Improved bremsstrahlung photon angular sampling in the EGS4 code system

Alex F. Bielajew*, Rahde Mohan† and Chen-Shou Chui†

November 14th, 1989

PIRS-0203

* Ionizing Radiation Standards
Institute for National Measurement Standards
National Research Council of Canada
Ottawa, Canada
K1A 0R6
Tel: 613-993-2197
Fax: 613-952-9865
internet: alex@irs.phy.nrc.ca

† Department of Medical Physics
Memorial Sloan-Kettering Cancer Center
New York, New York 10021

Abstract

One of the bremsstrahlung angular distribution formulae of Koch and Motz (1959) has been implemented in the EGS4 code. This report summarises the method by which the distribution is sampled and documents the revisions to the EGS4 system. A discussion of sampling efficiency and accuracy is given and variance reduction techniques that speed up the calculation of bremsstrahlung spectra are developed. **N. B.** An error in an early version of the code (before Oct 19, 1989) regarding the evaluation of the effective Z is fixed in this report. The "patch" to fix this error is given.

1 Introduction

The angular sampling of newly created bremsstrahlung photons in the EGS4 system (Nelson *et al.* 1985) may be oversimplified for some applications. The algorithm currently employed is to set all newly created photons in motion at a fixed angle with respect to the initiating electron direction. This fixed angle has the form $\Theta = 1/E_0$, where Θ is the scattering angle (in radians) and E_0 is the total energy of the initiating electron in units of $m_e c^2$, the rest mass of the electron (0.5110034 MeV). This angle represents the an estimate of the expected *average* scattering angle. The motivation for employing such an approximation was based on the following argument: At high energies the distribution is so strongly peaked in the forward direction that more accurate angular modeling will not significantly improve the shower development. At low energies, particularly in thick targets, multiple scattering of the initiating electron will greatly overwhelm any initial photon distribution. Therefore, the extra effort and computing time necessary to implement bremsstrahlung angular distributions was not considered worthwhile. It was recognised, however, that the above argument would break down for applications requiring thin target bremsstrahlung spectra.

What was surprising was the realisation that the assumption about multiple scattering dominance can be shown to break down *even for thick targets at low energies* (10 MeV or so) for narrow beams such as those employed in some medical linacs to produce photon beams for radiotherapy. As seen in figure 1, a 10 MeV thick tungsten target bremsstrahlung angular distribution shows about a 23% artefact in the forward direction with and without the Koch and Motz sampling introduced in this report. About 10% of the difference arises because 10% *fewer* photons emerge from the target when the Koch and Motz sampling is turned on. There is a differential self-absorption in the target depending on the angular distribution employed. Since Monte Carlo simulation of medical linacs is becoming routine, a more accurate sampling procedure is justified.

1.1 The Koch and Motz distribution

The formula employed for the angular sampling routine is 2BS of Koch and Motz (1959), which is the cross section, differential in photon energy and angle,

$$\mathrm{d}\sigma_{k,\Theta} = \frac{4Z^2 r_0^2}{137}\frac{\mathrm{d}k}{k} y \mathrm{d}y \left\{ \frac{16y^2 E}{(y^2+1)^4 E_0} - \frac{(E_0+E)^2}{(y^2+1)^2 E_0^2} + \left[\frac{E_0^2+E^2}{(y^2+1)^2 E_0^2} - \frac{4y^2 E}{(y^2+1)^4 E_0} \right] \ln M(y) \right\}, \tag{1}$$

where,

$$y = E_0\Theta;\ \frac{1}{M(y)} = \left(\frac{k}{2E_0 E}\right)^2 + \left(\frac{Z^{1/3}}{111(y^2+1)}\right)^2,$$

and, the following definitions for the variables apply:

k energy of the photon in units of $m_e c^2$

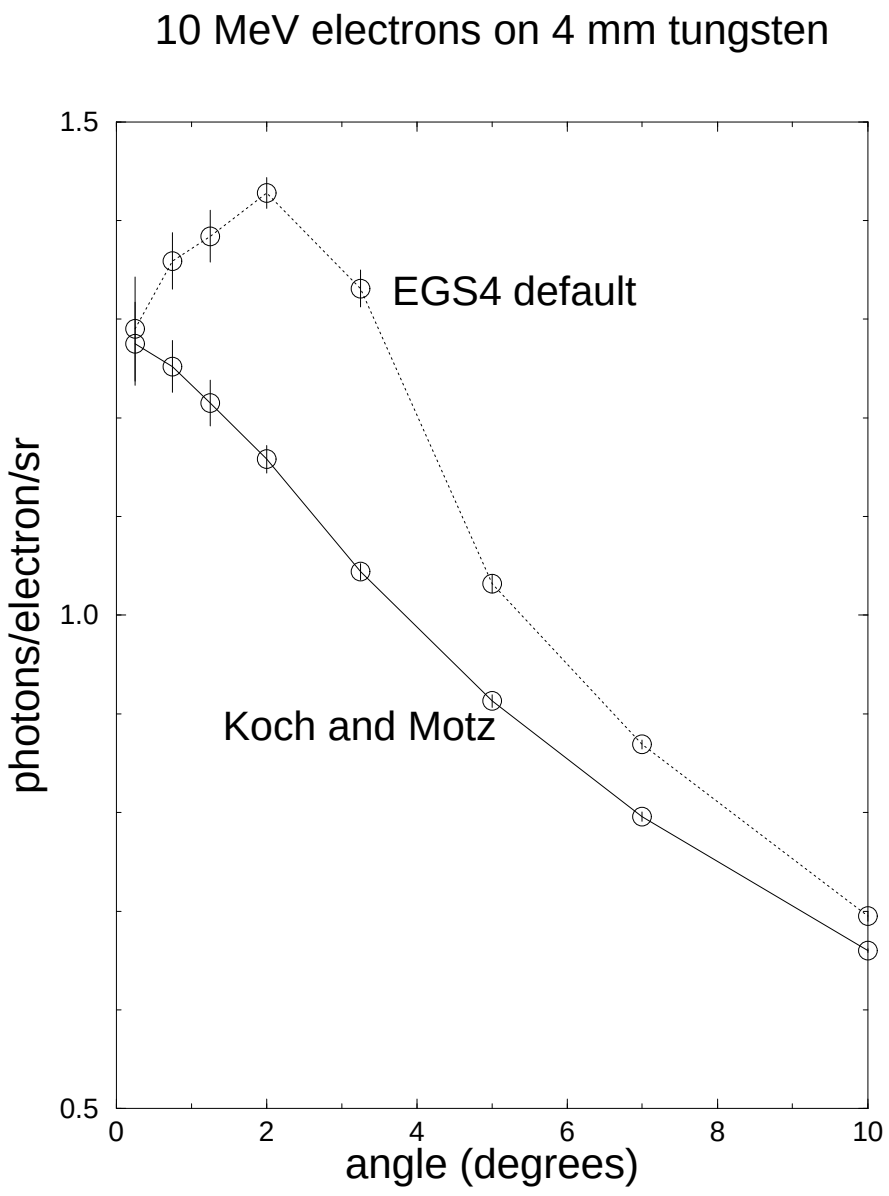

Figure 1: Total absolute bremsstrahlung angular distributions on the downstream side of a 4 m tungsten target. The data is normalised as photons/electron per unit solid angle. For this case, `AE=ECUT=1.511, AP=PCUT=0.1`. *(Figure provided by B A Faddegon, private communication.)*

Θ	angle between the outgoing photon and the incoming electron direction (in radians)
Z	atomic number of the target material
r_0	$\equiv e^2/m_e c^2$ (classical electron radius)
E_0, E	initial and final electron energy in units of $m_e c^2$

The following table, copied from the Koch and Motz article, outlines the essential approximations employed in the derivation of (1).

