Turbulence measurements from velocimeters on compliant mid-water moorings, part 2: motion correction

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ABSTRACT

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1. Introduction

- Brief history of ADVs

- Discussion of ADV v. ADP measurement

- ADVs have been difficult to deploy at mid-depths

- Availability of cheap+accurate IMUs (from smartphone tech) facilitates a new era of ADV measurement from moored platforms.

- Emphasize noise issue and spatial smearing of ADPs.

- Reference to 'Part 1'.

- This work developed for MHK, but has wider applications.

2. Measurements

This work is focused on measuring turbulence from ADVs that are deployed from non-stationary platforms and equipped with inertial motion sensors (IMU). The ADVs utilized for these measurements were all equipped with Microstrain 3DM-GX3-25 IMU sensors that captured all 6 components of the ADV motion (3 components of angular rotation and 3 components of linear acceleration), as well the orientation of the ADV pressure-case. The sampling of the motion sensor is tightly synchronized with the ADV measurements. The IMU measures its motion at 1kHz and uses internal signal integration (Kalman filtering) to output the motion signals at the same sample rate as the ADV's velocity measurements. This reduces aliasing of the IMU's motion measurements above the ADV's sample-rate (MicroStrain 2010). Cable-head ADVs were utilized throughout this work to allow for flexibility in the positioning of the ADV head relative to its pressure case.

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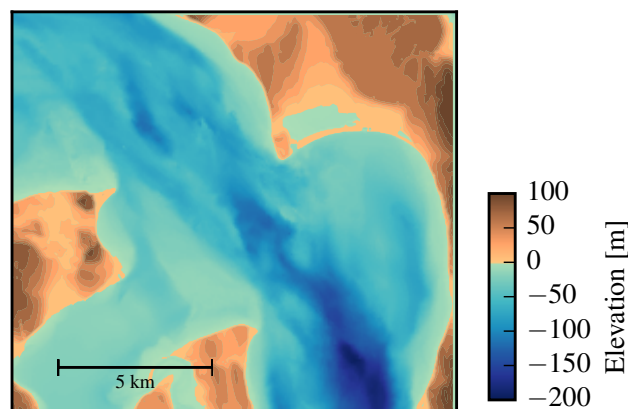E-mail: Levi.Kilcher@nrel.gov

FIG. 1. Bathymetry of Admiralty Inlet at Admiralty Head.

All measurements used in this work were made in Admiralty Inlet, Washington, approximately 500 meters (m) WSW of Admiralty Head–Fort Casey State Park–in 60 m of water depth at latitude 48.153 north and longitude 122.687 west (Figure 1). The site is approximately 6 kilometers (km) east of Port Townsend, and 1 km north of the Port Townsend – Coupeville ferry route. Admiralty inlet is the largest waterway connecting Puget Sound to the Strait of Juan de Fuca, and it possesses a large semi-diurnal tidal flow. This work utilizes data from three distinct deployment platforms: the 'tidal turbulence mooring', a 'StableMoor' buoy, and a simple sounding weight. Additional details, photos, and schematic diagrams of all three mooring systems can be found in Part 1.

a. Tidal Turbulence Mooring (TTM)

The 'tidal turbulence mooring' (TTM) is a simple mooring system with a 'strongback fin' suspended between a steel clump-weight anchor weighing 1200 kilograms (kg) when dry and a 0.93 m-diameter spherical steel buoy with a buoyancy of 320 kg. The ADV pressure cases were

clamped to one side of a 'strongback' fin and the ADV sensor head was positioned 10 cm in front of the fin's leading edge (Figure 2). The leading edge of the fin is fastened inline with the mooring line. This configuration was designed to work similar to a weather-vane, such that the drag on the fin held the ADV head upstream of the mooring components. This work utilizes data from two TTM deployments.

1) JUNE 2012 TTM DEPLOYMENT

The first was in June of 2012 at 48.15285 north, 122.68581 west. The mooring was in the water from 17:30 on the 12th until 14:30 on the 14th (local, i.e. pacific daylight time). Two Nortek ADVs were clamped to either side of the fin such that the axis of their cylindrical pressure-cases were parallel with the leading edge of the strong-back. The ADV heads were spaced 0.5 m apart vertically along the fin. Only one of these ADVs was equipped with an integrated IMU. This TTM also had an upward-looking acoustic Doppler profiler mounted on the mooring anchor.

Periods of time during which this mooring interfered with a beam of the Doppler profiler were identified by inspection of the profiler's acoustic amplitude signal. Periods during which one beam of the profiler had > 5% higher acoustic amplitude than the other beams were flagged as 'contaminated' and excluded from averaging. 5-minute averages in which more than 50% of the data was contaminated in this way were masked as invalid.

2) JUNE 2014 TTM DEPLOYMENT

The second TTM deployment was in 2014 from 06:00 on June 17 to 05:00 on June 19 (local time). The mooring was positioned at 48.15327 north, 122.68654 west. Two Nortek ADV-IMUs were mounted on this TTM, with their heads spaced 0.5 m apart along the fin. In this case the pressure-cases and ADV heads were inclined at an angle of 18° to the leading edge of the fin to account for mooring blow-down during strong currents (Figure 3). This change was made to reduce vibrational motion observed during the June 2012 deployment that was believed to be associated with the orientation of the pressure cases.

b. The StableMoor platform

The second deployment platform was a cylindrical, 'StableMoor', syntactic foam buoy (manufacturer: Deep Water Buoyancy) that was anchored to a clump weight that weighed 2700 lbs (Figure 4). The buoy is 3.5 m long and 0.45 m in diameter with a tail ring that is 0.76 m in diameter. The StableMoor weighs 295 kg in air, and has a buoyancy of 185 kg in water.

The StableMoor was ballasted to pitch upward a few degrees in zero-flow to avoid 'flying downward'. In the presence of an oncoming current the tail fins help to orient
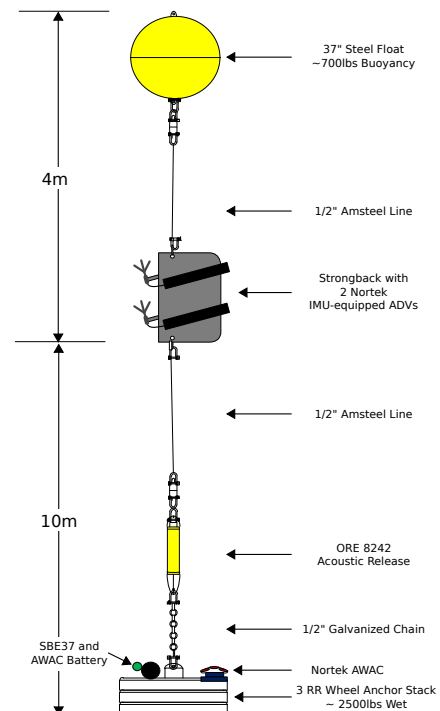

FIG. 2. Schematic diagram of the TTM, not to scale.

FIG. 3. TTM components on the deck of the R/V Jack Robertson. The TTM includes two ADVs, with pressure-cases mounted on opposite sides of the fin. The anchor stack includes a pop-up buoy for retrieval.

it into the flow. The anchor for this buoy is similar to that of the TTM, including an acoustic release so the mooring and anchor can be recovered separately.

The StableMoor platform has two primary advantages compared to the TTM. First, it is significantly more massive and hydro-dynamically stable than the TTM, which reduces the frequency of motions of the platform. The other major advantage of the StableMoor platform is that it is capable of supporting a bottom-tracking acoustic Doppler profiler, which provides an independent measure

of the platform's translational motion. Disadvantages of the StableMoor include: a) its size adds to the challenge of deployment and recovery, and b) it is significantly more expensive than the TTM system.

