# 量子计算
## —算法篇

# Quantum Computer

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# 介绍

**教程简介：**

- 面向对象：量子计算初学者
- 依赖课程：线性代数，解析几何，量子力学（非必需）

**知乎专栏：**
https://www.zhihu.com/column/c_1501138176371011584

**Github & Gitee 地址：**
https://github.com/mymagicpower/qubits
https://gitee.com/mymagicpower/qubits

# 初态制备

**初态制备**
指的是量子计算中初始量子态的构造，是量子计算的初始步骤。
以单比特为例：
在实际量子运算中，我们可以直接得到的默认量子态是基态 $|0\rangle$，通过 X 门可以得到基态 $|1\rangle$。
对于任给的目标叠加量子态，我们则需要构造相应的量子门组合来得到。
**从基态 $|0\rangle$ 出发制备任给目标叠加态的过程称为初态制备。**

**最大叠加态**
以两比特态空间为例，从 $|0\rangle^{\otimes 2}$ ( $|0\rangle\otimes|0\rangle$ ) 出发，对每个量子比特进行 H 门操作可以得到两比特空间中所有基态的均匀叠加。
类似地，在任意维态空间中，均可以借助 H 门从多维的 $|0\rangle$ 基态出发，得到所有基态均匀线性组合的量子态。
这种量子态称为最大叠加态，很多初始状态要求为最大叠加态，量子计算的并行性也有赖于此。

$$(H|0\rangle)^{\otimes n}=\frac{1}{\sqrt{2^n}}\sum_{x=0}^{2^n-1}|x\rangle$$

**纯态和混态**
非基态的量子态都为叠加态。叠加态又可以分为相干叠加和非相干叠加，分别称为纯态和混态。
如将态空间与Bloch球关联，球面上量子态为纯态，球体内的量子态为混态。另一种重要的区分方式为密度矩阵，混态的密度矩阵非对角元均为0。

# 量子纠缠

如果一个量子系统的量子态 $|\psi\rangle$ 可以表示成形如 $|\psi\rangle = |\psi_0\rangle \otimes |\psi_1\rangle$ 的两个量子系统的张量积形式，我们就将此量子态称为**直积态**。
而不能进行这种直积分解的量子态就是**纠缠态**。

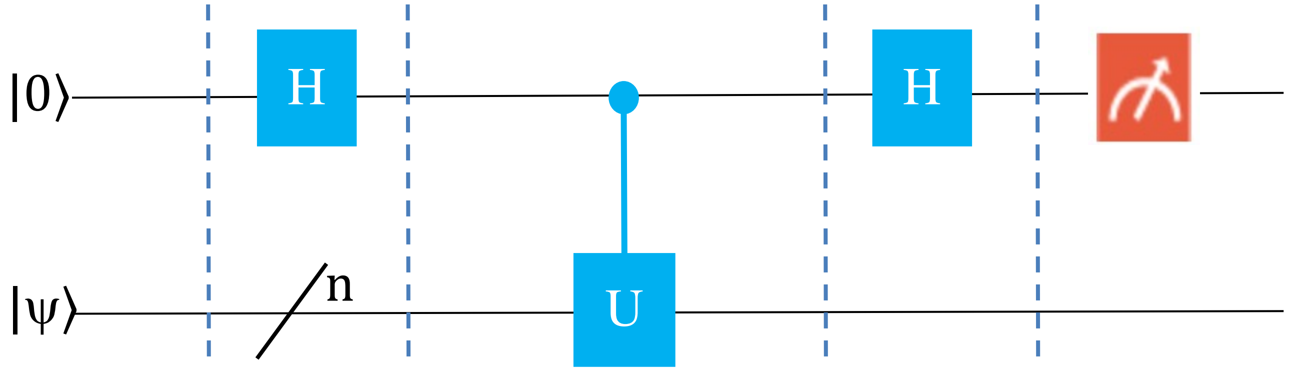
例如：
对两比特的Bell态 $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$，它不能写成两个单比特量子态的直积（张量积）形式。
量子纠缠态有超越经典关联的量子关联。为了发挥量子计算的并行性和高效性，量子计算的量子比特之间应当有着纠缠关联。

