choice of independent variables. These are calculated from the FDNS flow field. Only variables that are available to a RANS model at runtime are chosen to increase the portability of the resulting function F. The independent variables reflect those presently used in damping functions in the literature.[5] Also, the Taylor micro-scale is added, as this is the length scale of the smallest resolved eddies. It is easily calculated and has not been previously considered for hybrid closures of the type defined by Eq. 4. The length scales used in the regression are,

$$\begin{aligned} L_k &= \frac{\Delta}{\ell_k}, & \ell_k &= \left(\frac{\nu^3}{\varepsilon}\right)^{\frac{1}{4}} \quad \text{Kolmogorov length scale} \\ L_t &= \frac{\Delta}{\ell_t}, & \ell_t &= \frac{k^{\frac{3}{2}}}{\varepsilon} \quad \text{Integral length scale} \\ L_\lambda &= \frac{\Delta}{\ell_\lambda}, & \ell_\lambda &= \sqrt{\frac{10k\nu}{\varepsilon}} \quad \text{Taylor micro-scale} \end{aligned}$$

$$L_{TRRANS} = \max\left(\left(\frac{S}{1.25\Omega}\right), 1\right)^{-2}, \; L_{OES_S} = \frac{kS}{\varepsilon}, \qquad L_{OES_\Omega} = \frac{k\Omega}{\varepsilon} \; . \tag{7}$$

Note, $L_{TRRANS} = L_T$, $L_{OES_S} = L_S$ and $L_{OES_\Omega} = L_\Omega$ is used interchangeably. See, for example, the book[5] by Sagaut *et al.* for their original uses in TRRANS, OES, FSM, PANS and DES.

All six scales are non-dimensional, which means that the nondimensionality of F is trivially preserved. As a result the regression process can be handled by a state-of-the-art tool, without the need to enforce extra constraints. Gene Expression Programming[6] (GEP) is chosen for this study. GEP is an optimisation method which creates and evolves a population of functional forms in a mimicry of biological survival of the fittest. Using such a technique comes with the benefit of being able to treat the evolutionary process in an ensemble sense and thus ask deeper questions about the effectiveness of each of the variables listed in Eq. 7. For this study a look at the ensemble frequency, across the evolution of many disjoint populations, of each variable L_i is considered when deciding the optimal shape of F.

2.2. *Formulation using backward facing step flow data*

The flow behind a backward facing step is considered to formulate the shape of the function F. This flow presents a number of challenges for a locally formulated turbulence model, as the step induces non-local effects. That is for models with limited history effects, the propogation of the flow state from the step is challenging to capture correctly. The Reynolds number is $Re = 3,000$ based on step height. A previous study[7] looked at a simple attached flow. This study is aiming to look at the performance of the methodology using more complex training data. The reattachment position of the DNS is $x/h = 6.7$, where the step is located at $x/h = 0$. The plane $x/h = 6.5$, just before reattachment, is a region where the normal stress components are significant and $x/h = 10$ is a slice of the recovering boundary layer. These two locations present a reduction in the number of data-points to consider, yet still capture in essence the features of the flow field and therefore make good training data for the regression.

Figure 1 shows the plots of the length scales L_i for the FDNS at the locations $x/h = 6.5$ and $x/h = 10$ using fifty snapshots and spanwise averaging. Note L_k has been factored by 0.04 in order to fit conveniently into the plots. $y = 0$ corresponds to the lower surface and $y = 1$ is the height of the step.

Figure 1 also contains a plot of the idealised damping function at $x/h = 6.5$ and $x/h = 10$. This represents the training data for the regression. The echo of the shear layer can be seen — the location where F reaches its peak for the $x/h = 6.5$ profile. This peak has gone from the profile at $x/h = 10$ as the boundary layer has undergone some recovery.

Thirty independent runs of the evolutionary process, using both loca-

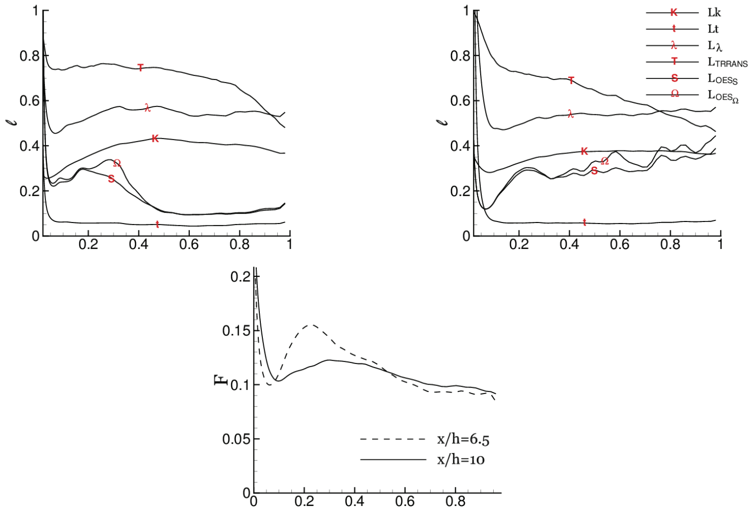

Fig. 1. Length scales for the backward facing step flow. Left: $x/h = 6.5$, Right: $x/h = 10$.

