

HYPPO

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1 Introduction

hyppo is an implicit coupled supersonic/hypersonic solver based on **foam-extend-4.1** and **Mutation++**. It computes the density ρ , the velocity ρU , the energy $\rho U E$ from Navier-Sokes equations using an implicit Rusanov scheme. The temperature, the pressure and the gas properties are obtained with different model explained further. A few test cases are available as tutorials.

2 Navier stokes equations

The solver can not be used to computes supersonic/hypersonic flows in rarefied regime. Instead it uses Navier-Stokes equation to solve the problem. The equations are discretized following the finite volume method.

2.1 Euler and Rusanov

The flux are obtained with a Rusanov scheme. By noting I the identity matrix, the Euler equations are written (1)

$$\frac{\partial U}{\partial t} + \nabla \cdot F(U) = 0 \quad (1)$$

where

$$\begin{cases} U &= (\rho; \rho \vec{V}; E) \\ F(U) &= (\rho \vec{V}; \rho \vec{V} \otimes \vec{V} + pI; (E + p)\vec{V}) \end{cases} \quad (2)$$

The Rusanov scheme (3) is a simple and robust scheme

$$\begin{cases} \frac{U_i^{n+1} - U_i^n}{\Delta t} + \frac{F_{i+\frac{1}{2}} - F_{i-\frac{1}{2}}}{\Delta x} = 0 \\ F_{i+\frac{1}{2}} = \frac{1}{2} \left(F(U_{i+1}) + F(U_i) - \beta_{i+\frac{1}{2}}(U_{i+1} - U_i) \right) \\ \beta_{i+\frac{1}{2}} = \max \{ |u_i| + c_i, |u_{i+1}| + c_{i+1} \} \end{cases} \quad (3)$$

The CFL is calculated thanks to $\beta_{i+0.5} \frac{\Delta t}{\Delta x} \leq C_{max}$. hyppo is an implicit solver, therefore a limited development of order 1 is performed on the function $F(U^{n+1})$ thus involving the calculation of a Jacobian by approximating the derivative of $F(U)$. This can be find in **numerics/rusanovFlux.H** [1].

Then, the equations are assembled in the folder **Eqns/** and assembled in a matrix solved with GMRES and a Cholesky preconditioner.

2.2 Viscous and diffusive terms

Viscous and diffusive terms are also taken into account. We therefore add the terms $div(\tau)$ and $div(\tau \vec{V} + q)$ respectively on $\rho \vec{V}$ and E to the right of the Navier-Stokes equations. These new members are also implicit and discretized using a centered scheme in **numerics/CenteredFluxViscosity.H**. The heat flux is given by the Fourier relation (4) and the viscosity tensor is defined by (5).

$$q = \lambda \nabla T \quad (4)$$

$$\tau = \mu \left(\nabla \vec{V} + (\nabla \vec{V})^T \right) - \frac{2}{3} \mu div(\vec{V}) I \quad (5)$$

The addition of these terms involves the calculation of velocity and temperature gradients. In order to overcome the possible non-orthogonality of the mesh, a diamond scheme [2] is implemented fig.1.

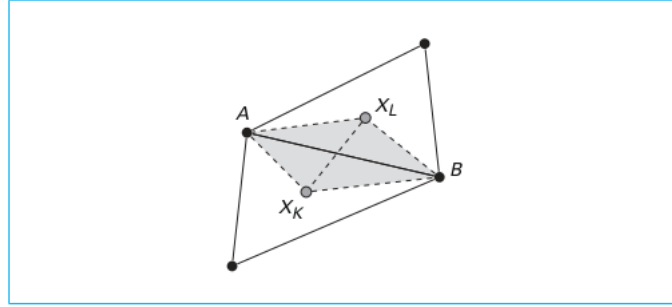


Figure 1: Principle of the 2D diamond scheme | R.Turpault

On the diagram 1, the gradient is known in two directions with :

$$\begin{cases} \nabla U.AB = U_B - U_A \\ \nabla U.KL = U_L - U_K \end{cases} \quad (6)$$

