

Minimal Walking Technicolor

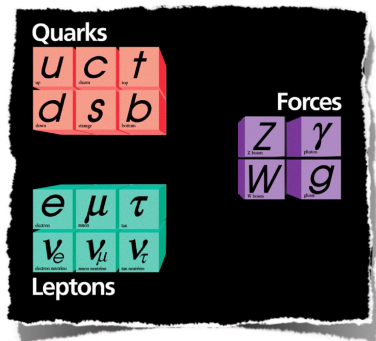
Liam Keegan

06/01/2010

Edinburgh University

[0910.4535] - Francis Bursa, Luigi Del Debbio, Claudio Pica, Thomas Pickup

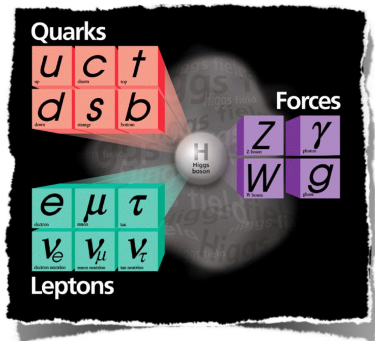
The Standard Model



Fermilab

- Standard Model is well verified experimentally
- Electroweak Symmetry breaking included (i.e. mass of Z/W bosons)
- But EWSB mechanism remains a mystery

The Higgs Mechanism



Fermilab

- Higgs mechanism will be tested at the LHC, but
 - Ad hoc: all fermion masses and mixings arbitrary parameters
 - Trivial: without new physics, Higgs decouples
 - Unnatural: quadratically sensitive to Planck scale, so requires fine tuning
- So thought to be an effective description of a more fundamental theory, e.g. SUSY, Technicolor, ...

Technicolor

- SM without Higgs already has some EW symmetry breaking.
- Quark condensate gives M_W of the order of the pion decay constant:

$$\langle \bar{u}_L u_R + \bar{d}_L d_R \rangle \neq 0 \rightarrow M_W = \frac{g F_\pi}{2} \sim 30 \text{ MeV}$$

- So why not have some more 'techni-quarks' that form a condensate at a higher scale ($F_\pi^{TC} \sim 250 \text{ GeV} \sim \Lambda_{TC}$)

Weinberg 78, Susskind 78

Extended Technicolor

- Add interactions between SM quarks and techni-quarks at some high scale Λ_{ETC}
- Get SM quark mass terms in effective low energy lagrangian:

Quark Masses

$$\frac{\langle \bar{\Psi}\Psi \rangle_{ETC} \bar{\psi}\psi}{\Lambda_{ETC}^2}$$

Dimopoulos, Susskind 79 - Eichten, Lane 80

Flavour Changing Neutral Currents

- But also get FCNC term:

Quark Masses

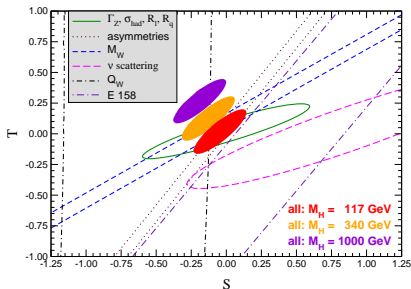
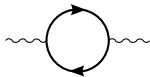
$$\frac{\langle \bar{\Psi}\Psi \rangle_{ETC} \bar{\psi}\psi}{\Lambda_{ETC}^2}$$

FCNC

$$\frac{\bar{\psi}\psi\bar{\psi}\psi}{\Lambda_{ETC}^2}$$

- Naively scaling up QCD leads to a problem:
- Need large $\Lambda_{ETC} \sim 1000 \text{ TeV}$ to suppress Flavour Changing Neutral Currents
- But this gives a strange quark mass that is ~ 50 times too small

