Qubits
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量子计算

Quantum Computing

—数学基础

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介绍

教程简介：

- 面向对象：量子计算初学者
- 依赖课程：线性代数，解析几何，量子力学（非必需）

知乎专栏：
https://www.zhihu.com/column/c_1501138176371011584

Github & Gitee 地址：
https://github.com/mymagicpower/qubits
https://gitee.com/mymagicpower/qubits

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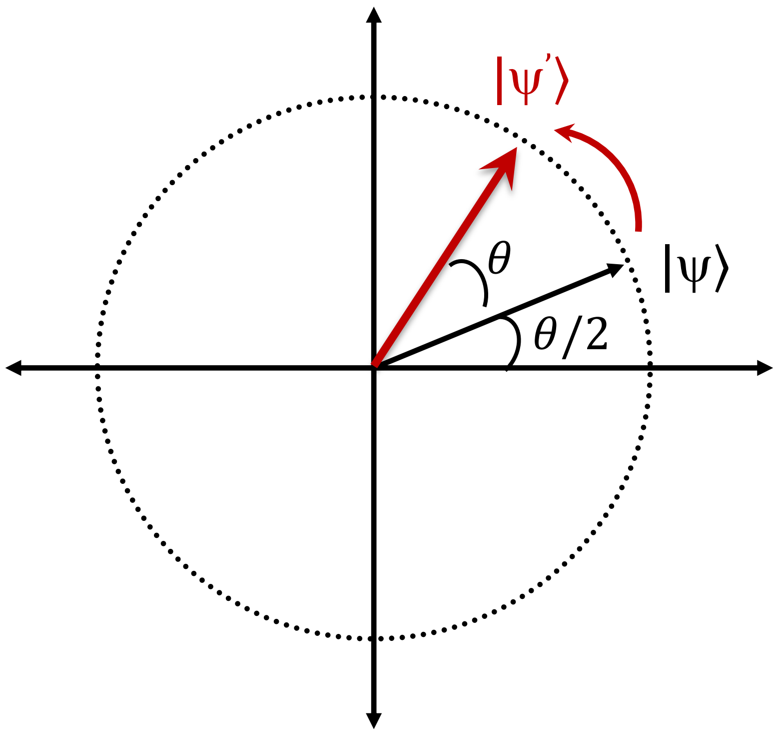
常用几何变换 - 逆时针旋转 θ



$$|\psi\rangle = \cos(\theta/2)\ |0\rangle + \sin(\theta/2)\ |1\rangle = \begin{bmatrix}\cos(\theta/2)\\ \sin(\theta/2)\end{bmatrix}$$

$$Q = \begin{bmatrix}\cos(\theta) & -sin(\theta)\\ \sin(\theta) & \cos(\theta)\end{bmatrix}$$

* 每次作用于向量，相当于逆时针旋转 θ

证明：

两角和与差的三角函数公式：

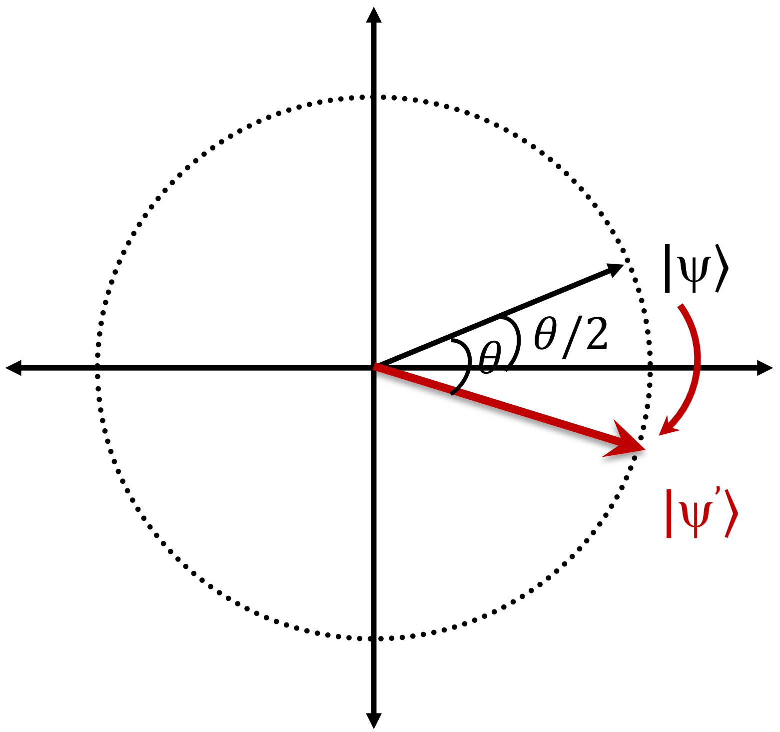
$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp sin\,\alpha\sin\beta$$