By solving the matrix system (7) we can find the gradient defined at the interface of the two cells.

$$\begin{pmatrix} x_L - x_K & y_L - y_K \\ x_B - x_A & y_B - y_A \end{pmatrix} \nabla U = \begin{pmatrix} U_L - U_K \\ U_B - U_A \end{pmatrix} \quad (7)$$

3 Simulation

This part presents the different options you can choose to solve your supersonic/hypersonic problem. Note that the constant properties and Sutherland model provide just a supersonic solver due to the assumptions of ideal gas. If you want a hypersonic solver you have to choose a real gas model. These options are available in the file **system/transportProperties**

3.1 Courant number

As the solver scheme is implicit we can increase the Courant number in order to solve the problem rapidly. The CFL is the same as the start during the first 100 iterations. After this point the CFL is increased of 20% each 10 iterations until the maximum Courant number specified by the user. The solver does not deal with high values. If the time step is too high and the energy become wrong so the solver decrease the time step until the iteration goes well.

3.2 Ideal gas

In this section the ideal gas models are described. The solver can deal with constant properties or can use a viscosity model in order to compute the viscosity as a function of the temperature.

3.2.1 Constant properties

The ideal gas model computes the supersonic flow with constant properties (viscosity, conductivity, heat capacities). To change these values you have to do it manually in the file **/constant/constantProperties**. Note that $C_v = \frac{C_p}{\gamma}$ and γ and $\lambda = \frac{C_p \mu}{Pr}$. Moreover, in this case, $T = \frac{e}{C_v}$ where e is the internal energy and the pressure is computed thanks to the perfect gas law.

Variable	Unit	Value
C_p	$J.K^{-1}$	1004
γ	-	1.4
μ	$Pa.s^{-1}$	1.85×10^{-5}
Pr	-	1

Table 1: Default values

3.2.2 Sutherland model

The Sutherland model gives an approximation of the viscosity as a function of the temperature following the next equation.

$$\mu(T) = \mu_{ref} \left(\frac{T}{T_{ref}} \right)^{3/2} \frac{T_{ref} + S}{T + S} \quad (8)$$

with $S = 110.4K$ $T_{ref} = 273.15K$ $\mu_{ref} = 1.715 \times 10^{-5} Pa.s^{-1}$.

Theoretically this model gives a good approximation from 170K to 1500K. Note that λ is still a function of μ . This model does not change the assumption of ideal gas so we still can compute the temperature as follow $T = \frac{e}{C_v}$.

3.3 Real gas

In this section the real gas models are described. The solver can read table in order to consider equilibrium chemistry. The format of tables is available in appendix. It is possible to use directly a finite rate model to compute the composition of the mixture. Note that this model is being implemented.

3.3.1 Equilibrium chemistry

This model takes into account the dependence of the heat capacities with the temperature and so does not consider a ideal gas anymore. The solver reads thermodynamic tables computed from Mutation++ [3] (von Karman Institute). With the density and the internal energy M++ gives the temperature, the pressure, the properties of the gas and even the specie composition of the gas.

The air models used are air5 (O_2 O N_2 N NO) or air11 (e- N N^+ O O^+ NO N_2 N_2^+ O^2 O^2^+ NO^+). To choose the model the user has just to specify the name of the file : air5.txt or air11.txt.

Values are interpolated from the table which is already generated in the correct location. If you want to generate your own table go to see the file **src/thirdParty/mutation++/examples/c++/air_equilibrium.cpp** or the appendix.

Note that this method is very dependent on the table and it requires to generate a fitted table to get correct results and so know the ranges for ρ , P and T .