	Approximation	Condition of validity
i)	Approximate screening potential	$(Ze/r)e^{-r/a}$
ii)	Born approximation	$(2\pi Z/137\beta_0)$, $(2\pi Z/137\beta) \ll 1$
iii)	Extreme relativistic	$E_0, E, k \gg 1$
iv)	Small angles	$\sin\Theta = \Theta$
v)	Approximate e^- angular integration	$\Theta < (Z^{1/3}/111E_0)$

It should be noted that only the angular distribution part of (1) is employed. The cross section differential in photon energy employed by the EGS4 code is far less restrictive (Approximation (iii) plus Thomas-Fermi screening factors). For the purposes of modeling electron linacs, the ultimate test of these approximations is comparison with experiment. In this regard, Koch and Motz present encouraging data (their figure 17) that exhibits excellent agreement between experiment and (1) for 4.54 MEV electrons on Au. Although use of (1) violates constraints ii), iii) and iv) in the cases they showed, the deviation was at worst 10% (at large angles) and usually much better. In particular, violating the Born approximation constraints seemed not especially deleterious to the comparison. The conditions of this experiment are similar to those used for medical linacs (6–50 MeV, high-Z targets) and therefore the employment of (1) is justified. At lower energies, use of (1) still needs to be demonstrated. At higher energies, the constraints are not so badly violated except for the Born approximation when high-Z materials are used. Again, experimental data will judge the suitability of (1) in this context.

2 Sampling procedure

To sample the photon angular distribution, a mixed sampling procedure is employed (Nelson *et al.* 1985). Since it is the angular distribution that is required, the overall normalisation of (1) is unimportant including any overall energy-dependent factors. The following expression for $p(y)$ is proportional to (1):

$$p(y)\mathrm{d}y = f(y^2)N_r g(y^2)\mathrm{d}y^2. \qquad (2)$$

Defining $x = y^2$,

$$f(x) = \frac{1 + 1/(\pi E_0)^2}{(x+1)^2}, \tag{3}$$

and,

$$g(x) = 2r - 3(1+r^2) - [4 + \ln m(x)][(1+r^2) - 4xr/(x+1)^2], \tag{4}$$

where,

$$r = E/E_0;\ m(x) = \left(\frac{1-r}{2E_0 r}\right)^2 + \left(\frac{Z^{1/3}}{111(x+1)}\right)^2.$$

Note that $1/E_0$(high frequency limit)$\leq r \leq 1$(low frequency limit). $N_{\rm r}$ is a normalisation constant which will be discussed later.

The function $f(x)$ will be used for direct sampling. It can be easily verified that this function is normalised correctly, (*i.e.* $\int_0^{(\pi E_0)^2} f(x)\mathrm{d}x = 1$) and the candidate scattering angle is easily found by inversion to be:

$$\hat{\Theta} = \frac{1}{E_0}\sqrt{\frac{\xi}{1 - \xi + 1/(\pi E_0)^2}}, \tag{5}$$

where ξ is a random number selected uniformly on the range (0,1) and the "caret" over Θ signifies that it is a quantity determined by random selection.

The function g is sampled using the rejection technique. In order to employ this technique, the optimum case is to have the location of the maximum of the function, $x_{\rm max}$ characterised allowing the most efficient determination $N_{\rm r} = g(x_{\rm max})^{-1}$. Failing this, the next best scenario is to overestimate $g(x_{\rm max})$. The closer this estimate is to the true maximum value, the more efficient the rejection technique will be. Unfortunately, characterising g in complete generality proved to be very difficult. The following observations were made, however. The maximum value of $g(x)$ occurs at either $x = 0$, $x = (\pi E_0)^2$ (*i.e.* at the minimum or maximum values of x), or in the vicinity of $x = 1$. Therefore, the rejection function normalisation was chosen to be:

$$N_{\rm r} = \{\max[g(0), g(1), g((\pi E_0)^2)]\}^{-1}. \tag{6}$$

To justify this prescription, consider the plots of f and g depicted in figures 2 and 3 spanning four decades of incident e^- kinetic energy, 100 keV, 1 MeV, 10 MeV and 100 MeV for two elements, $Z = 1$ (figure 2) and $Z = 125$ (figure 3), a fictional "superheavy" element. The Z-dependence of g is quite weak and the span of energies should suffice for many practical problems. Each figure contains six curves for g corresponding to $k = (0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1) \times k_{\rm max}$, where $k_{\rm max} = E_0 - 1$.

From these figures one sees the motivation for utilising the mixed sampling procedure. The function f, represented by a dashed line, is strongly peaked and hence is an unsuitable candidate for the rejection technique. However, it can be inverted, requiring only the calculation of a square root, and therefore, the direct technique may be applied efficiently. The function g, is much flatter and is a more reasonable candidate for the rejection technique.

