Adjoint Electron-Photon with ITS & Generalized Particle Concept

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Adjoint Electron-Photon with ITS

- Coupled electron-photon transport to solve Boltzmann-Fokker-Planck (BFP) equation

The adjoint BFP:

$$-\vec{\Omega}\cdot\vec{\nabla}\psi^\dagger + \sigma_t(\vec{r},E)\,\psi^\dagger = Q^\dagger(\vec{r},E,\vec{\Omega}) + \int_0^\infty dE' \int_0^{4\pi} d\vec{\Omega}'\,\sigma_s(\vec{r},E\to E',\vec{\Omega}\to\vec{\Omega}')\,\psi^\dagger(\vec{r},E',\vec{\Omega}')$$

$$+\frac{\alpha(\vec{r},E)}{2}\left(\frac{\partial}{\partial\mu}\left[(1-\mu^2)\frac{\partial}{\partial\mu}\psi^\dagger\right] + \frac{1}{1-\mu^2}\frac{d^2}{d\phi^2}\psi^\dagger\right) - \frac{\partial}{\partial E}\left[S(\vec{r},E)\,\psi^\dagger\right] + \left(\frac{\partial}{\partial E}S(\vec{r},E)\right)\psi^\dagger$$

- The 1st line contains the Boltzmann terms
- The 2nd line contains the Fokker-Planck terms
  - Represent electron interactions with small deflections or energy loss
- α is the restrictive momentum transfer & S is the restrictive stopping power

Forward-Adjoint Review

- Inner product gives us desired response

$$(\Psi, R) = \left(\Psi, Q^{\dagger}\right) = \lambda = \left(\Psi^{\dagger}, Q\right)$$

- Use adjoint source from solved from response function to calculate adjoint flux
- Forward transport is good for restricted source and broad response
- Adjoint transport is good for broad source and restricted response

Overview of Previous Methods

- ITS
  - System of Continuous-Energy electron-photon codes
  - No adjoint capability
  - Difficult to implement the adjoint method
  - The ACCEPT code used for complex 3D geometries
- CEPXS/ONELD
  - 1D electron/photon discrete ordinates transport
  - Multigroup-energy allows both forward and adjoint transport
  - Memory and geometry constrains limit it to 1D

Overview of Method

- MITS
  - The Multigroup/Continuous-Energy (hybrid) Integrated TIGER Series (ITS)
  - CEPXS/ONELD cross-section generator
  - ITS input/output and combinational geometry routines
  - Converts the multigroup-Legendre format cross sections into cumulative probability distributions for CM scattering
  - A particle’s group index is needed for Boltzmann scattering
  - A particle’s discrete energy is needed for Fokker-Planck energy loss

Comparison/Benchmark

- Similar to NOVICE the 1st electron Monte Carlo transport with adjoint capabilities
- Algorithm to solve BFP developed by Morel et al.
- Tested in MCNP
- Benchmarked in 1D and 3D to experimental data and other transport codes

Generalized Particle Concept for Adjoint Photon-Electron Transport

- Development of a generalized particle transport
- Applying transport for photon, electron, and positron transport
- Development of adjoint and forward cross sections
- Testing and comparison

Generalized Particle Transport

- Phase space is extended to include a discrete coordinate which corresponds with particle type
- Cross-section are modified to include the extended phase space
  - A new scattering cross section is developed
- Only one outgoing particle is tracked and weighted

Integral Transport Equations

- From the modified Boltzmann equation is used to solve in integral form using collision densities

$$\begin{cases} \chi^{+}(\boldsymbol{r}, E, \boldsymbol{\Omega}, \zeta) = \int T(\boldsymbol{r}' \to \boldsymbol{r} | E, \boldsymbol{\Omega}, \zeta) \psi^{+}(\boldsymbol{r}', E, \boldsymbol{\Omega}, \zeta) \, \mathrm{d}\boldsymbol{r}' \\ \psi^{+}(\boldsymbol{r}, E, \boldsymbol{\Omega}, \zeta) = \sum_{\zeta'=1}^{N} \iint C^{+}(E', \boldsymbol{\Omega}', \zeta' \to E, \boldsymbol{\Omega}, \zeta | \boldsymbol{r}) \times \chi^{+}(\boldsymbol{r}, E', \boldsymbol{\Omega}', \zeta') \, \mathrm{d}E' \, \mathrm{d}\boldsymbol{\Omega}' \\ \qquad + D(\boldsymbol{r}, E, -\boldsymbol{\Omega}, \zeta). \end{cases}$$

where

$$T(\boldsymbol{r}' \to \boldsymbol{r} | E, \boldsymbol{\Omega}, \zeta) = \frac{\Sigma_t(\boldsymbol{r}, E, \zeta) \exp(-\tau(\boldsymbol{r}' \to \boldsymbol{r} | E, \zeta)) \delta\left(\boldsymbol{\Omega} - \frac{\boldsymbol{r} - \boldsymbol{r}'}{|\boldsymbol{r} - \boldsymbol{r}'|}\right)}{|\boldsymbol{r} - \boldsymbol{r}'|^2}$$

Where $\chi^{\dagger}$ is the incoming collision density, $\Psi^{\dagger}$ is the outgoing, and τ is the optical distance

Integral Transport Equations

- The adjoint collision kernel is

$$C^{+}(E',\mathbf{\Omega}',\zeta' \to E,\mathbf{\Omega},\zeta|\boldsymbol{r}) = \frac{\Sigma(E,-\mathbf{\Omega},\zeta \to E',-\mathbf{\Omega}',\zeta'|\boldsymbol{r})}{\Sigma_t(\boldsymbol{r},E',\zeta')}$$

- The collision kernel must be weighted to take into account the single particle biasing

$$\sum_{\zeta'=1}^{N} \iint C(E,\mathbf{\Omega},\zeta \to E',\mathbf{\Omega}',\zeta'|\boldsymbol{r})\,\mathrm{d}E'\,\mathrm{d}\mathbf{\Omega}' = \frac{\sum_{m=1}^{\infty} m\Sigma_m(\boldsymbol{r},E,\zeta)}{\Sigma_t(\boldsymbol{r},E,\zeta)}$$

where Σ_m is the cross section of a process with m outgoing particles regardless of their type.

Implementing for photon-electron-positron transport

- Secondary particles with fixed energy could be transitioned toby introducing singular members
- 3 photon interactions were taken into account
  - Photoelectric effect
  - Compton scattering
  - Electron-positron pair generation
- CSDA and Moliere multiple scattering were used for electron transport

Cross-sections and testing

- For both the forward and adjoint cross sections were developed to include the extended phase space
- Forward and adjoint were compared
    - Results found to be within statistical error bars of each other