MathStats

Densities

$$f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)} = \frac{f(x,y)}{\int_{-\infty}^{\infty} f(x,y)\,dy}$$
$$F_{Y|X}(y|x) = \int_{-\infty}^{y} \frac{f(x,v)}{f_X(x)}\,dv$$

Expectation

$$\mathbb{E}[x^n] = \int_{-\infty}^{\infty} x^n f_X(x)$$
$$\mathbb{E}[Y] = \int_{-\infty}^{\infty} \mathbb{E}[Y|X=x] f_X(x)\,dx$$
$$\mathbb{E}[X^n] = \sum_{x:f(x)>0} x^n f(x)$$

Conditional expectation

$$\mathbb{E}[Y|X=x] = \int y f_{y|x}(y|x) dy$$
$$\mathbb{E}[g(Y)|X=x] = \int g(y) f_{y|x}(y|x) dy$$

Law of iterated expectation

$$\mathbb{E}_X[E_{Y|X}\{g(Y)|X\}] = E[g(Y)]$$

If $X \perp\!\!\!\perp Y$, then

$$E[g(Y)|X=x] = E[g(Y)]$$

Not well defined (cauchy)

$$E[X] = E[X_+] - E[X_-] = \infty - \infty$$

Well defined (cauchy)

$$E[|X|] = \infty$$

Basics

$$\mathrm{Cov}[X,Y] = \mathbb{E}[(X-\mathbb{E}[X])(Y-\mathbb{E}[Y])] = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$$
$$\rho(X,Y) = \frac{\mathrm{Cov}[X,Y]}{\sqrt{\mathrm{Var}[X]\cdot \mathrm{Var}[Y]}} = \frac{\mathrm{Cov}[X,Y]}{\sqrt{\sigma_X \sigma_Y}}$$

Expectation Algebra

$$\mathbb{E}[x^n] = \int_{-\infty}^{\infty} x^n f_x(x)$$

Variance Algebra

$$\mathrm{Var}[X+Y] = \mathrm{Var}[X] + 2\,\mathrm{Cov}[X,Y] + \mathrm{Var}[Y]$$
$$\mathrm{Var}[X-Y] = \mathrm{Var}[X] - 2\,\mathrm{Cov}[X,Y] + \mathrm{Var}[Y]$$
$$\mathrm{Var}[XY] = \mathbb{E}[X^2]\cdot\mathbb{E}[Y^2] - (\mathbb{E}[X]\cdot\mathbb{E}[Y])^2$$
$$\mathrm{Var}[X/Y] = \mathrm{Var}[X\cdot(1/Y)] = \mathrm{Var}[(1/Y)\cdot X]$$
$$\mathrm{Var}[X] = \mathrm{Cov}(X,X) = E[X^2] - E[X]^2$$
$$\mathrm{Var}[aX+bY] = a^2\,\mathrm{Var}[X] + b^2 Var[Y] + 2ab\,\mathrm{Cov}[X,Y]$$

Correlation

$$\mathrm{Corr}(X,Y) = \frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}(X)}\sqrt{Var(Y)}}$$

Calc

By parts

$$\int u\,dv = uv - \int v\,du$$

Chain rule

$$f(g(x))' = f'(g(x))g'(x)$$

Product rule

$$(u\cdot v)' = u'\cdot v + u\cdot v'$$

Quotient rule

$$(\frac{f(x)}{g(x)})' = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

Jacobian

$$\mathbb{J} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$
$$\iint_A g(x,y)\,dx\,dy = \iint_B g(x(u,v),y(u,v))|J(u,v)|\,du\,dv$$