This work uses data from two deployments of the StableMoor. During both deployments the buoy was equipped with a 1200 kHz RDI workhorse sentinel acoustic Doppler profiler that was oriented downward-looking to measure water velocity below the platform in 12 1-meter bins and measure buoy motion ('bottom tracking'), all at a 1 Hz sample rate.

1) 'NOSE MODE' STABLEMOOR DEPLOYMENT

The StableMoor was deployed with an ADV-IMU mounted at its nose from 11:21 on May 12 to 11:53 on May 13, 2015 (local time). This deployment was at 48.15277 north, 122.68623 west. In this configuration the sample volume of the ADV, is 10 cm forward of the nose and 20 cm above the centerline of the StableMoor (Figure 4, middle), which is assumed to be outside of the bow wave of the StableMoor buoy (NEED CITATION for: any ideas on how big the bow wave of this sort of thing is?). This configuration was designed to be the most stable platform for measuring turbulence from a moving platform.

2) 'WING MODE' STABLEMOOR DEPLOYMENT

The StableMoor buoy we used was customized to have an optional 1.5 m oval beam that holds two ADVs 0.53 m to either side of the buoy hull (Figure 4, bottom). The StableMoor was deployed in this configuration from 09:52 on May 11 to 06:23 on May 12, 2015 (local time). This configuration provides a simultaneous measurement of turbulence at two points laterally to the oncoming flow, which is a relatively unique mid-depth measurement.

c. Turbulence Torpedo

The turbulence torpedo is a simple sounding weight with an ADV head mounted forward of the nose, and the ADV pressure case strapped below. This platform was deployed on May 14, 2015 for 37 minutes starting at 07:41 local time. Position of deployment not mentioned in deck-log. Can we get this from ship data? Does this matter?. This measurement was made from a davit that hung the system from the side of the ship to a depth of approximately 25 m. The primary logistical advantages of this platform are its compact size, low cost, and the flexibility to perform spatial transects.

d. Coordinate system and turbulence averaging

Unless stated otherwise, vector quantities in this work are in a fixed 'principal-axes' coordinate system that is aligned with the bi-directional tidal flow: positive u is

FIG. 4. Top: Alex DeKlerk checks to ensure that the StableMoor buoy is properly fastened to its anchor; the RDI workhorse ADCP can be seen in the rear instrument bay. A bridle is draped across the top of the buoy for deployment and recovery, and a small marker buoy fastened to the tail is useful during recovery. Middle: a close-up of the Stable-Moor in the 'nose mode' configuration shows the ADV head and the top of its pressure case. Bottom: the StableMoor buoy in 'wing mode', floating on the ocean surface with the marker buoy trailing behind. The ADV heads mounted at the ends of the cross-beam are protruding above the waterline.

in the direction of ebb (310° True), positive w is verti-

Fig. 5. The turbulence platform showing details of the ADV head and pressure case configuration. The head-cable was taped out of the way shortly after taking this photo.

cally upward, and v is the cross-stream component in a right-handed coordinate system. The full velocity vector, $\vec{u} = (\tilde{u}, \tilde{v}, \tilde{w})$, is separated into a mean and turbulent component as $\vec{\tilde{u}} = \vec{\bar{u}} + \vec{u}$, where the over-bar denotes a 5 minute average. Turbulence kinetic energy, tke $= \overline{u^2} + \overline{v^2} + \overline{w^2}$, and Reynold's stresses, $\overline{uv}$, $\overline{uw}$, $\overline{vw}$ are computed by averaging over the 5-minute window. Throughout this work we use $\bar{U} = (\bar{u}^2 + \bar{v}^2)^{1/2}$ to denote the mean horizontal velocity magnitude.

All spectra, $S\{x\}(f) = |\mathscr{F}\{x(t)\}|^2$, and cross-spectra, $C\{x,y\}(f) = \mathrm{real}(\mathscr{F}\{x(t)\}\mathscr{F}\{y(t)\})$, are computed using NumPy fast Fourier transform routines (van der Walt et al. 2011). Here, $\mathscr{F}\{x(t)\}$ denotes the fast Fourier transform of a signal $x(t)$. Time series', e.g. $x(t)$, are linearly detrended and Hanning windowed prior to computing $\mathscr{F}\{x\}$ to reduce spectral reddening.

Throughout the remainder of this work the dependence of S and C on f is implied (e.g. $S\{x\}(f)$ is hereafter $S\{x\}$), and for other variables the dependence on t is implied. Spectra and cross-spectra are normalized to preserve variance: $\int S\{u\}\mathrm{d}f = \overline{u^2}$, and $\int C\{u,v\}\mathrm{d}f = \overline{uv}$. The notations $S\{\vec{u}\} = (S\{u\}, S\{v\}, S\{w\})$ and $C\{\vec{u}\} = (C\{u,v\}, C\{u,w\}, C\{v,w\})$ denote the set of spectra and cross-spectra for each velocity component and pairs of components, respectively.

Turbulence dissipation rates are computed as,

$$\varepsilon = \frac{1}{\bar{U}}\left(\alpha \left\langle (S\{u\} + S\{v\} + S\{w\})f^{5/3}\right\rangle_{fIS}\right)^{3/2} \quad (1)$$

Where $\alpha = 0.5$, and $\langle\rangle_{fIS}$ denotes an average over the inertial-subrange of the velocity spectra and where the signal-to-noise ratio is small (Lumley and Terray 1983; Sreenivasan 1995). Throughout this work we take this average from 0.3 to 1 Hz for the u and v components, and 0.3 to 3 Hz for the w component.

3. Methodology

The essential approach of this methodology is to estimate time-series of velocity on a compliant mooring by obtaining an independent estimate of ADV head motion and removing that motion from the measured signal. Nortek offers an ADV that is equipped with an IMU that measures the linear acceleration, $\vec{a}$, rotational-motion, $\vec{\omega}$, and orientation matrix, $\mathbf{R}$, of the ADV pressure case (body) in the earth reference frame. So long as the ADV head is rigidly connected to the ADV pressure case, it is possible to utilize the IMU motion signals to calculate the motion of the ADV head, and remove it from the measured velocity signal. The ADV head motion, is calculated as the sum of rotational and translational motion:

$$\begin{aligned}\vec{u}_{\mathrm{h}} &= \vec{u}_{\omega} + \vec{u}_a + \vec{u}_{\mathrm{low}} \\ &= \mathbf{R}^{\mathrm{T}} \cdot \vec{\omega}^*(t) \times \vec{\ell}^* + \int \{\vec{a}(t)\}_{HP(f_a)}\mathrm{d}t + \vec{u}_{\mathrm{low}}\end{aligned} \quad (2)$$

Here '$*$' superscripts denote quantities in the ADV's local coordinate system, and $\vec{\ell}^*$ is the vector from the IMU to the ADV head. $\mathbf{R}^{\mathrm{T}}$—the inverse of the orientation matrix—rotates vectors from the IMU to the earth reference frame. The notation $\{\vec{a}\}_{HP(f_a)}$ indicates that the IMU's accelerometer signal is high-pass filtered (in the earth's stationary reference frame) at a chosen filter-frequency, f_a. This is necessary because accelerometers have low-frequency noise, sometimes referred to as 'bias-drift' (Barshan and Durrant-Whyte 1995; Bevly 2004; Gulmammadov 2009).