S, T Parameters

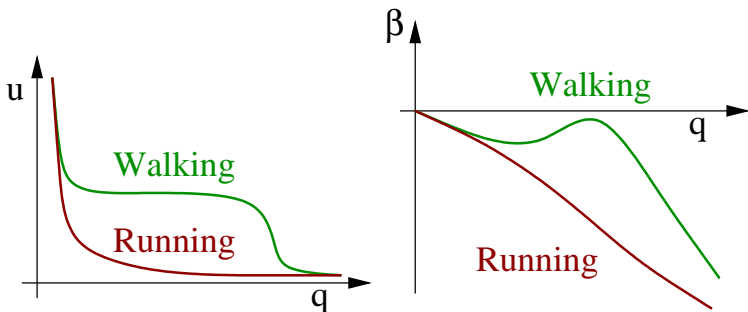


Particle Data Group 2008

- S, T parameters measure deviation from SM caused by new physics
- Naive QCD scaling gives $\sim 2\sigma$ disagreement with experiment
- Perturbative estimate:

$$S = \frac{1}{6\pi} \frac{N_f}{2} d(R) = 0.16$$

Walking Technicolor Cartoon



Walking Technicolor Quark Masses

$$\langle \bar{\Psi}\Psi \rangle_{ETC} = \langle \bar{\Psi}\Psi \rangle_{TC} \exp \left(\int_{\Lambda_{TC}}^{\Lambda_{ETC}} \gamma(\mu) d \ln \mu \right)$$

- In QCD this gives logarithmic enhancement:

$$\langle \bar{\Psi}\Psi \rangle_{ETC} = \log \left(\frac{\Lambda_{ETC}}{\Lambda_{TC}} \right)^{\gamma} \langle \bar{\Psi}\Psi \rangle_{TC}$$

- But a walking coupling gives power enhancement:

$$\langle \bar{\Psi}\Psi \rangle_{ETC} = \left(\frac{\Lambda_{ETC}}{\Lambda_{TC}} \right)^{\gamma} \langle \bar{\Psi}\Psi \rangle_{TC}$$

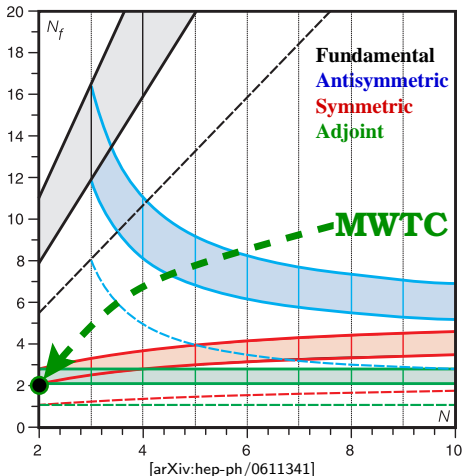
Walking Technicolor S Parameter

- Walking seems to reduce S parameter compared to running case.
- And other sectors of the theory, such as new leptons, are expected to contribute negatively

Dietrich, Sannino, Tuominen [arXiv:hep-ph/0505059]

- But ideally this also needs to be studied non-perturbatively

Minimal Walking Technicolor



- Simplest interesting model: MWT
- 2 dirac fermions transforming under the adjoint representation of $SU(2)$

Saninno, Tuominen
[arXiv:hep-ph/0405209]

Prediction for anomalous dimension

Conjectured all orders beta function

$$\beta(g) = \frac{g^3}{(4\pi)^2} \frac{\beta_0 - \frac{2}{3} T(r) N_f \gamma(g^2)}{1 - \frac{g^2}{8\pi^2} C_2(G) \left(1 + \frac{2\beta'_0}{\beta_0}\right)}$$

$$\beta_0 = \frac{11}{3} C_2(G) - \frac{4}{3} T(r) N_f, \quad \beta'_0 = C_2(G) - T(r) N_f$$

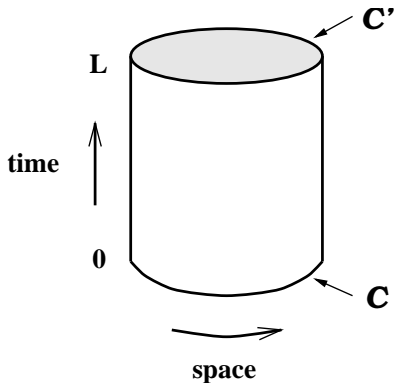
- For MWT this predicts anomalous dimension $\gamma = 0.75$ at fixed point
- This is a scheme-independent quantity at a fixed point

Ryttov, Sannino [arXiv:0711.3745]

Scheme dependence

- Walking/Running of coupling is scheme dependent
- Want to measure physical, scheme independent quantities:
 - **Existence** of fixed point
 - **Anomalous mass dimension** at the fixed point

Schrodinger Functional



($L \times L \times L$ box with periodic b.c.)

