MEMS Inertial Measurement Units
Lecture 20

- Accelerometer: Principle of Operation
- Accelerometer: Euler and Centripetal Acceleration Compensation
- Gyroscope: Principle of Operation
- Noise Model
- ~~Allan Variance Analysis~~
- ~~Magnetometer: Principle of Operation~~
- ~~Magnetometer: Soft-Iron and Hard-Iron Calibration~~

MEMS IMU
Lecture 20

- Micro Electro-Mechanical Sensors
- An IMU unit typically consists of 3 × 3 axes of sensors: 3 one-axis accelerometers, 3 one-axis gyroscopes, 3 one-axis magnetometers (and, maybe a barometer)
- Accelerometers and Gyroscopes are inertial sensors: measure the motion of an object (acceleration and angular velocity respectively) with respect to an inertial reference frame.

Accelerometer - Principle of Operation

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- A MEMS accelerometer is a small mass suspended by a spring
- The mass is known as the proof mass and the direction that the mass is allowed to move is known as the sensitivity axis.
- When an accelerometer is subjected to a linear acceleration along the sensitivity axis, the acceleration causes the proof mass to shift to one side, with the amount of deflection proportional to the acceleration

Accelerometer - Principle of Operation

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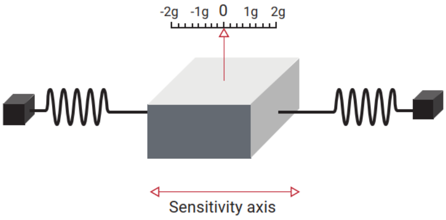
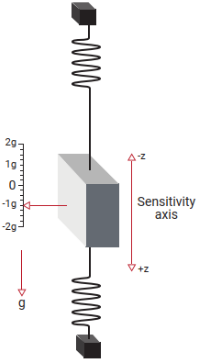


a) Horizontal



b) Vertical

Figures from “Inertial Navigation Primer, VectorNav Library”

Accelerometer - Principle of Operation

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- Thus a 3-axis accelerometer measures/outputs the linear acceleration due to motion as well as the "pseudo acceleration" caused by gravity, plus some noise

$$\mathbf{a}_{s,i}^{s} = \begin{bmatrix} \tilde{a}_x \\ \tilde{a}_y \\ \tilde{a}_z \end{bmatrix} = \mathbf{a}_{s,i}^{s} - \mathbf{g} + \boldsymbol{noise}$$

- The measurement corresponds to the linear acceleration of the object the IMU is fixed to wrt the inertial frame, expressed in the sensors's coordinates;

Accelerometer - Removal of "Angular" Forces

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- The sensor is attached to a body-frame. It is the body frames linear acceleration we are interested in, namely $\mathbf{a}_{b,i}$ and not $\mathbf{a}_{s,i}$.
- These two quantities are identical if a) body-frame b is not rotating wrt to the inertial frame or b) if the sensor-frame s is placed at the center of rotation of the body frame (b)
- A rotating frame of reference is a special case of a non-inertial reference, namely is a frame that is rotating relative to an inertial reference frame. All non-inertial reference frames exhibit fictitious forces. Rotating reference frames are characterized by three: the centrifugal force, the Coriolis force, and, for non-uniformly rotating reference frames, the Euler force. So we need to eliminate these forces from the IMU reading.

Accelerometer - Removal of “Angular” Forces

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$$\mathbf{a}_{s,i} = \mathbf{a}_{b,i} + \underbrace{\omega_{b,i} \times (\omega_{b,i} \times \mathbf{r})}_{\text{centrifugal acc.}} + \underbrace{2\omega_{b,i} \times \mathbf{v}_{s,b}}_{\text{Coriolis acc.}} + \underbrace{\dot{\omega}_{b,i} \times \mathbf{r}}_{\text{Euler acc.}},$$

where $\omega_{b,i}$ is the angular velocity of the body-frame b wrt to the inertial frame i, and $\mathbf{r}$ is the vector connecting the origin of the sensor s-frame with the center of rotation of the b-frame. Since the sensor is bolted to the body-frame, $\mathbf{v}_{s,b} = 0$;

Accelerometer Measurement Correction

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$$\mathbf{a}_{b,i} = \tilde{\mathbf{a}}_{s,i} + \mathbf{g} - \omega_{b,i} \times (\omega_{b,i} \times \mathbf{r}) - \alpha_{b,i} \times \mathbf{r}$$

To perform the correction numerically, all the vector quantities must be resolved in a particular frame. And also, the correction cannot be done by the sensor alone, as it depends on other state measurements (or estimation) and general system configuration. So this is normally done in the system software.

Gyroscope - Principle of Operation

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- A MEMS gyroscope typically uses a “tuning-fork” configuration in which two (2) masses are connected by a spring. When an angular rate is applied, the Coriolis force on each mass acts in the opposite direction and the resulting change in capacitance is directly proportional to the angular velocity. This is an 1-axis determination.
- However, when a linear acceleration is applied, the two masses moves in the same direction, resulting in no change in capacitance and a measured angular rate of zero.
- This configuration minimizes a gyroscope’s sensitivity to linear acceleration from instances of shock, vibration, and tilt.

