EE2012/ST2334 Discussion 1

Liu Zichen

14 Feb, 2019

1. [**sample space**] The color of a single pixel on the screen can usually be controlled by R, G, B components. Suppose the intensity value of R, G, B can only be taken from $\{0, 1, 2\}$.
   (a) What's the sample space of the color of a single pixel?
   (b) If a low-resolution image consists of 28×28 pixels, what's the size of sample space for such an image?
2. [**event probability**] Think of an event that has probability $p(x) = \frac{\pi}{4}$.
3. [**Law**] Write down the De Morgans Law for 3 event case.
4. [**counting**] How the multiplication principle and addition principle can be represented using a tree diagram? Come up with an example to show a $(2 \times 6 \times 2)$ and a $(2 + 3)$ case.
5. [**permutation**] $P(n, k) = \frac{n!}{(n-k)!}$. Explain the equation on the left. Does the order matter in permutation?
   When do we use $(n-1)!$ to calculate permutation?
   When do we use $P(n, (n_1, n_2, \ldots, n_k)) = \frac{n!}{n_1! n_2! \cdots n_k!}$ to calculate permutation?
6. [**combination**] $\left(\begin{array}{c} n \\ k \end{array} \right) = \frac{n!}{k!(n-k)!}$. Explain the equation on the left. What's the relationship between combination and permutation? Does the order matter in combination?
7. [**probability properties**] Use Venn Diagram to show $\Pr(A \cup B) = \Pr(B) + \Pr(A) - \Pr(A \cap B)$. To generalize, what's $\Pr(A \cup B \cup C)$?
8. [**conditional probability**] $\Pr(B|A) = \frac{\Pr(A \cap B)}{\Pr(A)}$. Explain the equation on the left. Can you make intuitive explanation for the joint probability $\Pr(A \cap B) = \Pr(A) \Pr(B|A)$?
9. [**law of total probability or marginalization**] $\Pr(B) = \sum_{i=1}^{n} \Pr(B \cap A_i) = \sum_{i=1}^{n} \Pr(A_i) \Pr(B|A_i)$.
10. [**independence**] $\Pr(A \cap B) = \Pr(A) \Pr(B)$. What condition must be satisfied for the equation on the left to hold?
    Research on mutually independent and pair-wise independent.