- Finite size renormalisation scheme
- Can be defined in continuum and on lattice
- Scale $\mu \sim 1/L$
- Dirichlet timelike bcs
- Constant gauge fields C, C'

Boundary Conditions

Boundary gauge fields

$$\begin{aligned} U(x, k)|_{t=0} &= \exp[\eta\tau_3 a/iL] \\ U(x, k)|_{t=L} &= \exp[(\pi - \eta)\tau_3 a/iL] \end{aligned}$$

These induce a background chromoelectric field in the bulk with strength parametrised by η , we work at $\eta = \pi/4$.

Fermionic boundary conditions

$$\begin{aligned} P_+ \psi &= 0, \quad \bar{\psi} P_- = 0 & \text{at } t = 0 \\ P_- \psi &= 0, \quad \bar{\psi} P_+ = 0 & \text{at } t = L \end{aligned}$$

These allow simulation directly at zero mass, $P_{\pm} = (1 \pm \gamma_0)/2$.

Coupling

Define a coupling as the response of the system to perturbations of the background gauge field configuration.

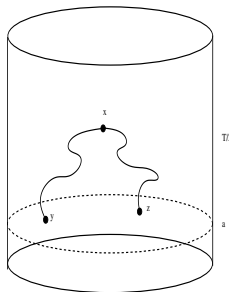
SF Coupling

$$\bar{g}^2(L) = k \left\langle \frac{\partial S}{\partial \eta} \right\rangle^{-1}$$

$$k = -24 \left(\frac{L}{a} \right)^2 \sin \left[\left(\frac{a}{L} \right)^2 (\pi - 2\eta) \right] \sim -12\pi$$

chosen such that $\bar{g}^2 = g_0^2$ to leading order in perturbation theory.

PCAC Mass



SF bcs allow simulation directly at zero mass, which we define using the Partially Conserved Axial Current:

PCAC Mass

$$am(x_0) = \frac{\frac{1}{2}(\partial_0 + \partial_0^*)f_A(x_0)}{2f_P(x_0)}$$

$$f_A(x_0) = -1/12 \int d^3y d^3z \langle \bar{\psi}(x_0) \gamma_0 \gamma_5 \tau^a \psi(x_0) \bar{\zeta}(y) \gamma_5 \tau^a \zeta(z) \rangle$$

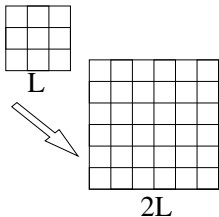
$$f_P(x_0) = -1/12 \int d^3y d^3z \langle \bar{\psi}(x_0) \gamma_5 \tau^a \psi(x_0) \bar{\zeta}(y) \gamma_5 \tau^a \zeta(z) \rangle$$

Naive Scaling



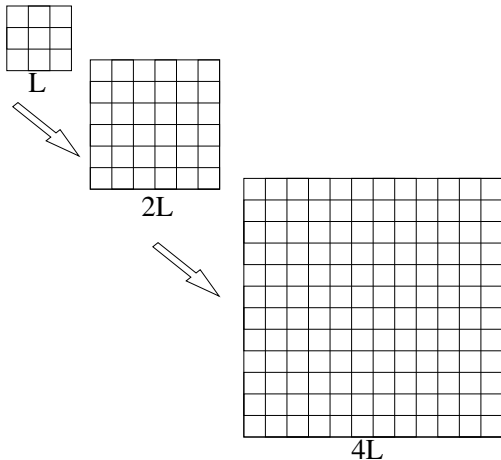
- Naive scaling: measure on $L_0, 2L_0, 4L_0, \dots, 2^n L_0$
- Corresponds to scales $\mu_0, \frac{1}{2}\mu_0, \frac{1}{4}\mu_0, \dots, 2^{-n}\mu_0$
- But cpu time scales as $\sim L^5$, and we want to simulate over a large range ($\sim 10^3$) of scales
- So naive scaling method no good

