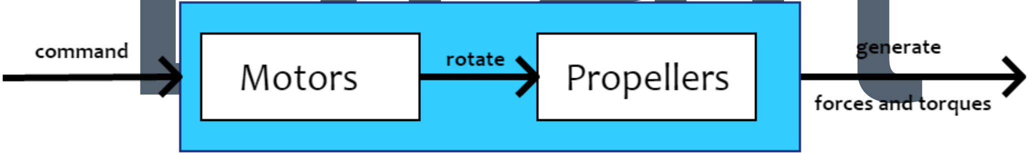
System Identification Results

Lecture 8

Draft



Command

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- The code on the electronics (firmware/embedded code) sends a digital command to the motors, that is then transformed to an analog PWM (Pulse Width Modulation signal) that commands the coreless motors of the Crazyflie
- The digital command consists of four 16-bit integer numbers (from 0 to 65535), one for each motor.

Mapping functions

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- Given command from 0 to 65535, how fast does a propeller rotate ?
- How much thrust does a rotating propeller produce ?
- How much torque does a rotating propeller produce ?
- What are the aerodynamic forces on the quadrotor in flight ?

Assumptions

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- Given command from 0 to 65535, we assume that the motors and propellers respond instantaneously thus we do not look at the transitory response
- The aerodynamic forces are due mainly the drag
- We are going to use results from Julian Förster bachelor thesis at ETH Zurich, which is uploaded on the lecture's resources

Parameters

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- Mass of the quadrotor: $m = 0.028$ kg
- Radius of the quadrotor (length from center of mass to the propeller center) $r = 0.045$
- Inertia

$$\mathbf{I}^b = \begin{bmatrix} 16.571710 & -0.830806 & -0.718277 \\ -0.830806 & 16.655602 & -1.800197 \\ -0.718277 & -1.800197 & 29.261652 \end{bmatrix} 10^{-6} \cdot \text{kg} \cdot \text{m}^2$$

Rotor Static Mappings

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- Mapping from input command to the motor/propeller angular velocity:

$$\omega_{r,i} = [illegible]040[illegible]521 \cdot \text{cmd}_i + 38[illegible]8359 \text{ [rad/s]} \quad (1)$$

- Mapping from input command to the motor/propeller thrust:

$$f_{\text{thrust},i} = [illegible].130295 \cdot 10^{-11} \cdot \text{cmd}_i^2 + 1.03263310^{-6} \cdot \text{cmd}_i + 5.484560 \cdot 10^{-4} \text{ [N]} \quad (2)$$

- Mapping from motor thrust to motor torque

$$\tau_i = 0.005964552 \cdot f_{\text{thrust},i} + 1.563383 \cdot 10^{-5} \text{ [N·m]} \quad (3)$$

Drag model

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$$\boldsymbol{f}_a^b \;\; K_{\text{aero}} \left(\sum^{4} \;\; {}_{r,i} \right) \tag{4}$$

$$\text{where } K_{\text{aero}} = \begin{bmatrix} -10.2506 & -0.3177 & -0.4332 \\ -0.3177 & -10.2506 & -0.4332 \\ -7.7050 & -7.7050 & -7.5530 \end{bmatrix} 10^{-7}$$

Forces and Torque Model

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- Force in body frame

$$\boldsymbol{f}^b = \left[0,\quad 0,\quad \textstyle\sum_{i=1}^{4} f_{\text{thrust},i}(\text{cmd}_i)\right]^T = \boldsymbol{f}_z^b \tag{5}$$

- Torque in body frame, for the plus-configuration

$$\boldsymbol{\tau}^b = \left[(f_{\text{thrust},2} - f_{\text{thrust},4}) \cdot r,\quad (f_{\text{thrust},3} - f_{\text{thrust},1}) \cdot r,\quad -\tau_1 - \tau_3 + \tau_2 + \tau_4\right]^T \tag{6}$$

- Torque in body frame, for the cross-configuration

$$\boldsymbol{\tau}^b = \Big[(f_{\text{thrust},2} + f_{\text{thrust},3} - f_{\text{thrust},1} - f_{\text{thrust},4})\frac{\sqrt{2}}{2} \cdot r, \tag{7}$$

$$(f_{\text{thrust},3} + f_{\text{thrust},4} - f_{\text{thrust},2} - f_{\text{thrust},1})\frac{\sqrt{2}}{2} \cdot r, -\tau_1 - \tau_3 + \tau_2 + \tau_4\Big]^T \tag{8}$$

Simplified Model

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A simplified model where we consider the command to thrust and torque models to be quadratic in the rotor angular velocity

$$\text{[illegible]}_{r,i},\ c_T = \text{[illegible]}.903 \cdot \text{[illegible]}^{-8} \tag{9}$$

$$\tau_i = c_Q \omega^2_{r,i},\ c_Q = c_T \cdot 0.005964552 = 1.246 \cdot 10^{-10} \tag{10}$$

Forces and Torque Simplified Model

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Plus configuration:

$$\begin{bmatrix} F_z^b \\ \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} = \underbrace{\begin{bmatrix} c_T & c_T & c_T & c_T \\ 0 & r \cdot c_T & 0 & -r \cdot c_T \\ -r \cdot c_T & 0 & r \cdot c_T & 0 \\ -c_Q & c_Q & -c_Q & c_Q \end{bmatrix}}_{\Gamma} \begin{bmatrix} \omega_{r,1}^2 \\ \omega_{r,2}^2 \\ \omega_{r,3}^2 \\ \omega_{r,4}^2 \end{bmatrix} \tag{11}$$

Forces and Torque Simplified Model

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Cross configuration

$$\begin{bmatrix} f_z^b \\ \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} = \underbrace{\begin{bmatrix} c_T & c_T & c_T & c_T \\ -r\frac{\sqrt{2}}{2}c_T & r\frac{\sqrt{2}}{2}c_T & r\frac{\sqrt{2}}{2}c_T & -r\frac{\sqrt{2}}{2}c_T \\ -r\frac{\sqrt{2}}{2}c_T & -r\frac{\sqrt{2}}{2}c_T & r\frac{\sqrt{2}}{2}c_T & r\frac{\sqrt{2}}{2}c_T \\ -c_Q & c_Q & -c_Q & c_Q \end{bmatrix}}_{\Gamma} \begin{bmatrix} \omega_{r,1}^2 \\ \omega_{r,2}^2 \\ \omega_{r,3}^2 \\ \omega_{r,4}^2 \end{bmatrix} \tag{12}$$