# Hadamard Test

Hadamard Test量子线路的主要作用：
对任给的幺正变换 $U$ 和量子态 $|\psi\rangle$ ，可以给出该**幺正算符 U 在量子态上的投影期望 $\langle\psi|U|\psi\rangle$** 。即可以通过测量一个辅助比特（ ancilla qubit ）来方便地得到一个**幺正算符 $U$ 对于一个量子态 $|\psi\rangle$ 的平均值。**

$$\langle\psi|U|\psi\rangle = \underbrace{\mathrm{Re}\,\langle\psi|U|\psi\rangle}_{\text{实部}} + \underbrace{\mathrm{Im}\,\langle\psi|U|\psi\rangle}_{\text{虚部}}\,\mathrm{i}$$

其中的实部对应的量子线路图为：




整个量子线路可以视为，对两个寄存器中量子比特组成的一个n+1维量子态 $|0\rangle|\psi\rangle$ ，进行量子门操作组合：

$$Q=(H\otimes I^{\otimes n})(\mathrm{Ctrl}-U)(H\otimes I^{\otimes n})$$

其中 Ctrl−$U$ 表示基于幺正算符 $U$ 的受控门

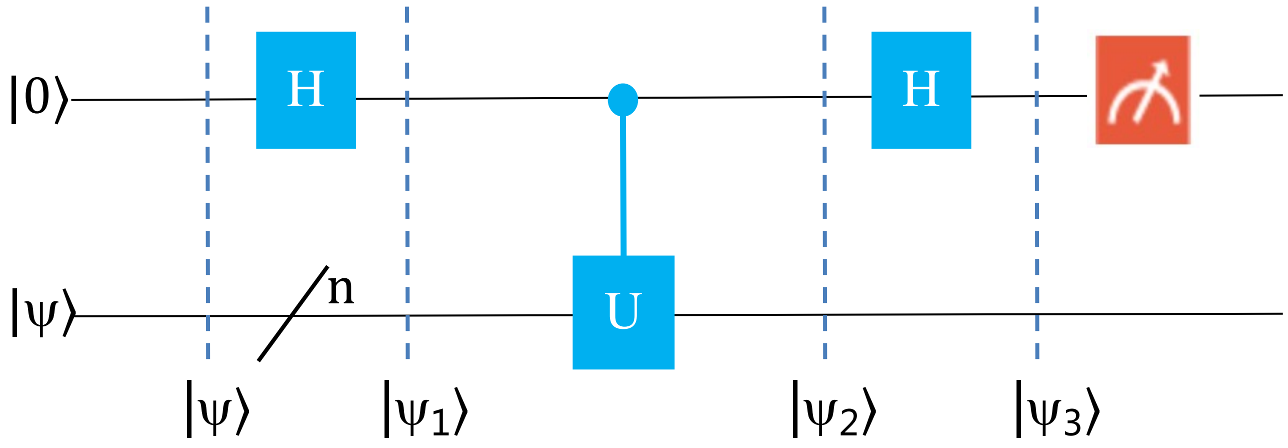
# Hadamard Test – 测量前的状态（实部）

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$

**H 门作用在基态：**

$$H|0\rangle = \frac{1}{\sqrt{2}}\left(\,|0\rangle + |1\rangle\,\right)$$

$$H|1\rangle = \frac{1}{\sqrt{2}}\left(\,|0\rangle - |1\rangle\,\right)$$




$$\begin{aligned}|\psi_1\rangle &= H|0\rangle \otimes |\psi\rangle\\ &= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\otimes|\psi\rangle\\ &= \frac{1}{\sqrt{2}}(|0\rangle|\psi\rangle + |1\rangle|\psi\rangle)\end{aligned}$$

