Chapter 8 Recursion

Ground Rules

- Switch off your handphone and pager
- Switch off your laptop computer and keep it
- No talking while lecture is going on
- No gossiping while the lecture is going on
- Raise your hand if you have question to ask
- Be on time for lecture
- Be on time to come back from the recess break to continue the lecture
- Bring your lecturenotes to lecture

Iteration versus Recursion

- Most of the time, you can express a problem more elegantly using recursion.
- E.g. summation of numbers from 1 to n (in iterative form)

```
sum(n)  =  1 + 2 + … + (n-1) + n
```

$$= \sum_{i=1}^{n} i$$

```
        =  for (i=1;i<=n;i++)
              sum = sum+i;
           return sum;
```

In Recursion Form

- Summation of numbers from 1 to n using *recursion*.

```
sum(n)  =   1 + 2 + 3 +(n-1)+ n

        =  { 1              if (n==1)
           { sum(n-1) + n   if (n>1)

        =   if (n==1) return 1;
            else return sum(n-1) + n;
```


Recursion - basic idea

- In top-down design, you break up a problem into simpler sub-problems.
- In recursion, one or more of these sub-problems are simpler instances of the original problem.
- In practice, these algorithms can be implemented by methods calling themselves.

Another Example of Recursion

- Product of numbers from 1 to n using recursion.

```
factorial(n) = n*(n-1)*(n-2)*...*2*1

             =  { 1                  if (n==1)
                { n*factorial(n-1)   if  (n>1)

             =   if (n==1) return 1;
                 else return n*factorial(n-1);
```


Visualizing execution of a program containing recursion

- With non-recursive programs, it is natural to visualize execution by imagining control stepping through the source code.
- This can be confusing for programs containing recursion.
- Instead, it is useful to imagine each call of a method generating a copy of the method, so that if the same method is called several times, several copies are present.

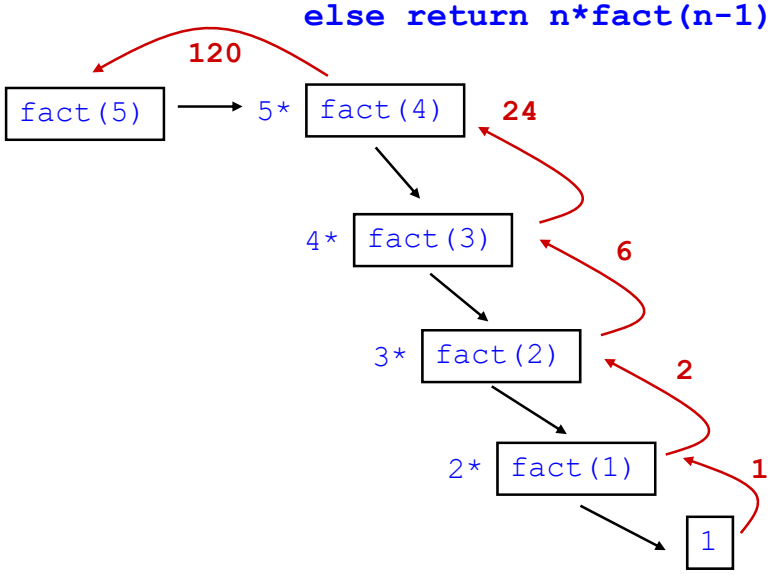
Scope

- When the method is called
  - caller is suspended,
  - "state" of caller saved in stack (LIFO – Last in first out),
  - new space allocated for variables of new method.
- With recursive call, same things happen.

How Recursion Works?

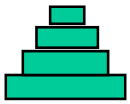
- Given.

```
fact(n) =   if (n==1) return 1;
            else return n*fact(n-1);
```



