S&T2024 Bridging Physics (Mock Test) for Checkpoint 3

1 hour and 30 minutes

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains five questions. It comprises 10 printed pages.

2. Answer ALL questions.

3. This is a 100% CLOSED BOOK examination. Formula list is provided

4. Scientific calculator is allowed to be used, but students are NOT allowed to bring any programmable calculator, machine translator, or dictionary, in electronic form or in paper form, to the Examination Hall. Graphic calculator is a programmable calculator thus it cannot be brought to the Examination Hall.

5. If your calculator has the translation function, the calculator cannot be brought to the Examination Hall.

6. Students are also NOT allowed to bring any paper to the Examination Hall.

7. Students are also NOT allowed to share the use of calculators.

8. You do NOT have to begin each question with a fresh page.

9. All numerical values given in this paper are perfectly accurate. You have to show the intermediate working steps in your answers. The final answer can have 2 decimal places if needed.

~~~~~~

1. Consider a particle of mass m = 3.00 kg moving in the xy-plane. The x- and y-coordinates of the particle are functions of time t, and are given by

$$x(t) = Ae^{-bt}\cos\left(ct^2\right), \text{ and, } \quad y(t) = Ae^{-bt}\sin\left(ct^2\right),$$

where A = 5.00 m, b = 0.750 s^{-1} and c = 2.50 rad s^{-2}.

(i) Determine the x-component and y-component of the velocity of the particle, i.e., $v_x(t)$ and $v_y(t)$ respectively. **(10 marks)**

(ii) Compute the distance of the particle from the origin and its velocity at time t. Hence briefly in words, describe the motion of the particle. **(10 marks)**
~~~~~~

2. (i) A banked circular highway curve is designed for traffic moving at 95.0 km/h. The radius of the curve is 210 m, and the angle of banking measured from the horizontal is θ.

(a) The design of the circular highway curve allows a car travelling at 95.0 km/h to negotiate the turn without the need of friction. That is, a car travelling at 95.0 km/h can safely round the curve even on wet ice during winter. What is the correct angle θ of banking of the highway?

(b) If traffic is moving along the highway at 52.0 km/h on a stormy day with no wet ice, what is the minimum coefficient of static friction between tires and road that will allow cars to negotiate the turn without sliding down the banked curve?

(c) If the wet ice has disappeared and the road surface is rough with this value of coefficient of static friction calculated in (b), what is the greatest speed of cars at which the curve can be negotiated without sliding?

(10 marks)

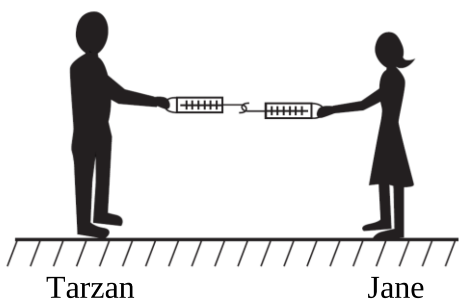
(ii) A 110-kg man lowers himself to the ground from a height of 12.0 m by holding on to a rope passed over a fixed frictionless pulley and attached to a 74.0 kg sandbag.

(a) With what speed does the man hit the ground?

(b) Is there anything he could do to reduce the speed with which he hits the ground? Briefly justify your answer.

(5 marks)

3. Tarzan of weight 700 N and Jane of weight 500 N stand on rough surface with the coefficients of friction $\mu_s = 0.5$ and $\mu_k = 0.45$. Each of them holds an identical spring scale and pulls against each other as shown in the diagram. Tarzan applies a force and the reading shown on his spring scale is 320 N.



(i) What will be the reading on the spring scale of Jane?

(ii) Will Jane be pulled towards Tarzan? If no, explain the reason. If yes, calculate the acceleration of Jane.

(iii) Jane is replaced by Anita who is heavier than Tarzan. Knowing that Anita has a weight of 750 N, Tarzan exerts a larger force and the reading on his scale is 360N.

(a) Will Anita be pulled towards Tarzan? If no, explain the reason. If yes, calculate the acceleration of Anita.

(b) Will Tarzan remain stationary when he exerts the force of 360 N? Explain the reason.