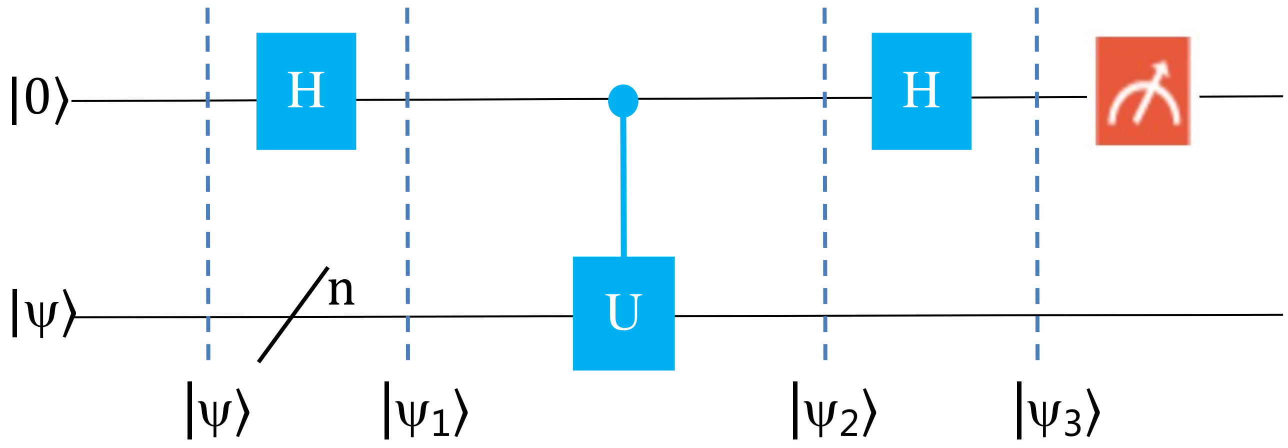
$$\begin{aligned}|\psi_2\rangle &= (\mathrm{Ctrl}-U)\,|\psi_1\rangle\\ &= \frac{1}{\sqrt{2}}(\mathrm{Ctrl}-U)(|0\rangle|\psi\rangle + |1\rangle|\psi\rangle\,)\end{aligned}$$

因为 Ctrl−$U$ 表示基于幺正变换 $U$ 的受控门，只有控制位为 $|1\rangle$ 时，才会作用于 $|\psi\rangle$，所以有：

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle|\psi\rangle + |1\rangle U|\psi\rangle)$$

# Hadamard Test – 测量前的状态（实部）

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$




**H 门作用在基态：**

$$H|0\rangle = \frac{1}{\sqrt{2}}\,(\,|0\rangle + |1\rangle\,)$$
$$H|1\rangle = \frac{1}{\sqrt{2}}\,(\,|0\rangle - |1\rangle\,)$$

$$\begin{aligned}
|\psi_3\rangle &= H|\psi_2\rangle \\
&= \frac{1}{\sqrt{2}}(H|0\rangle|\psi\rangle + H|1\rangle U|\psi\rangle) \\
&= \frac{1}{\sqrt{2}}(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|\psi\rangle + \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)U|\psi\rangle) \\
&= |0\rangle\frac{|\psi\rangle + U|\psi\rangle}{2} + |1\rangle\frac{|\psi\rangle - U|\psi\rangle}{2} \\
&= |0\rangle\frac{I+U}{2}|\psi\rangle + |1\rangle\frac{I-U}{2}|\psi\rangle
\end{aligned}$$

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# Hadamard Test – 测量计算（实部）

$$|\psi_3\rangle = |0\rangle \frac{I+U}{2}|\psi\rangle + |1\rangle \frac{I-U}{2}|\psi\rangle$$