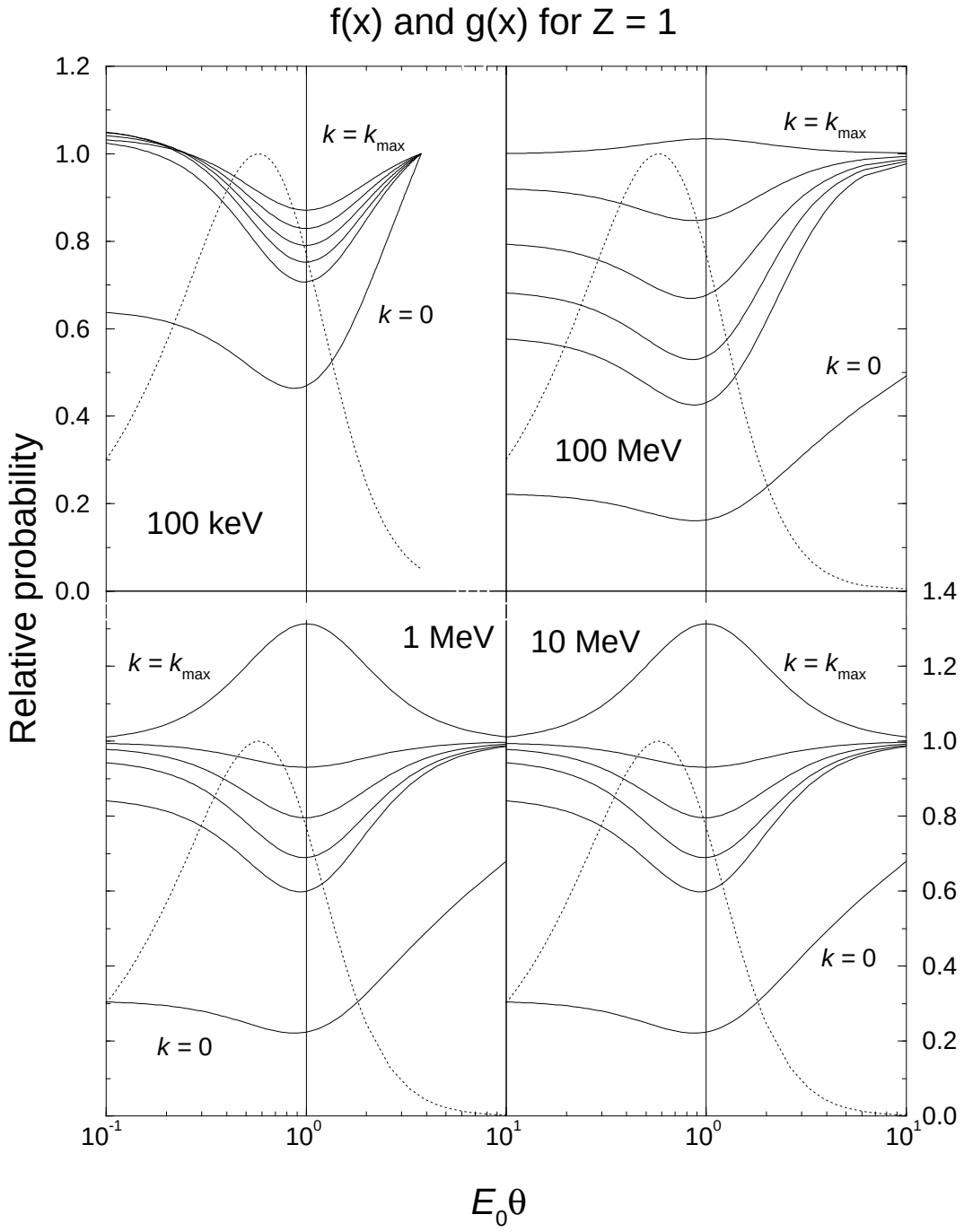

Figure 2: The functions f (dashed line) and g (solid lines) for 100 keV, 1 MeV, 10 MeV and 100 MeV for H ($Z = 1$). Six curves for g are depicted for each incident energy corresponding to $k = (0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1) \times k_{\max}$.

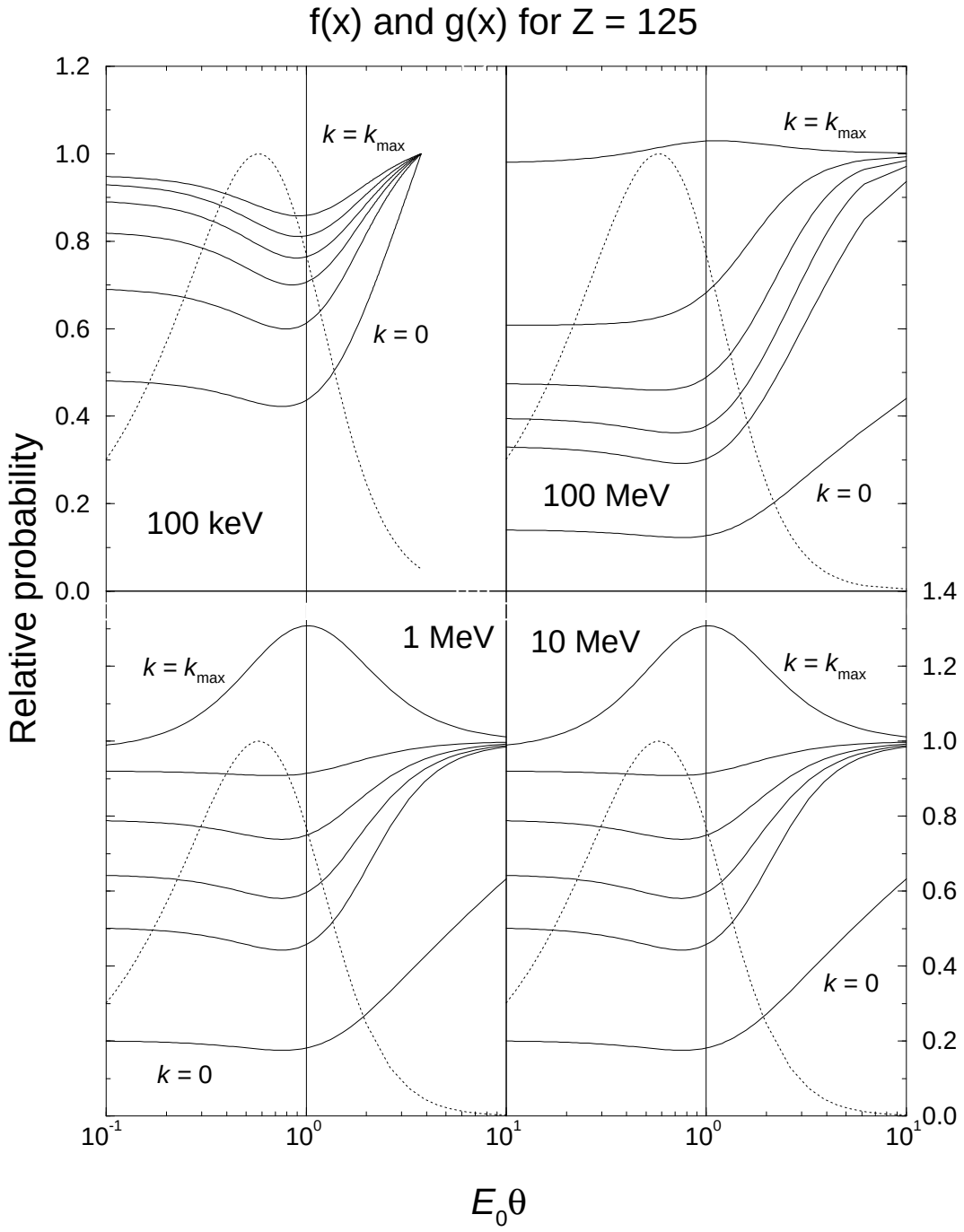

Figure 3: The functions f (dashed line) and g (solid lines) for 100 keV, 1 MeV, 10 MeV and 100 MeV for a fictional superheavy element ($Z = 125$). Six curves for g are depicted for each incident energy corresponding to $k = (0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1) \times k_{\max}$.