Gyroscope - Principle of Operation

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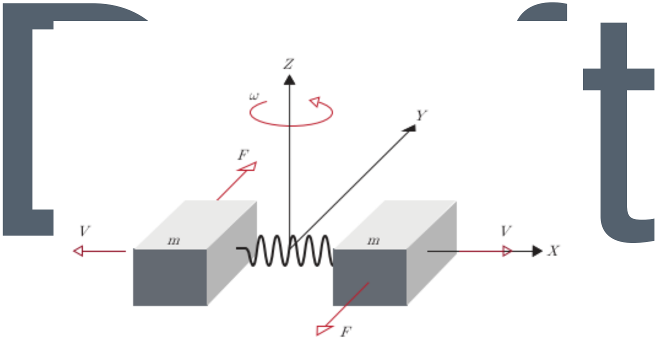

1-axis gyroscope tunning-fork configuration

Gyroscope - Principle of Operation
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- A 3-axis gyroscope measures/outputs the angular velocity due to motion wrt to an inertial measurement frame (i.e. it implicitly measures the earth's rotation as well) and some noise:

$$\tilde{\omega}^s_{s,i} = \begin{bmatrix} \tilde{\omega}_x \\ \tilde{\omega}_y \\ \tilde{\omega}_z \end{bmatrix} = \omega^s_{s,e} + \omega^s_{e,i} + \textit{noise}$$

- The measurement corresponds to the angular velocity vector expressed in the sensors's coordinates;
- $\omega_{e,i} = 7.2921159 \times 10^{-5}\ \mathrm{rad/s}$, the earth's rotation rate, is below the noise level of many "low-end" gyroscopes;

Noise Characteristics of Acc & Gyro

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- First, MEMS sensors can provide measurements at a high output frequency, e.g. 500 — 1000 Hz. The system-specific processing software (e.g. navigational software) cannot typically process measurements this fast, it will for example use only a measurement every 100 Hz;
- For the in-between measurements not to be lost, the sensors have typically different options to perform fast on-board filtering (e.g. averaging, linear filters, even Kalman Filtering) and supply processed measurements at the requested lower output frequency;
- This is important to know and setup properly, as for example it affects the noise characteristics of the output

Noise Characteristics of Acc & Gyro

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It is typical to assume that a measurement from a calibrated sensor is affected by:

- random white noise, thus with mean zero with variance σ_v^2;
- plus a slowly moving random bias (accumulated white noise, or random walk).

Noise Characteristics of Acc & Gyro

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- The slowly moving random bias is a dynamic process defined by $b_{k+1} = b_k + w$, where w is random white noise, with constant variance $\Delta t \sigma_w^2$;
- If the initial bias b_0 has mean zero and variance of σ_{b0}^2, then b at time t has still mean zero but increased variance of $\sigma_{bt}^2 = \sigma_{b0}^2 + t\sigma_w^2$;
- Accs and gyros sensor should be kept entirely still (from both linear and angular motion) at restart. This allows to reset the bias component. This also means that σ_{b0}^2 is related to σ_v^2.

Noise Characteristics of Acc & Gyro
Lecture 20

- A raw acc- or gyro- measurement is thus a random variable defined as:

$$\tilde{m}(t) = m(t) + v + b(t);\ \mathrm{E}[\tilde{m}(t)] = m(t);\ \mathrm{Var}[\tilde{m}(t)] = \sigma_v^2 + \sigma_{b0}^2 + t\sigma_w^2$$

- An averaged down-sampled acc- or gyro- measurement is a random variable that

$$\tilde{m}(t) = \frac{1}{n}\sum_{k=1}^{n} m(t_0 + k\Delta t) + v + b(t_0 + k\Delta t);\ \mathrm{E}[\tilde{m}(t)] = \frac{1}{n}\sum_{k=1}^{n} m(t_0 + k\Delta t);$$

$$\mathrm{Var}[\tilde{m}(t)] = \ldots$$

- If other algorithms are used for down-sampling, it can be that the sensor itself can provide for each measurement an estimation of the noise variance, or we can use the averaged formulas above as our noise estimation;

Noise Characteristics of Acc & Gyro
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Reading noise specifications from data sheets.

- Would be great if sensors would output the measurement together with is estimated covariance. They mostly do not do this yet.
- The do not even give σ_v and σ_w, but other values that need to be processed [illegible] to obtain the σ_v and σ_w (which we would need in a Kalman Filter algorithm for example)
- Noise density is a proxy for σ_v, i.e. $\sigma_v = \text{ND}\sqrt{\text{freq}}$;
- The σ_w can be obtained from running an Allan Variance analysis on about 24 to 48 hours of data measurements.

Well done so far!