测量 $|0\rangle$ ：

$$(|0\rangle\langle 0| \otimes I)\,|\psi_3\rangle = (|0\rangle\langle 0| \otimes I)(|0\rangle \frac{I+U}{2}|\psi\rangle + |1\rangle \frac{I-U}{2}|\psi\rangle)$$

$$= |0\rangle\langle 0|0\rangle \frac{I+U}{2}|\psi\rangle + |0\rangle\langle 0|1\rangle \frac{I-U}{2}|\psi\rangle$$

$$= \frac{1}{2}|0\rangle \otimes (I+U)|\psi\rangle$$

测量的概率：

$$\mathrm{Prob}(0) = ||\frac{1}{2}|0\rangle \otimes (I+U)|\psi\rangle||^2 = \frac{1}{4}|||0\rangle||^2\,||(I+U)|\psi\rangle||^2$$

$$= \frac{1}{4}\langle\psi|(I+U^\dagger)(I+U)|\psi\rangle = \frac{1}{4}(\langle\psi| + \langle\psi|U^\dagger)(|\psi\rangle + U|\psi\rangle)$$

$$= \frac{1}{4}(\langle\psi|\psi\rangle + \langle\psi|U|\psi\rangle + \langle\psi|U^\dagger|\psi\rangle + \langle\psi|U^\dagger U|\psi\rangle)$$

$$= \frac{1}{4}(2 + \langle\psi|U|\psi\rangle + \langle\psi|U^\dagger|\psi\rangle) = \frac{1}{4}(2 + \langle\psi|U|\psi\rangle + \langle\psi|U|\psi\rangle^*)$$

$$= \frac{1+Re(\langle\psi|U|\psi\rangle)}{2}$$

公式：

$\langle 0|1\rangle = 1 \quad \langle 0|1\rangle = 0$

$(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$

$||\psi\rangle|^2 = |\psi\rangle^\dagger|\psi\rangle = \langle\psi|\psi\rangle$

$(A \otimes B)^\dagger = A^\dagger \otimes B^\dagger$

$\langle e_j|A|e_k\rangle = \langle e_k|A^\dagger|e_j\rangle^*$

$\langle u|A|v\rangle = \langle A^\dagger u|v\rangle = \langle v|A^\dagger|u\rangle^*$

$Re(\langle\psi|U|\psi\rangle)$

幺正算符 $U$ 在量子态 $\psi$ 上投影期望的实部

# Hadamard Test – 测量计算（实部）

那么：

$\text{Prob}(1) = 1 - \text{Prob}(0) = \frac{1-Re(\langle\psi|U|\psi\rangle)}{2}$

经量子线路，如果测量结果为 $|0\rangle$，则让输出为1，如果测量结果为 $|1\rangle$，则让输出为 -1，那么期望值为：

$$\text{E(M)} = \sum_m m\text{P}(m)$$

$$= 1 * \text{Prob}(0) + (-1) * \text{Prob}(1)$$

$$= \frac{1+Re(\langle\psi|U|\psi\rangle)}{2} - \frac{1-Re(\langle\psi|U|\psi\rangle)}{2}$$

