Qubits
qubits.top

量子计算

—基础篇

Quantum Computing

网址：www.qubits.top

Calvin Tang

179209347@qq.com

介绍

教程简介：

- 面向对象：量子计算初学者
- 依赖课程：线性代数，量子力学（非必需）

知乎专栏：
https://www.zhihu.com/column/c_1501138176371011584

Github & Gitee 地址：
https://github.com/mymagicpower/qubits
https://gitee.com/mymagicpower/qubits

*** 版权声明：**

- 仅限用于个人学习
- 禁止用于任何商业用途

介绍

教程简介：

- 面向对象：量子计算初学者
- 依赖课程：线性代数，量子力学（非必需）

知乎专栏：
https://www.zhihu.com/column/c_1501138176371011584

Github & Gitee 地址：
https://github.com/mymagicpower/qubits
https://gitee.com/mymagicpower/qubits

*** 版权声明：**

- 仅限用于个人学习
- 禁止用于任何商业用途

RX(θ) 门

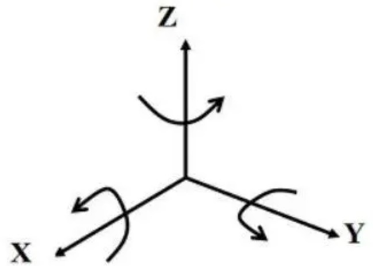
RX门由Pauli-X 矩阵作为生成元生成，其矩阵形式为：

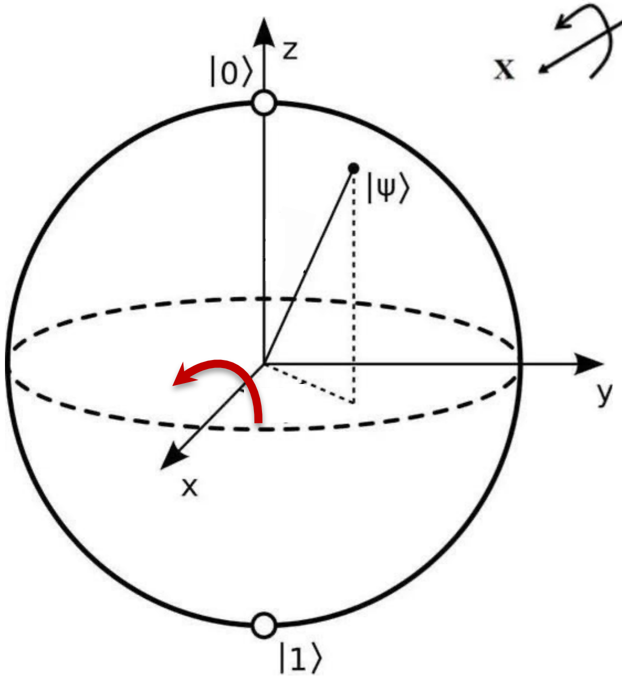
$$X = \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$R_x(\theta) = e^{-i\theta X/2} = \cos(\theta/2)\, I - i\sin(\theta/2) X$$

$$= \begin{bmatrix} \cos(\theta/2) & -i\sin(\theta/2) \\ -i\sin(\theta/2) & \cos(\theta/2) \end{bmatrix}$$

其量子线路符号：





RX操作将原来的态上绕X轴逆时针旋转 θ 角。

RX(θ) 门

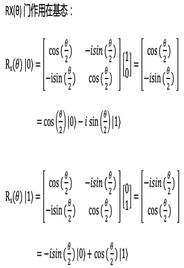
RX(θ) 门作用在基态：

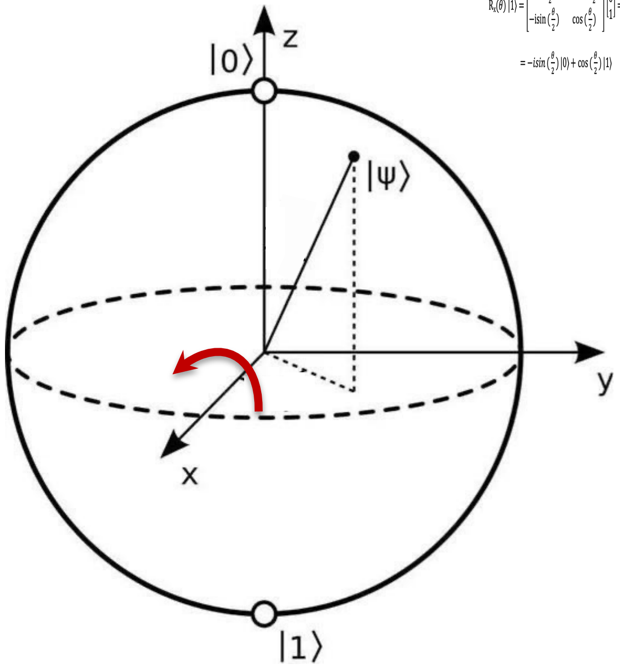
$$R_x(\theta)\,|0\rangle = \begin{bmatrix} \cos\left(\frac{\theta}{2}\right) & -isin\left(\frac{\theta}{2}\right) \\ -\mathrm{isin}\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right) \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\left(\frac{\theta}{2}\right) \\ -isin\left(\frac{\theta}{2}\right) \end{bmatrix}$$