$$\begin{aligned}
|\psi'\rangle &= Q\,|\psi\rangle \\
&= \begin{bmatrix}\cos(\theta) & -sin(\theta)\\ \sin(\theta) & \cos(\theta)\end{bmatrix}\begin{bmatrix}\cos(\theta/2)\\ \sin(\theta/2)\end{bmatrix} \\
&= \begin{bmatrix}\cos(\theta)\cos(\theta/2) - sin(\theta)\sin(\theta/2)\\ \sin(\theta)\ \cos(\theta/2)\ + \cos(\theta)\sin(\theta/2)\end{bmatrix} \\
&= \begin{bmatrix}\cos(\theta/2+\theta)\\ \sin(\theta/2+\theta)\end{bmatrix}
\end{aligned}$$

常用几何变换 - 顺时针旋转 θ

$$Q=\begin{bmatrix} \cos(\theta) & sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$$

* 每次作用于向量，相当于顺时针旋转 θ



$$|\psi\rangle=\cos(\theta/2)\ |0\rangle+\sin(\theta/2)\ |1\rangle=\begin{bmatrix} \cos(\theta/2) \\ \sin(\theta/2) \end{bmatrix}$$

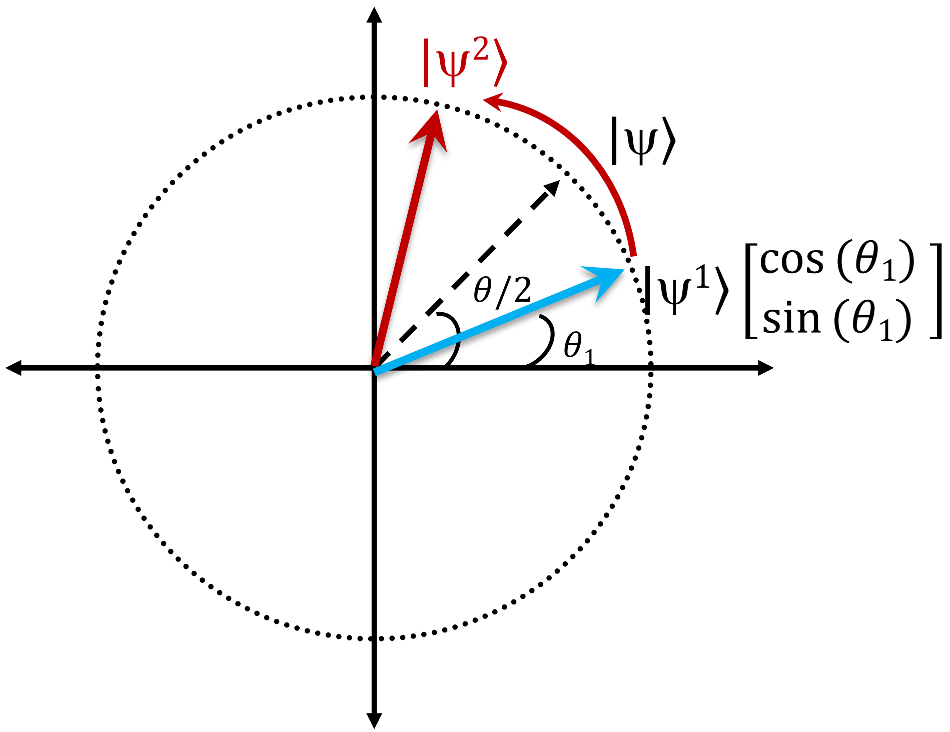
证明：

两角和与差的三角函数公式：

$$\sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\cos\alpha\sin\beta$$
$$\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp sin\,\alpha\sin\beta$$

$$\begin{aligned}|\psi'\rangle &= Q\,|\psi\rangle \\ &= \begin{bmatrix} \cos(\theta) & sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}\begin{bmatrix} \cos(\theta/2) \\ \sin(\theta/2) \end{bmatrix} \\ &= \begin{bmatrix} \cos(\theta)\cos(\theta/2)+sin(\theta)\sin(\theta/2) \\ -\sin(\theta)\ \cos(\theta/2)\ +\cos(\theta)\sin(\theta/2) \end{bmatrix} \\ &= \begin{bmatrix} \cos(\theta/2-\theta) \\ \sin(\theta/2-\theta) \end{bmatrix}\end{aligned}$$

