EE2012/ST2334 Discussion 3*

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- 1. [2-D random variable] Let E be an experiment and S a sample space associated with E. Let X and Y be two functions each assigning a real number to each $s \in S$. We call (X,Y) a two-dimensional random variable (or random vector). Its **range** is $R_{X,Y} = \{(x,y)|x = X(s), y = Y(s), s \in S\}$. The definition can be extended to higher dimension, and they can be defined for both discrete and continuous random variable.
- 2. [Joint probability function]

Discrete:

$$f_{X,Y}(x_{i}, y_{j}) = \Pr(X = x_{i}, Y = y_{j})$$

$$f_{X,Y}(x_{i}, y_{j}) \ge 0 \text{ for all } (x_{i}, y_{j}) \in R_{X,Y}$$

$$\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} f_{X,Y}(x_{i}, y_{j}) = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \Pr(X = x_{i}, Y = y_{j}) = 1$$
(1)

Continuous: change \sum to \int .

3. [Marginal distribution] Recall the law of total probability (discrete case):

$$Pr(A) = \sum_{n} Pr(A \cap B_n)$$

$$= \sum_{n} Pr(A|B_n) Pr(B_n)$$
(2)

If (X,Y) is a 2-D discrete random variable, and its joint probability is $f_{X,Y}(x,y)$, the marginal distributions are:

$$f_X(x) = \sum_y f_{X,Y}(x,y)$$

$$f_Y(y) = \sum_x f_{X,Y}(x,y)$$
(3)

4. [Conditional distribution]

$$f_{Y|X}(y|x) = \frac{f_{X,Y}(x,y)}{f_X(x)}, \text{ if } f_X(x) > 0$$
 (4)

5. [Independence]

$$P(A \cap B) = P(A)P(B) \iff P(A) = \frac{P(A \cap B)}{P(B)} = P(A|B)$$
(5)

6. [Expectation]

$$E[g(X,Y)] = \begin{cases} \sum_{x} \sum_{y} g(x,y) f_{X,Y}(x,y), & \text{for Discrete RV's} \\ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) f_{X,Y}(x,y) dx dy, & \text{for Continuous RV's} \end{cases}$$
 (6)

- (a) A special case is that when $g(X,Y) = (X \mu_X)(Y \mu_Y)$, the expectation is the **covariance** of (X,Y). Cov $(X,Y) = E[(X \mu_X)(Y \mu_Y)] = E(XY) \mu_X \mu_Y$.
- (b) If X and Y are independent, cov(X,Y) = 0. But cov(X,Y) = 0 does **NOT** imply independence.
- (c) $Cov(aX + b, cY + d) = ac Cov(X, Y), V(aX + bY) = a^2V(X) + b^2V(Y) + 2ab Cov(X, Y)$
- 7. [Correlation coefficient]

$$\rho_{X,Y} = \frac{\text{Cov}(X,Y)}{\sqrt{V(X)}\sqrt{V(Y)}} \tag{7}$$

- (a) $-1 \le \rho_{X,Y} \le 1$
- (b) $\rho_{X,Y}$ measures the degree of linear relationship between X and Y.
- (c) If X and Y are independent, $rho_{X,Y} = 0$. But $rho_{X,Y} = 0$ does **NOT** imply independence.

^{*}Some examples from bookmarked pages of the lecture notes