(Hint: Model Tarzan, Jane and Anita as point mass.) **(15 marks)**

4. Alice and Tom are playing a game in which they try to hit a small box on the floor with a marble of mass m fired from a spring-loaded gun that is mounted on a table of height H. The target box is 2.20 m horizontally from the edge of the table as shown in Figure 1.

Figure 1

Alice compresses the light spring 1.10 cm, but the marble falls 27.0 cm short as shown in Figure 2.

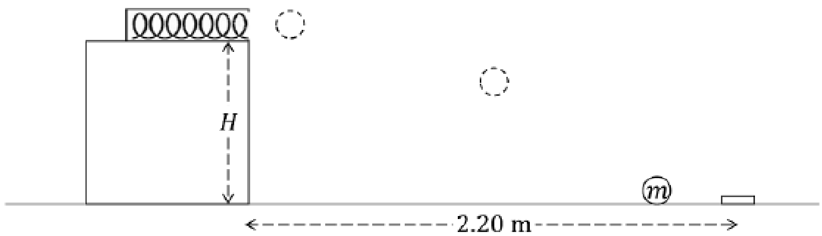
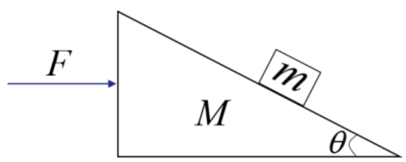


Figure 2

(i) Neglect air resistance. Determine the initial horizontal velocity v_{0x} of the marble, when it just loses contact with the spring. Express your answer in terms of g and H.
(ii) Neglect energy loss due to friction. Hence, find the force constant k of the spring. Express your answer in terms of m, g, and H.
(iii) How far should Tom compress the spring to score a hit?
(iv) If the spring has a nonzero mass, how would it affect your answer to (iii)? Briefly give your qualitative justifications in words.

(20 marks)

5. A right triangular wedge of mass M and angle θ, supporting a block of mass m on its inclined plane, rests on a horizontal table. When a finite horizontal force F is applied to the system as shown in the figure (front view),



(i) What is the acceleration of the block with mass m, as observed by a non-moving person in a Cartesian coordinate system (Give your answers in two components a_x and a_y) ?
(ii) From your answer in (i), derive the asymptotic value of the acceleration of the block if the mass of the wedge tends to a very large value.

Assume that all contacts are frictionless. **(30 marks)**

- End –

Formula List

Logarithm:

$$\log a + \log b = \log(a \times b)$$
$$\log a - \log b = \log\left(\frac{a}{b}\right)$$
$$(\log x) > 0,\ \text{if } x > 1.$$
$$(\log 1) = 0.$$
$$(\log x) < 0,\ \text{if } 0 < x < 1.$$
$$(\log x)\ \text{is undefined if } x < 0.$$

Trigonometry:

$$\sin = \frac{opposite}{hypotenuse}$$
$$\cos = \frac{adjacent}{hypotenuse}$$
$$\tan = \frac{opposite}{adjacent} = \frac{\sin}{\cos}$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$\cos 2\theta = 1 - 2\sin^2\theta$$
$$\cos 2\theta = 2\cos^2\theta - 1$$
$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Differentiation:

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$
$$\frac{d}{dx}\left(e^x\right) = e^x$$
$$\frac{d}{dx}\left(e^{f(x)}\right) = e^{f(x)} \times \frac{d}{dx}\left(f(x)\right)$$
$$\frac{d}{dx}(\sin x) = \cos x$$
$$\frac{d}{dx}(\cos x) = -\sin x$$
$$\frac{d}{dx}\left(f(x)\times g(x)\right) = f(x)\times\left(\frac{d}{dx}g(x)\right) + \left(\frac{d}{dx}f(x)\right)\times g(x)$$

Integration:

$$\int \frac{f'(x)}{f(x)}\,dx = \ln\left|f(x)\right| + c$$

Vector:

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}}$$
$$\vec{\mathbf{A}} \bullet \vec{\mathbf{B}} = |\mathbf{A}|\times|\mathbf{B}|\times\cos\theta$$

Linear kinematic equations:

$$v_f = v_i + at$$
$$\Delta x = v_i t + \frac{1}{2}at^2$$
$$\Delta x = \frac{(v_i + v_f)}{2}\times t$$
$$v_f^2 = v_i^2 + 2a\Delta x$$

