MathStats

Densities

$$f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)} = \frac{f(x,y)}{\int_{-\infty}^{\infty} f(x,y)\,dy}$$
$$F_{Y|X}(y|x) = \int_{-\infty}^{y} \frac{f(x,v)}{f_X(x)}\,dv$$

Expectation

$$\mathbb{E}[x^n] = \int_{-\infty}^{\infty} x^n f_X(x)$$
$$\mathbb{E}[Y] = \int_{-\infty}^{\infty} \mathbb{E}[Y|X=x] f_X(x)\,dx$$
$$\mathbb{E}[X^n] = \sum_{x:f(x)>0} x^n f(x)$$

Basics

$$\mathrm{Cov}[X,Y] = \mathbb{E}[(X-\mathbb{E}[X])(Y-\mathbb{E}[Y])] = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$$
$$\rho(X,Y) = \frac{\mathrm{Cov}[X,Y]}{\sqrt{\mathrm{Var}[X]\cdot\mathrm{Var}[Y]}} = \frac{\mathrm{Cov}[X,Y]}{\sqrt{\sigma_X\sigma_Y}}$$

Expectation Algebra

$$\mathbb{E}[x^n] = \int_{-\infty}^{\infty} x^n f_x(x)$$

Variance Algebra

$$\mathrm{Var}[X+Y] = \mathrm{Var}[X] + 2\,\mathrm{Cov}[X,Y] + \mathrm{Var}[Y]$$
$$\mathrm{Var}[X-Y] = \mathrm{Var}[X] - 2\,\mathrm{Cov}[X,Y] + \mathrm{Var}[Y]$$
$$\mathrm{Var}[XY] = \mathbb{E}[X^2]\cdot\mathbb{E}[Y^2] - (\mathbb{E}[X]\cdot\mathbb{E}[Y])^2$$
$$\mathrm{Var}[X/Y] = \mathrm{Var}[X\cdot(1/Y)] = \mathrm{Var}[(1/Y)\cdot X]$$
$$\mathrm{Var}[X] = \mathrm{Cov}(X,X) = E[X^2] - E[X]^2$$
$$\mathrm{Var}[aX+bY] = a^2\,\mathrm{Var}[X] + b^2 Var[Y] + 2ab\,\mathrm{Cov}[X,Y]$$

Calc

By parts

$$\int u\,dv = uv - \int v\,du$$

Jacobian

$$\mathbb{J} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$
$$\iint_A g(x,y)\,dx\,dy = \iint_B g(x(u,v),y(u,v))|J(u,v)|\,du\,dv$$

Distributions

Distributions arising from the Normal

$$\Gamma(\frac{1}{2},\frac{k}{2}) \to \chi^2_k \to t_k \overset{D}{=} \frac{\mathcal{N}(O,1)}{\sqrt{\chi^2_k/k}} \to F_{m,k} \overset{D}{=} \frac{\chi^2_m/m}{\chi^2_k/k}$$
$$F_{1,k} \overset{D}{=} t_k^2$$

Gamma

$$X \sim \Gamma(\lambda,p) \iff \lambda X \sim \Gamma(1,p)$$
$$X \sim \Gamma(\lambda,p) \overset{c>0}{\Longrightarrow} cX \sim \Gamma(\frac{\lambda}{c},p)$$
$$Theorem(\Gamma(\lambda,p)+\Gamma(\lambda,q) \overset{\perp\!\!\!\perp}{=} \Gamma(\lambda,p+q)):$$
$$\begin{cases} X_1 \sim \Gamma(\lambda,p) \\ X_2 \sim \Gamma(\lambda,q) \\ X_1 \perp\!\!\!\perp X_2 \end{cases} \Longrightarrow \begin{cases} Y_1 = X_1 + X_2 \sim \Gamma(\lambda,p+q) \\ Y_2 = \frac{X_1}{X_1+X_2} \sim \mathrm{Beta}(p,q) \\ Y_1 \perp\!\!\!\perp Y_2 \end{cases}$$

Chi Squared

Definition

if $\{Z_1,\ldots,Z_k\} \overset{i.i.d.}{\sim} \mathcal{N}(0,1)$, then the distribution of

$$V = Z_1^2 + \cdots + Z_k^2$$

is called the χ^2_k distribution with k degrees of freedom PDF of chi^2_k:

$$f_V(v) = \frac{v^{(k-2)/2}e^{-v/2}}{2^{k/2}\Gamma(k/2)} \sim \Gamma(\frac{1}{2},\frac{k}{2}), v \geq 0$$

Chi squared distribution with k=2 is a gamma/exp with the following params

$$\chi^2_2 = \Gamma(\frac{1}{2},1) = \mathrm{Exp}(\frac{1}{2})$$

T Distribution

if

$$Z \sim \mathcal{N}(0,1)$$
$$V \sim \chi^2_k$$
$$Z \perp\!\!\!\perp V$$

then

$$Q = \frac{Z}{\sqrt{\frac{V}{k}}}$$

which is the Student's t distribution with k degrees of freedom PDF of t_k:

$$f_Q(q) = c_1(1+\frac{q^2}{k})^{-(k+1)/2}, q \in \mathbb{R}$$

where $c_1 > 0$ is a constant

Linear Algebra

Determinant

$$\begin{array}{ccc|cc} + & + & + & & \\ a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \\ - & - & - & & \end{array}$$