Integrating $\vec{a}$ to estimate $\vec{u}_a$ amplifies the bias-drift noise at low-frequencies, which dramatically reduces the signal-to-noise ratio at those time scales (Figure 3). The high-pass filtering reduces this noise so that it does not contaminate motion correction, but real motion that exists at these frequencies is still lost in the low signal-to-noise ratio (Egeland 2014). This means that low-frequency motion is not well resolved by the IMU, and so there is a residual low-frequency translational motion, $\vec{u}_{\mathrm{low}}$, that needs to be measured independently—or at the very least considered—when using motion corrected ADV-IMU data.

The choice of high-pass filter for low-frequency accelerometer noise depends on the flow conditions of the measurement, and the platform that is being used. In particular, filter-selection involves a trade off between filtering-out the bias-drift noise while not filtering-out measured motion that is unresolved by an independent measurement of $\vec{u}_{\mathrm{low}}$. If an independent measure of low-frequency motion is available it can be used to increase the accuracy of $\vec{u}_{\mathrm{h}}$ at low-frequency. Note that, to avoid double counting, $\vec{u}_{\mathrm{low}}$ should be estimated by applying the complimentary low-pass filter to the independent measurement of low-frequency motion.

With this estimate of ADV head motion it is straightforward to correct the measured velocity, $\vec{u}_{\mathrm{m}}$, to estimate the velocity in the earth's inertial reference frame:

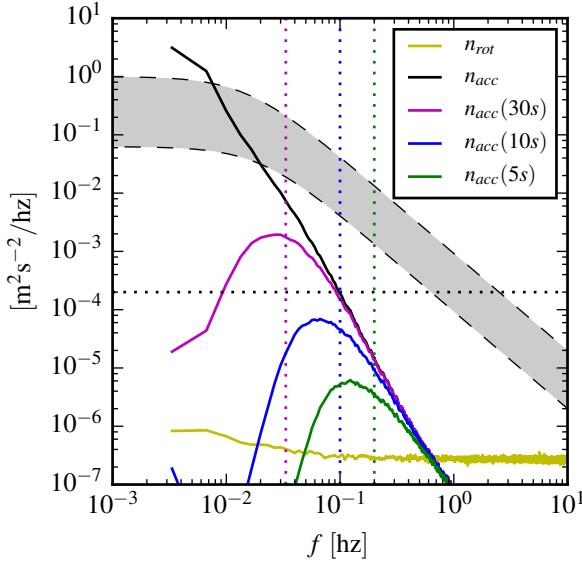$$\vec{u}(t) = \vec{u}_{\mathrm{m}}(t) + \vec{u}_{\mathrm{h}}(t) \qquad . \qquad (3)$$

FIG. 6. Spectra of $\vec{u}_\omega$ (yellow) and $\vec{u}_a$ signals from the Microstrain IMU sitting on a motionless table. The $\vec{u}_a$ signals are unfiltered (black), and high-pass filtered at 30s (magenta), 10s (blue), 5s (green). Vertical dotted lines indicate the filter frequency. The black horizontal dotted line indicates the noise-level of a Nortek Vector ADV configured to measure $\pm$4m/s. The shaded region indicates the range of spectra presented herein ($0.002 < $ tke < 0.03 m^2/s^2, 1e-5 $< \varepsilon <$ 5e-4 W/kg).
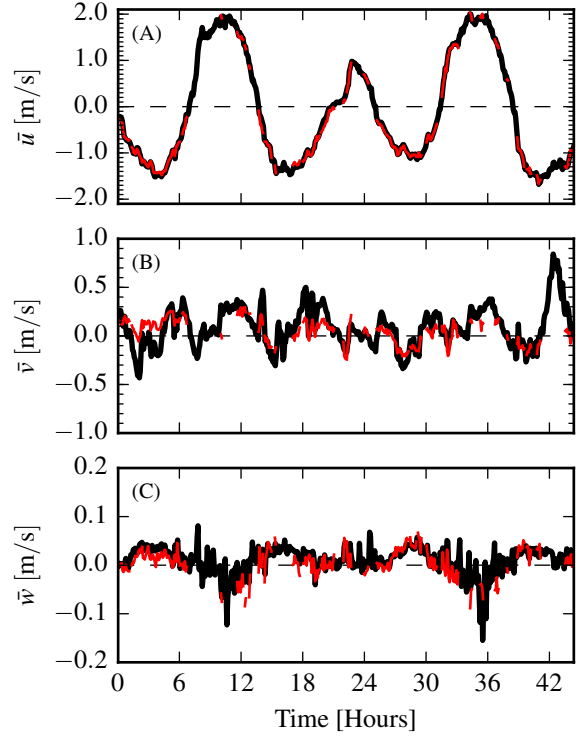

FIG. 7. Time series of tidal velocity at Admiralty Head from TTM measurements (black), and an acoustic Doppler profiler (red). The profiler measurements–taken at the same depth as the ADV on the TTM—were contaminated by acoustic reflection from the strongback fin when it was inline with one of the profiler's beams. Note that the vertical scale on the three axes vary by more than an order of magnitude; the small ticks in A and B are equivalent to the ticks in C.

Note here that the '+'-sign is correct because head motion, $\vec{u}_h$, induces a measured velocity in the opposite direction of the head motion itself ($\vec{u}_m = \vec{u} - \vec{u}_h$).

For the TTM and Turbulence Torpedo we utilize $f_a = 0.0333Hz$ (30 second period), and assume that $\vec{u}_{low} = 0$. For the StableMoor $f_a = 0.2Hz$ (5 second period). The bottom-track velocity was low-pass filtered at this frequency to provide an estimate of $\vec{u}_{low}$, and $\vec{a}$ was high-pass filtered at this frequency. We use 4-pole, bi-directional (zero-phase), Hanning filters for all filtering operations.

Additional details on motion correction–including a detailed accounting of the distinct coordinate systems of the IMU, ADV pressure case, and ADV head–can be found in Kilcher et al. (2016). Open-source Python tools for performing motion correction of ADV-IMU data–including scripts that write processed data in Matlab and tabulated formats–are available at http://lkilcher.github.io/dolfyn/.

4. Results

a. Mean velocity

Figure 7 shows a comparison of $\vec{u}$ measured by an ADV-IMU mounted on a TTM, to that of an upward-looking acoustic Doppler profiler mounted on the TTM anchor. This shows excellent agreement between the ADV and Doppler profiler measurements of velocity. The $\bar{u}$, $\bar{v}$ and

$\bar{w}$ components have a root-mean-square error of 0.05, 0.13 and 0.03 m/s, respectively. While it is important to note that their is some discrepancy between ADP and ADV measured velocities (especially in $\bar{v}$, which is most likely due to incomplete motion correction), the agreement between the magnitude and direction of these independent velocity measurements indicates that moored ADV-IMUs provide a reliable estimate of velocity in the Earth's reference frame.

