MathStats

Densities

$$f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)} = \frac{f(x,y)}{\int_{-\infty}^{\infty} f(x,y)\,dy}$$
$$F_{Y|X}(y|x) = \int_{-\infty}^{y} \frac{f(x,v)}{f_X(x)}\,dv$$

Moments

kth order moment
$$m_k = \mathbb{E}[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) = \sum_{x:f_X(x)>0} x^k f_X(x)$$
kth order central moment
$$\sigma_k = \mathbb{E}[(X-m_1)^k]$$

Expectation

$$\mathbb{E}[g(X,Y)] = \sum_y \sum_x g(x,y) f_{X,Y}(x,y)$$
$$\mathbb{E}[Y] = \int_{-\infty}^{\infty} \mathbb{E}[Y|X=x] f_X(x)\,dx$$
If $X \geq 0$ (continuous or discrete)
$$\mathbb{E}[X] = \int_0^{\infty} (1 - F_X(x))\,dx$$
Conditional expectation
$$\mathbb{E}[Y|X=x] = \int y f_{y|x}(y|x) dy$$
$$\mathbb{E}[g(Y)|X=x] = \int g(y) f_{y|x}(y|x) dy$$
Law of iterated expectation
$$\mathbb{E}_X[E_{Y|X}\{g(Y)|X\}] = \mathbb{E}[g(Y)]$$
$$\mathbb{E}[h(X)g(Y)] = \mathbb{E}[h(X)\mathbb{E}[g(Y)|X]]$$
$$\mathbb{E}[Y] = \mathbb{E}[\mathbb{E}[Y|X]] = \sum_x \mathbb{E}[Y|X=x] f_X(x)$$
If $X \perp\!\!\!\perp Y$, then
$$\mathbb{E}[g(Y)|X=x] = E[g(Y)]$$
$$\mathrm{Var}[X] = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$

Basics

$$\mathrm{Cov}[X,Y] = \mathbb{E}[(X-\mathbb{E}[X])(Y-\mathbb{E}[Y])] = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$$
$$\rho(X,Y) = \frac{\mathrm{Cov}[X,Y]}{\sqrt{\mathrm{Var}[X]\cdot\mathrm{Var}[Y]}} = \frac{\mathrm{Cov}[X,Y]}{\sqrt{\sigma_X \sigma_Y}}$$

Expectation Algebra

$$\mathbb{E}[X] = \mathbb{E}[X_+] - \mathbb{E}[X_-] \text{ if } X_+ \geq 0$$
$$\mathbb{E}[|X|] = \mathbb{E}[X_+] + \mathbb{E}[X_-] \text{ if } X_- \geq 0$$
$$\mathbb{E}[a+bX] = a + b\mathbb{E}[X]$$
$$\mathbb{E}[aX+bY+c] = a\mathbb{E}[X] + b\mathbb{E}[Y] + c$$
If $X \perp\!\!\!\perp Y$, then
$$\mathbb{E}[XY] = \mathbb{E}[X]\mathbb{E}[Y]$$

Variance Algebra

$$\mathrm{Var}[X+Y] = \mathrm{Var}[X] + 2\,\mathrm{Cov}[X,Y] + \mathrm{Var}[Y]$$
$$\mathrm{Var}[X-Y] = \mathrm{Var}[X] - 2\,\mathrm{Cov}[X,Y] + \mathrm{Var}[Y]$$
If $X \perp\!\!\!\perp Y$, then
$$\mathrm{Var}[X+Y] = \mathrm{Var}[X] + \mathrm{Var}[Y]$$
If $X \perp\!\!\!\perp Y$, then
$$\mathrm{Var}[X-Y] = \mathrm{Var}[X] - \mathrm{Var}[Y]$$
$$\mathrm{Var}[XY] = \mathbb{E}[X^2]\cdot\mathbb{E}[Y^2] - (\mathbb{E}[X]\cdot\mathbb{E}[Y])^2$$
$$\mathrm{Var}[X/Y] = \mathrm{Var}[X\cdot(1/Y)] = \mathrm{Var}[(1/Y)\cdot X]$$
$$\mathrm{Var}[X] = \mathrm{Cov}(X,X) = E[X^2] - E[X]^2$$
$$\mathrm{Var}[aX+bY] = a^2\,\mathrm{Var}[X] + b^2 Var[Y] + 2ab\,\mathrm{Cov}[X,Y]$$