For most cases, one sees that the maximum occurs at either endpoint. Only for the high frequency limit for 1 MeV and higher, the maximum occurs in the vicinity of $x = 1$. In fact, the maximum occurs so close to $x = 1$ that it is not worthwhile to attempt to find the location of the maximum. The extra computer time required is not justified given the minor amount of "clipping" that will occur in these cases.

To account for electron-electron bremsstrahlung and to consider materials other than pure elements, the Z in (1) has to be estimated. The dependence of the angular distributions on Z is weak and the following estimate has been effected:

$$Z_{\text{eff}} = \sqrt{\sum_{i=1}^{N_{\text{e}}} p_i Z_i (Z_i + 1)}, \tag{7}$$

where N_{e} is the number of elements in the material, p_i is the proportion by number of the i^{th} element of the material properly normalised so that $\sum_{i=1}^{N_{\text{e}}} p_i = 1$, and Z_i is the atomic number of the i^{th} element. The factor $Z_i(Z_i + 1)$ is intended to account for electron-electron bremsstrahlung in an approximate fashion. Equation (7) is therefore used for all materials, elemental (*i.e.* $N_{\text{e}} = 1$) or not. **N. B.** In the pre-report code distribution (before Oct 19, 1989) the "square root" was missing in (7). The change is documented in the code listing given in the appendix to this report.

2.1 Sampling algorithm

The following algorithm is employed to sample the angular distribution:

1. Pick a candidate $\hat{\Theta}$ from (5).
2. Calculate N_{r} from (6).
3. Evaluate $g_{\text{test}} = N_{\text{r}} g((E_0 \hat{\Theta})^2)$.
4. Pick a random number ξ on the interval (0,1).
5. If $\xi \leq g_{\text{test}}$ accept $\hat{\Theta}$, else, go back to step 1.

This is essentially the process executed by the selection macros described in the Appendix except for some short cuts that save computing time.

2.2 Sampling efficiency

Given the initial values of E_0 and k, the theoretical efficiency of the above sampling technique can be expressed:

$$\epsilon(E_0, k, Z) = \frac{1}{N_{\text{r}}} \int_0^{(\pi E_0)^2} \mathrm{d}x f(x) g(x). \tag{8}$$

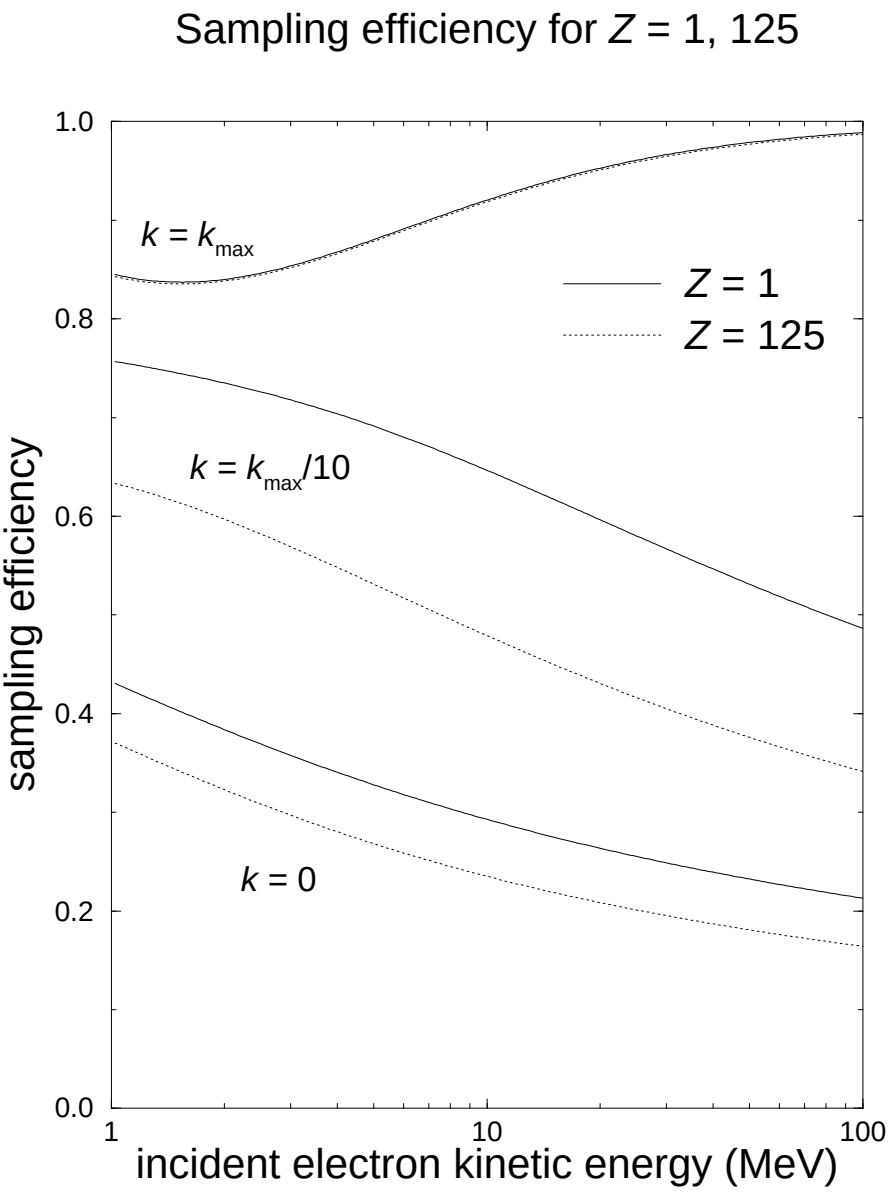

Figure 4: Sampling efficiency (equation 8) for energies from 100 keV to 100 MeV for $Z = 1$ and 125, and $k = 0, k_{\max}/10$, and $k_{\max}$.

This sampling efficiency is plotted in figure 4 for energies from 100 keV to 100 MeV for $Z = 1$ and 125, and $k = 0, k_{\max}/10$, and $k_{\max}$. One notes that the efficiency is worst for $k = 0$ and improves greatly by $k_{\max}/10$. The ultimate test of sampling efficiency is determined by the degradation of overall timing. This is discussed in a later section.

2.3 Sampling accuracy

This section concerns the incorporation of the new sampling routine into the EGS4 code and the verification that the sampling procedure works as expected. This is accomplished by comparing Monte Carlo sampled angular distributions with the theoretical expression (2). Two examples are presented. In figure 5a, angular distributions are plotted for 100 keV electrons in C ($Z = 6$) for the limits of zero ($k = 0$) and maximum ($k = k_{\max}$) photon energies. The histograms are sampled distributions from the Monte Carlo code and the smooth curves are from (1). A similar comparison is given in figure 5b except for 10 MeV electrons in W ($Z = 74$). Evidently, the sampling routines are working as expected. (A specialised user code `BREMTEST` was written for this purpose. It calls the sampling subroutine `BREMS` directly, avoiding particle transport and allows the user to pick the fraction of incident energy going into the bremsstrahlung photon.)

