Calculating Moments Using Sloan's Algorithm

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Moments

Legendre

$$f_l = \int_{-1}^{1} f(\mu) P_l(\mu) d\mu$$

Gauss

$$M_n = \int_{-1}^{1} \mu^n f(\mu) d\mu = \sum_{l=0}^{\text{inf}} \frac{2l+1}{2} f_l \int_{-1}^{1} \mu^n P_l(\mu) d\mu$$

Radau

$$M_n^* = M_n - M_{n+1}$$

Negative Weights

Variance

If the Variance is negative a negative weight will be preoduced

$$\sigma_i^2 = N_i / N_{i-1}$$

Therefore the variance is negative if the normalization factor N is negative.

Normalization Factor

$$N_i = \sum_{k=0}^{i} A_{i,k} M^*_{k+i}$$

Where $A_{i,k}$ are the coefficients of the orthogonal polynomial Q_n

The n=2 case

The abscissa for $x_n = 1$. Sloan shows (B-57) that the weight of the last abscissa is:

$$w_n = 1 - \sum_{i=1}^{n-1} \frac{N_{i-1}}{Q_{i-1}(1)Q_i(1)}$$

Plugging in the recursion relationship for $Q_i(x)$ (B-117)
and requiring $w_n > 0$ and solving for the mean coefficient,μ_{n-1} we get:

$$\mu_{n-1}^{max} \leq 1 - \sigma_{n-2}^2 \frac{Q_{n-3}(1)}{Q_{n-2}(1)} - G_{n-2}(1)$$

Where (B-143b):

$$G_{n-2}(1) = \frac{N_{n-2}}{[Q_{n-2}(1)]^2} \left[1 - \sum_{k=1}^{n-2} \frac{N_{k-1}}{Q_{k-1}(1)Q_k(1)} \right]^{-1}$$

The n=2 case continued

For the $n = 2$ case the reduces to:

$$\mu_1^{max} \leq 1 - \sigma_0^2 \frac{0}{Q_0(1)} - G_0(1)$$

Where:

$$G_0(1) = \frac{N_0}{[Q_0(1)]^2} = N_0 = M_0^* = f_0 - f_1 = 1 - f_1$$

Therefore sloan requires:

$$\mu_1 \leq f_1 \rightarrow \frac{M_1^*}{M_0^*} \leq f_1$$

$$\frac{-\frac{1}{3} + f_1 - \frac{2}{3} f_2}{1 - f_1} \leq f_1$$

$$2f_2 + 1 \geq 3f_1^2$$