

# Update: Electron Mode in FRENSE

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## Hybrid Multigroup/Continuous-Energy Monte Carlo using Boltzmann-Fokker-Planck

- Advantages
- Boltzmann-Fokker-Planck Equation (BFP)
- Modifications to BFB
- Solution to Modified BFG
- Monte Carlo Method
- Adjoint
- Other Possible Adjoint Methods



- The same basic multigroup cross-section data can be used for forward and adjoint calculations.
- The adjoint transport model is nearly identical to the forward making implementation easy
- The transport equation is generalized for Monte Carlo transport of neutral and charged particles.  
They implement for electrons and photons.

$$\begin{aligned}\Omega \cdot \nabla \psi + \sigma_t \psi = & \int_0^\infty \int_0^{2\pi} \int_{-1}^{+1} \sigma_s(E' \rightarrow E, \mu_0) \times \psi(\mu', \phi', E') d\mu' d\phi' dE' \\ & + \frac{\alpha}{2} \left\{ \frac{\partial}{\partial \mu} \left[ (1 - \mu^2) \frac{\partial \psi}{\partial \mu} \right] + \frac{1}{1 - \mu^2} \frac{\partial^2 \psi}{\partial \phi^2} \right\} + \frac{\partial}{\partial E} [S\psi] \\ & + Q\end{aligned}$$

- The Boltzmann Operator treats the large-angled or "smooth" component of the cross-section
- The Fokker-Planck Operator treats the forward-peaked or "singular" component of the cross-section

## Continuous-Scattering Operator

$$F_{\alpha}\psi = \frac{\alpha}{2} \left\{ \frac{\partial}{\partial \mu} \left[ (1 - \mu^2) \frac{\partial \psi}{\partial \mu} \right] + \frac{1}{1 - \mu^2} \frac{\partial^2 \psi}{\partial \phi^2} \right\}$$

- Constructed so the mean change in angle cosine per path length is equal to the restricted momentum transfer

$$\Delta\mu/\text{path length} = \text{restricted momentum transfer}$$

## Continuous-Slowing Down Operator

$$\frac{\partial}{\partial E}[S\psi]$$

- Constructed so the mean change in energy per path length is equal to the restricted stopping power

$$\Delta E/\text{path length} = \text{restricted stopping power}$$

Let:

$$\lim_{\mu_s \rightarrow 1} B_\alpha \psi = F_\alpha \psi$$

Where:

$$B_\alpha \psi = \int_0^{2\pi} \int_{-1}^{+1} \sigma_a(E, \mu_0) \psi(\mu', \phi', E) d\mu' d\phi' - \sigma_a \psi$$

- Eigenvalues are equal at limit
- High-order eigenvalues become more approximate and are underestimated
- Error for higher order flux moments can be ignored if they are large compared to temporal and spatial scale lengths
- Holds for condensed history where the scale lengths are large compared to mfp

Expand the cross-sections using Legendre polynomials

$$\hat{\sigma}_s(E' \rightarrow E, \mu_o) = \sum_{l=0}^L \frac{2l+1}{4\pi} \sigma_s^{(l)}(E' \rightarrow E) P_l(\mu_o)$$

Where:

$$\sigma_s^{(l)}(E' \rightarrow E) = 2\pi \int_{-1}^{+1} \sigma_s(E' \rightarrow E, \mu_o) P_l(\mu_o) d\mu_o$$

# Hybrid Multigroup/Continuous-Energy Approximation

- Break energy up into  $N$  groups such that for group  $g$ :

$$E_{g+1/2} < E < E_{g-1/2}$$

- Radau quadratures are used to get the weighted least-squares fits in energy for:

The Smooth Component Cross-Sections ( $\sigma$ )

The Restricted Momentum Transfers ( $\alpha$ )

The Restricted Stopping Power ( $S$ )

Replace the parameter,  $f$  with  $\tilde{f}$

$$\tilde{f}(E) = \sum_{g=1}^N f_g B_g(E) \quad \text{Where} \quad B_g(E) = \begin{cases} 1 & E \in (E_{g+1/2}, E_{g-1/2}) \\ 0 & \text{Otherwise} \end{cases}$$