3.3.2 Finite rate chemistry

Instead of the previous model, the finite rate model consider a reaction velocity for the chemical reactions. It is more accurate but adds stiffness. In order to compute and get correct results, the equations (9) have to be solved in an fully implicit and coupled way. Once the densities computed, the temperature, the pressure and the gas properties are updated with M++ thanks to densities and the energy of the mixture.

$$\frac{\partial \rho_s}{\partial t} + \nabla \cdot (\rho_s U) + \nabla (D_s \nabla \rho_s) = \dot{\omega}_s \quad (9)$$

To transform the equation fully implicit, the specie production rates have to be implicit.

$$\dot{\omega}^{n+1} = \dot{\omega}^n + J(\rho^n)(\rho^{n+1} - \rho^n) \quad (10)$$

where $\rho = (\rho_1, \dots, \rho_n)^T$ and J the jacobian matrix.

$$J(\rho^n) = \begin{pmatrix} \frac{\partial \dot{\omega}_1}{\partial \rho_1} & \frac{\partial \dot{\omega}_1}{\partial \rho_2} & \dots & \frac{\partial \dot{\omega}_1}{\partial \rho_n} \\ \frac{\partial \dot{\omega}_2}{\partial \rho_1} & \dots & \dots & \frac{\partial \dot{\omega}_2}{\partial \rho_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial \dot{\omega}_n}{\partial \rho_1} & \dots & \dots & \frac{\partial \dot{\omega}_n}{\partial \rho_n} \end{pmatrix}$$

Note that the jacobian is homogenous of degree zero, wich means : $J(\lambda U) = J(u)$ with $\lambda \in \mathbb{R}_+^*$.

3.4 Turbulence model

This option allow you to compute your supersonic/hypersonic flow taking into account turbulence using Baldwin-Lomax model [4]. This model compute the turbulent viscosity with the following equation.

$$\mu_t = \begin{cases} \rho l^2 |\omega| & y \leq y_{cross} \\ KC_{CP} \rho F_{WAKE} F_{KLEB} & y_{cross} < y \end{cases} \quad (11)$$

$$\left\{ \begin{array}{ll} F_{WAKE} & = \min \left(y_{max} F_{max}; C_{wk} y_{max} \frac{u_{DIF}^2}{F_{max}} \right) \\ F(y) & = |\omega| y \left(1 - \exp\left(\frac{-y^+}{A^+}\right) \right) \\ F_{KLEB}(y) & = \left(1 - 5.5 \left(\frac{y C_{KLEB}}{y_{max}} \right)^6 \right)^{-1} \\ u_{DIF} & = \max(\sqrt{u_i u_i}) - \min(\sqrt{u_i u_i}) \\ K & = 0.0168 \\ C_{cp} & = 1.6 \\ C_{wk} & = 0.25 \\ C_{KLEB} & = 0.3 \end{array} \right. \quad (12)$$

The introduction of a bug in foam-extend 4.1 makes it unusable in many cases. The cellDistFuncs.C file in foam-extend 4.1 must be replaced by the one in version 4.0.

The Baldwin Lomax model also need a good refinement at wall with y^+ value equal to 1 in the first cell. For the moment this model is not parallelizable and has boundary layer detection problems. In order to overcome the problems of boundary layer detection made by enthalpy variation, this model is only usable for simulations in front of a body.

In order to avoid catching the shock as a boundary layer, the user defines a time after which the turbulence is activated and the distance in meters that should not be exceeded to avoid catching the shock.

Note that this model needs to be improved.

4 Tutorials

The following table shows the different tutorials. Note that the test cases was not computed in parallel. Some pictures are available in appendix to show what the user is supposed to obtain.

File name	Mach	Altitude	Model	Running time
halfSpherePG	10	20km	Ideal gas	15s
halfSphereEquil	10	20km	Equilibrium air5	1min
halfSphereMach50	56	50km	Equilibrium air11	6min

Table 2: Test cases

4.1 Run a tutorial

To run a tutorial of hyppo you need to install **foam-extend-4.1** , **OpenFOAM-7** and **PATO**. If you have already these softwares you can follow the following instructions.

How to run a tutorial ?