b. TTM spectra

As discussed in detail in Part 1 the mooring motion of the TTM, $S\{\vec{u}_h\}$, has a peak at 0.1 to 0.2 Hz from swaying of the mooring that is most likely driven by eddy-shedding from the spherical buoy (Figure 8, red lines). There is also higher-frequency broad-band motion that is associated with fluttering of the strongback fin around the mooring line. Both of these motions are especially energetic in the v-component spectra, because this is the direction in which the TTM mooring system is most unstable. As
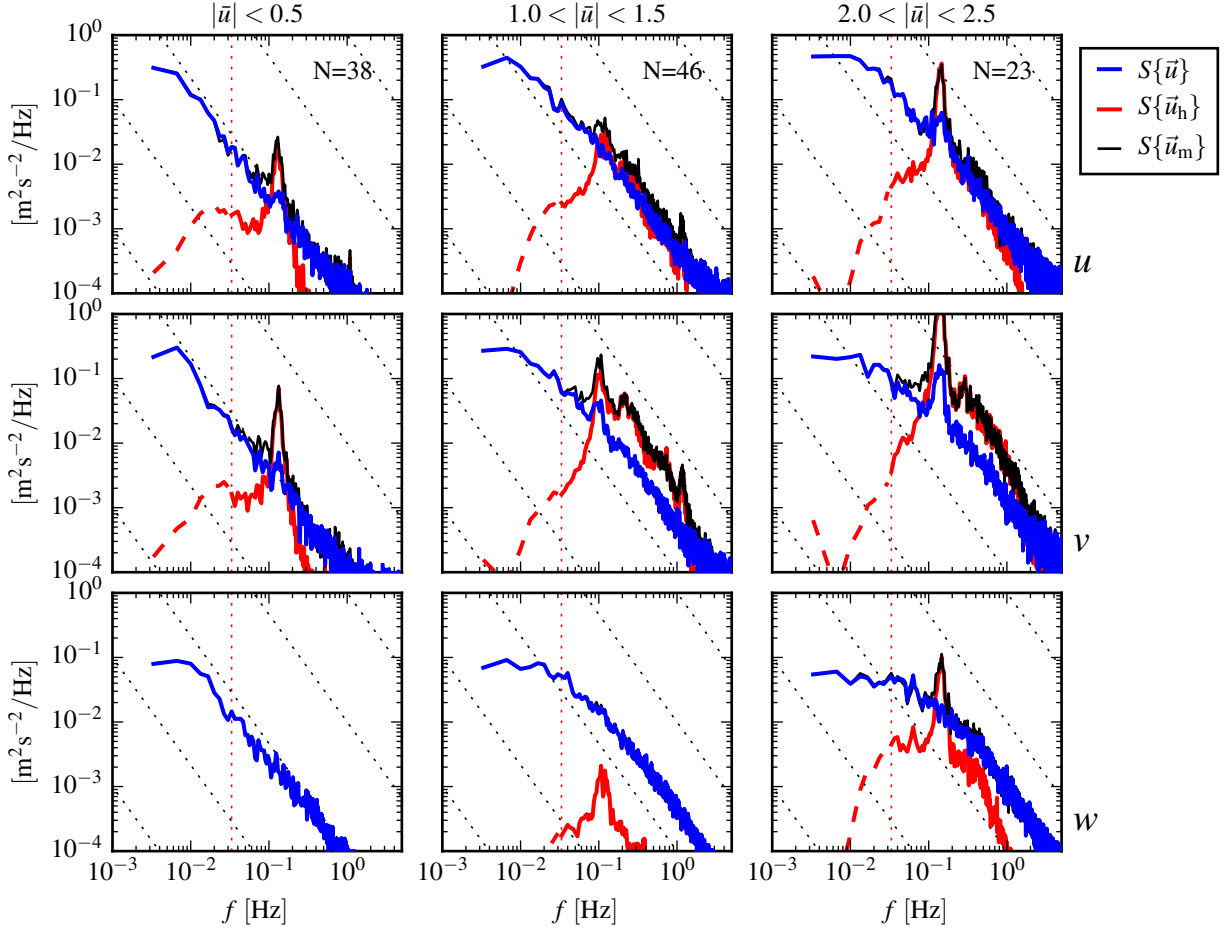
FIG. 8. Turbulence spectra from the June 2014 TTM deployment. Each column is for a range of streamwise velocity magnitudes (indicated at top). The rows are for each component of velocity (indicated to the lower-right of the right column). The uncorrected spectra are in black and the corrected spectra are blue, and the spectra of ADV head motion, $\vec{u}_h$, is red (also indicated in the legend). The vertical red dotted line indicates the filter frequency applied to the IMU accelerometers when estimating $\vec{u}_h$; below this frequency $S\{\vec{u}_h\}$ is plotted as a dashed line. Diagonal black dotted lines indicate a $f^{-5/3}$ slope. The number of spectral-ensembles, N, in each column is indicated in the top row.

is expected from fluid-structure interaction theory the amplitude of these motions increases with increasing mean velocity (Morison et al. 1950). Should we move this sentence and reference to Part 1, or leave it in here?

The mooring motion contaminates the uncorrected ADV-measurements of velocity, $S\{\vec{u}_m\}$, whenever the amplitude of the motion is similar to or greater than the amplitude of the turbulence. Fortunately, much of this motion can be removed using the IMU's motion signals as detailed in section 3. Lacking an independent measurement of turbulence velocity at this site, we interpret the agreement of these spectra with turbulence theory as evidence of the success of the method. In particular, at high-frequencies ($f > 0.3$ Hz) for each mean-flow speed the spectra decay with a $f^{-5/3}$ slope and have equal amplitude across the velocity components. These results are consistent with

Kolmogorov's (1941) theory of isotropic turbulence, and are consistent with other measurements of turbulence in energetic tidal channels from stationary platforms (Kolmogorov 1941; Walter et al. 2011; Thomson et al. 2012; McMillan et al. 2016).

At low frequencies, the spectra tend to become roughly constant (especially at higher flow speeds), which is also consistent with previous works. Note here, that the very-low magnitude of $S\{\vec{u}_h\}$ at low frequencies is partially a result of filtering the IMU's accelerometer signal when calculating $\vec{u}_a$. The true low-frequency spectrum of ADV-head motion is unknown (indicated using a dashed line below f_a). A comparison of $S\{\vec{u}\}$ measured by the TTM to that measured by the ADP—during the June 2012 deployment—are in agreement at low-frequencies (not shown). This suggests that the assumption that $\vec{u}_{low} =$

0 at these frequencies, at this site, for this platform is justified—even if $S\{\vec{u}_h\}$ is not as low as indicated in Figure 8.

As successful as motion correction is, some of the motion contamination persists in $S\{\vec{u}\}$. This is most notable in $S\{v\}$ at the highest flow speeds (> 2.0 m/s): a peak at 0.15 Hz is an order of magnitude larger than a spectral fit to the other frequencies would indicate. This persistent motion contamination is evident to a lesser degree in $S\{u\}$ for $|u| > 2$ m/s, and in $S\{v\}$ at lower flow speeds. $S\{w\}$ appears to have no persistent motion contamination because the amplitude of the motion in this direction is much lower than for the other two components. For these measurements, $S\{w_h\}$ is so low that w-component motion correction is only necessary when $|u| > 2$ m/s.

The amplitude of the persistent motion contamination peaks in $S\{v\}$ at 0.15 Hz are a factor of 5 to 10 times smaller than the amplitude of the ADV head motion itself. This suggests the Microstrain IMU can be used to effectively correct for mooring motion at 0.15 Hz when the amplitude of that motion is less than 3 times the amplitude of the real turbulence spectrum. Where we have chosen a value of 3 as a conservative estimate of motion correction's effectiveness.

This reveals an ancillary benefit of the IMU measurements: in addition to the primary benefit of correcting for mooring motion, they can also be used to identify and screen-out persistent motion contamination. For example, one of the most common uses of turbulence spectra is for the calculation of ε and tke. For these purposes, based on the relative amplitudes of the 0.15 Hz peaks, we assume that persistent motion contamination is likely where $S\{\vec{u}_h\}/S\{\vec{u}\} > 3$ and exclude these regions from spectral fits.

In the present case, for the u and w spectra, this criteria only excludes a narrow range of frequencies at the 0.15 Hz motion peak for some cases. This criteria is more restrictive of the v-component spectra at high frequencies for $\bar{U} > 1.0$ m/s, but this may be acceptable because the amplitude of the spectrum at these frequencies—i.e. in the isotropic inertial subrange—should be equal to that of u and w (Kolmogorov 1941).