$$= \cos\left(\frac{\theta}{2}\right)|0\rangle - i\sin\left(\frac{\theta}{2}\right)|1\rangle$$

$$R_x(\theta)\,|1\rangle = \begin{bmatrix} \cos\left(\frac{\theta}{2}\right) & -isin\left(\frac{\theta}{2}\right) \\ -\mathrm{isin}\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right) \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -isin\left(\frac{\theta}{2}\right) \\ \cos\left(\frac{\theta}{2}\right) \end{bmatrix}$$

$$= -isin\left(\frac{\theta}{2}\right)|0\rangle + \cos\left(\frac{\theta}{2}\right)|1\rangle$$





RY(θ) 门

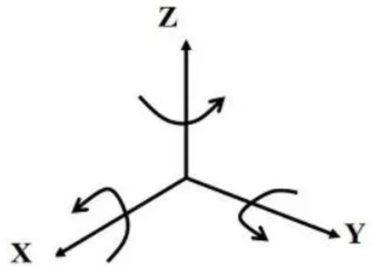
RY门由Pauli-Y 矩阵作为生成元生成，其矩阵形式为：

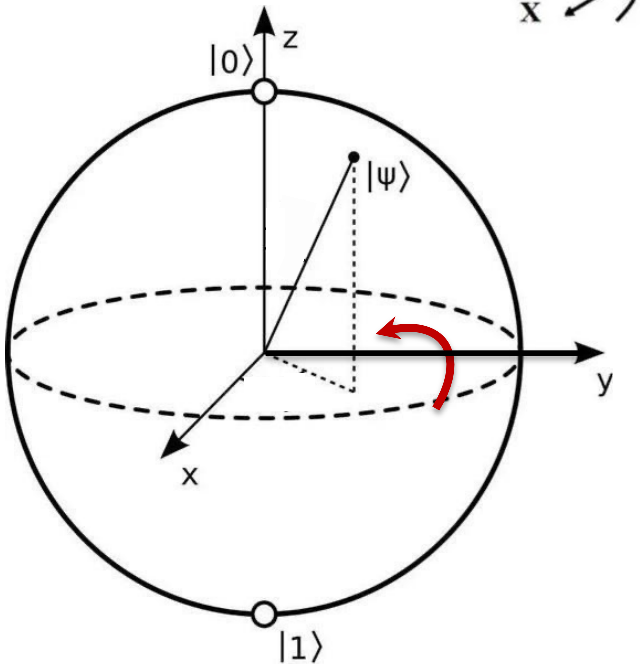
$$Y = \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$R_y(\theta) = e^{-i\theta Y/2} = \cos(\theta/2)\, I - i \sin(\theta/2) Y$$

$$= \begin{bmatrix} \cos(\theta/2) & -sin(\theta/2) \\ \sin(\theta/2) & \cos(\theta/2) \end{bmatrix}$$