- Open a terminal in the case folder
- Source 'YOUR_PATH/openfoam7/etc/bashrc'
- blockMesh
- Open a new terminal in the same location
- Source 'YOUR_PATH/foam-extend-4.1/etc/bashrc'
- Source 'YOUR_PATH/PATO/PATO-dev-2.3.1/bashrc'
- hyppo

References

- [1] *Développement d'un solveur compressible vectoriel dans OpenFOAM pour les applications aérospatiales*, A.Figuin, 2019.
- [2] *Des schémas numériques pour la mécanique des fluides compressibles*, R.Turpault, 2020.
- [3] *Développement de méthodes numériques et étude des phénomènes couplés d'écoulement, de rayonnement, et d'ablation dans les problèmes d'entrée atmosphérique*, James Scoggins, 2017.
- [4] *Thin Layer Approximation and Algebraic Model for Separated Turbulent Flows*, B.S. Baldwin and H.Lomax, 1978.

A Table air5

1	rho	rhoe[J/m3]	e[J/kg]	c[m/s]	T[K]	P[Pa]	Cp[J/kg-K]	Cv[J/kg-K]
2	0.01	-2750	-275000	120.227	35.8256	103.247	1008.68	720.483
3	0.01	-2350	-235000	191.975	91.3439	263.247	1008.68	720.483
4	0.01	-1950	-195000	243.422	146.862	423.247	1008.68	720.486
5	0.01	-1550	-155000	285.745	202.378	583.24	1008.78	720.586
6	0.01	-1150	-115000	322.507	257.869	743.161	1009.44	721.246
7	0.01	-750	-75000	355.325	313.263	902.804	1011.42	723.222
8	0.01	-350	-35000	385.049	368.435	1061.81	1015.35	727.159
9	0.01	50	5000	412.195	423.226	1219.71	1021.52	733.328
10	0.01	450	45000	437.129	477.478	1376.06	1029.79	741.596
11	0.01	850	85000	460.151	531.066	1530.5	1039.74	751.549
12	0.01	1250	125000	481.525	583.905	1682.77	1050.85	762.658
13	0.01	1650	165000	501.482	635.955	1832.78	1062.6	774.411
14	0.01	2050	205000	520.222	687.212	1980.5	1074.58	786.383
15	0.01	2450	245000	537.916	737.697	2125.99	1086.45	798.257
16	0.01	2850	285000	554.704	787.444	2269.36	1098.02	809.822
17	0.01	3250	325000	570.703	836.499	2410.73	1109.14	820.95
18	0.01	3650	365000	586.009	884.908	2550.25	1119.77	831.578
19	0.01	4050	405000	600.7	932.717	2688.03	1129.88	841.69
20	0.01	4450	445000	614.84	979.969	2824.2	1139.5	851.303
mu[kg/m-K]	L[W.m-1.K-1]	Y_N	Y_0	Y_N0	Y_N2	Y_02		
6.71234e-06	0.00984287	0	0	3.02131e-133	0.767082	0.232918		
1.07181e-05	0.0156895	1.33324e-266	4.15468e-139	1.59179e-52	0.767082	0.232918		
1.35904e-05	0.0198857	7.98067e-165	1.39655e-85	7.81916e-33	0.767082	0.232918		
1.59536e-05	0.0233414	7.06258e-119	2.06482e-61	6.03225e-24	0.767082	0.232918		
1.80085e-05	0.026362	1.02691e-92	1.20348e-47	6.91387e-19	0.767082	0.232918		
1.99659e-05	0.0292804	7.73008e-76	9.16296e-39	1.26596e-15	0.767082	0.232918		
2.21979e-05	0.0326705	4.52458e-64	1.43208e-32	2.38078e-13	0.767082	0.232918		
2.44012e-05	0.0361159	2.02978e-55	5.1367e-28	1.11783e-11	0.767082	0.232918		
2.65933e-05	0.0396552	8.30684e-49	1.55554e-24	2.11598e-10	0.767082	0.232918		
2.87706e-05	0.0432878	1.33451e-43	8.56893e-22	2.14171e-09	0.767082	0.232918		
3.09442e-05	0.0470213	2.10739e-39	1.39168e-19	1.38439e-08	0.767082	0.232918		
3.28547e-05	0.050448	5.99827e-36	9.16833e-18	6.42609e-08	0.767082	0.232917		
3.46169e-05	0.0537183	4.66426e-33	3.0496e-16	2.3215e-07	0.767082	0.232917		
3.63527e-05	0.057	1.32935e-30	5.97844e-15	6.90881e-07	0.767082	0.232917		
3.80684e-05	0.0602903	1.718e-28	7.72672e-14	1.76491e-06	0.767082	0.232917		
3.97696e-05	0.063588	1.17839e-26	7.1526e-13	3.98966e-06	0.767081	0.232915		
4.14612e-05	0.066894	4.82987e-25	5.04799e-12	8.16574e-06	0.767079	0.232913		
4.31477e-05	0.070211	1.29538e-23	2.84928e-11	1.53999e-05	0.767075	0.232909		
4.48332e-05	0.0735429	2.43972e-22	1.33496e-10	2.71285e-05	0.76707	0.232903		