常用几何变换 – 镜像



$$|\psi\rangle = \cos(\theta/2)\ |0\rangle + \sin(\theta/2)\ |1\rangle = \begin{bmatrix} \cos(\theta/2) \\ \sin(\theta/2) \end{bmatrix}$$

$$Q = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ \sin(\theta) & -\cos(\theta) \end{bmatrix}$$

* 关于通过原点、方向和水平轴夹角为 $\theta/2$ 直线镜像;

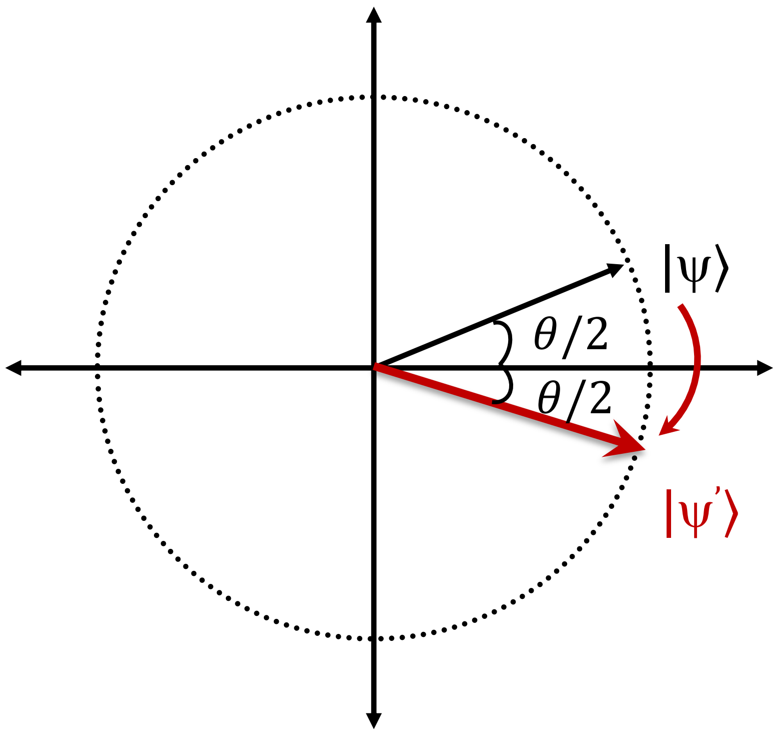
证明：

$$\begin{aligned} |\psi^2\rangle &= Q\ |\psi^1\rangle \\ &= \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ \sin(\theta) & -\cos(\theta) \end{bmatrix} \begin{bmatrix} \cos(\theta_1) \\ \sin(\theta_1) \end{bmatrix} \\ &= \begin{bmatrix} \cos(\theta)\cos(\theta_1) + \sin(\theta)\sin(\theta_1) \\ \sin(\theta)\ \cos(\theta_1) - \cos(\theta)\sin(\theta_1) \end{bmatrix} \\ &= \begin{bmatrix} \cos(\theta - \theta_1) \\ \sin(\theta - \theta_1) \end{bmatrix} \end{aligned}$$

关于通过原点、方向和水平轴夹角为 $\theta/2$ 直线镜像，可以理解为逆时针旋转 $2\left(\frac{\theta}{2} - \theta_1\right)$，则：

$$|\psi^2\rangle = \begin{bmatrix} \cos\left(\theta_1 + 2\left(\frac{\theta}{2} - \theta_1\right)\right) \\ \sin\left(\theta_1 + 2\left(\frac{\theta}{2} - \theta_1\right)\right) \end{bmatrix} = \begin{bmatrix} \cos(\theta - \theta_1) \\ \sin(\theta - \theta_1) \end{bmatrix}$$