$$= Re(\langle\psi|U|\psi\rangle)$$

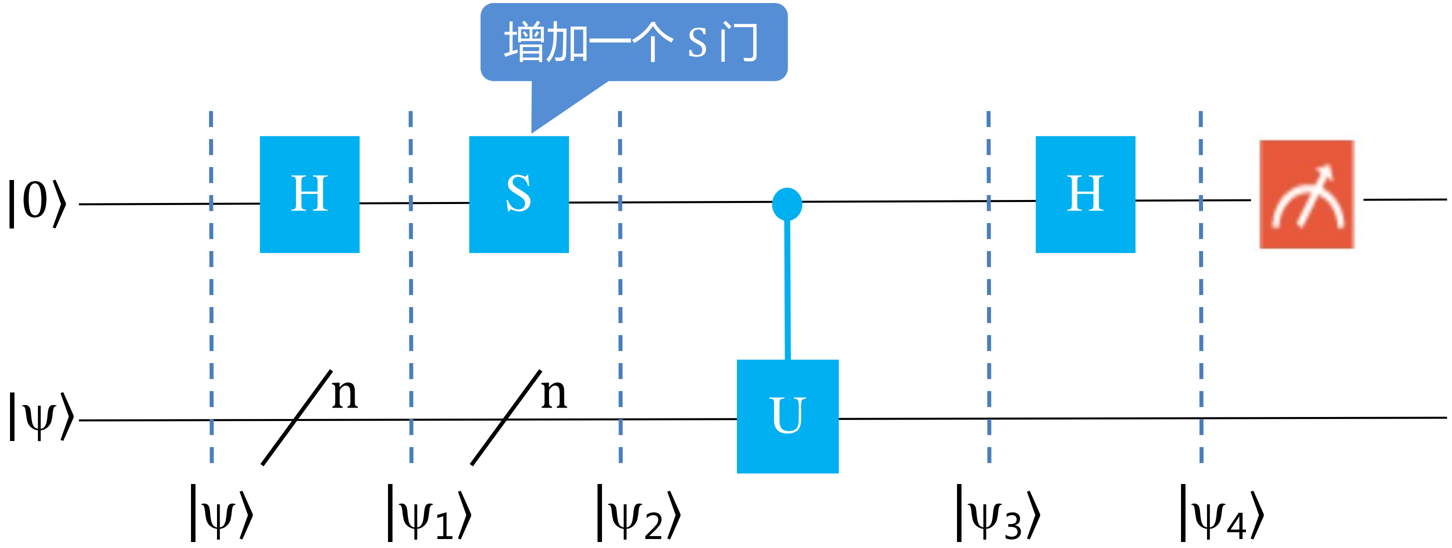
其为幺正算符 $U$ 在量子态 $\psi$ 上投影期望的实部。

# Hadamard Test – 测量前的状态（虚部）

$H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}$ $H|0\rangle = \frac{1}{\sqrt{2}}(\ |0\rangle + |1\rangle\ )$

$H|1\rangle = \frac{1}{\sqrt{2}}(\ |0\rangle - |1\rangle\ )$

$S = \begin{bmatrix}1 & 0\\0 & i\end{bmatrix}$




所以有：

$|\psi_1\rangle = H|0\rangle \otimes |\psi\rangle$

$$
\begin{aligned}
|\psi_2\rangle &= SH|0\rangle \otimes |\psi\rangle \\
&= \begin{bmatrix}1 & 0\\0 & i\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} \otimes |\psi\rangle \\
&= \frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix} \otimes |\psi\rangle \\
&= \frac{1}{\sqrt{2}}(|0\rangle\otimes|\psi\rangle + i\,|1\rangle \otimes |\psi\rangle)
\end{aligned}
$$

$$
\begin{aligned}
|\psi_3\rangle &= (\mathrm{Ctrl}-U)\ |\psi_2\rangle \\
&= \frac{1}{\sqrt{2}}(\mathrm{Ctrl}-U)(|0\rangle\otimes|\psi\rangle + i\,|1\rangle \otimes |\psi\rangle\ )
\end{aligned}
$$

因为 Ctrl−$U$ 表示基于幺正变换 $U$ 的受控门，只有控制位为 $|1\rangle$ 时，才会作用于 $|\psi\rangle$，所以有：

$$|\psi_3\rangle = \frac{1}{\sqrt{2}}(\ |0\rangle|\psi\rangle + i|1\rangle \otimes U|\psi\rangle\ )$$