Simple harmonic motion:

$$x(t) = r_0\cos(\theta_0 + \omega_0 t)$$
$$v_x(t) = -r_0\omega_0\sin(\theta_0 + \omega_0 t)$$
$$a_x(t) = -r_0\omega_0^2\cos(\theta_0 + \omega_0 t)$$
$$= -\omega_0^2 x(t)$$
$$\omega^2 = \frac{k}{m}$$
$$T = 2\pi\sqrt{\frac{m}{k}}$$
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$
$$v_{max} = \sqrt{\frac{k}{m}}A$$
$$a_{max} = \frac{k}{m}A$$

Retarding force:

$$R = -bv$$

Projectile motion:

$$y = -\frac{g}{2{v_{0x}}^2}\left(x - x_0\right)^2 + \frac{v_{0y}}{v_{0x}}\left(x - x_0\right) + y_0 \text{, or}$$

$$y = x\tan\theta - \frac{gx^2}{2{v_0}^2}\left(1 + \tan^2\theta\right)$$

Uniform circular motion:

$$\theta = \frac{\text{arc length}}{\text{radius}} = \frac{s}{r}$$

$$\omega = \frac{d\theta}{dt} = \frac{2\pi}{T}$$

$$v = r\omega$$

$$a_c = r\omega^2 = \frac{v^2}{r}$$

Non-uniform circular motion:

$$a_t = \frac{dv}{dt}$$

$$a = \sqrt{a_c^2 + a_t^2}$$

Relative velocity:

$$\vec{\mathbf{V}}_{P/A} = \vec{\mathbf{V}}_{P/B} + \vec{\mathbf{V}}_{B/A}$$

Newton's second law:

$$\sum \vec{F} = m\vec{a}$$

Newton's third law:

$\vec{F}_{12} = -\vec{F}_{21}$, and the two forces act on the opposite body.

Frictional force:

$f_s \le \mu_s n,$ and impending motion occurs when $f_s = \mu_s n.$

$$f_k = \mu_k n$$

Translational kinetic energy:

$$KE_{\text{translational}} = \frac{1}{2}mv^2$$

Gravitational potential energy:

$$\Delta PE = mg(y_2 - y_1)$$
$$= mgh$$

Spring

$$F_{\text{spring}} = -kx$$

Elastic $PE_{\text{spring}} = \frac{1}{2}kx^2$

Work-energy theorem:

$$W = \Delta KE = KE_f - KE_i$$

Work done (constant force):

$$W = \vec{F} \bullet \Delta\vec{r} = F\Delta r\cos\theta$$

Work done (varying force):

$$W = \int_{x_i}^{x_f} \vec{F} \bullet d\vec{r}$$

Power:

$$P = \frac{dW}{dt} = F \times v$$

Linear momentum:

(also valid for *y* and *z*)

$$p_x = mv_x$$

$$F_x = \frac{dp_x}{dt}$$

$$F_{\text{average, }x} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} F_x(t)\, dt$$

Conservation of linear momentum:

$$\vec{p}_{1,i} + \vec{p}_{2,i} = \vec{p}_{1,f} + \vec{p}_{2,f}$$

Elastic collision:

$$v_{A1} - v_{B1} = -(v_{A2} - v_{B2})$$

$$v_{A2} = \left(\frac{m_A - m_B}{m_A + m_B}\right)v_{A1} + \left(\frac{2m_B}{m_A + m_B}\right)v_{B1}$$

$$v_{B2} = \left(\frac{m_B - m_A}{m_A + m_B}\right)v_{B1} + \left(\frac{2m_A}{m_A + m_B}\right)v_{A1}$$

Center of mass
(also valid for *y* and *z*)

$$x_{\text{cm}} = \frac{1}{M}\sum mx, \text{ where } M = \sum m, \text{ or}$$
$$= \frac{1}{M}\int x\, dm$$
$$v_{\text{cm},x} = \frac{1}{M}\sum mv_x$$
$$a_{\text{cm},x} = \frac{1}{M}\sum ma_x$$

Simple Harmonic Motion:

$$a_x = -\omega^2 x$$
$$\omega^2 = \frac{k}{m}$$
$$T = 2\pi\sqrt{\frac{m}{k}}$$
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$
$$v_{max} = \sqrt{\frac{k}{m}}A$$
$$a_{max} = \frac{k}{m}A$$