2.4 Timing comparisons

With respect to timing degradation, the worst performance was found in thin-target applications where electrons were "forced" to produce bremsstrahlung photons. Two examples were investigated: 100 keV electrons incident on a thin slab of C and 10 MeV electrons incident on a thin slab of W. The electrons were forced to produce bremsstrahlung photons at the surface of the slabs and then not allowed to interact again. The photons were also not allowed to interact. This simulates photon production from a vanishingly thin target. The timing degradations were about 20% for the low energy example and about 30% for the high energy example. (The user code `BREMTEST` was used for this example, avoiding all particle transport.)

For a thick target case, the degradation is not so great because the Monte Carlo code spends a good deal of time doing electron transport. For the thick target example given in figure 1, (`AE=ECUT=1.511`, `AP=PCUT=0.1`, 10 MeV electrons incident on 4 mm tungsten, PRESTA default algorithm (Bielajew and Rogers 1987), `NBRSPL=30` (see next section and the Appendix)), the timing degradation was only 13%, despite a bremsstrahlung enhancement of 30. For normal transport (no bremsstrahlung enhancement) the timing degradation would be a fraction of a percent.

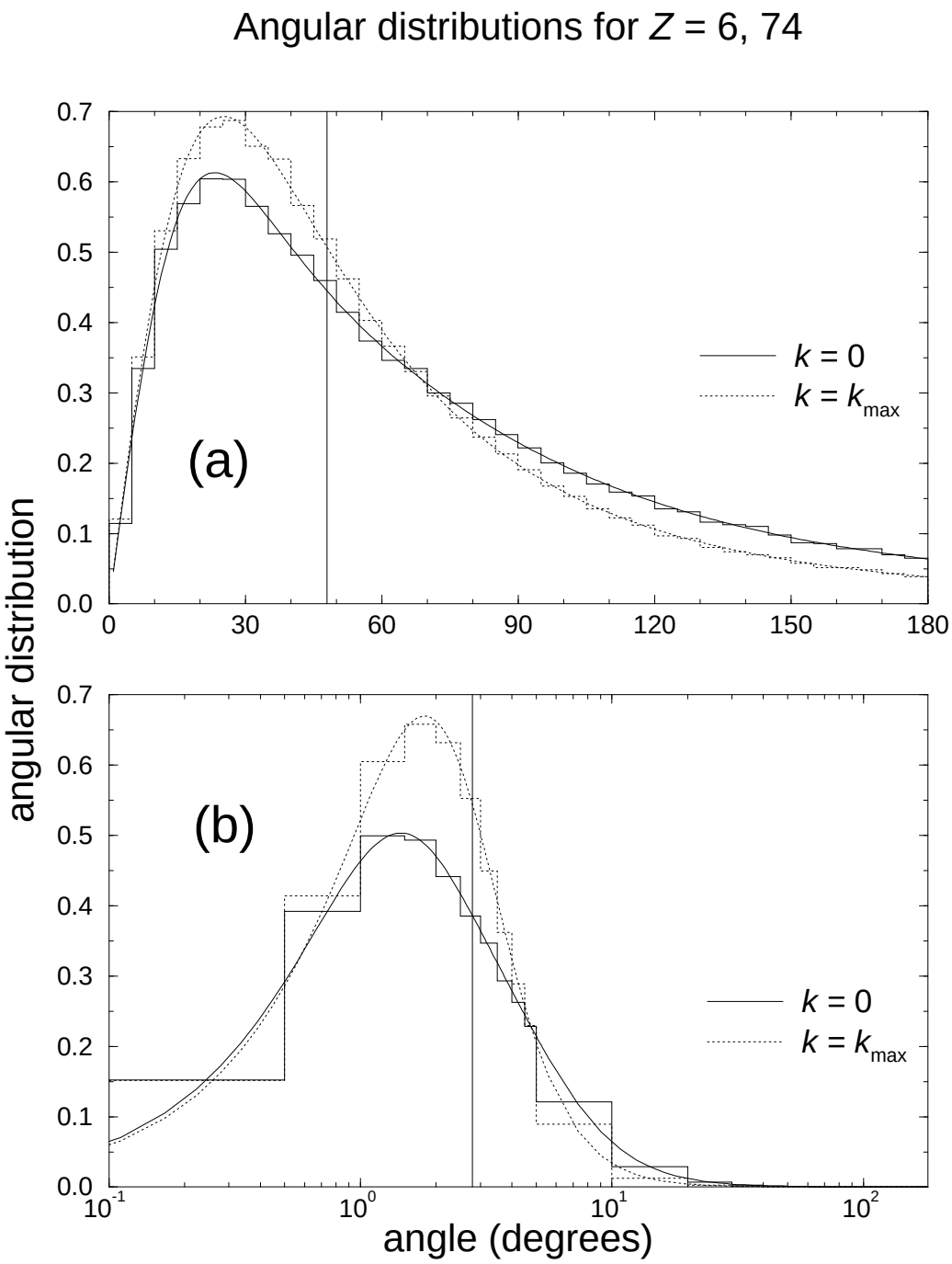

Figure 5: **(a)** Angular distributions $(yf(y^2)g(y^2)/\int_0^{\pi E_0} y\mathrm{d}yf(y^2)g(y^2))$ for 100 keV electrons in C ($Z = 6$) for $k = 0$ and $k = k_{\max}$, minimum and maximum photon energies. The histograms are sampled distributions from the Monte Carlo code and the smooth curves are from equation (1).
(b) Same as **(a)** except for 10 MeV electrons in W ($Z = 74$). The vertical line at $\Theta = 1/E_0$ is the constant angle chosen by default in EGS4.

3 Variance reduction technique—bremsstrahlung splitting

This section describes a variance reduction technique that has been found useful for determining bremsstrahlung spectra, particularly for low energy problems.

The technique works as follows: Normal electron transport is done until the point at which the bremsstrahlung interaction is sampled. Instead of creating just one photon, N photons are created with "weight" $1/N$ relative to the electron's weight. Both the energy and angular distributions are sampled N times and N new photons are placed on the particle "stack". The energy of the electron is reduced by the energy of only *one* of the photons (the first one). Although this violates detailed energy conservation, on average it is still conserved. This way of deducting energy preserves the full energy straggling of the electron's history. The newly created photons are placed on top of the stack so that the particle transport logic handles them before the electron thereby inhibiting stack "overflow". Care is taken never to exceed the stack limits and N is reduced dynamically or splitting is even "shut off" if the stack limit is approached. For high energy applications where one is interested primarily in bremsstrahlung spectra, one may gain efficiency by applying "Russian Roulette" products of pair production and Compton scatterings with the same weighting scheme thereby avoiding needless electron transport. (A fraction $1 - 1/N$ particles are "thrown away" while the fraction $1/N$ are kept with their weights increased by a factor N. Thus all electrons in the problem will have the same weight and all photons will have weight $1/N$ relative to the electron weight.)