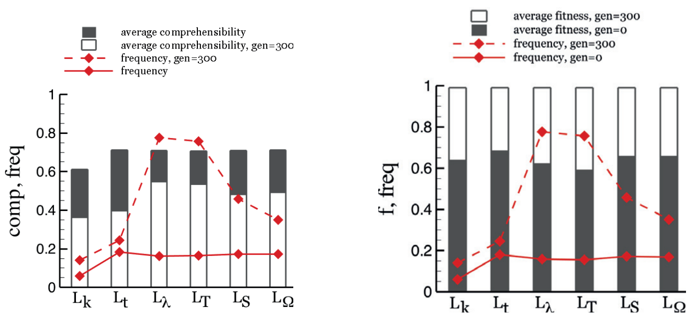

Fig. 2. Ensemble statistics for evolutionary system. Left: comprehensibility, Right: fitness.

tions as training data, are performed to gather ensemble statistics for the final state of the evolutionary system. During the evolutionary process, each candidate solution in the population is given a fitness and comprehensibility score bound between 0 and 1. The fitness is a measure of how closely an individual function matches the training data (1 is an exact correspondence), whilst the comprehensibility is a measure of how long its mathematical expression is (1 is a function consisting of just one mathematical symbol). The top 2% of the population based on fitness, is plotted in Fig. 2. Bars correspond to either the fitness or comprehensibility, given that a function contains L_i. L_λ and L_T are the most frequent and produce the simplest functions (not mutually exclusive phenomenon). Evident from the average fitness after 300 generations, the algorithm is capable of finding good functional forms for F from any of the variables L_i. However, good solutions containing L_k, L_t, L_S and L_Ω are much rarer, hinting at an underlying complexity required in the functional form.

The simplicity and major preference of the variables L_λ and L_T are the driving forces behind an optimisation including only these variables. An optimal solution from evolutionary runs including only L_T and L_λ is,

$$F = CL_\lambda L_T = C\frac{\Delta}{\ell_\lambda}\max\left[\left(\frac{S}{1.25\Omega}\right), 1\right]^{-2}. \tag{8}$$

This is not the final form every time, rather a function is produced that easily simplifies to this form by cancelling negligible terms.

2.3. *Implementation to form a hybrid closure*

In order to turn Eq. 8 into a fully working hybrid model, it must be modified to ensure proper boundedness between 0 and 1. The lower bound is trivially maintained, but to ensure $F \leq 1$, Eq. 8 is modified,

$$F = C\min\left(\frac{\Delta}{\ell_\lambda}, 1.0\right)\max\left[\left(\frac{S}{1.25\Omega}\right), 1\right]^{-2}. \tag{9}$$

Before the calibration of the coefficient, near wall treatment[3] is added,

$$F = \min\left(\frac{\tilde{F}}{1-f_b}, 1\right), \tag{10}$$

where $\tilde{F}$ is the damping function Eq. 9. f_b is the damping function from the SST turbulence model[8] designed to be 0 at the edge of the boundary layer and 1 at the wall. This achieves RANS at the wall and LES outside of the boundary layer. This is vital to the success of the hybrid simulation, not only for near wall cost requirements, but also for predictive performance.[5] The underlying RANS model chosen is the explicit algebraic stress model (EASM) of Speziale, Sarkar and Gatski[9] (SSG). The coefficient was calibrated using a channel flow of friction Reynolds number 550. By matching the shear stress component the coefficient is found to be $C = 0.8$.