Retarding force:
$R = -bv$

Rotation

$$\theta = \frac{s}{r}$$
$$\omega = \frac{d\theta}{dt}$$
$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$
$$v_{\text{tangential}} = r\omega$$
$$a_{\text{centripetal}} = r\omega^2 = \frac{v^2}{r}$$
$$a_{\text{tangential}} = r\alpha$$

Rotational kinematic equations:

$$\omega_f = \omega_i + \alpha t$$
$$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$$
$$\Delta\theta = \frac{(\omega_i + \omega_f)}{2} \times t$$
$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

Moment of inertia:

$$I = \sum mr^2$$
$$I = \int r^2 dm$$

Parallel axis theorem

$$I = I_{cm} + Md^2$$

Rotational kinetic energy:

$$KE_{\text{rotational}} = \frac{1}{2}I\omega^2$$

Moment of inertia:

$$I = \sum m_i r_i^2$$
$$I = \int r^2 dm$$

Moment of inertia of hoop or thin cylindrical shell:

$$I_{CM} = MR^2$$

Moment of inertia of hollow cylinder:

$$I_{CM} = \frac{1}{2}M\left(R_1^2 + R_2^2\right)$$

Moment of inertia of solid cylinder:

$$I_{CM} = \frac{1}{2}MR^2$$

Moment of inertia of rectangular plate:

$$I_{CM} = \frac{1}{12}M\left(a^2 + b^2\right)$$

Moment of inertia of thin rod about its CM:

$$I_{CM} = \frac{1}{12} ML^2$$

Moment of inertia of thin rod about its end:

$$I = \frac{ML^2}{3}$$

Moment of inertia of solid sphere:

$$I_{CM} = \frac{2}{5} MR^2$$

Moment of inertia of spherical shell:

$$I_{CM} = \frac{2}{3} MR^2$$

Torque and Moment of force:

$$\tau = F \times r \times \sin\theta$$

$$\sum \tau = I\alpha$$

Work due to rotation:

$$\Delta W = \tau \times \Delta\theta$$

$$W = \int_{\theta_1}^{\theta_2} \tau \, d\theta$$

Power due to rotation:

$$P = \tau \times \omega$$

Pure rolling:

$$v_{cm} = r\omega$$

$$a_{cm} = r\alpha$$

$$KE_{\text{total}} = \frac{1}{2} M v_{\text{cm}}^2 + \frac{1}{2} I_{\text{cm}} \omega^2$$

Conservation of Angular momentum:

$$\sum_{\substack{\text{Before}\\ \text{collision}}} I\omega = \sum_{\substack{\text{After}\\ \text{collision}}} I\omega$$

Avogadro's number:

$$N_A \approx 6.022 \times 10^{23} \text{ particles/mole}$$

Ideal gas constant:

$$R \approx 8.314 \text{ J mole}^{-1} \text{ K}^{-1}$$

Boltzmann constant:

$$k = \frac{R}{N_A} \approx 1.381 \times 10^{-23} \text{ J/K}$$

Ideal gas equation:

$$pV = nRT$$

Speed of a molecule

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

Mole

$$n = \frac{N}{N_A}$$

$$n = \frac{m}{M}$$

Absolute temperature

$$T = T_{\text{Celsius}} + 273.15$$

Specific heat *c* and molar heat *C*

$$Q = Mc(T_2 - T_1)$$
$$= nC(T_2 - T_1)$$

Latent heat *L*

$$Q = L \times M$$

First law of thermodynamics

$$\Delta U = Q - W$$

Internal energy for *N* molecules

monatomic: $U = N \times \frac{3}{2} kT$

diatomic: $U = N \times \frac{5}{2} kT$

Relationship between C_p and C_v for an ideal Gas

$$C_p - C_v = R$$

Isochoric process

monatomic: $C_V = \frac{3}{2}R$

diatomic: $C_V = \frac{5}{2}R$

C_p is undefined for isochoric process.

Isobaric process

monatomic: $C_p = \frac{5}{2}R$

diatomic: $C_p = \frac{7}{2}R$

C_V is undefined for isobaric process.