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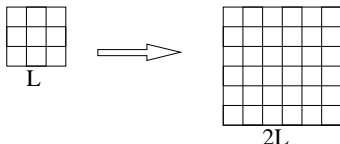
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Step Scaling



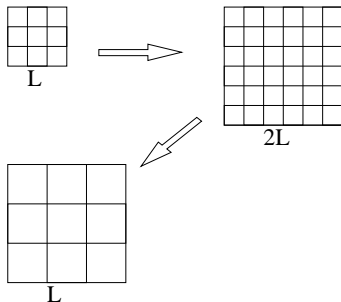
- Step scaling - only need $L, 2L$
- $\bar{g}^2(\beta, L) = u$
- $u' = \bar{g}^2(\beta, 2L)$
- Now tune bare parameters until $\bar{g}^2(\beta', L) = u'$
- Repeat

Step Scaling



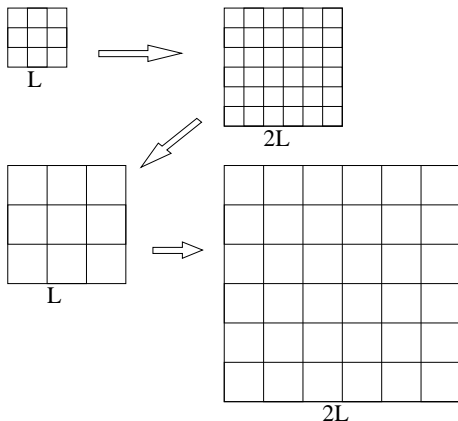
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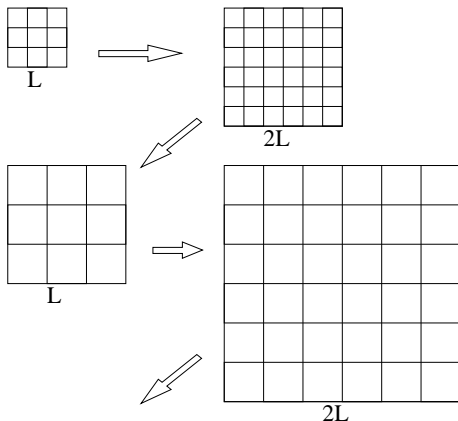
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Step Scaling

- This method was used by the ALPHA collaboration
- Can cover an arbitrary range of scales
- But each step requires retuning β, κ , which is time consuming
- And each step must be done sequentially, can't parallelise the runs

Interpolation Method

- Interpolation function method - just measure \bar{g}^2 at a range of β for each L and interpolate:

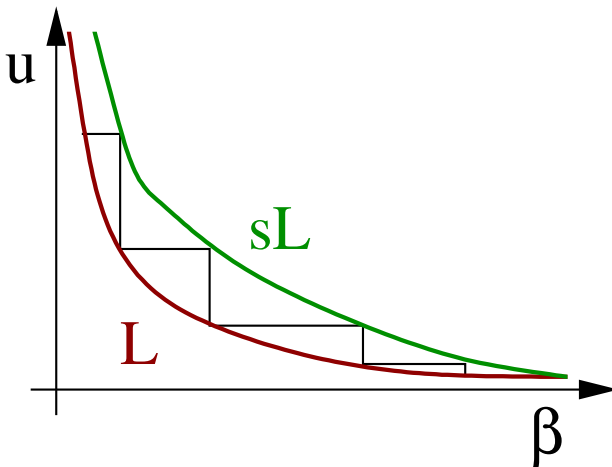
Coupling interpolation function

$$\frac{1}{\bar{g}^2(\beta, L/a)} = \frac{\beta}{2N} \sum_{i=0}^n c_i \left(\frac{2N}{\beta} \right)^i$$

- All simulations can be done in **parallel**, and no need for constant retuning
- However the choice of interpolation function introduces a new source of **systematic error**

method first used by Appelquist et. al. [arXiv:0901.3766]

