IV

Theory of Shock Waves

14

On the Possibility of Rarefaction Shock Waves*

A shock wave is the surface of a sudden very sharp variation in the motion and state (pressure, density, etc.) of a material, which moves with respect to this material.

The relations between the state of the material before the wave passes (I), the properties of the wave, and the state after the wave passes (II) are easily obtained from the laws of conservation of mass, momentum, and energy. These relations are symmetrical with respect to the quantities describing I and II, and are equally suited to description of the transition from I to II and the reverse transition from II to I.

Zemplen [1], considering an ideal gas with constant specific heat, showed that entropy is not conserved in a shock wave: it rises with increased pressure and falls with decreased pressure. From this follows the so-called Zemplen theorem of the impossibility of rarefaction shock waves.

In fact, just one year earlier Jouguet [2] gave an expression for the entropy change in a small-amplitude shock wave,

$$S_{\mathrm{II}} - S_{\mathrm{I}} = \frac{1}{12}\frac{1}{T}\left(\frac{\partial^2 v}{\partial p^2}\right)_S (p_{\mathrm{II}} - p_{\mathrm{I}}),$$

whence follows, for an ideal gas and for all substances with a positive second derivative $(\partial^2 v/\partial p^2)_S$, the assertion of the impossibility of rarefaction shock waves.

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E. Jouguet also proved in a general form that this thermodynamic criterion of the possibility of compressive shock waves

$$S_{\mathrm{II}} > S_{\mathrm{I}} \quad \text{for} \quad p_{\mathrm{II}} > p_{\mathrm{I}},$$

and the impossibility of rarefaction shock waves

$$S_{\mathrm{II}} < S_{\mathrm{I}} \quad \text{for} \quad p_{\mathrm{II}} < p_{\mathrm{I}},$$

is identically related to the relation between the speed of sound before and after passage of a wave and the speed of the wave relative to the corresponding states ($c_{\mathrm{I}} > D_{\mathrm{I}}$, $c_{\mathrm{II}} > D_{\mathrm{II}}$ for $p_{\mathrm{II}} > p_{\mathrm{I}}$ and $c_{\mathrm{I}} > D_{\mathrm{I}}$, $c_{\mathrm{II}} < D_{\mathrm{II}}$ for $p_{\mathrm{II}} < p_{\mathrm{I}}$). This interrelation may be viewed as a mechanical condition of the possibility of compression shock waves and the impossibility of rarefaction shock waves.

Our point is that, without a doubt, there exist substances in nature for which, under certain conditions of pressure and temperature, $(\partial^2 p/\partial v^2)_S < 0$ holds.

In this case, it follows from the general theory that in such a state (sharp) rarefaction shock waves will exist, while compression shock waves will be diffuse and ill-defined, with a width proportional to the path traveled by the wave.

For proof we note that the isotherm has an inflection point at the critical point, i.e., $(\partial^2 p/\partial v^2)_T = 0$ for

$$p = p_{\mathrm{cr}}, \qquad v = v_{\mathrm{cr}}, \qquad T = T_{\mathrm{cr}}.$$

It is easy to verify that on the isotherm corresponding to the critical temperature $T = T_{\mathrm{cr}}$ there is a finite region $(\partial^2 p/\partial v^2)_T < 0$ bounded on one side by the critical point and on the other by some point at a lower pressure and higher specific volume. This region, like the entire isotherm at $T = T_{\mathrm{cr}}$, lies completely outside the condensation region. Analogous regions are present on other isotherms which are close to the critical one for $T < T_{\mathrm{cr}}$ and $T > T_{\mathrm{cr}}$; taken together, they form in the p,v-plane a region in which $(\partial^2 p/\partial v^2)_T < 0$. Part of this region obviously lies outside the condensation region since the entire isotherm $T = T_{\mathrm{cr}}$ lies outside the condensation. In this region, in which $(\partial^2 p/\partial v^2)_T < 0$, the quantity $(\partial^2 v/\partial p^2)_T$ is also negative since

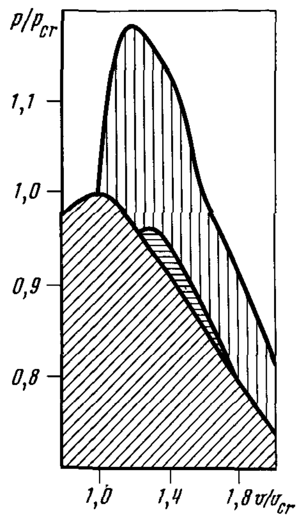

Fig. 1

$$\left(\frac{\partial^2 v}{\partial p^2}\right)_T = -\left(\frac{\partial v}{\partial p}\right)_T^3 \left(\frac{\partial^2 p}{\partial v^2}\right)_T$$

In the stable phases, $(\partial v/\partial p)_T < 0$, and the signs of $(\partial^2 p/\partial v^2)_T$ and $(\partial^2 v/\partial p^2)_T$ coincide.

For the problem in question, it is the isentropic derivative, $(\partial^2 v/\partial p^2)_S < 0$, that is important, not the isothermic one.

However, the greater the internal molecular specific heat, the closer the adiabate is to the isotherm. We may therefore assert that for sufficiently large specific heat outside the condensation region, the region of interest, $(\partial^2 v/\partial p^2)_S < 0$, probably exists. The arguments given above are completely general, but at the same time they do not allow us to determine the required value of the specific heat without experiment.

Using the very rough assumptions of a constant specific heat c_v and strict accuracy of the Van-der-Waals equations, numerical calculations were performed which implied that the minimum value of the specific heat for which the desired region appears is equal to c_v =20 cal/mole · deg. The figure above shows in the p,v-plane the condensation region (diagonal lines), the region $(\partial^2 v/\partial p^2)_T < 0$ (vertical lines), and the region $(\partial^2 v/\partial p^2)_S < 0$ at $c_v = 40$ cal/mole · deg (horizontal lines).

F. E. Yudin participated in the calculations.

This problem is considered in detail in the monograph "Theory of Shock Waves and Introduction to Gasdynamics" [3].

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References

1. *Zemplen G.*—C. r. Bull. chim. Soc. **141**, 712 (1905); **142**, 142 (1906).
2. *Jouguet E.*—C. r. Bull. chim. Soc. **138**, 1685 (1904); **138**, 786 (1904).
3. *Zeldovich Ya. B.* Teoriïa udarnykh voln i vvedenie v gazodinamiku [Theory of Shock Waves and Introduction to Gasdynamics]. Moscow, Leningrad: Izd-vo AN SSSR, 186 p. (1946).

Commentary

It is shown in this paper that, contrary to previous common belief, under certain special, but still fully feasible experimental conditions rarefaction shock waves can exist. In particular, this situation should certainly occur in gases with a sufficiently large specific heat c_v near the critical point of fluid–vapor transition. In recent years the prediction made by Ya.B. has been conclusively confirmed by experiment.[1] Later Ya.B. considered the peculiarities of the state near the critical point which may occur in a rapid, "shock" expansion.[2]

[1] *Kutateladze S. S., Borisov Al. A., Borisov A. A., Nakoryakov V. E.*—Doklady AN SSSR **252**, 595–598 (1978)

[2] *Zeldovich Ya. B.*—ZhETF **80**, 2111–2112 (1981).