Isothermal process

$$W = nRT\ln\left(\frac{V_f}{V_i}\right)$$

Adiabatic process

$$\gamma = \frac{C_p}{C_V}$$

monatomic: $\gamma = \frac{5}{3}$

diatomic: $\gamma = \frac{7}{5}$

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$p_1^{1-\gamma}T_1^{\gamma} = p_2^{1-\gamma}T_2^{\gamma}$$

Thermal efficiency of a heat engine

$$e = 1 - \frac{|Q_C|}{|Q_H|}$$

Coefficient of performance of a heat pump for cooling

$$\mathrm{COP} = \frac{|Q_C|}{|Q_H| - |Q_C|}$$

Entropy

$$\Delta S = \int_{\substack{\text{initial}\\ \text{state}}}^{\substack{\text{final}\\ \text{state}}} \frac{dQ}{T} \geq 0$$

Coulomb's Law:

$$F = \frac{kq_1q_2}{r^2}$$

where

$k = 8.9875\times10^9\ \mathrm{Nm^2/c^2}$

Electric field:

$$E = \frac{F}{q_0}$$

Electric field for point charge:

$$E = \frac{kq}{r^2}$$

Electric field at a distance along central axis of charged ring:

$$E = \frac{kQx}{(x^2+a^2)^{3/2}}$$

Electric potential:

$$V = \frac{U}{q_0}$$

Electric potential in uniform field:

$$\Delta V = -Ed$$

Electric potential for point charge:

$$V = \frac{kq}{r}$$

Electric potential at a distance along central axis of charged ring:

$$V = \frac{kQ}{\sqrt{x^2+a^2}}$$

Electric potential energy (for multiple point charges):

$$U = k\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$$

Electric current:
$$I = \frac{dQ}{dt}$$
$$I_{av} = nqv_d A$$

Ohm's Law:
$$I = \frac{\Delta V}{R}$$

Resistance of ohmic conductor:
$$R = \rho \frac{L}{A}$$

Resistivity:
$$\rho = \rho_0[1 + \alpha(T - T_0)]$$

Electric power:
$$P = I\Delta V = I^2 R = \frac{\Delta V^2}{R}$$

Electric energy:
$$U = P\Delta t$$

Terminal voltage:
$$\Delta V = \varepsilon - Ir$$

Resistors in series:
$$R_{eq} = R_1 + R_2 + R_3 + \cdots$$

Resistors in parallel:
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

Kirchhoff's Junction Rule:
$$\Sigma I_{in} = \Sigma I_{out}$$

Kirchhoff's Loop Rule:
$$\Sigma \Delta \mathrm{V} = 0$$

Magnetic force on a moving charge:
$$F_B = qvB\sin\theta$$

Magnetic force on a current carrying conductor:
$$F = IlB\sin\theta$$

Radius of moving charged particle in uniform magnetic field:
$$r = \frac{mv}{qB}$$

Angular speed:
$$\omega = \frac{qB}{m}$$

Period:
$$T = \frac{2\pi m}{qB}$$

Velocity selector:
$$v = \frac{E}{B}$$

Magnetic field due to long straight conductor:
$B = \frac{\mu_0 I}{2\pi a}$ where

$\mu_0 = 4\pi \times 10^{-7}$ Tm/A

Magnetic field at center of circular wire loop:
$$B = \frac{\mu_0 I}{2R}$$

Magnetic field at a distance along central axis of current-carrying ring:
$$B = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$$

Magnetic force between parallel conductors:
$$F = \frac{\mu_0 I_1 I_2 l}{2\pi a}$$

Magnetic field inside toroid:
$$B = \frac{\mu_0 N I}{2\pi r}$$

Magnetic field inside solenoid:
$$B = \mu_0 n I$$

Magnetic flux:

$$\Phi_B = BA\cos\theta$$

Faraday's Law:

$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

Motional emf:

$$I = \frac{Blv}{R}$$

Power input:

$$P = F_{app}v = IlBv$$

Mass

Electron: 9.1094×10^{-31} kg

Proton: 1.6726×10^{-27} kg

Neutron: 1.6749×10^{-27} kg

Charge

Electron: -1.6022×10^{-19} C

Proton: $+1.6022\times10^{-19}$ C

Neutron: 0 C