Interpolation Method



Coupling Step Scaling Function

Lattice step scaling function

$$\Sigma(u, s, a/L) = \bar{g}^2(g_0, sL/a) \Big|_{\bar{g}^2(g_0, L/a)=u}$$

- Start on L^4 lattice where $\bar{g}^2 = u$
- Go to $(sL)^4$ lattice and measure $\bar{g}^2 = \Sigma$

Coupling Step Scaling Function

Continuum step scaling function

$$\sigma(u, s) = \lim_{a/L \rightarrow 0} \Sigma(u, s, a/L)$$

- Repeat for different lattice spacings a/L
- Extrapolate to the continuum $a/L \rightarrow 0$

Coupling Step Scaling Function

Relation to continuum beta-function

$$-2 \log s = \int_u^{\sigma(u,s)} \frac{dx}{\sqrt{x} \beta(\sqrt{x})}$$

- Integrated β -function
- $\sigma(u, s) = u$ corresponds to a fixed point ($\beta = 0$)

Boundary Conditions

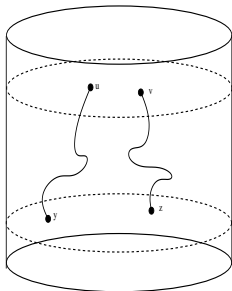
- To measure the anomalous mass dimension we use different boundary gauge fields, where $\eta = 0$

Boundary gauge fields

$$U(x, k)|_{t=0} = \exp[\eta\tau_3 a/iL] = 1$$

$$U(x, k)|_{t=L} = \exp[(\pi - \eta)\tau_3 a/iL] = 1$$

Z_P



Pseudoscalar density renormalisation constant

$$Z_P(L) = \frac{\sqrt{3f_1}}{f_P(L/2)}$$

$$f_1 = -1/12L^6 \int d^3u d^3v d^3y d^3z \langle \bar{\zeta}'(u) \gamma_5 \tau^a \zeta'(v) \bar{\zeta}(y) \gamma_5 \tau^a \zeta(z) \rangle$$

- f_1 correlator included to cancel boundary renormalisation factors

Mass Step Scaling Function

Lattice step scaling function

$$\Sigma_P(u, s, a/L) = \frac{Z_P(g_0, sL/a)}{Z_P(g_0, L/a)} \Big|_{\bar{g}^2(L)=u}$$

- Start on L^4 lattice where $\bar{g}^2 = u$, measure Z_P
- Go to $(sL)^4$ lattice and measure new Z_P then take ratio

Mass Step Scaling Function

Continuum step scaling function

$$\sigma_P(u, s) = \lim_{a/L \rightarrow 0} \Sigma_P(u, s, a/L)$$

- Repeat for different lattice spacings a/L
- Extrapolate to the continuum $a/L \rightarrow 0$

Mass Step Scaling Function

Relation to Beta-function

$$\sigma_P(u) = \left(\frac{u}{\sigma(u)} \right)^{(d_0/(2\beta_0))} \exp \left[\int_{\sqrt{u}}^{\sqrt{\sigma(u)}} dx \left(\frac{\gamma(x)}{\beta(x)} - \frac{d_0}{\beta_0 x} \right) \right]$$

Anomalous Dimension

- We define an estimator for γ :

Estimator for γ

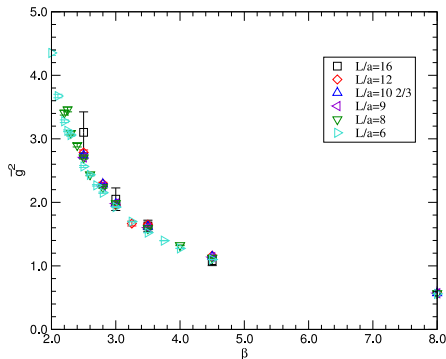
$$\hat{\gamma}(u) = -\frac{\log |\sigma_P(u, s)|}{\log |s|}$$

- At a fixed point, $\hat{\gamma} = \gamma$
- Away from a fixed point $\hat{\gamma}$ will deviate from γ
- The difference increases as γ develops a sizeable dependence on the energy scale

