EE2012/ST2334 Discussion 2

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1. **[random variable]** Random variable is a function that maps the sample space to a real value space.

$$X : S \to \mathbb{R} \in R_X$$

   All the possible real values form a set called "range" R_X. Recall the pixel example in discussion 1, if I want a random variable to describe the grey level (use the simple average method: $I_{grey} = (R + G + B)/3$) of a single pixel, what's the range R_X?

2. **[equivalent events]** Choose any one pair of equivalent events from (1) and explain. Can you see why their probabilities are equal?

3. **[PMF / PDF]** For a random variable (remember it's a function) X, if its range R_X is finite or countably infinite, we call X **discrete** RV. On the other hand, if R_X is an interval or a collection of intervals, we call X **continuous** RV.

   For discrete RV, we use **probability mass function (PMF)** to describe the probability distribution of X; for continuous, we use **probability density function (PDF)** to describe the probability distribution of X.

   An ***important*** note is that PMF is a proper probability, but PDF is **NOT** a proper probability! Can you see why?

4. **[CDF]** $F(x) = \Pr(X \le x)$. PDF for continuous random variable can be otained by $f(x) = \frac{dF(x)}{dx}$ is the derivative exists.

5. **[Expectation]** Expectation tells us the average (i.e., expected) value of some function $f(x)$ taking into account the distribution of x.

$$E[f(x)] = \sum_x f(x)p(x)$$
$$E[f(x)] = \int f(x)p(x)dx$$

   - If $f(x) = x$
     - $E[f(x)] = E(x) = \mu_x$, the "mean of x"
     - If we observe x many (infinite) times and average, we get μ_x
   - If $f(x) = (x - \mu_x)^2$
     - $E[f(x)] = E[(x - \mu_x)^2] = \sigma_x^2$
     - $\sigma_x^2 = Var(x)$ called "variance"; σ_x called "standard deviation"
     - If we observe x many (infinite) times and average square of difference between each observation and μ_x, we get σ_x^2
     - Measure how likely x is going to be far away from mean

6. **[properties]** For mean: $E(aX + b) = aE(X) + b$. For variance: $V(X) = E\left(X^2\right) - [E(X)]^2$, $V(aX + b) = a^2V(X)$.

7. **[Chebyshevs Inequality]** $\Pr(|X - \mu| > k\sigma) \le \frac{1}{k^2}$, given mean μ and variance σ for a random variable, and $\mathbf{k} > \mathbf{0}$. Another form is $\Pr(|X - \mu| \le k\sigma) \ge 1 - \frac{1}{k^2}$.