Agreement of the v-component spectral amplitude with that of u and w at frequencies > 0.3 Hz indicates that motion correction is effective at those frequencies even when $S\{\vec{u}_h\}/S\{\vec{u}\} > 3$. This suggests that our screening threshold is excessively conservative at those frequencies, and that a more precise screening threshold is frequency dependent. For example, it might take into account the f^3 character of the noise in $S\{\vec{u}_a\}$ (Figure 3). For the purposes of this work the $S\{\vec{u}_h\}/S\{\vec{u}\} < 3$ threshold for spectral fits is sufficient, and detailed characterization of the IMU's motion- and frequency-dependent noise level is left for future work.

c. StableMoor Spectra

The spectra of the stablemoor motion has a broader peak with a maximum amplitude that is approximately half the frequency of the TTM spectral peak (Figure 9). The motion of this platform also does not have high-frequency 'sub-peaks' or other high-frequency broadbanded excitation (Part 1). Is the following in part 1: These characteristics of the motion are most-likely due to the more massive and hydro-dynamically streamlined properties of the platform.

Like the TTM, the motion-corrected spectra from the StableMoor are consistent with turbulence theory and previous observations. Most importantly, there is an improvement in the quality of the motion corrected spectra compared to the TTM. In particular the persistent motion contamination peaks appear to be completely removed. That is, this measurement system provides an accurate estimate of the turbulence spectra at this location from low frequencies to more than 1Hz—well into the inertial sub-range—for all three components of velocity.

Note that this level of accuracy can not be obtained without the independent estimate of $\vec{u}_{low}$. If we assume that $\vec{u}_{low} = 0$ a similar plot to Figure 9 (not shown) reveals persistent motion-contamination peaks and troughs in the u- and v-spectra regardless of the choice of f_a. This indicates that the low-frequency motion of the StableMoor is below a threshold where the IMU's signal to noise ratio is high enough to resolve its motion. In other words, compared to the TTM, the StableMoor platform provides a more accurate measurement of turbulence when it includes an independent measure of $\vec{u}_{low}$ (here a bottom-tracking ADCP), but it does no better—and perhaps worse—when it doesn't.

d. Torpedo spectra

The u and v motion of the turbulence 'torpedo' is broadbanded and the w motion has a narrow peak at 0.3 Hz (Figure 10). Because $\vec{u}_h$ is estimated using $f_a = 0.0333 Hz$ and assuming $\vec{u}_{low} = 0$ its spectra rolls-off quickly below f_a. A better estimate of $\vec{u}_{low}$ could be obtained by accounting for ship motion, but this has not been done here. ← Should we do this?

Motion correction of the torpedo data appears to effectively remove a motion from $S\{w\}$ at 0.3Hz, and straightens out $S\{v\}$ between 0.04 and 0.6Hz. $S\{u\}$ is relatively unimproved by motion correction, apparently because the torpedo motion is smaller than the turbulence in this direction. At frequencies below f_a, $S\{u\}$ and $S\{v\}$ increase dramatically. This suggests that unresolved low-frequency motion of the torpedo is contaminating the velocity measurements at these frequencies. It may be possible to correct for some of this using a measurement of the ship's motion as a proxy for the torpedo's low-frequency motion,
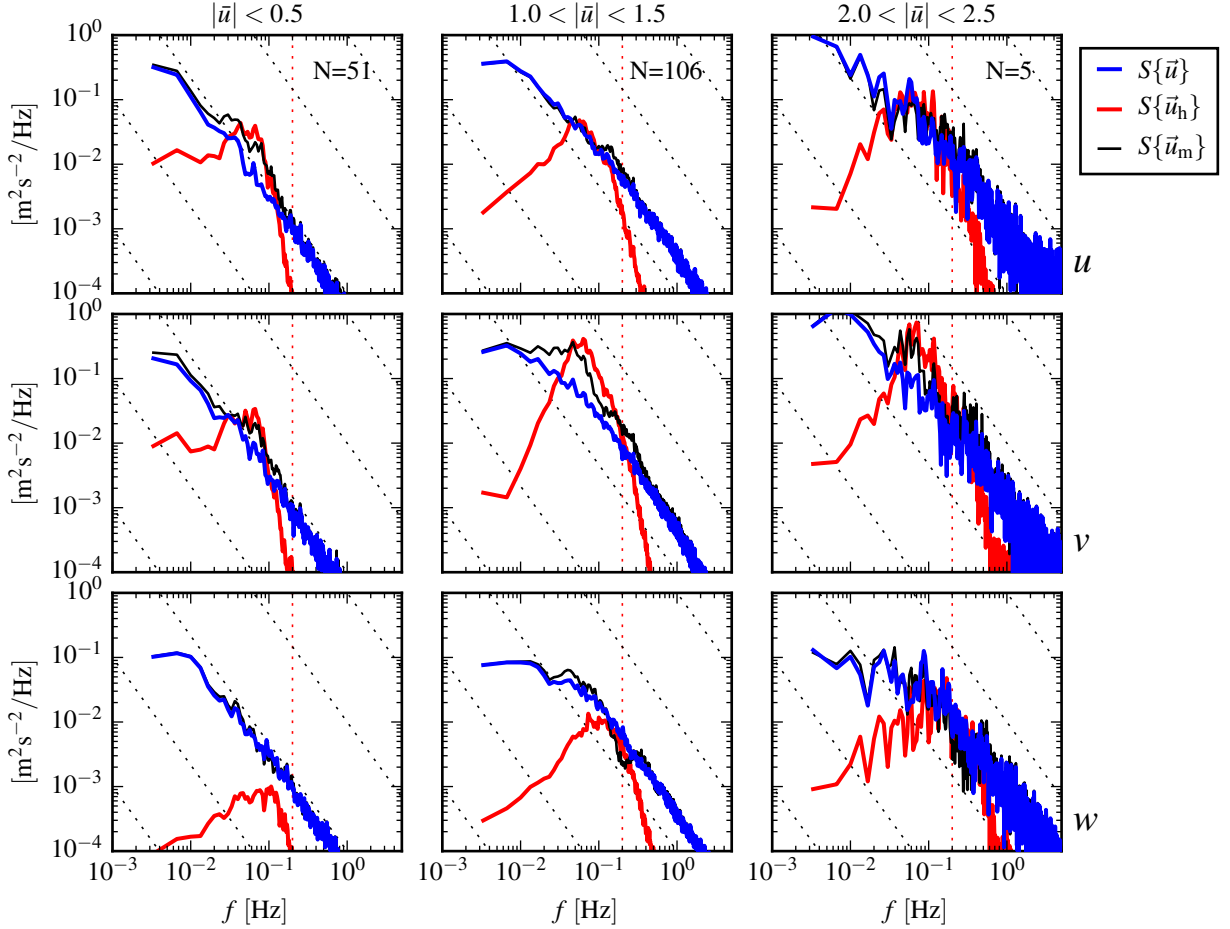
FIG. 9. Turbulence spectra from the StableMoor buoy. The axes-layout and annotations are identical to Figure 8, except that $S\{\vec{u}_h\}$ is plotted as a solid line at all frequencies because it is measured at all frequencies.

but this has not been done. Still, above f_a, the torpedo appears to provide a reliable estimate of spectral amplitude in the inertial subrange and can therefore be used to estimate ε. Considering the simplicity of the platform it may be a useful option for quantifying this essential turbulence quantity in a variety of scenarios. If a GPS is positioned above it, it may be capable of providing even more.

e. Cross-spectra

Inspection of cross-spectra from TTM measurements demonstrates that motion correction can reduce motion contamination to produce reliable estimates of velocity cross-spectra (Figure 11). At low flow speeds (left column), cross-spectra between components of $\vec{u}_h$ (i.e. between components of head-motion, red) are small compared to correlated velocities. As the velocity magnitude increases (center, and right columns), the swaying motion of the TTM at 0.15 Hz appears as a peak in the amplitude of the cross-spectra of $\vec{u}_h$ and $\vec{u}_m$ (black) for all three

components of cross-spectra (rows). Fortunately, motion correction reduces the amplitude of this peak dramatically so that $C\{\vec{u}\}$ (blue) is small at 0.15 Hz compared to lower frequencies. Furthermore, the fact that the standard deviation of $C\{\vec{u}\}$ is also relatively small at 0.15 Hz suggests that motion correction is effective for each spectral window, not just in their mean.