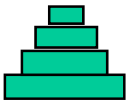
Tower of Hanoi

initial state



A B C

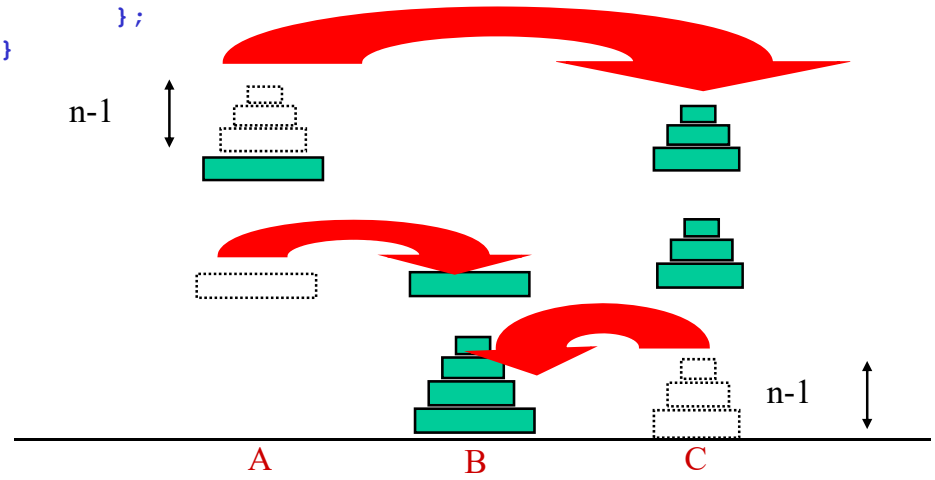
final state



A B C

Tower of Hanoi (move n disc from A to B)

```
void tower (int n,char A,char B,char c)
{
  if (n==1) move(A,B); (take the disc from A to B)
  else {
          tower(n-1,A,C,B); (move n-1 disc from A to C)
          move(A,B); (take the disc from A to B)
          tower(n-1,C,B,A); (move n-1 disc from C to B)
       };
}
```

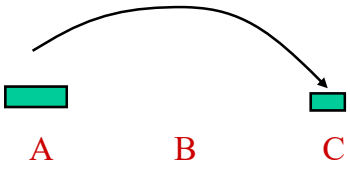


If we were to move 2 discs from A to B

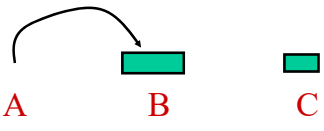
initial state

-We will need 3 steps:

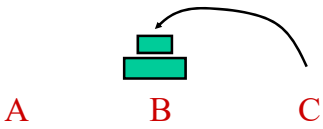
Move one disc on top from A → C



Move one disc on top from A → B



- Move one disc on top from C → B



T (4, A, B, C)

T (3, A, C, B) ⇒ T (3, A, C, B)

T (2, A, B, C) ⇒ T (2, A, B, C)

A➔ C

A➔B

C➔B

A➔ C

T (2, B, C, A) ⇒ T (2, B, C, A)

B➔ A

B➔C

A➔C

A ➔B

T (3, C, B, A) ⇒ T (3, C, B, A)

T (2, C, A, B) ⇒ T (2, C, A, B)

C➔ B

C➔A

B➔A

C➔B

T (2, A, B, C) ⇒ T (2, A, B, C)

A➔ C

A➔B

C➔B

```
void tower (int n,char A,char B,char c)
{
  if (n==1) move(A,B); (take the disc from A to B)
  else {
          tower(n-1,A,C,B); (move n-1 disc from A to C)

          move(A,B); (take the disc from A to B)

          tower(n-1,C,B,A); (move n-1 disc from C to B)
       };
}
```

A➔ C
A➔B
C➔B
A➔C
B➔ A
B➔C
A➔C
A➔B
C➔ B
C➔A
B➔A
C➔B
A➔ C
A➔B
C➔B

Recursion - how to

Ask the following

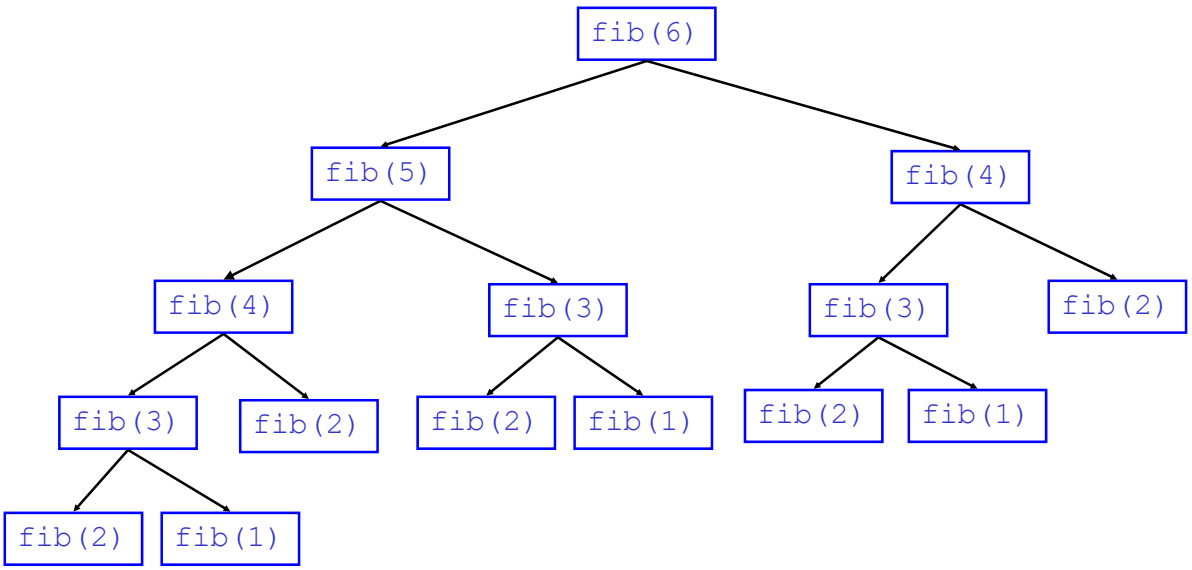
- How can you solve the problem using the solution of a "simpler" instance of the problem?
- Can you be sure to have a "simplest" input? (If so, include separate treatment of this case.)
- Can you be sure to reach the "simplest" input?