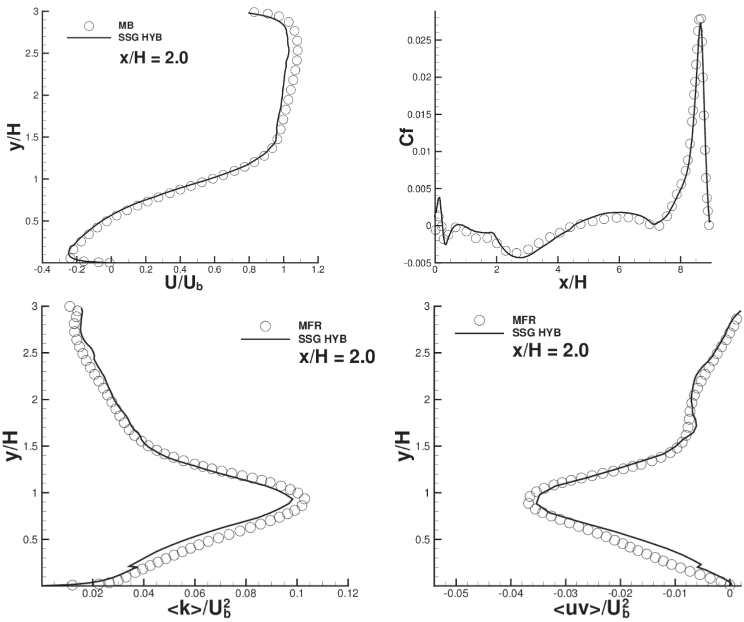

Fig. 3. 2D Periodic Hills. Top left: Mean streamwise velocity. Top right: Friction Coefficient. Bottom left: Total turbulent kinetic energy. Bottom right: Total shear stress. Symbols denote reference LES data.[10,11]

3. Application to Periodic Hills

The new closure has been tested on the classical periodic hills geometry. This is a well studied case and is a good validation study for new turbulence closures. In Fig. 3, results on a relatively coarse grid of 800,000 cells, are shown against two highly resolved LES datasets (MB)[10] and (MFR)[11] which agree well with experimental values.[10] The location $x/h = 2.0$ is particularly challenging as this profile crosses the reverse flow region, free shear layer and the bulk flow. In general the hybrid methodology with the new damping function is in excellent agreement with the reference data for every variable.

One cause for concern is the kink present in the second order statistics in the lower surface boundary layer. This could be caused by an unforeseen interaction with the shielding function which effectively forces a boundary condition on F. This should be investigated and incorporated into the regression process.

4. Conclusion and Outlook

A new damping function for hybrid RANS/LES built using a novel approach for turbulence modelling has been presented and applied to the well known periodic hills test case, with good agreement to the reference data.

This method can be improved by simultaneously considering more datasets that include complex flow structures with differing order filters so that a damping function suitable for a wide range of flow problems can be found. Further, specialised damping functions can be generated for flow problems that contain a dominant feature such as transitional, massively separated or turbo-machinery flows.

References

1. C. G. Speziale, Turbulence modeling for time-dependent RANS and VLES: A review, *AIAA J.* **36**, 173 (1998).
2. P. R. Spalart, W.-H. Jou, M. Strelets and S. R. Allmaras, Comments on the feasibility of LES for wings, and on a Hybrid RANS/LES approach, in *Advances in DNS/LES*, eds. C. Liu and Z. Liu, Gayden Press (1997).
3. M. Weinmann, R. D. Sandberg and C. Doolan, Tandem cylinder flow and noise predictions using a hybrid RANS/LES approach, *International J. Heat and Fluid Flow* **50**, 263 (2014).
4. M. Germano, Comment on "Turbulence Modeling for Time-dependent RANS and VLES: A Review", *AIAA J.* **36** (1998).
5. P. Sagaut, S. Deck and M. Terracol, *Multiscale and multiresolution approaches in turbulence* World Scientific (2013).
6. C. Ferreira, Gene Expression Programming: A New Adaptive Algorithm for Solving Problems, *Complex Systems* **13**, 87 (2001).
7. J. Weatheritt and R. D. Sandberg, Use of Symbolic Regression for construction of Reynolds-stress damping functions for Hybrid RANS/LES, in *53rd AIAA Aerospace Sciences Meeting*, AIAA SciTech, American Institute of Aeronautics and Astronautics, January (2015).
8. F. R. Menter, Zonal Two Equation k-ω Turbulence Models for Aerodynamic Flows, *AIAA*-93-2906 (1993).
9. C. G. Speziale, S. Sarkar and T. B. Gatski, Modelling the pressure strain correlation of turbulence: an invariant dynamical systems approach, *J. Fluid Mech.* **227**, 245 (1991).
10. M. Breuer, N. Peller, C. Rapp and M. Manhart, Flow over periodic hills - Numerical and experimental study in a wide range of Reynolds numbers, *Comput. and Fluids* **38**, 433 (2009).

11. J. Fröhlich, C. Mellen, W. Rodi, L. Temmerman and M. Leschziner, Highly resolved large-eddy simulation of separated flow in a channel with streamwise periodic constrictions, *J. Fluid Mech.* **526**, 16 (2005).