Figure 2: Examples of table air5

B Table air11

1	rho	rhoe[J/m3]	e[J/kg]	c[m/s]	T[K]	P[Pa]	Cp[J/kg-K]	Cv[J/kg-K]	mu[kg/m-K]	L[W.m-1.K-1]	
2	0.0001	-25	-250000	168.685	70.5246	2.03247	1008.68	720.483	9.41776e-06	0.0137906	
3	0.0001	25	250000	540.061	743.955	21.4403	1087.92	799.723	3.65682e-05	0.0574109	
4	0.0001	75	750000	708.798	1325.28	38.1938	1202.17	913.929	5.4355e-05	0.0939371	
5	0.0001	125	1.25e+06	809.456	1821.95	52.588	1532.27	1228.87	6.7489e-05	0.163487	
6	0.0001	175	1.75e+06	843.675	2109.77	61.6337	2942.22	2531.07	7.49531e-05	0.421152	
7	0.0001	225	2.25e+06	871.739	2264.13	67.4426	4615.86	4035.83	7.88964e-05	0.728666	
8	0.0001	275	2.75e+06	898.52	2371.77	72.208	5996.04	5242.36	8.19029e-05	0.981852	
9	0.0001	325	3.25e+06	924.7	2460.13	76.6057	6907.83	6014.22	8.45212e-05	1.14738	
10	0.0001	375	3.75e+06	951.278	2540.92	80.928	7243.14	6275.14	8.69842e-05	1.20752	
11	0.0001	425	4.25e+06	979.64	2622	85.385	6913.45	5960.41	8.94526e-05	1.14966	
12	0.0001	475	4.75e+06	1012.4	2712.37	90.2393	5868.32	5027.98	9.21202e-05	0.96809	
13	0.0001	525	5.25e+06	1056.28	2829.29	96.0174	4188.94	3537.75	9.5368e-05	0.678673	
14	0.0001	575	5.75e+06	1130.27	3019.57	104.146	2405.36	1944.54	0.00010023	0.37245	
15	0.0001	625	6.25e+06	1219.41	3347.6	116.473	1768.64	1381.35	0.000100087	0.271449	
16	0.0001	675	6.75e+06	1248.95	3665.37	128.287	2328.44	1907.91	0.000115585	0.396945	
17	0.0001	725	7.25e+06	1264.31	3880.3	136.855	3340.82	2840.7	0.000120686	0.617013	
18	0.0001	775	7.75e+06	1280.36	4030.36	143.434	4489.1	3884.97	0.000124175	0.867922	
19	0.0001	825	8.25e+06	1296.65	4144.16	148.927	5667.97	4943.71	0.000126502	1.1237	
20	0.0001	875	8.75e+06	1312.65	4235.92	153.775	6839.94	5983.45	0.000128385	1.37716	
	Y_e-	Y_N	Y_N+	Y_O	Y_O+	Y_NO	Y_N2	Y_N2+	Y_O2	Y_O2+	Y_NO+
	0	0	0	6.7812e-180	0	6.67989e-68	0.767082	0	0.232918	0	6.59948e-217
1.74895e-49	2.5395e-29	8.61447e-79	8.40558e-14	1.16776e-57	7.82773e-07	0.767082	2.28808e-57	0.232917	2.02159e-34	1.59235e-21	
