# State Estimation ...

## Lecture 18



... in a more general probabilistic framework:

- Bayes Filter (BF)
- KF Derivation as a particular form of the BF
- Particle Filter
- Ensemble KF
- Cubature KF

# Bayes Filter
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We will consider the following generic non-linear, (invariant), discrete-time dynamical system with noise,

$$\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k, \mathbf{u}_k) + \mathbf{w}_k$$
$$\mathbf{y}_k = \mathbf{h}(\mathbf{x}_k) + \mathbf{v}_k$$

# Bayes Filter

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- Time Update Equation:

$$\underbrace{\Pr(\mathbf{x}_{k+1}|\mathbf{Y}_k)}_{\text{predictive distribution}} = \int_{\mathbb{R}^n} \underbrace{\Pr(\mathbf{x}_{k+1}|\mathbf{x}_k)}_{\text{prior distribution}} \underbrace{\Pr(\mathbf{x}_k|\mathbf{Y}_k)}_{\text{updated posterior distribution}} d\mathbf{x}_k$$

- Measurement Update Equation:

$$\underbrace{\Pr(\mathbf{x}_{k+1}|\mathbf{Y}_{k+1})}_{\text{updated posterior distribution}} = \frac{1}{z_{k+1}} \underbrace{\Pr(\mathbf{x}_{k+1}|\mathbf{Y}_k)}_{\text{predictive distribution}} \underbrace{l(\mathbf{y}_{k+1}|\mathbf{x}_{k+1})}_{\text{likelyhood function}},$$

where $z_{k+1}$ is the normalization constant defined as $z_{k+1} = \int_{\mathbb{R}^n} \Pr(\mathbf{x}_{k+1}|\mathbf{Y}_k))l(\mathbf{y}_{k+1}|\mathbf{x}_{k+1})d\mathbf{x}_{k+1}$.

# Bayes Filter
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- The prior distribution term $\mathrm{Pr}(\mathbf{x}_{k+1}|\mathbf{x}_k)$ is related only to the dynamic equations,
- The likelihood function term $l(\mathbf{y}_{k+1}|\mathbf{x}_{k+1})$, is related only to the measurement equations .
- The exact computation of the predictive distribution $\mathrm{Pr}(\mathbf{x}_{k+1}|\mathbf{Y}_k)$ is generally not possible, except for a couple of special cases. Both the probability prior and the integration are a problem.

# Bayes Filter + KF Derivation

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Starting from the Bayes Filter definition it is possible to derive the Linear Kalman Filter equations in the known form

# Particle Filter
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- It is possible to represent a probability distribution as a (large) collection of sample points or particles
- How to extract a number *p* of particles from a given probability distribution: easy, if we know the CDF
- It is possible to implement the Bayesian filter using a particle-based approach

# Particle Filter

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- Step 1: Extract a number $p$ of particles from/to represent the posterior distribution $\Pr(\mathbf{x}_k|\mathbf{Y}_k)$, i.e. the set of particles $\{\mathbf{x}_k^1, \ldots, \mathbf{x}_k^p\}$
- Step 2: To approximate the predictive distribution $\Pr(\mathbf{x}_{k+1}|\mathbf{Y}_k)$, each previous particle/sample is propagated through the dynamics (including extracting the random process noise), to obtain the set $\{\mathbf{x}^1_{k+1|k}, \ldots, \mathbf{x}^p_{k+1|k}\}$. This is a Monte-Carlo approximation.
- Step 3: How to integrate a new observation $\mathbf{y}_{k+1}$ and update the set of particles. The key technique for doing this is Importance Sampling.

# Ensemble KF & Cubature KF

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- Both assume underlying Gaussian distributions
- Cubature [Arasaratnam & Haykin, 2009]

Well done!