其量子线路符号：





RY操作将原来的态上绕Y轴逆时针旋转θ角。

RY(θ) 门

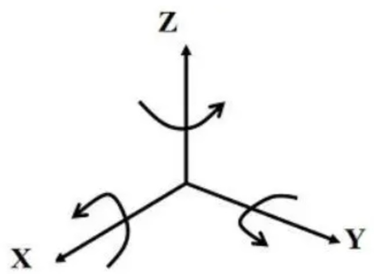
RY(θ) 门作用在基态：

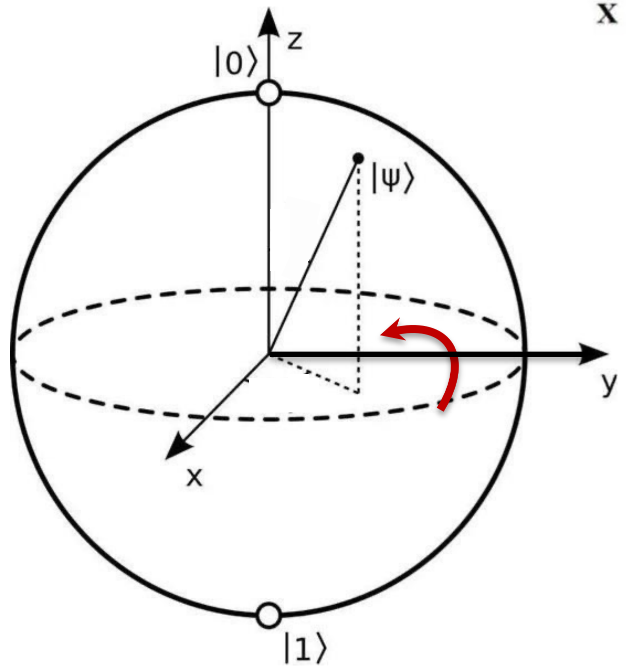
$$R_y(\theta)\,|0\rangle = \begin{bmatrix} \cos(\theta/2) & -sin\,(\theta/2) \\ \sin(\theta/2) & \cos(\theta/2) \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos(\theta/2) \\ \sin(\theta/2) \end{bmatrix}$$

$$= \cos\left(\frac{\theta}{2}\right)|0\rangle + \sin\left(\frac{\theta}{2}\right)|1\rangle$$

$$R_y(\theta)\,|1\rangle = \begin{bmatrix} \cos(\theta/2) & -sin\,(\theta/2) \\ \sin(\theta/2) & \cos(\theta/2) \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -sin\,(\theta/2) \\ \cos(\theta/2) \end{bmatrix}$$

$$= -\sin\left(\frac{\theta}{2}\right)|0\rangle + \cos\left(\frac{\theta}{2}\right)|1\rangle$$





RY(θ) 门 - 重要性质

两角和与差的三角函数公式：

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp sin\,\alpha\sin\beta$

$$Q = R_y(\theta) = \begin{bmatrix} \cos(\theta/2) & -sin(\theta/2) \\ \sin(\theta/2) & \cos(\theta/2) \end{bmatrix}$$

$$Q^2 = \begin{bmatrix} \cos(\theta/2) & -sin(\theta/2) \\ \sin(\theta/2) & \cos(\theta/2) \end{bmatrix}\begin{bmatrix} \cos(\theta/2) & -sin(\theta/2) \\ \sin(\theta/2) & \cos(\theta/2) \end{bmatrix} = \begin{bmatrix} \cos^2(\theta/2) - sin^2(\theta/2) & -2\cos(\theta/2)\,sin(\theta/2) \\ 2\cos(\theta/2)\,sin(\theta/2) & \cos^2(\theta/2) - sin^2(\theta/2) \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta/2 + \theta/2) & -sin(\theta/2 + \theta/2) \\ \sin(\theta/2 + \theta/2) & \cos(\theta/2 + \theta/2) \end{bmatrix}$$

$$Q^3 = \begin{bmatrix} \cos(\theta) & -sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}\begin{bmatrix} \cos(\theta/2) & -sin(\theta/2) \\ \sin(\theta/2) & \cos(\theta/2) \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta)\cos(\theta/2) - sin(\theta)\ sin(\theta/2) & -\cos(\theta)\sin(\theta/2) - sin(\theta)\ cos(\theta/2) \\ \sin(\theta)\cos(\theta/2) + cos(\theta)\ sin(\theta/2) & -\sin(\theta)\sin(\theta/2) + cos(\theta)\ cos(\theta/2) \end{bmatrix}$$

$$= \begin{bmatrix} \cos(3\theta/2) & -sin(3\theta/2) \\ \sin(3\theta/2) & \cos(3\theta/2) \end{bmatrix}$$

....

$$Q^n = \begin{bmatrix} \cos(n\theta/2) & -sin(n\theta/2) \\ \sin(n\theta/2) & \cos(n\theta/2) \end{bmatrix}$$

RY(θ) 门 - 举例（ α 和 β 都为实数 ）

$$Q = R_y(\theta) = \begin{bmatrix} \cos(\theta/2) & -sin(\theta/2) \\ \sin(\theta/2) & \cos(\theta/2) \end{bmatrix}$$