常用几何变换 -关于横轴镜像对称



$$|\psi\rangle = \cos(\theta/2)\ |0\rangle + \sin(\theta/2)\ |1\rangle = \begin{bmatrix}\cos(\theta/2)\\ \sin(\theta/2)\end{bmatrix}$$

$$Q=\begin{bmatrix}\cos(0) & \mathit{sin}(0)\\ \sin(0) & -\cos(0)\end{bmatrix}$$

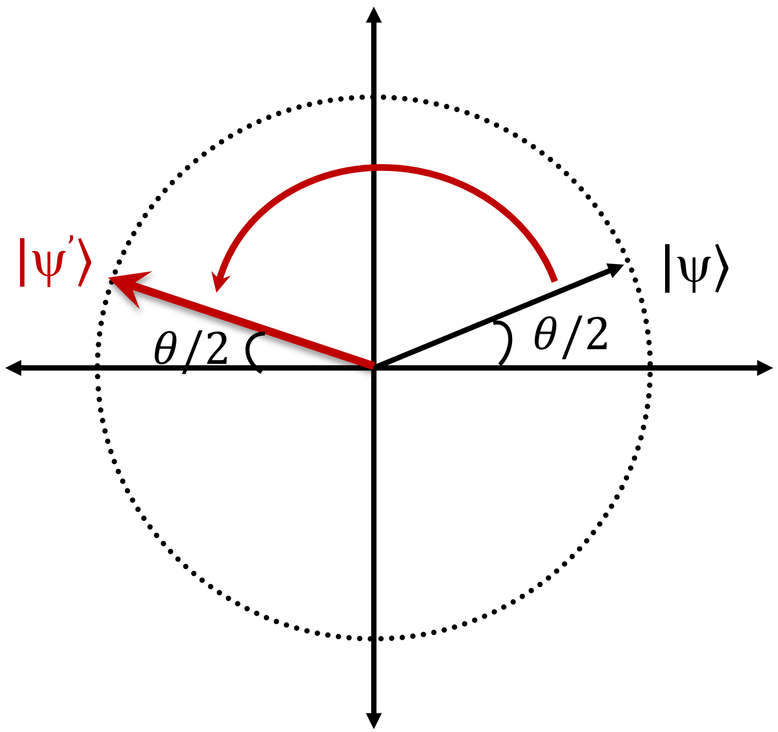
$$Q=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$$

* 作用于向量，相当于关于横轴镜像

证明：

$$\begin{aligned}|\psi'\rangle &= Q\,|\psi\rangle\\ &= \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}\cos(\theta/2)\\ \sin(\theta/2)\end{bmatrix}\\ &= \begin{bmatrix}\cos(\theta/2)\\ -\sin(\theta/2)\end{bmatrix}\end{aligned}$$

常用几何变换 -关于纵轴镜像对称



$$|\psi\rangle = \cos(\theta/2)\ |0\rangle + \sin(\theta/2)\ |1\rangle = \begin{bmatrix} \cos(\theta/2) \\ \sin(\theta/2) \end{bmatrix}$$

$$Q = \begin{bmatrix} \cos(2\frac{\pi}{2}) & \sin(2\frac{\pi}{2}) \\ \sin(2\frac{\pi}{2}) & -\cos(2\frac{\pi}{2}) \end{bmatrix}$$

$$Q = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

* 作用于向量，相当于关于纵轴镜像

证明：

$$\begin{aligned} |\psi'\rangle &= Q\,|\psi\rangle \\ &= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta/2) \\ \sin(\theta/2) \end{bmatrix} \\ &= \begin{bmatrix} -\cos(\theta/2) \\ \sin(\theta/2) \end{bmatrix} \end{aligned}$$

常用几何变换 – 镜像

令 $|\psi\rangle = \begin{bmatrix} \cos(\theta/2) \\ \sin(\theta/2) \end{bmatrix}$

则 $|\psi\rangle\langle\psi| = \begin{bmatrix} \cos(\theta/2) \\ \sin(\theta/2) \end{bmatrix} \begin{bmatrix} \cos(\theta/2) & \sin(\theta/2) \end{bmatrix} = \begin{bmatrix} \cos^2(\theta/2) & \cos(\theta/2)\,sin(\theta/2) \\ \cos(\theta/2)\,sin(\theta/2) & sin^2(\theta/2) \end{bmatrix}$

则 $2|\psi\rangle\langle\psi| - I = \begin{bmatrix} 2\cos^2(\theta/2) - 1 & 2\cos(\theta/2)\,sin(\theta/2) \\ 2\cos(\theta/2)\,sin(\theta/2) & 2\,sin^2(\theta/2) - 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & sin(\theta) \\ \sin(\theta) & -\cos(\theta) \end{bmatrix}$

- $2|\psi\rangle\langle\psi| - I$ ，相当于关于 $|\psi\rangle$ 镜像
- $|\psi\rangle$ 为镜像轴

Thank

You