These results indicate that motion-corrected TTM velocity measurements can be used to obtain reliable estimates of turbulence Reynold's stresses, which are the integral of the cross-spectra. Without motion correction, Reynold's stress estimates would be contaminated by the large peaks in the cross-spectra that are due to the swaying and fluttering motion of the TTM vane.

A similar investigation of StableMoor cross-spectra (not shown) indicates that cross-spectral motion contamination is much lower amplitude than for the TTM. The low-frequency (< 0.3 Hz) 'swimming' motion of that platform produces minimal cross-spectral signal, and the rel-
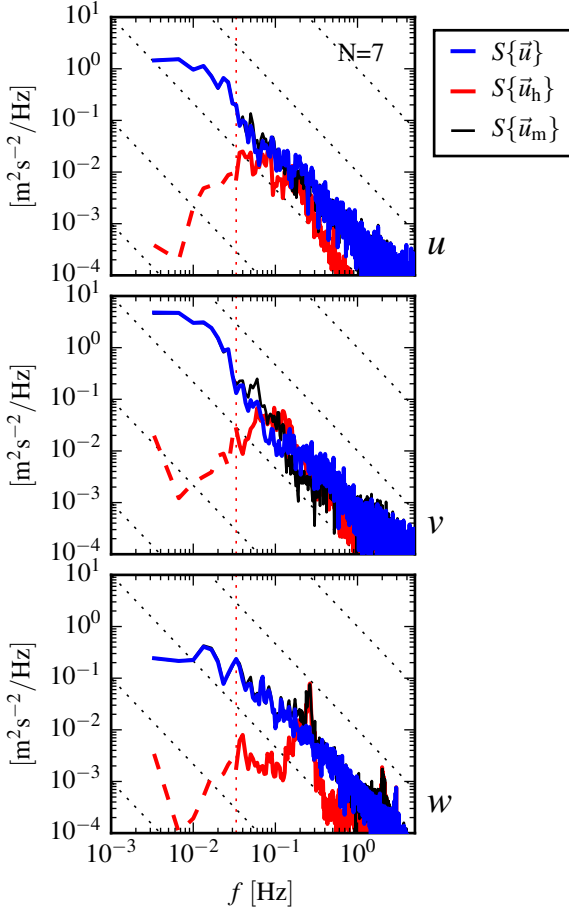
FIG. 10. Turbulence spectra from the turbulence torpedo during a 35 minute period when the mean velocity was 1.3 m/s. Annotations and line colors are identical to Figure 8.

ative large-mass of the platform minimizes the kinds of higher-frequency swaying/fluttering that creates large values of cross-spectral head-motion. Thus, the StableMoor platform also produces reliable estimates of Reynold's stresses, which are presumed to be improved by motion correction.

5. Discussion

The beginning of the previous section presented a comparison of $\vec{u}$ measured by a TTM-mounted ADV, to measurements from a co-located ADP. This demonstrated that the IMU provides a reliable estimate of the ADV's orientation and that this can be utilized to estimate mean velocity in the earth's reference frame. Turbulence velocity estimates from the same ADP are also in agreement with low-frequency TTM turbulence estimates (not shown), but the ADP does not resolve turbulence at the scales where motion contamination is strongest (0.1 to 1.0 Hz).

Ideally, moored motion-corrected turbulence velocity measurements would be validated against simultaneous independent validated measurements of turbulence velocity at the same scales, exact time and exact location. Accomplishing this, however, involves significant technical challenges not easily overcome—most notably the difficulty of measuring turbulence at the same point as the moving ADV. A slightly less ideal but much more realistic confirmation of the methodology might involve comparing the statistics of moored turbulence measurements to that from a nearby fixed platform, or a fixed platform placed at the same location at a different time (e.g. the 'TTT' platform described in Thomson et al. 2012). Unfortunately, to our knowledge, these measurements have not yet been made.

Lacking a relevant, fixed, independent turbulence measurement to compare to it is instructive to demonstrate the degree to which the moored measurements are consistent with turbulence theory and other turbulence measurements in similar flow environments. The previous section showed that the shape of the turbulence velocity spectra from moored ADVs is consistent with Kolmogorov's theory of locally isotropic turbulence, which has been observed consistently in turbulence measurements for decades (Kolmogorov 1941; Grant et al. 1962; McMillan et al. 2016). In particular, we observed an isotropic subrange—an $f^{-5/3}$ spectral slope, and equal amplitude spectra between components—that is driven by anisotropic turbulence at longer time-scales (Figures 8, 9, 10). This is interpreted as the first indication that the measurement systems presented are capable of accurately resolving turbulence. The degree to which uncorrected spectra were corrected toward this theoretical and observationally confirmed shape is interpreted as a measure of the improvement of the spectral estimates by motion correction.

Figure 12 presents a time-series of the mean velocity (A) and several turbulence statistics that were measured during the June 2014 TTM deployment. This figure shows the evolution of the flow through Admiralty Inlet during 1.5 tidal cycles. The tke (B), Reynold's stresses (C), dissipation and one component of turbulence production (D) grow and strengthen with ebb or flood, then subside during slack tide. This component of turbulence production is:

$$P_{uz} = \frac{\partial \bar{u}}{\partial z} \overline{uw} \qquad . \qquad (4)$$