Simulation details

- Simulated on lattices of size $L=6,8,12,16$
- β in range 2.0 – 8.0
- Limited by bulk phase transition at $\beta \sim 2.0$
- ~ 1000 configurations on the largest lattices

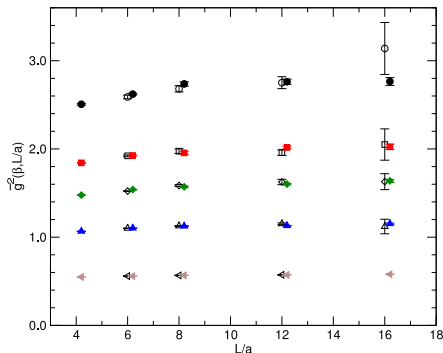
Coupling Data



- Not much variation with L
- Very good agreement with independent results

Hietanen, Rummukainen,
Tuominen [arXiv:0904.0864]

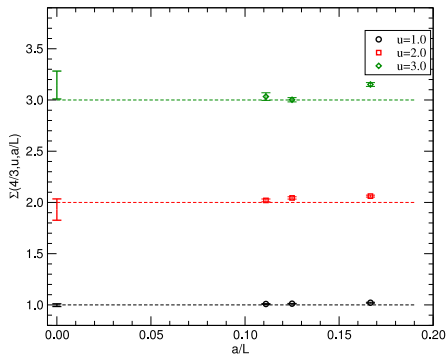
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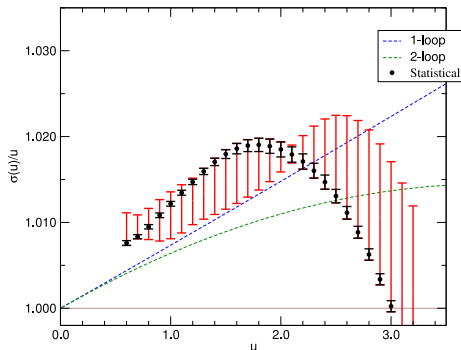
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Continuum Extrapolation



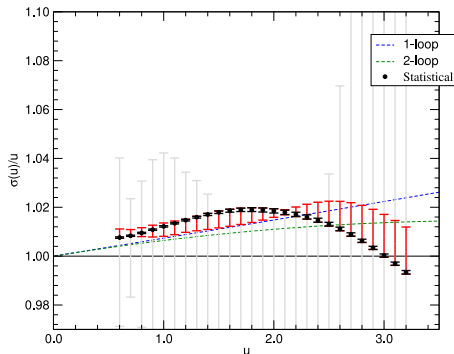
- No clear a/L dependence
- This is our largest source of error
- Continuum values consistent with zero within errors

Running Coupling



- Coupling runs very slowly
- Looks like there may be a fixed point at $u \sim 3$
- But once we include systematic errors the signal is swamped

Running Coupling

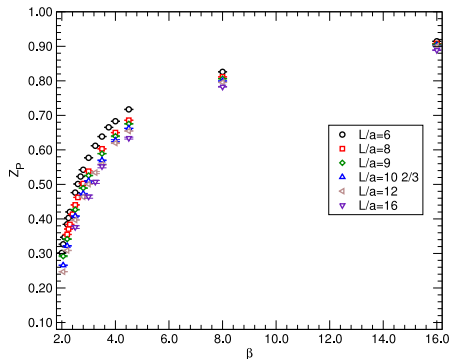


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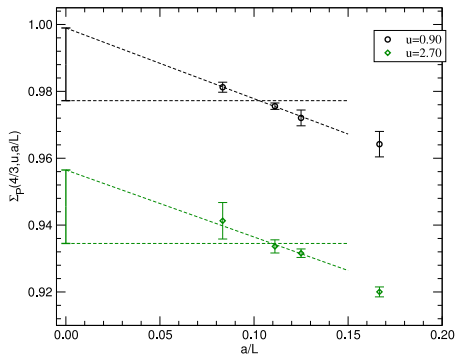
- Simulated on lattices of size $L=6,8,12,16$
- β in range 2.0 – 16.0
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Z_P Data