For an example of "bremsstrahlung splitting", a report is in preparation for which a preprint will be provided by the author on request (Faddegon *et al.* 1989). At SLAC, splitting factors as high as 300 have been employed to good advantage in low energy problems (W R Nelson, private communication 1989).

4 Acknowledgements

One of us (AFB) acknowledges B. A. Faddegon for providing one of the figures in addition to suggesting the splitting technique and helping "debug" the initial version as well as D. W. O. Rogers for providing a useful critique of this report.

Appendix A Code changes for the EGS4 system

One way to get correct versions of the sampling macros is to write to the author (AFB) and request them. They are also included in the general distribution of EGS4 from SLAC. The following changes were made to the EGS4 system:

In SUBROUTINE HATCH, part of EGS4.MORTRAN:

```
                                .
                                .
                                .
"   NOW PUT OUT LINES SHOWING COMPOSITION OF MEDIUM"
"THE FOLLOWING LINE WAS CHANGED TO STORE THE ELEMENTAL COMPOSITION AFB 88/05/31"
"$UINPUT(KMPI)(MBUF(I),I=1,5),RHO(IM),NE;"
$UINPUT(KMPI)(MBUF(I),I=1,5),RHO(IM),NNE(IM);
(5A1,5X,F11.0,4X,I2);
"THE FOLLOWING LINE WAS CHANGED AS WELL AFB 88/05/31"
"$UOUTPUT(KMPO)(MBUF(I),I=1,5),RHO(IM),NE;"
$UOUTPUT(KMPO)(MBUF(I),I=1,5),RHO(IM),NNE(IM);
(5A1,',RHO=',1PG11.4,',NE=',I2,',COMPOSITION IS :');
"THE FOLLOWING LINE WAS CHANGED AS WELL AFB 88/05/31"
"DO IE=1,NE["
DO IE=1,NNE(IM)[
    "THE FOLLOWING LINE, COMMENTED OUT, WAS THE OLD WAY OF READING IN "
    "THE ELEMENTAL COMPOSITION OF EACH MEDIUM. THE INFORMATION WAS NOT"
    "PASSED ON TO EGS. IN THE PRESENT VERSION IT IS READ IN AND STORED"
    "IN COMMON BREMPR. AFB 88/05/31.                                  "
    "READ(KMPI,:BYTE:)MBUF;WRITE(KMPO,:BYTE:)MBUF;"
    $UINPUT(KMPI)
    (MBUF(I),I=1,6),(ASYM(IM,IE,I),I=1,2),
    ZELEM(IM,IE),WA(IM,IE),PZ(IM,IE),RHOZ(IM,IE);
    (6A1,2A1,3X,F3.0,3X,F9.0,4X,F12.0,6X,F12.0);
    $UOUTPUT(KMPO)
    (MBUF(I),I=1,6),(ASYM(IM,IE,I),I=1,2),
    ZELEM(IM,IE),WA(IM,IE),PZ(IM,IE),RHOZ(IM,IE);
    (6A1,2A1,',Z=',F3.0,',A=',F9.3,',PZ=',1PE12.5,',RHOZ=',1PE12.5);
    ]
                                .
                                .
                                .

"   USE STANDARD DENSITY FOR REGIONS NOT SPECIALLY SET UP"
IF (RHOR(JR).EQ.0.0)[RHOR(JR)=RHO(MD);]
]]

"BREMSSTRAHLUNG ANGULAR DISTRIBUTION INITIALIZATION - DEFAULT IS NULL"
"NEXT LINE ADDED AFB 88/05/31"
```

```
$INITIALIZE-BREMS-ANGLE;

"   SETUP IS NOW COMPLETE"
                                .
                                .
                                .
```

