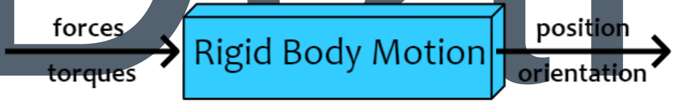
# Summary of Rigid Body Motion Eqs

Lecture 7



Draft



# Summary of Rigid Body Motion Eqs

Lecture 7 | Rigid Body Motion Eqs



## Differential Eqs. of Motion with the Rotation Matrix

Draft

$$\dot{\boldsymbol{p}}^e = \boldsymbol{v}^e \tag{1a}$$

$$\dot{\boldsymbol{v}}^e = \frac{1}{m}\boldsymbol{f}_{total,ext} = \frac{1}{m}\mathbf{R}_b^e \boldsymbol{f}_{total}^b \tag{1b}$$

$$\dot{\mathbf{R}}_b^e = \mathbf{R}_b^e \left[\omega^b\right]_\times \tag{1c}$$

$$\dot{\omega}^b = \left(\mathbf{J}^b\right)^{-1}\left(-\left[\omega^b\right]_\times \mathbf{J}^b\omega^b + \tau_c^b\right) \tag{1d}$$

# Summary of Rigid Body Motion Eqs





## Differential Eqs. of Motion with the Unit Quaternion

Draft

$$\dot{\boldsymbol{p}}^e = \boldsymbol{v}^e \tag{2a}$$

$$\dot{\boldsymbol{v}}^e = \frac{1}{m}\boldsymbol{f}^e_{total,ext} = \frac{1}{m}\mathbf{R}(\boldsymbol{q})\boldsymbol{f}^b_{total} \tag{2b}$$

$$\dot{\boldsymbol{q}} = \begin{bmatrix} \dot{s} \\ \dot{\mathbf{v}} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -\boldsymbol{v}^T \\ s\mathbf{I}_3 + [\mathbf{v}]_\times \end{bmatrix}\omega^b = \frac{1}{2}\begin{bmatrix} -v_1\omega^b_x - v_2\omega^b_y - v_3\omega^b_z \\ s\omega^b_x - v_3\omega^b_y + v_2\omega^b_z \\ v_3\omega^b_x + s\omega^b_y - v_1\omega^b_z \\ -v_2\omega^b_x + v_1\omega^b_y + s\omega^b_z \end{bmatrix} \tag{2c}$$

$$\dot{\omega}^b = \left(\mathbf{J}^b\right)^{-1}\left(-\left[\omega^b\right]_\times \mathbf{J}^b\omega^b + \tau^b_c\right) \tag{2d}$$

# Summary of Rigid Body Motion Eqs

Lecture 7 | Rigid Body Motion Eqs



Draft

## Differential Eqs. of Motion with the Unit Quaternion

where

$$\mathbf{R}(\boldsymbol{q}) = \left[-\mathbf{v} \quad s\mathbf{I}_3 + [\mathbf{v}]_\times\right] \begin{bmatrix} -\mathbf{v}^T \\ s\mathbf{I}_3 + [\mathbf{v}]_\times \end{bmatrix}$$

$$= 2 \begin{bmatrix} s^2 + v_1^2 - 0.5 & v_1 v_2 - s v_3 & v_1 v_3 + s v_2 \\ v_1 v_2 + v_3 s & s^2 + v_2^2 - 0.5 & -s v_1 + v_2 v_3 \\ v_1 v_3 - s v_2 & v_2 v_3 + v_1 s & s^2 + v_3^2 - 0.5 \end{bmatrix}$$

# Params, Inputs, Initial Conditions

## Lecture 7 | Rigid Body Motion Eqs



Draft

- Parameters: $m$ - the mass of the object, $\mathbf{I}^b$ - the [illegible] inertia matrix
- Inputs: forces $\boldsymbol{f}_{total}$ [illegible] and torques about the center of mass $\boldsymbol{\tau}_C$
- Example of initial conditions: quadrotor object at the origin of the e-frame, b-frame aligned with the e-frame, and the quadrotor is at rest:

$$\boldsymbol{p}^e = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{v}^e = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{R}_b^e = \mathbf{I}_3 \text{ and } \boldsymbol{q} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{\omega}^b = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

# Simple Integration Technique

## Lecture 7 | Rigid Body Motion Eqs



Draft

- Given a differential equation $\dot{y}(t) = f(y, u, t)$, where $y(t)$ is unknown, $u(t)$ is a known input function and the initial condition $y(0) = y_0$ is known also, we can obtain an approximation of $y(\Delta t)$, $y(k\Delta t)$ as

$$\tilde{y}(k\Delta t) = \tilde{y}((k-1)\Delta t) + \dot{y}(t)\Delta t$$

$$\boxed{\tilde{y}(k\Delta t) = \tilde{y}((k-1)\Delta t) + f(y(k\Delta t), u(k\Delta t), k\Delta t)\Delta t}$$

$$\Rightarrow \tilde{y}(\Delta t) = y_0 + f(y_0, u(0), 0)\Delta t, \quad \tilde{y}(2\Delta t) = \tilde{y}(\Delta t) + f(\tilde{y}(\Delta t), u(\Delta t), \Delta t)\Delta t$$

$$\tilde{y}(3\Delta t) = \tilde{y}(2\Delta t) + f(\tilde{y}(2\Delta t), u(2\Delta t)2\Delta t)\Delta t, \ldots$$

- The global error $|y(k\Delta t) - \tilde{y}(k\Delta t)|$ is proportional to $\Delta t$
- The smaller $\Delta t$, the better the approximation