Where $\partial \bar{u}/\partial z$ is computed from the two ADV's on the TTM. The highest values of ε and P_{uz} occur at the peak of the ebb or flood, which is in agreement with other measurements in tidal channels. The agreement of the magnitude of P_{uz} with ε at those times suggests a local production-dissipation balance that is often observed in tidally forced channels (Trowbridge et al. 1999; Stacey
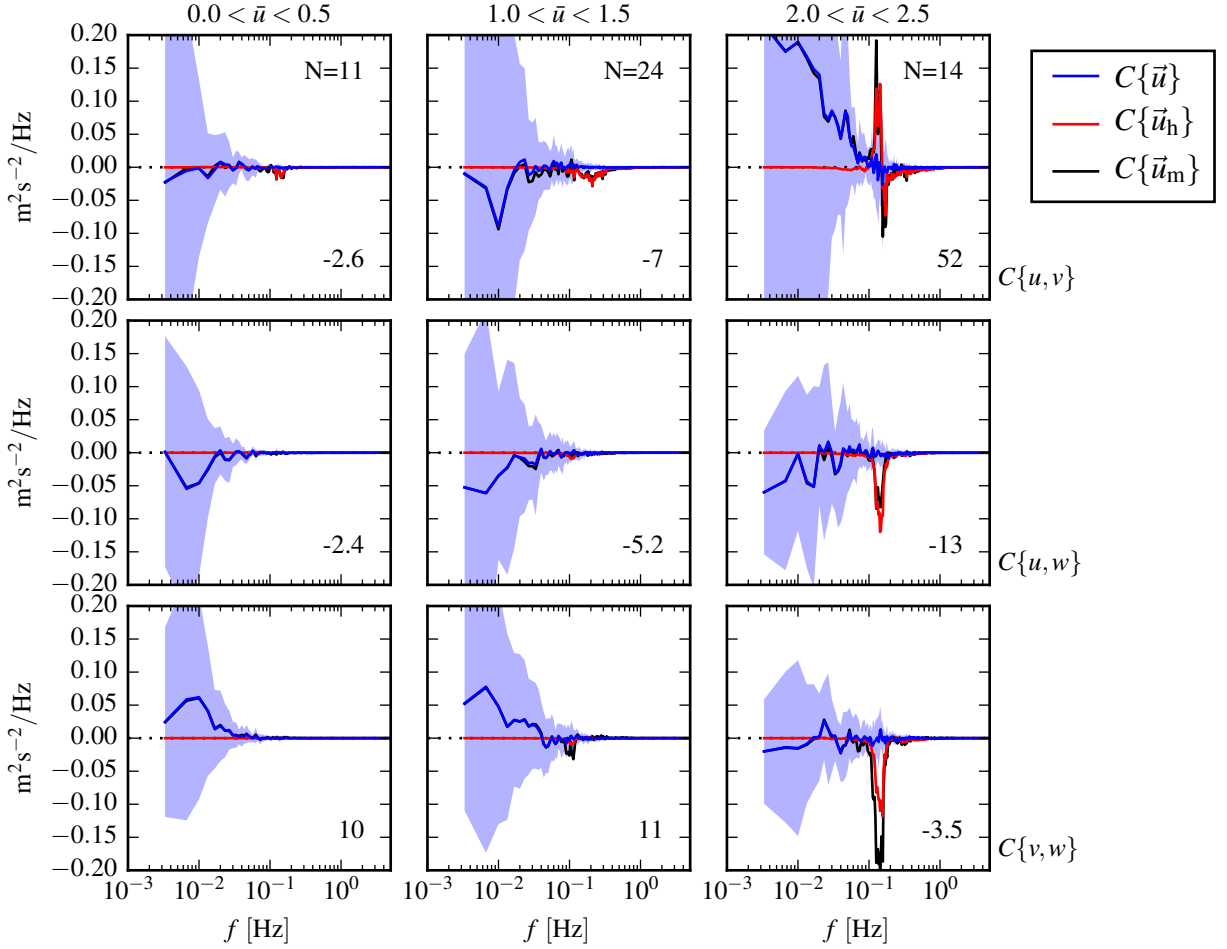
FIG. 11. The real part of the cross-spectral density between velocity components measured by the TTM. The upper-row is the *u-v* cross-spectral density, the middle-row is the *u-w* cross-spectral density, and the bottom-row is the *v-w* cross-spectral density. The columns are for different ranges of the stream-wise mean velocity magnitude (indicated above the top row). The blue line is the cross-spectrum between components of motion-corrected velocity, the red line is the cross-spectrum between components of head-motion, and the black line is the cross-spectrum between components of uncorrected velocity. The light-blue shading indicates one standard deviation of the C for the motion corrected cross-spectral density. N is the number of spectral ensembles in each column. The number in the lower right corner of each panel is the motion-corrected Reynold's stress (integral of the blue line) in units of 1e-4 m^2s^{-2}.

et al. 1999; McMillan et al. 2016). At other times the value of P_{uz} is insufficient to balance ε or is negative.

Inspection of the negative P_{uz} values reveals that most of them are due to a reversed sign of $\overline{uw}$ rather than a reversed sign of $\partial u / \partial z$ (i.e. when compared to the sign of u). This suggests that uncertainty in $\overline{uw}$ may be contributing to discrepancies between P_{uz} and ε. It is also possible that other terms of the tke equation are important, such as other components of production, advection terms, or turbulent transport terms.

Figure 13 compares individual values of P_{uz} with ε directly. Given the assumptions implicit in this comparison, and the discussion above, the agreement between P_{uz} and ε is an encouraging result that suggests the turbulent boundary reaches the depth of these measurements (10 m)

during the highest flow speeds. This result is further supported by a comparison of $\bar{U}$ with ε (Figure 14). Here we see a $\varepsilon \propto \bar{U}^3$ dependence that is again suggestive of bottom boundary layer physics (Trowbridge 1992; Nash et al. 2009). At lower flow speeds, ε deviates from this relationship, which suggests that the boundary layer is no longer the dominant physical process at the depth of these measurements.
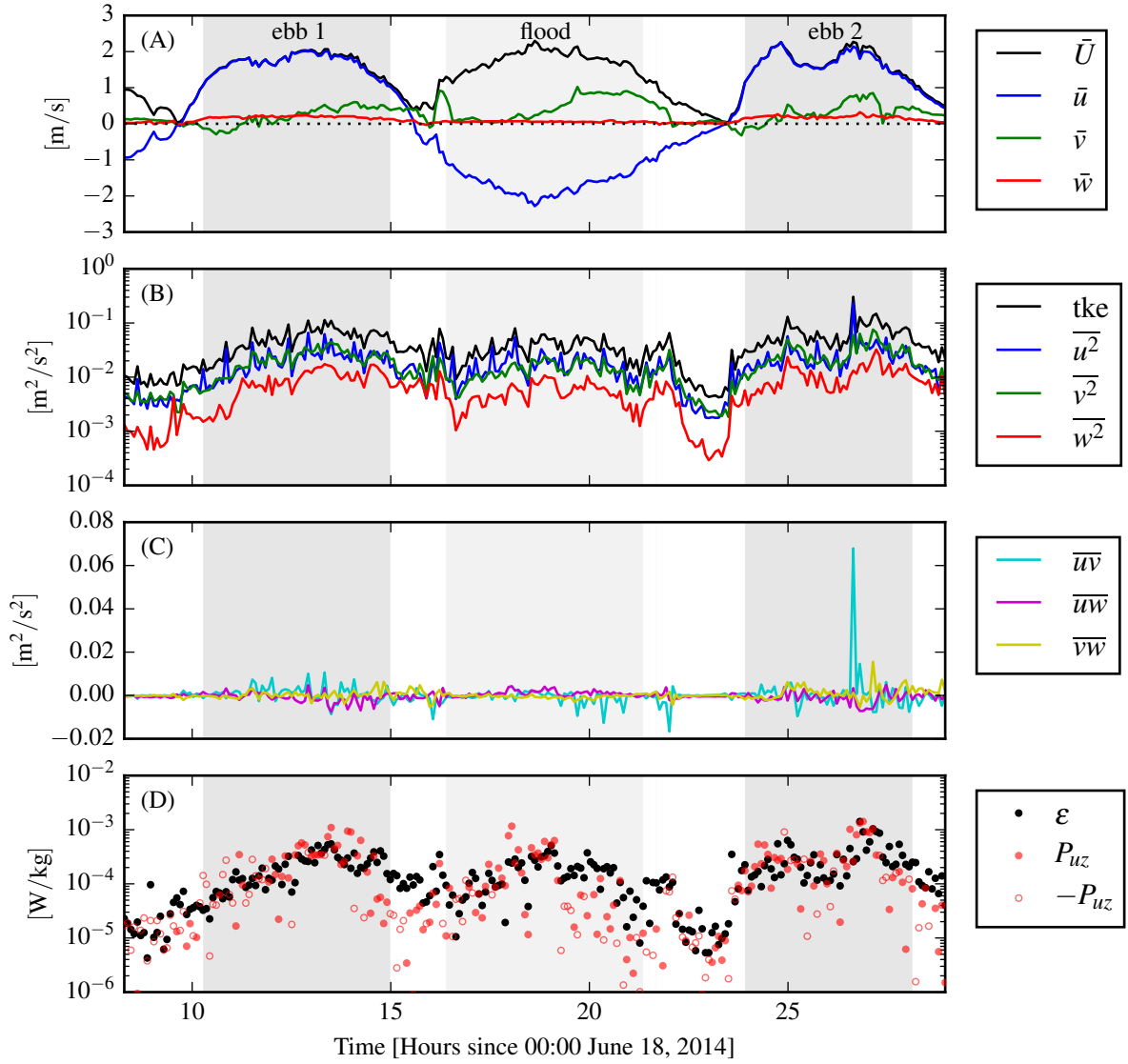
6. Conclusion