Correlation

$$\mathrm{Corr}(X,Y) = \frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}(X)}\sqrt{Var(Y)}}$$

Calc

Chain rule
$$f(g(x))' = f'(g(x))g'(x)$$
Product rule
$$(u\cdot v)' = u'\cdot v + u\cdot v'$$
Quotient rule
$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$
Indeterminate Forms
$\frac{0}{0}$, $\frac{\pm\infty}{\pm\infty}$, $\infty-\infty$, $0*\infty$, 0^0, 1^∞, ∞^0
Determinate Forms
$\infty+\infty=\infty$, $-\infty-\infty=-\infty$, $0^\infty=0$, $0^{-\infty}=\infty$, $\infty*\infty=\infty$

Series

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$$
$$\sum_{k=1}^{\infty} a^k = \frac{a}{1-a} \text{ where } |a|<1$$
$$\sum_{k=1}^{\infty} k(1-p)^{k-1} = \frac{1}{p^2} = p^{-2}$$
$$\sum_{k=2}^{\infty} k(k-1)(1-p)^{k-2} = \frac{2}{p^3}$$

Binomial Coefficient

$$\binom{n}{k} = \binom{n}{n-k}$$
$$\binom{n}{k} = \binom{n-1}{k-1}\binom{n}{k} \text{ if } k>0$$
$$(n+1)\binom{r}{n+1} = r\binom{r-1}{n}$$
$$\binom{n}{r+a}\binom{r+a}{r} = \binom{n}{r}\binom{n-r}{a}$$
$$\frac{1}{k+1}\binom{n}{k} = \binom{n+1}{k+1}\frac{1}{n+1}$$

Distribution Functions

$$\lim_{x\to-\infty} F(x) = 0 \quad \lim_{x\to\infty} F(x) = 1$$
$$F(x) \leq F(y) \text{ if } x<y$$
$$\lim_{h\downarrow 0} F(x+h) = F(x)$$
$$F(x) = \int_{-\infty}^{x} f(t)\,dt \text{ where } f(t)\geq 0 \text{ and } \int_{-\infty}^{\infty} f(t)\,dt = 1$$
$$F_{X,Y}(x,y) = \mathbb{P}(X\leq x, Y\leq y)$$
$$F_{\boldsymbol{X}}(\boldsymbol{x}) = \mathbb{P}(\boldsymbol{X}\leq\boldsymbol{x}) = \mathbb{P}(X_1\leq x_1,\ldots,X_k\leq x_k)$$
$$F_{\boldsymbol{X}}(\boldsymbol{x}) = \int_{-\infty}^{x_1}\cdots\int_{-\infty}^{x_k} f(t_1,\cdots,t_k)\,dt_1\cdots dt_k$$

Random Variables

$$X:\Omega\to\mathbb{R}$$
$$\{\omega\in\Omega : X(\omega)\leq x\}\in\mathcal{F}$$
$$F(x) = \mathbb{P}(X\leq x) = \mathbb{P}(\{\omega : X(\omega)\leq x\})$$
$$\mathbb{P}(X>x) = 1-F(x)$$
$$\mathbb{P}(x<X\leq y) = F(y) - F(x)$$
$$\mathbb{P}(X<x) = \lim_{n\to\infty} F(x-\frac{1}{n})$$
$$\mathbb{P}(X=x) = \mathbb{P}(X\leq x) - \mathbb{P}(X<x) = F(x) - \lim_{n\to\infty} F(x-\frac{1}{n})$$

PDF

Joint PDF
$$F(x,y) = \mathbb{P}(X\leq x, Y\leq y) = \int_{-\infty}^{x}\left\{\int_{-\infty}^{y} f(u,v)\,dv\right\}\,du$$
$$f(x,y) = \frac{\partial^2}{\partial y\partial x}F(x,y)$$
Marginal PDF
$$f_X(x) = \int_{-\infty}^{+\infty} f(x,v)\,dv,\ f_Y(y) = \int_{-\infty}^{+\infty} f(u,y)\,du$$

Convolution

Let $Z = X+Y$
$$f_Z(z) = \sum_x f_X(x) f_{Y|X}(z-x|x) = \sum_y f_Y(y) f_{X|Y}(z-y|y)$$
If $X \perp\!\!\!\perp Y$, then
$$f_Z(z) = \sum_x f_X(x) f_Y(z-x) = \sum_y f_Y(y) f_X(z-y)$$