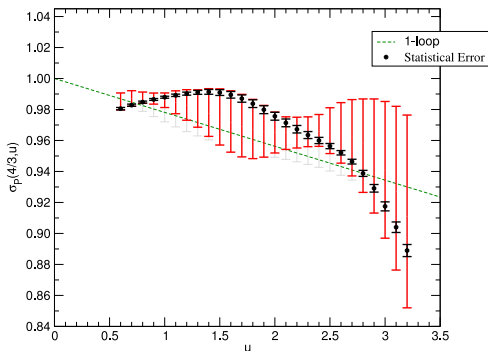
- Clear dependence on L
- Much easier quantity to measure: less noisy and smaller autocorrelation time

Continuum Extrapolation



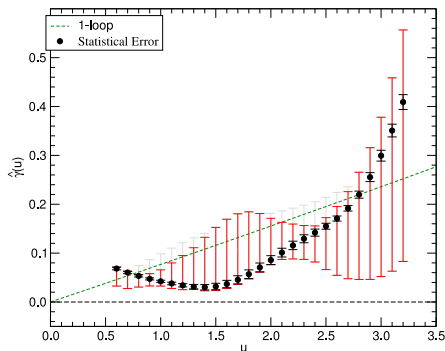
- Clear variation with a/L
- Can perform a continuum extrapolation

Sigma P



- Smaller errors than σ
- Consistent with one-loop perturbative prediction

Mass Anomalous Dimension



- $\hat{\gamma}$ is well determined
- Consistent with one-loop prediction
- Smaller than desired for phenomenology
- But is sensitive to the location of the fixed point

Conclusion

- Difficult to measure fixed point in coupling
- Have full control over statistical and systematic errors
- But can only say running is consistent with zero everywhere within errors
- Can determine mass anomalous dimension well as a function of coupling
- But only scheme-independent at a fixed point

Improvement

- We used an unimproved action, with $\mathcal{O}(a)$ discretisation errors
- Could use an improved action, remaining errors $\mathcal{O}(a^2)$
- This would significantly reduce our main source of error - the systematic error in our continuum extrapolation

MCRG

- Monte Carlo RG methods used a lot in 80's
- Less control over finite size effects
- But allows a determination of γ in a different scheme
- If we are at a fixed point then this is a scheme-independent quantity

Summary

- We present the first measurement of the mass anomalous dimension in Minimal Walking Technicolor.
- This is a phenomenologically important quantity, but is sensitive to the location of a fixed point, which needs better statistics and/or techniques to determine.
- Many complementary approaches are required to study these theories:
- Schrodinger Functional scaling studies, Monte Carlo Renormalisation Group methods, Spectral studies, ...

Particle content of MWT

- Fermionic content:
 - (U,D) techni-quark doublet
 - (N,E) new lepton doublet
 - composite techniquark-technigluon doublet
- Composite Higgs from techni-pion

LHC Phenomenology

- details depend on choice of ETC model
- then construct low energy EFT for LHC

Frandsen, Sannino, et. al. [arXiv:0710.4333v1] [arXiv:0809.0793v1]

Dark Matter candidate

- lightest technibaryon is a cold dark matter candidate
- TIMP: Technicolour Interacting Massive Particle
- iTIMP: lightest weak isotriplet technibaryon
- Prospects for discovery/exclusion from both dark matter experiments and LHC

Frandsen, Sannino [arXiv:0911.1570]

SF Coupling

- Choose background field B which is classical minimum of system, so fields close to B will dominate effective action

$$\Gamma[B] \equiv -\ln \mathcal{Z}[C, C'] = -\ln \left| \int D[\psi] D[\bar{\psi}] D[U] e^{-S} \right|$$

- Perturbative expansion

$$\Gamma[B] = \frac{1}{g_0^2} \Gamma_0[B] + \Gamma_1[B] + g_0^2 \Gamma_2[B] + \dots$$

- Choose $\Gamma' \equiv \partial\Gamma/\partial\eta$ as observable, then can define a renormalised coupling as

$$\bar{g}^2 = \Gamma'_0/\Gamma' = k \left\langle \frac{\partial S}{\partial \eta} \right\rangle^{-1} = g_0^2 + \mathcal{O}(g_0^4)$$