FIG. 12. Time-series of mean velocities (A), turbulence energy and its components (B), Reynold's stresses (C), and turbulence dissipation rate (D) measured by the TTM during the June, 2014 deployment. Shading indicates periods of ebb ($\bar{u} > 1.0$, grey), and flood ($\bar{u} < -1.0$, lighter grey).
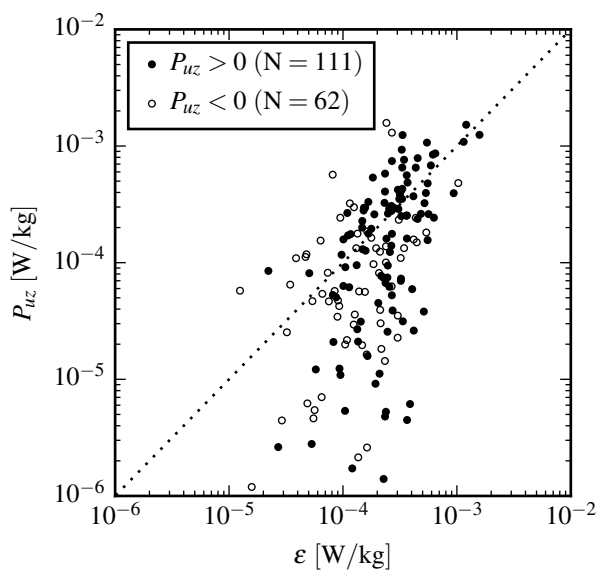
FIG. 13. P_{uz} vs. ε during the June 2014 TTM deployment for values of $|u| > 1$ m/s. Values of 'negative' production are indicated as open circles.

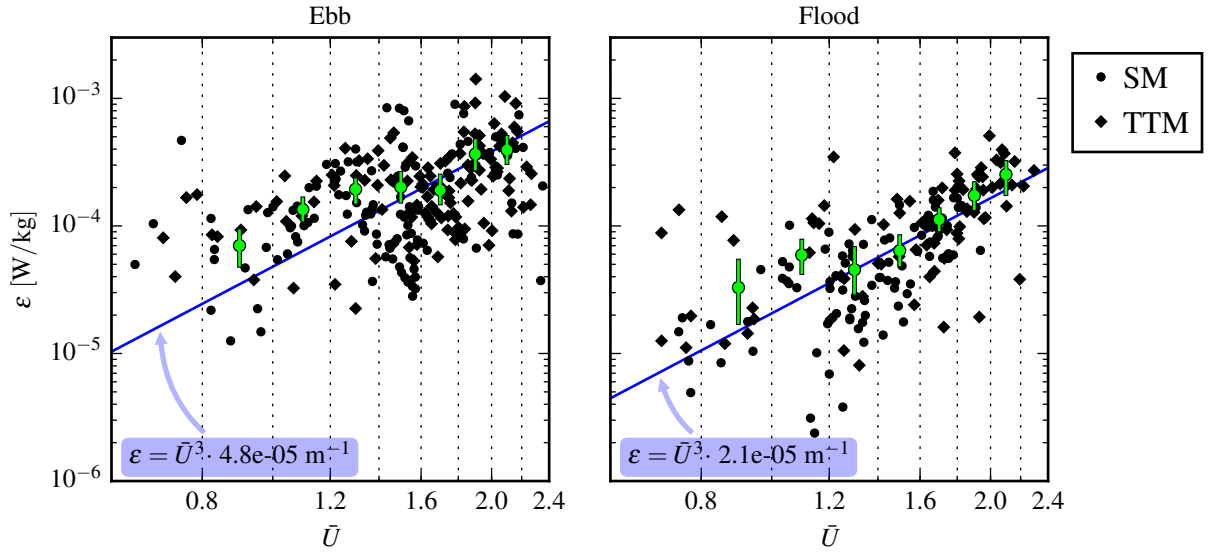FIG. 14. A log-log plot of ε versus $\bar{U}$ for the June 2014 TTM (diamonds) and May 2015 StableMoor (dots) deployments, during ebb (left) and flood (right). Black points are 5 minute averages. Green dots are mean values within speed bins of 0.2 m s^{-1} width that have at least 10 points (50 minutes of data); their vertical bars are 95% bootstrap confidence intervals. The blue line shows a U^3 slope, where the proportionality constant (blue box) is calculated by taking the log-space mean of ε/U^3.

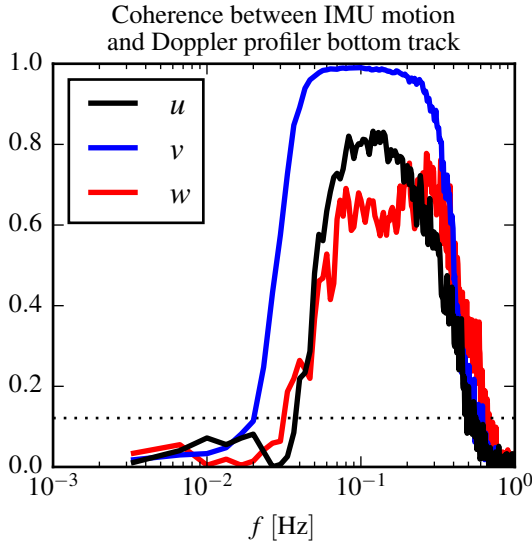## Coherence between IMU motion and Doppler profiler bottom track

FIG. A1. Coherence between IMU-measured motion of StableMoor buoy and ADP bottom track velocity for $1.0 < \bar{U} < 1.5$. The vertical dotted line indicates the 95% confidence level for the 102 spectral windows in this estimate.

APPENDIX

A1. Comparing StableMoor $\vec{u}_{\text{low}}$ to IMU $\vec{u}_{\text{h}}$

In order to understand better understand the IMU's signal-to-noise ratio, it is instructive to compare the motion of the StableMoor buoy from the ADP bottom track measurements, u_{BT}, to the IMU's estimates of ADP motion. To do this, we compute the IMU's estimate of ADP motion using equation (2), and replacing ℓ^* with the vector that points from the IMU to the ADP head. We then linearly interpolated the ADP measurements onto the times of the ADV-IMU measurements.

The coherence between these two signals is high and statistically significant over 1.5 decades (from 0.03 to 0.8 Hz). The v component has the highest coherence, 98%, because this is the direction that has the most motion and therefore these estimates have a higher signal to noise ratio. The u and w components have slightly lower coherence, 80% and 65%, respectively, which indicates the *convolved* signal to noise ratio of these measurements.

On the low-frequency side, our interpretation is that the signal to noise ratio of the IMU increases dramatically below 0.03 Hz, resulting in low coherence. On the high-frequency side Doppler-noise in the ADP measurements contaminates its estimates of motion, causing the decrease in coherence at 0.8 Hz. A comparison of the phase between these signals shows that there is no lag between the measurements (not shown).

These results help to inform the selection of zero-lag filters used to estimate $\vec{u}_{\text{low}}$ from u_{BT}. In particular, by

selecting 0.2 Hz, we target the middle of the coherence peak between the two measurements. Furthermore, the rapid decrease in coherence below 0.03 Hz provides an objective measure of the performance of the frequency at which IMU measured velocity becomes unreliable in the flow conditions we observed.

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