$f_g$  is the weighted group average of  $f(E)$  using Radau quadratures



$$\begin{aligned}\Omega \cdot \nabla \psi + \tilde{\sigma}_t \psi = & \int_E^{E_{1/2}} \int_0^{2\pi} \int_{-1}^{+1} \tilde{\sigma}_s(E' \rightarrow E, \mu_0) \psi(\mu', \phi', E') d\mu' d\phi' dE' \\ & + \int_0^{2\pi} \int_{-1}^{+1} \tilde{\sigma}_\alpha(\mu_o) \psi(\mu', \phi') d\mu' d\phi' - \tilde{\sigma}_\alpha(\mu_o) \psi \\ & + \frac{\partial}{\partial E} [\tilde{S} \psi] + Q\end{aligned}$$

- The Boltzmann Operator reduces to Standard Multigroup method.
- Exponential distribution of path lengths (compared fixed path length for condensed history).
- Accuracy depends on: # of groups, Order of Legendre expansion,  $\mu_s$

Let  $E_p$  be the energy of a particle in group  $g$

- The total group cross-section is the sum of the smooth-component Boltzmann and continuous-scattering cross-sections:

$$\sigma_g^{total} = \sigma_{t,g} + \sigma_{\alpha,g} \quad \text{Where} \quad \sigma_{\alpha,g} = \frac{\alpha}{1 - \mu_s}$$

- $\sigma_g^{total}$  is used to find the distance to next collision,  $D_c$
- $D_c$  is compared to the distance to material,  $D_m$ , and distance to energy,  $D_e = \frac{E_p - E_{g+1/2}}{S_g}$
- The new energy is:

$$E_p^{new} = E_p^{old} - S_g D_c$$

Can either have a smooth-component Boltzmann or continuous-scattering reaction with probabilities:

$$P_B = \frac{\sigma_{t,g}}{\sigma_g^{total}} \quad \text{and} \quad P_\alpha = \frac{\sigma_{\alpha,g}}{\sigma_g^{total}}$$

- If  $P_\alpha$  is selected a new direction for the particle is randomly sampled based on a polar scattering angle with cosine equal to  $\mu_s$ .
- If  $P_B$  is selected the particle is removed and  $M$  new particle are generated at the collision site.
- Multiplication Factor

$$M = \frac{1}{\sigma_g^{total}} \int_{E_{g+1/2}}^{E_{g-1/2}} \sigma_s^{(0)}(E' \rightarrow E) dE' = \frac{1}{\sigma_{t,g}} \sum_{k=g}^N \sigma_{s,g \rightarrow k}^{(0)}$$



Average  $M$  must be preserved

- Let  $M = \text{Integer} + \text{Remainder} = I + R$
- Create  $I$  or  $I + 1$  particles with probability  $1.0 - R$  or  $R$ .

Energy

- Particles generated in group  $g$  has an energy range of  $E_{g+1/2} < E < E_{g-1/2}$
- Randomly sample energy from a uniform distribution.

Angle

- Sample angle based on the discrete Radau distributions.
- Separate Radau distribution for each smooth-component Boltzmann group-to-group transfer.

# Adjoint Multigroup/Continuous Energy BFP Equation

$$-\Omega \cdot \nabla \psi^\dagger + \tilde{\sigma}_t \psi^\dagger =$$

$$\begin{aligned} & \int_{E_{N+1/2}}^E \int_0^{2\pi} \int_{-1}^{+1} \tilde{\sigma}_s(E \rightarrow E', \mu_0) \psi^\dagger(\mathbf{r}, \mu', \phi', E') d\mu' d\phi' dE' \\ & + \int_0^{2\pi} \int_{-1}^{+1} \tilde{\sigma}_\alpha(\mu_0) \psi^\dagger(\mu', \phi') d\mu' d\phi' - \tilde{\sigma}_\alpha(\mu_0) \psi^\dagger \\ & - \frac{\partial}{\partial E} [\tilde{S} \psi^\dagger] + \frac{\partial S}{\partial E} \psi^\dagger + Q^\dagger \end{aligned}$$

- Let the dot product be:

$$[f, h] = \sum_{g=1}^N f_g h_g \frac{1}{\Delta E_g}$$

- The adjoint cross-section is then:

$$\sigma_{s,k \rightarrow g}^{(I)} = \sigma_{s,g \rightarrow k}^{(I)} \frac{\Delta E_g}{\Delta E_k}$$

- 1980 - Adjoint Electron Transport in the CSDA
  - Goudsmit and Saunderson Scattering
- 1995 - Adjoint Electron-Photon Transport using BFS in ITS
  - Multigroup/Continuous Energy
- 1996 - Adjoint Multigroup/Continuous Energy BFP Equation
- 2005 - Generalized Particle for Couple Adjoint  $\gamma$  -  $e^-$  -  $e^+$  Transport
  - CSDA using Molière's multiple scattering