The following macros should be inserted at the top of your user code to perform the new angular sampling. The sampling is implemented via an option, setting `IBRDST=1`.

```
                                .
                                .
                                .
%C80
"-------------------------------------------------------------------"
"  BREMSSTRAHLUNG ANGLE SELECTION AND SPLITTING MACROS              "
"-------------------------------------------------------------------"


"THESE MACROS ARE EXPLAINED IN NRCC REPORT #PIRS0203"
"BY BIELAJEW, MOHAN AND CHUI                       "

"THIS REDEFINITION IS REQUIRED TO INCLUDE THE COMIN BREMPR IN ELECTR"
REPLACE {$COMIN-ELECTR;} WITH {;COMIN/DEBUG,BOUNDS,BREMPR,ELECIN,
  EPCONT,MEDIA,MISC,PATHCM,STACK,THRESH,UPHIIN,
  UPHIOT,USEFUL,USER,RANDOM/;}
;

"MACRO TO INITIALIZE DATA FOR BREMSSTRAHLUNG PRODUCTION               "
"THE QUANTITY ZBRANG IS ( (1/111)*Zeff**(1/3) )**2                    "
"WHERE Zeff IS DEFINED IN EQUATION (7) OF PIRS0203                    "
"THIS MACRO GOES IN SUBROUTINE HATCH                                  "
"                                                                     "
"FIXED BUG IN ZBRANG MAKING THE EXPONENT **(1/3) RATHER THAN (2/3)    "
"                                        AFB 89/10/19                 "
"                                                                     "
REPLACE {$INITIALIZE-BREMS-ANGLE;} WITH {;
    IF(IBRDST.EQ.1)[
        DO IM=1,NMED[
            ZBRANG(IM)=0.0;PZNORM=0.0;
            DO IE=1,NNE(IM)[
                ZBRANG(IM)=
                  ZBRANG(IM)+PZ(IM,IE)*ZELEM(IM,IE)*(ZELEM(IM,IE)+1.0);
                PZNORM=PZNORM+PZ(IM,IE);
                ]
                ZBRANG(IM)=(8.116224E-05)*(ZBRANG(IM)/PZNORM)**(1./3.);
            ]
        ]
}
;

"THE FOLLOWING REPLACES THE EGS4 DEFAULT $SET-BREMS-ANGLE MACRO   "
"IT'S USE REQUIRES AN ASSOCIATE MACRO $SET-BREM-REJECTION-FUNCTION"
```

```
"DEFINED BELOW                                                    "
"                                                                 "
"USAGE: IBRDST=0 => EGS4 DEFAULT ANGLE SELECTION                  "
"       IBRDST=1 => KOCH AND MOTZ (1959) EQ. 2BS ANGLE SELECTION  "
"                                                                 "
REPLACE {$SET-BREMS-ANGLE;} WITH {;
    IF(IBRDST.EQ.0)[
        "DEFAULT EGS4 ANGLE SELECTION"
        THETA=RM/EIE;
        ]
    ELSEIF(IBRDST.EQ.1)[
        "KOCH AND MOTZ (1959) EQUATION 2BS ANGLE SELECTION"
        "ZBRANG=( (1/111)*Zeff**(1/3) )**2"
        ZTARG=ZBRANG(MEDIUM);
        "TTEIE = TOTAL INITIAL ELECTRON ENERGY IN ELECTRON REST MASS UNITS"
        TTEIE=EIE/RM;
        "TTESE = TOTAL FINAL ELECTRON ENERGY IN ELECTRON REST MASS UNITS"
        TTESE=ESE/RM;
        "THIS IS THE RATIO (r IN PIRS0203)"
        ESEDEI=TTESE/TTEIE;
        "MAXIMUM VALUE OF (THETA*TTEIE)**2"
        Y2MAX=(PI*TTEIE)**2;
        "THE FOLLOWING THREE STATEMENTS DEFINE QUANTITES REQUIRED"
        "BY THE $SET-BREM-REJECTION-FUNCTION MACRO               "
        RJARG1=(1.0+ESEDEI**2);
        RJARG2=3.0*RJARG1-2.0*ESEDEI;
        RJARG3=((1.0-ESEDEI)/(2.0*TTEIE*ESEDEI))**2;
        $SET-BREM-REJECTION-FUNCTION(REJMIN,0.0E0);
        $SET-BREM-REJECTION-FUNCTION(REJMID,1.0E0);
        $SET-BREM-REJECTION-FUNCTION(REJMAX,Y2MAX);
        "ESTIMATE MAXIMUM OF THE REJECTION FUNCTION"
        "FOR LATER USE BY THE REJECTION TECHNIQUE  "
        REJTOP=MAX(REJMIN,REJMID,REJMAX);
        LOOP[
          "SAMPLE THE DIRECT PART, FUNCTION F(X) OF PIRS0203"
          "PICK A CANDIDATE Y**2 (X IN PIRS0203)           "
          $RANDOMSET Y2TST;
          Y2TST=Y2TST/(1.0-Y2TST+1.0/Y2MAX);
          "EVALUATE THE REJECTION FUNCTION AT Y2TST"
          $SET-BREM-REJECTION-FUNCTION(REJTST,Y2TST);
          $RANDOMSET RTEST;
          "LOOP UNTIL REJECTION TECHNIQUE ACCEPTS Y2TST"
          ]UNTIL(RTEST.LE.(REJTST/REJTOP));
        "CONVERT THE SUCCESSFUL CANDIDATE Y2TST TO AN ANGLE IN RADIANS"
        THETA=SQRT(Y2TST)/TTEIE;
        ]
}
;
```

```
"THIS IS THE FUNCTION G(X) OF PIRS0203                "
"THE RESULT IS RETURNED IN {P1} AS A FUNCTION OF {P2}"
"I.E. {P1}=G({P2}) WHERE {P2}=X                       "
"                                                     "
REPLACE {$SET-BREM-REJECTION-FUNCTION(#,#);} WITH {;
Y2TST1=(1.+{P2})**2;
{P1}= (4.+LOG(RJARG3+ZTARG/Y2TST1))*(4.*ESEDEI*{P2}/Y2TST1-RJARG1)+RJARG2;
}
;

"THESE MACROS REDEFINE THE COMMON BLOCK BREMPR (A STANDARD EGS4 COMMON) "
"TO INCLUDE ELEMENTAL DATA REQUIRED BY THE SAMPLING ROUTINES           "
"                                                                      "
PARAMETER $MXEL=50; "MAXIMUM # OF ELEMENTS IN A MEDIUM (FROM PEGS4)"

REPLACE {;COMIN/BREMPR/;} WITH
   {;COMMON/BREMPR/
     $LGN(DL(6,$MXMED)/1,2,3,4,5,6/),
     $LGN(ALPHI,BPAR,DELPOS(2,$MXMED)),
     ASYM($MXMED,$MXEL,2),
     $LGN(WA,PZ,ZELEM,RHOZ($MXMED,$MXEL)),
     PWR2I($MXPWR2I),
     $LGN(DELCM,NNE,ZBRANG($MXMED)),
     IBRDST;
     $TYPE ASYM;
    }
                                .
                                .
                                .
```