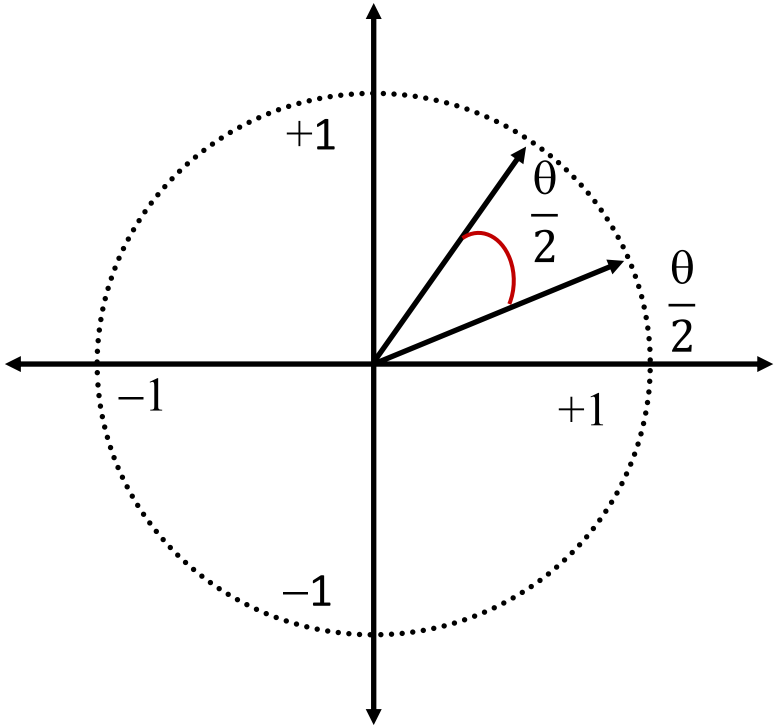
Q 作用在量子态 $|\psi\rangle = \cos(\theta/2)\ |0\rangle + \sin(\theta/2)\ |1\rangle = \begin{bmatrix} \cos(\theta/2) \\ \sin(\theta/2) \end{bmatrix}$

$$Q^1 |\psi\rangle = Q^1 \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \cos(\theta/2) & -sin(\theta/2) \\ \sin(\theta/2) & \cos(\theta/2) \end{bmatrix} \begin{bmatrix} \cos(\theta/2) \\ \sin(\theta/2) \end{bmatrix} = \begin{bmatrix} \cos(\theta/2 + \theta/2) \\ \sin(\theta/2 + \theta/2) \end{bmatrix}$$

$$Q^2 |\psi\rangle = \begin{bmatrix} \cos(2\theta/2 + \theta/2) \\ \sin(2\theta/2 + \theta/2) \end{bmatrix}$$

....

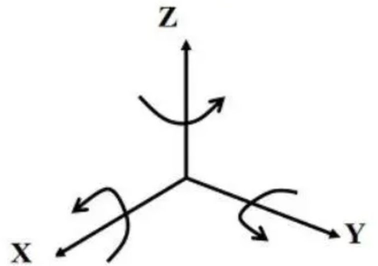
$$Q^n |\psi\rangle = \begin{bmatrix} \cos((n+1)\theta/2) \\ \sin((n+1)\theta/2) \end{bmatrix} = \cos((n+1)\theta/2)\,|0\rangle + \sin((n+1)\theta/2)\,|1\rangle$$



* 每次作用于量子态(向量)，相当于逆时针旋转 $\frac{\theta}{2}$

RZ(θ) 门

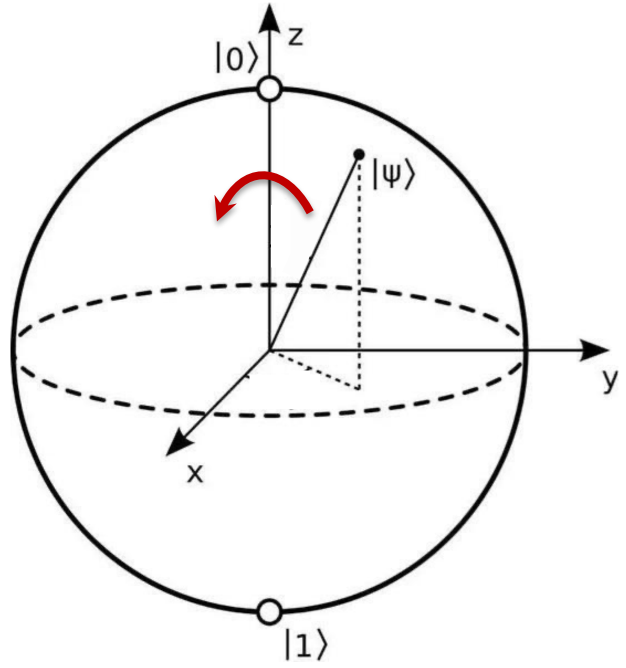
RZ 门又称为相位转化门(phase-shift gate)，由Pauli-Z 矩阵作为生成元生成，其矩阵形式为：