3.52969e-26	8.81111e-15	5.84449e-44	3.74074e-06	1.72467e-32	0.000499111	0.76685	5.39663e-34	0.232648	1.73894e-21	3.74003e-14	
3.76484e-17	1.03754e-09	1.24549e-32	0.00170219	1.53367e-24	0.00470939	0.764884	1.1627e-26	0.228704	8.27577e-18	2.05392e-12	
3.44523e-15	7.24168e-08	3.64709e-27	0.0151702	2.55028e-20	0.0102871	0.76228	1.20177e-22	0.212262	3.71658e-15	1.87953e-10	
2.37909e-14	4.51851e-07	8.56584e-25	0.0372573	1.66185e-18	0.0137876	0.760646	6.67842e-21	0.188309	4.88116e-14	1.29788e-09	
7.75197e-14	1.40621e-06	2.58515e-23	0.0625539	2.1781e-17	0.0159087	0.759655	8.25141e-20	0.161881	2.27997e-13	4.22891e-09	
1.8609e-13	3.31478e-06	3.45475e-22	0.089267	1.49576e-16	0.0171084	0.759093	5.62058e-19	0.134528	6.93283e-13	1.01516e-08	
3.86276e-13	6.89113e-06	3.21745e-21	0.116628	7.61162e-16	0.0175646	0.758877	2.9469e-18	0.106924	1.69841e-12	2.10719e-08	
7.53859e-13	1.37279e-05	2.68549e-20	0.144135	3.45484e-15	0.0173067	0.75899	1.4329e-17	0.0795548	3.72243e-12	4.11237e-08	
1.48032e-12	2.81941e-05	2.52416e-19	0.171236	1.63701e-14	0.0162374	0.759475	7.66709e-17	0.0530231	7.85788e-12	8.07521e-08	
3.21954e-12	6.68165e-05	3.83496e-18	0.196877	1.02312e-13	0.014075	0.760445	5.93468e-16	0.028536	1.74314e-11	1.75627e-07	
9.5536e-12	0.000235754	2.15801e-16	0.217988	1.41953e-12	0.0103215	0.762028	1.24873e-14	0.0094253	4.81049e-11	5.21154e-07	
4.45615e-11	0.00147643	8.09133e-14	0.228216	6.26046e-11	0.00593304	0.762835	1.12184e-12	0.00153682	1.82684e-10	2.43079e-06	
1.51166e-10	0.00635607	9.31027e-12	0.23059	1.27698e-09	0.00371069	0.75899	4.14098e-11	0.000344924	5.14619e-10	8.24399e-06	
3.08365e-10	0.014832	1.48657e-10	0.231269	7.43939e-09	0.00280445	0.750934	3.40374e-10	0.000143756	9.42971e-10	1.68075e-05	
4.84762e-10	0.0253087	8.62652e-10	0.231574	2.28308e-08	0.00233894	0.74067	1.29312e-09	8.24774e-05	1.38858e-09	2.63991e-05	
6.68031e-10	0.0368768	3.00302e-09	0.231749	5.07015e-08	0.00205013	0.729232	3.32524e-09	5.56072e-05	1.83107e-09	3.6338e-05	
8.53895e-10	0.0491025	7.81e-09	0.231866	9.36235e-08	0.00184854	0.717095	6.84164e-09	4.11041e-05	2.26705e-09	4.6383e-05	

