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Implementation of the Reverse/Adjoint Monte Carlo Method into Geant4

Backwards Integration Method

- The phase space coordinates (position, direction, and energy) of a particle arriving at the detector are sampled at random.
- Weight factors are used when one of several possible interactions must be selected at random.
- The source distribution at the originating source and the particle weight are then combined to give the correct flux at the detector.

T.M. Jordan, An adjoint charged particle transport method, E.m.p. Consultants Report, 1976.

Simple Forward-Adjoint Example

- How would bold particle track be implemented?

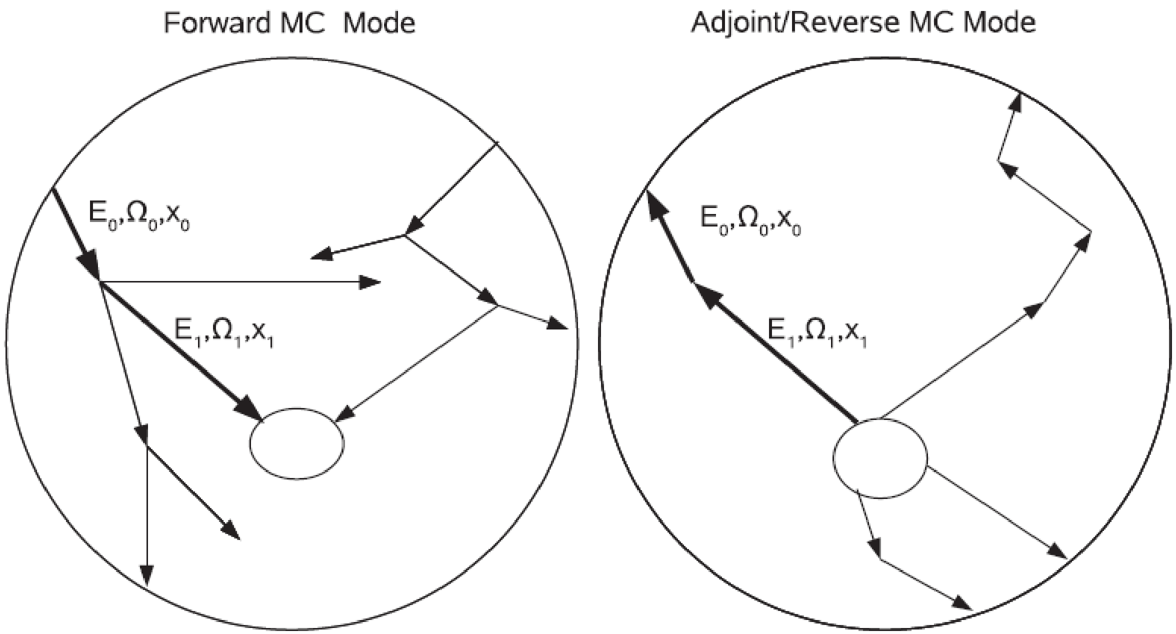

Fig. 1. Schematic view of the forward and reverse Monte Carlo integration methods. In the forward MC method primary particles are generated randomly on the external surface (big circle) and tracked randomly in the geometry. Only a limited part of the tracks reach the sensitive region (delimited by the small circle) where signals are registered. In the reverse Monte Carlo mode primary adjoint particles are generated on the boundary of or in the sensitive region and are tracked backward till the external surface with reverse processes acting on the particles. By this way the computing time is reserved only to tracks that are contributing to the detector signals.

Forward Monte Carlo Integration

- Probability primary particle *i* $(E_0\ \Omega_0)$ travels x_o then produces particle *j* $(E_1\ \Omega_1)$.

$$e^{-\int_0^{x_0} \Sigma_{i,t}(E_0,x)\,dx} N_A \frac{\partial^2 \sigma(E_0)}{\partial E_1 \partial \Omega}\, dx\, dE_1\, d\Omega_1$$

- Probability primary particle *j* $(E_1\ \Omega_1)$ travels x_1 without interaction.

$$e^{-\int_0^{x_1} \Sigma_{j,t}(E_1,x)\,dx}$$

Forward Particle Weight

- The normalized weight of the bold track is then given by

$$W = W_s e^{-\int_0^{x_0} \Sigma_{i,t}(E_0,x)\,dx} N_A \frac{\partial^2 \sigma(E_0)}{\partial E_1 \partial \Omega}\, dx\, dE_1\, d\Omega_1 e^{-\int_0^{x_1} \Sigma_{j,t}(E_1,x)\,dx}.$$

- Where W_s is the weight of the primary particle

$$W_s = f(E_0,\Omega_0)\, dE_0\, d\Omega_0 \cos\theta_s\, dS_s$$

- Where $f(E_0\ \Omega_0)$ is the differential directional flux of the primary source.