Fibonacci numbers

- Fibonacci series is a sequence where the first two numbers are 1, and a number in the sequence is the sum of the previous two numbers, i.e., 1, 1, 2, 3, 5, 8,…
- Naïve method for calculating the *n*th Fibonacci number recursively:

```
int fib(int n)
{
   if (n <= 2)
      return 1;
   else
      return fib(n-1)+fib(n-2);
}
```


Tracing Fibonacci Calls



Verify the Output of function f:

```
int f (int x)
{
  if (x>9) return 9;
  else if (x>5) return 5;
       else return 3+f(x+1);
}
```

```
f(1) = 20
f(2) = 17
f(3) = 14
f(7) = ?
f(199) = ?
```


Complete the following function in iterative form to produce the integral value of `this1` in reverse magnitude, i.e. `reverse (1234)` will return the integral value `4321`.

```
int reverse (int this1)
{
    :
    :
  return ..;
}
```

Answer:

```
int reverse (int this1)
{
  int sum=0, remainder;

  while (this1 !=0)
  {
    remainder = this1%10;
    sum = sum*10 + remainder;
    this1 = this1/10;
  }

  return sum;
}
```

Now write the function in recursive form!

```
int reverse (int this1)
{
  int sum=0, remainder;

  while (this1 !=0)
  {
    remainder = this1%10;
    sum = sum*10 + remainder;
    this1 = this1/10;
  }

  return sum;
}
```

reverse (1234) = 4321.

Answer:

```
int recur (int this1)
{
  static int sum=0;
  int remainder;

  if (this1==0) return 0;
  else
  {
    remainder = this1%10;
    sum = sum*10 + remainder;
    recur(this1/10);
  }

  return sum;
}
```

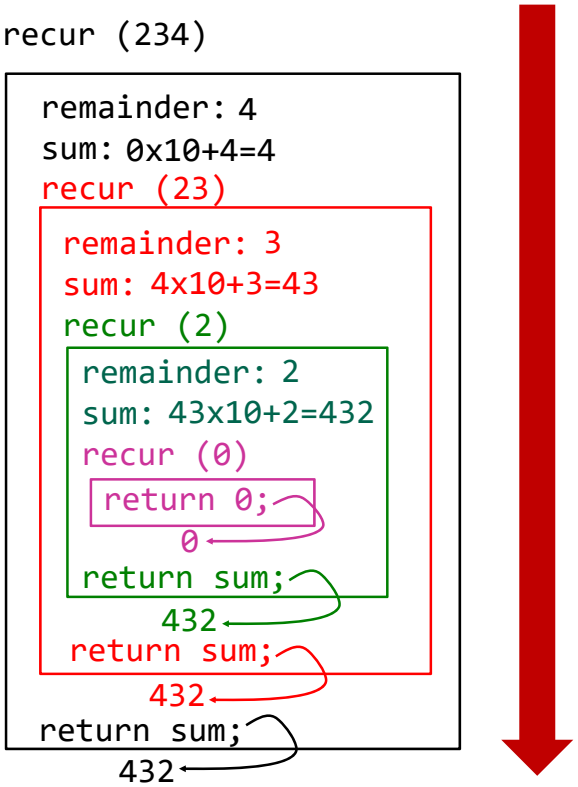
recur (1234) = 4321.


```
int recur (int this1)
{
  static int sum=0;
  int remainder;

  if (this1==0) return 0;
  else
  {
    remainder = this1%10;
    sum = sum*10 + remainder;
    recur(this1/10);
  }

  return sum;
}
```

recur (234) = 432.



```
int recur (int this1)
{
  static int sum=0;
  int remainder;

  if (this1==0) return 0;
  else
  {
    remainder = this1%10;
    sum = sum*10 + remainder;
    recur(this1/10);
  }

  return sum;
}
```

recur (1234) = 4321.

```
int recur2 (int this1)
{
  static int sum=0;
  int remainder;

  if (this1==0) return 0;
  else
  {
    recur2(this1/10);
    remainder = this1%10;
    sum = sum*10 + remainder;
  }

  return sum;
}
```

recur2 (1234) = 1234.


```
int recur2 (int this1)
{
  static int sum=0;
  int remainder;

  if (this1==0) return 0;
  else
  {
    recur2(this1/10);
    remainder = this1%10;
    sum = sum*10 + remainder;
  }

  return sum;
}
```

recur2 (1234) = 1234.