MathStats

Densities

$$f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)} = \frac{f(x,y)}{\int_{-\infty}^{\infty} f(x,y)\,dy}$$
$$F_{Y|X}(y|x) = \int_{-\infty}^{y} \frac{f(x,v)}{f_X(x)}\,dv$$

Moments

kth order moment
$$m_k = \mathbb{E}[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) = \sum_{x: f_X(x)>0} x^k f_X(x)$$
kth order central moment
$$\sigma_k = \mathbb{E}[(X-m_1)^k]$$

Expectation

$$\mathbb{E}[Y] = \int_{-\infty}^{\infty} \mathbb{E}[Y|X=x] f_X(x)\,dx$$
Conditional expectation
$$\mathbb{E}[Y|X=x] = \int y f_{y|x}(y|x)dy$$
$$\mathbb{E}[g(Y)|X=x] = \int g(y) f_{y|x}(y|x)dy$$
Law of iterated expectation
$$\mathbb{E}_X[E_{Y|X}\{g(Y)|X\}] = \mathbb{E}[g(Y)]$$
If $X \perp\!\!\!\perp Y$, then
$$\mathbb{E}[g(Y)|X=x] = E[g(Y)]$$

Basics

$$\mathrm{Cov}[X,Y] = \mathbb{E}[(X-\mathbb{E}[X])(Y-\mathbb{E}[Y])] = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$$
$$\rho(X,Y) = \frac{\mathrm{Cov}[X,Y]}{\sqrt{\mathrm{Var}[X]\cdot \mathrm{Var}[Y]}} = \frac{\mathrm{Cov}[X,Y]}{\sqrt{\sigma_X \sigma_Y}}$$

Expectation Algebra

$$\mathbb{E}[X] = \mathbb{E}[X_+] - \mathbb{E}[X_-] \text{ if } X_+ \geq 0$$
$$\mathbb{E}[|X|] = \mathbb{E}[X_+] + \mathbb{E}[X_-] \text{ if } X_- \geq 0$$

Variance Algebra

$$\mathrm{Var}[X+Y] = \mathrm{Var}[X] + 2\,\mathrm{Cov}[X,Y] + \mathrm{Var}[Y]$$
$$\mathrm{Var}[X-Y] = \mathrm{Var}[X] - 2\,\mathrm{Cov}[X,Y] + \mathrm{Var}[Y]$$
$$\mathrm{Var}[XY] = \mathbb{E}[X^2]\cdot\mathbb{E}[Y^2] - (\mathbb{E}[X]\cdot\mathbb{E}[Y])^2$$
$$\mathrm{Var}[X/Y] = \mathrm{Var}[X\cdot(1/Y)] = \mathrm{Var}[(1/Y)\cdot X]$$
$$\mathrm{Var}[X] = \mathrm{Cov}(X,X) = E[X^2] - E[X]^2$$
$$\mathrm{Var}[aX+bY] = a^2\,\mathrm{Var}[X] + b^2 Var[Y] + 2ab\,\mathrm{Cov}[X,Y]$$

Correlation

$$\mathrm{Corr}(X,Y) = \frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}(X)}\sqrt{Var(Y)}}$$

Calc

Chain rule
$$f(g(x))' = f'(g(x))g'(x)$$
Product rule
$$(u\cdot v)' = u'\cdot v + u \cdot v'$$
Quotient rule
$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$
Indeterminate Forms
$\frac{0}{0}$, $\frac{\pm\infty}{\pm\infty}$, $\infty-\infty$, $0*\infty$, 0^0, 1^∞, ∞^0
Determinate Forms
$\infty+\infty=\infty$, $-\infty-\infty=-\infty$, $0^\infty=0$, $0^{-\infty}=\infty$, $\infty*\infty=\infty$

Series

$$\sum_{n=0}^{\infty}\frac{x^n}{n!} = e^x$$

Binomial Coefficient

$$\binom{n}{k} = \binom{n}{n-k}$$
$$\binom{n}{k} = \binom{n-1}{k-1}\left(\frac{n}{k}\right)$$
$$(n+1)\binom{r}{n+1} = r\binom{r-1}{n}$$
$$\binom{n}{r+a}\binom{r+a}{r} = \binom{n}{r}\binom{n-r}{a}$$

Distribution Functions

$$\lim_{x\to-\infty} F(x) = 0 \quad \lim_{x\to\infty} F(x) = 1$$
$$F(x) \leq F(y) \text{ if } x < y$$
$$\lim_{h\downarrow 0} F(x+h) = F(x)$$
$$F_{X,Y}(x,y) = \mathbb{P}(X\leq x, Y\leq y)$$
$$F_{\boldsymbol{X}}(\boldsymbol{x}) = \mathbb{P}(\boldsymbol{X}=\boldsymbol{x}) = \mathbb{P}(X_1\leq x_1,\ldots,X_k\leq x_k)$$

Random Variables

$$\mathbb{P}(X > x) = 1 - F(x)$$
$$\mathbb{P}(x < X \leq y) = F(y) - F(x)$$
$$\mathbb{P}(X = x) = F(x) - lim_{t\uparrow x}F(t)$$