Adjoint Monte Carlo Integration

Adjoint Cross Sections

- The forward and adjoint double differential cross sections are equivalent with initial and final energy switched.

$$\frac{\partial^2 \sigma^+(E)}{\partial E' \partial \Omega} = \frac{\partial^2 \sigma(E')}{\partial E \partial \Omega}$$

- Following the forward process the adjoint weight will be

$$W = W_d e^{-\int_0^{x_1} \Sigma_{j,t}^+(E_1,x)\,dx} N_A \frac{\partial^2 \sigma(E_0)}{\partial E_1 \partial \Omega}\, dx\, dE_0 d\Omega_0 e^{-\int_0^{x_0} \Sigma_{i,t}^+(E_0,x)\,dx} W_n$$

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Adjoint Particle Weight

- W_n is the normalization factor to the external flux which is the differential directional flux, $f(E_0\ \Omega_0)$
- W_d is the weight correction associated with sampling of the adjoint primaries on the detector surface
  - Dependent on particle spectrum at detector surface, which is unknown without running the forward model.
  - For the spectrum, *I(E)*, bias with a 1/E spectrum to reduce variance
  - Uniform position distribution
  - Cosine law distribution for angle

$$W_d = n\frac{I(E_1)\,dE_1}{\int_{E_{min}}^{E_{max}} I(E)\,dE}\,\frac{\cos\theta_d\,d\Omega_1}{2\pi\int_0^{\pi}\cos\theta\sin\theta\,d\theta}\,\frac{dS_d}{\int_{A_d} dS_d}W_{prim} = n\frac{dE_1}{E_1\log\frac{E_{max}}{E_{min}}}\,\frac{\cos\theta_d\,d\Omega_1}{\pi}\,\frac{dS_d}{A_d}W_{prim}$$

Adjoint Weight Correction and Biasing

- The probability for a particle to service over a step distance *x* should be the same for both adjoint and forward particles
- To insure this, a weight correction is added after every step

$$W_{step} = e^{-\int_0^x \Sigma_t(E,x)\,dx} / e^{-\int_0^x \Sigma_t^+(E,x)\,dx}$$

- Similarly, if the differential cross section is biased a weight correction must be added

$$W_{dcs} = \frac{\partial^2 \sigma(E)}{\partial E' \partial \Omega} \bigg/ \frac{\partial^2 \sigma_{bias}(E)}{\partial E' \partial \Omega}$$

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Charged Particles Continuous Energy Loss/Gain

- In forward transport charged particles are continuously losing energy, this is accounted for by a net energy loss at each step
- Similarly, in the adjoint transport charged particles are continuously gaining energy
- Therefore, another weight correction has to be added

$$W_{dE} = \frac{dE'}{dE} = \frac{dE'/dx}{dE/dx}$$

Implementation

- How is the adjoint implemented in Geant4?

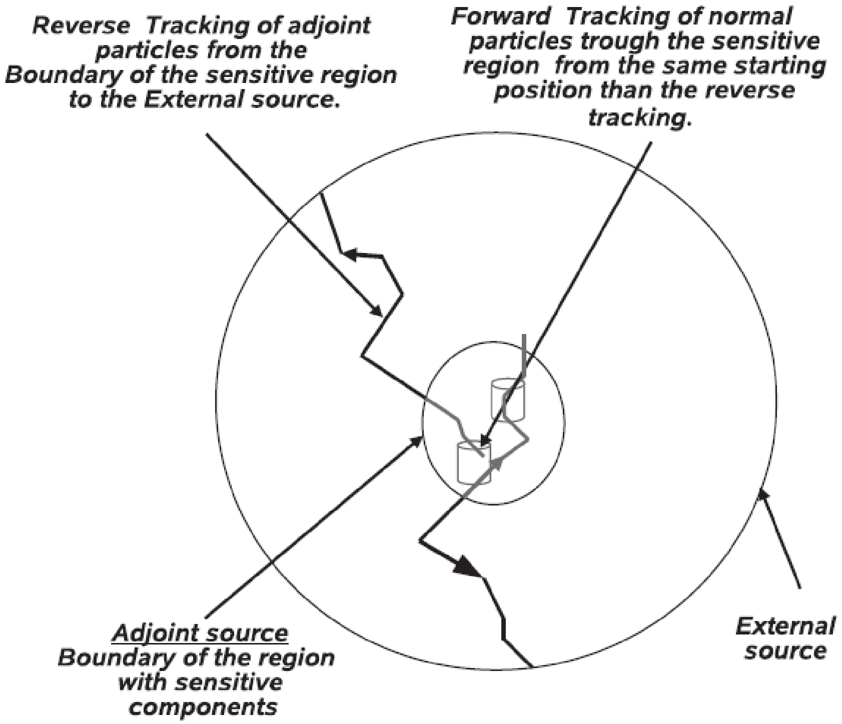

Fig. 2. Schematic view of the way we have implemented the reverse MC method in Geant4. See text for details.

Implementation

- A adjoint source region is selected with all sensitive structures inside
- From the adjoint source to the external source adjoint transport is performed
- Inside the adjoint source forward transport is performed
- Allows for implementation within Geant4 with the list change of existing code

Questions?