The following macros allow bremsstrahlung splitting to be done. They also should be inserted near the beginning of the user code. In addition to defining several new variables in COMIN/BREMPR, the size of the "stack" (*i.e.* $MXSTACK) should be set to a value large enough to accommodate the increased number of photons. There are three user-defined inputs to this macro, IBRSPL=0 (no splitting) or IBRSPL=1 (splitting on), NBRSPL (# of photons created per interaction) and FBRSPL=1./FLOAT(NBRSPL) which is used for split photon weight adjustment. For high energy applications where we are interested primarily in bremsstrahlung spectra, we "Russian Roulette" products of pair production and Compton scatterings with the same weighting scheme to prevent stack overflow and needless electron transport. (A fraction 1-FBRSPL particles are "thrown away" while the fraction FBRSPL are kept with their weights increased by a factor 1/FBRSPL.) If the number of these second order electrons were not reduced, they would also produce second order bremsstrahlung that would overflow the stack.

```
                                .
                                .
                                .
"-------------------------------------------------------------"
"  BREMSSTRAHLUNG SPLITTING MACROS                           "
"-------------------------------------------------------------"

"MAXIMUM NUMBER OF PHOTONS TO MAKE PER BREM INTERACTION"
"NOT USED DIRECTLY IN THE SPLITTING MACROS BUT MAY BE USED BY THE"
"USER FOR INITIALIZATION OR DEFAULTS"

REPLACE {$MAXBRSPLIT} WITH {100}
;

"$MXSTACK MUST BE INCREASED BY AT LEAST $MAXBRSPLIT  "
"THIS IS A SUGGESTION, USER SHOULD CHANGE AS REQUIRED"

REPLACE {$MXSTACK} WITH {125}
;

"THIS MACRO REDEFINES THE COMMON BLOCK BREMPR (A STANDARD EGS4 COMMON)  "
"TO INCLUDE ELEMENTAL DATA REQUIRED BY THE SAMPLING ROUTINES AS WELL AS "
"PARAMETERS REQUIRED BY THE SPLITTING ROUTINES                          "
"                                                                       "
;
REPLACE {;COMIN/BREMPR/;} WITH
   {;COMMON/BREMPR/
     $LGN(DL(6,$MXMED)/1,2,3,4,5,6/),
     $LGN(ALPHI,BPAR,DELPOS(2,$MXMED)),
     ASYM($MXMED,$MXEL,2),
     $LGN(WA,PZ,ZELEM,RHOZ($MXMED,$MXEL)),
     PWR2I($MXPWR2I),
```

```
     $LGN(DELCM,NNE,ZBRANG($MXMED)),
     IBRDST,IBRSPL,NBRSPL,FBRSPL;
     $TYPE ASYM;
    }

; "BUFFER FLUSH"

"THIS MACRO EXCHANGES TWO POSITIONS ON THE STACK"
"NB: LATCH IS A NON-STANDARD STACK VARIABLE     "
"    REMOVE IT IF IT CAUSES PROBLEMS            "
"                                               "
REPLACE {$EXCHANGE-STACK(#,#);} WITH {;
FDUMMY = U({P2});     U({P2})     = U({P1});     U({P1})     = FDUMMY;
FDUMMY = V({P2});     V({P2})     = V({P1});     V({P1})     = FDUMMY;
FDUMMY = W({P2});     W({P2})     = W({P1});     W({P1})     = FDUMMY;
FDUMMY = E({P2});     E({P2})     = E({P1});     E({P1})     = FDUMMY;
FDUMMY = WT({P2});    WT({P2})    = WT({P1});    WT({P1})    = FDUMMY;
IDUMMY = IQ({P2});    IQ({P2})    = IQ({P1});    IQ({P1})    = IDUMMY;
"LATCH IS NON-STANDARD"
IDUMMY = LATCH({P2}); LATCH({P2}) = LATCH({P1}); LATCH({P1}) = IDUMMY;
}
;

"THIS MACRO PLACES ADDITIONAL BREMSSTRAHLUNG PHOTONS ON THE STACK   "
"RESETTING PARTICLE WEIGHTS TO MAKE THE GAME FAIR. THREE USER INPUTS"
"ARE REQUIRED:                                                      "
"                                                                   "
"IBRSPL = 0 => NO ADDITIONAL BREMSSTRAHLUNG PHOTONS (DEFAULT)       "
"       = 1 => PERFORM BREMSSTRAHLUNG SPLITTING                     "
"NBRSPL = NUMBER OF BREMSSTRAHLUNG PHOTONS CREATED/INTERACTION      "
"FBRSPL = 1/NBRSPL (USED TO ADJUST THE PARTICLE WEIGHTS)            "
"                                                                   "
"NBRSPL AND FBRSPL ARE CHANGED DYNAMICALLY IF STACK OVERFLOW MIGHT  "
"OCCUR                                                              "
"                                                                   "
"THIS MACRO IS INVOKED AFTER THE FIRST CALL THE SUBROUTINE BREMS    "
"                                                                   "
REPLACE {$PARTICLE-SELECTION-BREMS;} WITH {;
    IF(IBRSPL.EQ.1)[
        "SPLITTING HAS BEEN REQUESTED"
        IF((NBRSPL.GT.1).AND.((NP+NBRSPL).GE.$MXSTACK))[
            "STACK OVERFLOW IMMINENT, REDUCE NBRSPL, RAISE FBRSPL"
            LOOP[
                OUTPUT $MXSTACK,NBRSPL,(2*NBRSPL+1)/3;
                ('0*** WARNING ***. STACK SIZE = ',I4,' MIGHT OVERFLOW'/
                 '                 NBRSPL BEING REDUCED, ',I4,'-->',I4/);
                NBRSPL=(2*NBRSPL+1)/3;
                FBRSPL=1.0/FLOAT(NBRSPL);
```

```
        IF(NBRSPL.EQ.1)[
            "STACK IS TOO SMALL TO ALLOW SPLITTING, SHUT IT OFF"
            OUTPUT $MXSTACK;
            ('0*** WARNING ***. STACK SIZE = ',I4,' IS TOO SMALL'/
             '                 BREMSSTRAHLUNG SPLITTING NOW SHUT OFF');
            IBRSPL=0;
            ]
        "KEEP LOOPING UNTIL NBRSPL IS SMALL ENOUGH"
        ]UNTIL((NP+NBRSPL).LT.$MXSTACK);
    ]
"SHUFFLE THE ELECTRON TO THE TOP OF THE STACK"
"NPSTRT IS A POINTER TO THE ORIGINAL LOCATION OF THE ELECTRON"
IF(IQ(NP).EQ.0)[NPSTRT=NP-1;$EXCHANGE-STACK(NP,NP-1);]
ELSE[NPSTRT=NP;]
"ADJUST THE WEIGHT OF THE INITIAL PHOTON"
WT(NP-1)=WT(NP-1)*FBRSPL;
"STORE THE ENERGY OF THE INITIAL PHOTON"
FRSTBR=E(NP-1);
"RESTORE THE ELECTRON'S INITIAL ENERGY BECAUSE THE INTERACTION"
"REDUCED IT"
E(NP)=E(NP)+E(NP-1);
"TELL AUSGAB THAT A BREMSSTRAHLUNG INTERACTION HAS OCCURRED"
$AUSCALL($BREMAUSA);
"INITIALIZE THE SPLITTING COUNTER"
ICSPLT=1;
WHILE(ICSPLT.LT.NBRSPL)[
    "LOOP NBRSPL-1 TIMES (TOTAL NUMBER OF PHOTONS = NBRSPL)"
    ICSPLT=ICSPLT+1;
    "TELL AUSGAB THAT A BREMSSTRAHLUNG INTERACTION WILL OCCURRED"
    $AUSCALL($BREMAUSB);
    "SAMPLE THE BREMSSTRAHLUNG INTERACTION"
    CALL BREMS;
    "SHUFFLE THE ELECTRON TO THE TOP OF THE STACK"
    IF(IQ(NP).EQ.0)[$EXCHANGE-STACK(NP,NP-1);]
    "ADJUST THE PHOTON WEIGHT"
    WT(NP-1)=WT(NP-1)*FBRSPL;
    "RESTORE THE ELECTRON'S INITIAL ENERGY"
    E(NP)=E(NP)+E(NP-1);
    "TELL AUSGAB THAT A BREMSSTRAHLUNG INTERACTION HAS OCCURRED"
    $AUSCALL($BREMAUSA);
    "END OF THE SPLITTING LOOP"
    ]
"RESTORE THE ELECTRON'S ENERGY TO WHAT IT HAD AFTER THE"
"FIRST INTERACTION"
E(NP)=E(NP)-FRSTBR;
"PUT THE ELECTRON BACK TO IT'S ORIGINAL STACK LOCATION"
"THIS WILL PREVENT OVERFLOW BECAUSE USUALLY THE PHOTON"
"HAS LOWER ENERGY"
```

```
    $EXCHANGE-STACK(NPSTRT,NP);
    ]
;}
```

.
.
.

References

Bielajew A F and Rogers D W O 1987 **PRESTA**—the **P**arameter **R**educed **E**lectron-**S**tep **T**ransport **A**lgorithm for electron Monte Carlo transport *Nuc. Instr. and Meth.* **B18**,165–81

Faddegon B A, Ross C K and Rogers D W O 1990 Forward-directed bremsstrahlung of 10–30 MeV electrons incident on thick targets of Al and Pb *Med. Phys.* **17**, 773–85

Koch H W and Motz J W 1959 Bremsstrahlung cross-section formulas and related data *Rev. Mod. Phys.* **31**, 920–55

Nelson W R, Hirayama H, and Rogers D W O 1985 The EGS4 Code System *Stanford Linear Accelerator Report SLAC-265 (Stanford, CA 94305)*