Figure 3: Examples of table air11

C Tutorials

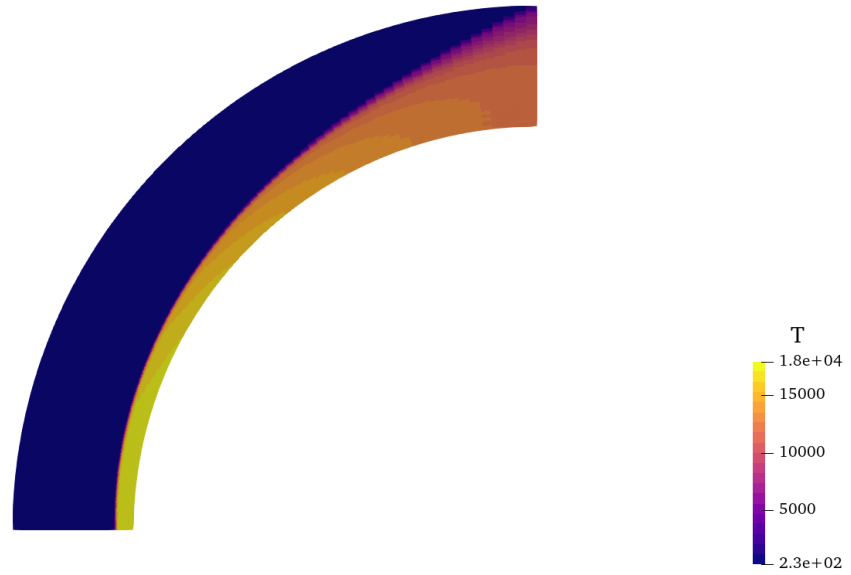


Figure 4: Temperature at Mach 56 and 50km

D Solver development

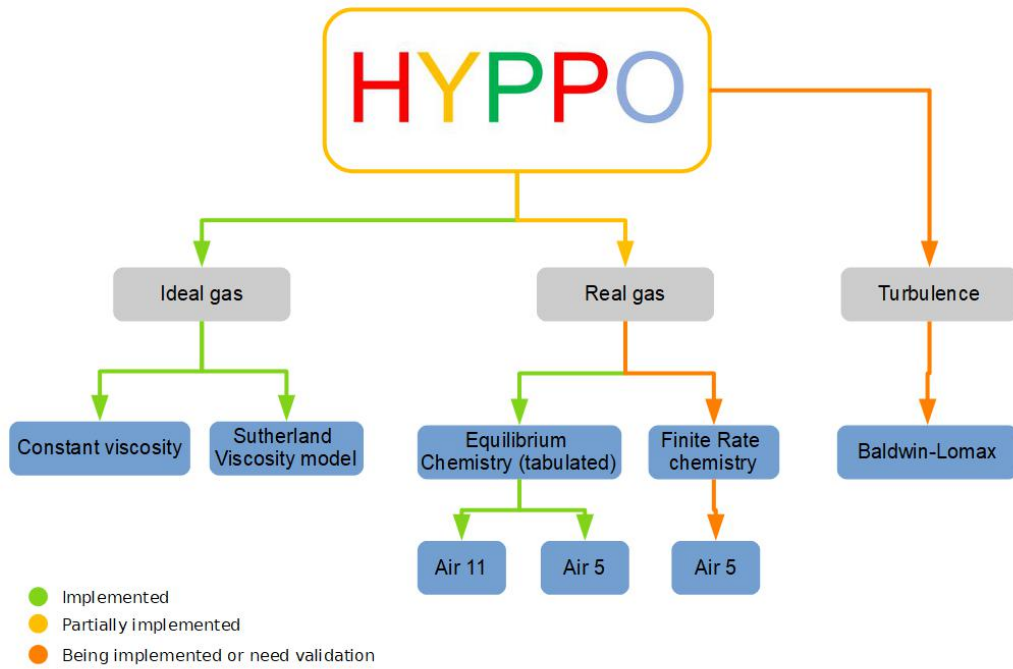


Figure 5: hyppo development scheme