$$R_z(\theta) = e^{-i\theta Z/2} = \cos(\theta/2)\, I - i \sin(\theta/2) Z$$

$$= \begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix} = e^{-i\theta/2} \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix}$$

$$Z = \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$



其量子线路符号：

RZ门作用在基态：

$$R_z(\theta)\,|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

$$R_z(\theta)\,|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ e^{i\theta} \end{bmatrix} = e^{i\theta}\,|1\rangle$$

RZ 操作将原来的态上绕 Z 轴逆时针旋转θ角。不会导致概率振幅的变化，只会改变相位。

RZ(θ) 门

由于$e^{-i\theta/2}$是一个全局相位，其没有物理意义，只考虑单门，则可以省略该参数。于是，RZ门矩阵可简写为：

$$\mathrm{R_z}(\theta) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix}$$

RZ 操作将原来的态上绕 Z 轴逆时针旋转θ角。不会导致概率振幅的变化，只会改变相位。

因为：

① $|\psi\rangle = r_0|0\rangle + r_1 e^{i\varphi}|1\rangle = \cos(\theta)|0\rangle + \sin(\theta)\, e^{i\varphi}|1\rangle = \begin{bmatrix} \cos(\theta) \\ \sin(\theta)\, e^{i\varphi} \end{bmatrix}$

② $\mathrm{R_z}(\theta) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix}$

可得：

$$\mathrm{R_z}(\theta)\,|\psi\rangle = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix} \begin{bmatrix} \cos(\theta) \\ \sin(\theta)\, e^{i\varphi} \end{bmatrix} = \begin{bmatrix} \cos(\theta) \\ e^{i\theta} e^{i\varphi} \sin(\theta) \end{bmatrix} = \begin{bmatrix} \cos(\theta) \\ e^{i(\theta+\varphi)} \sin(\theta) \end{bmatrix}$$

RZ(θ) 门

$$X = \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

RZ(θ) 门其它性质：

$$\begin{aligned} \mathrm{X}\mathrm{R_z}(\theta)\mathrm{X} &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & e^{i\theta} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} e^{i\theta} & 0 \\ 0 & 1 \end{bmatrix} \\ &= e^{i\theta} \begin{bmatrix} 1 & 0 \\ 0 & e^{-i\theta} \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & e^{-i\theta} \end{bmatrix} \text{（舍去全局相位）} \end{aligned}$$

即：

$$\mathrm{X}\mathrm{R_z}(\theta)\mathrm{X} = \mathrm{R_z}(-\theta) = \mathrm{R_z}(\theta)^{-1} = \mathrm{R_z}(\theta)^{+}$$

$$\mathrm{R_z}(\theta)^{+} = (\cos(\theta/2)\, \mathrm{I} - \mathrm{i}\sin(\theta/2)\ \mathrm{Z})^{+} = \cos(\theta/2)\, \mathrm{I} + \mathrm{i}\sin(\theta/2)\ \mathrm{Z} = e^{i\theta Z/2}$$

RZ(θ) 门 – T 门, S 门, Z 门

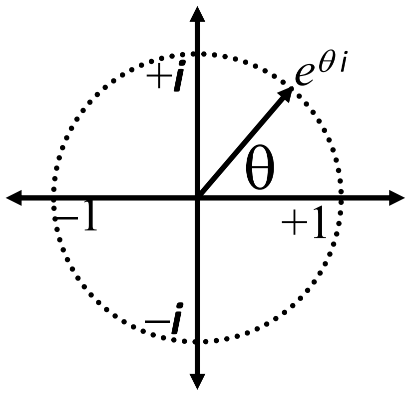
公式：$R_z(\theta) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix}$

$$T = R_z(\pi/4) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$

$$S = R_z(\pi/2) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

$S = T^2 \quad 45° + 45° = 90°$

$$Z = R_z(\pi) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$



$e^{\pi i/2} = i$
$e^{\pi i} = -1$ (欧拉恒等式)
$e^{3\pi i/2} = -i$
$e^{2\pi i} = e^{0} = 1$

Thank You