MathStats

Densities

$$f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)} = \frac{f(x,y)}{\int_{-\infty}^{\infty} f(x,y)\,dy}$$

$$F_{Y|X}(y|x) = \int_{-\infty}^{y} \frac{f(x,v)}{f_X(x)}\,dv$$

Expectation

$$\mathbb{E}[x^n] = \int_{-\infty}^{\infty} x^n f_X(x)$$

$$\mathbb{E}[Y] = \int_{-\infty}^{\infty} \mathbb{E}[Y|X=x] f_X(x)\,dx$$

$$\mathbb{E}[X^n] = \sum_{x:f(x)>0} x^n f(x)$$

Basics

$$\mathrm{Cov}[X,Y] = \mathbb{E}[(X-\mathbb{E}[X])(Y-\mathbb{E}[Y])] = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$$

$$\rho(X,Y) = \frac{\mathrm{Cov}[X,Y]}{\sqrt{\mathrm{Var}[X]\cdot\mathrm{Var}[Y]}} = \frac{\mathrm{Cov}[X,Y]}{\sqrt{\sigma_X\sigma_Y}}$$

Expectation Algebra

$$\mathbb{E}[x^n] = \int_{-\infty}^{\infty} x^n f_x(x)$$

Variance Algebra

$$\mathrm{Var}[X+Y] = \mathrm{Var}[X] + 2\,\mathrm{Cov}[X,Y] + \mathrm{Var}[Y]$$

$$\mathrm{Var}[X-Y] = \mathrm{Var}[X] - 2\,\mathrm{Cov}[X,Y] + \mathrm{Var}[Y]$$

$$\mathrm{Var}[XY] = \mathbb{E}[X^2]\cdot\mathbb{E}[Y^2] - (\mathbb{E}[X]\cdot\mathbb{E}[Y])^2$$

$$\mathrm{Var}[X/Y] = \mathrm{Var}[X\cdot(1/Y)] = \mathrm{Var}[(1/Y)\cdot X]$$

$$\mathrm{Var}[X] = \mathrm{Cov}(X,X) = E[X^2] - E[X]^2$$

$$\mathrm{Var}[aX+bY] = a^2\,\mathrm{Var}[X] + b^2\,Var[Y] + 2ab\,\mathrm{Cov}[X,Y]$$

Jacobian

$$\mathbb{J} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

$$\iint_A g(x,y)\,dx\,dy = \iint_B g(x(u,v),y(u,v))|J(u,v)|\,du\,dv$$

Calc

$$\int u\,dv = uv - \int v\,du$$

Distributions

Chi squared distribution with k=2 is a gamma/exp with the following params

$$\chi_2^2 = \Gamma(\frac{1}{2},1) = \mathrm{Exp}(\frac{1}{2})$$

T Distribution

$$Z \sim \mathcal{N}(0,1)$$

$$V \sim \chi_k^2$$

$$Z \perp V$$

Linear Algebra

Determinant

$$\begin{array}{ccc|cc} + & + & + & & \\ a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \\ - & - & - & & \end{array}$$