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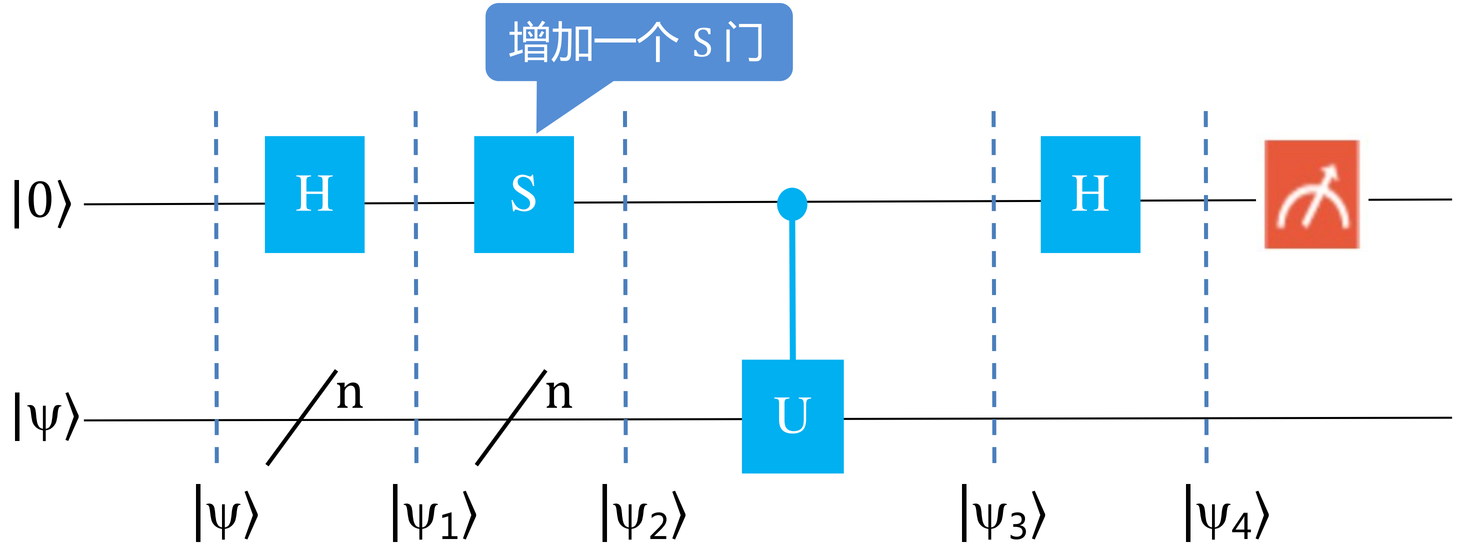
# Hadamard Test – 测量前的状态（虚部）

$H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$

**H 门作用在基态：**

$H|0\rangle = \frac{1}{\sqrt{2}}\left( |0\rangle + |1\rangle \right)$

$H|1\rangle = \frac{1}{\sqrt{2}}\left( |0\rangle - |1\rangle \right)$




$$
\begin{aligned}
|\psi_4\rangle &= H|\psi_3\rangle \\
&= \frac{1}{\sqrt{2}}\left( H|0\rangle\otimes|\psi\rangle + iH|1\rangle \otimes U|\psi\rangle \right) \\
&= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\otimes |\psi\rangle + i\,\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \otimes U|\psi\rangle \right) \\
&= \frac{1}{2}(|0\rangle + |1\rangle) \otimes |\psi\rangle + i\,(|0\rangle - |1\rangle) \otimes U|\psi\rangle\,)
\end{aligned}
$$

# Hadamard Test – 测量计算（虚部）

$|\psi_4\rangle = \frac{1}{2}(|0\rangle + |1\rangle) \otimes |\psi\rangle + i\,(|0\rangle - |1\rangle) \otimes U|\psi\rangle\,)$

测量 $|0\rangle$ ：

$$
\begin{aligned}
(\,|0\rangle\langle 0| \otimes I\,)\,|\psi_4\rangle &= (\,|0\rangle\langle 0| \otimes I\,)\,(\,\tfrac{1}{2}(|0\rangle + |1\rangle) \otimes |\psi\rangle + i\,(|0\rangle - |1\rangle) \otimes U|\psi\rangle\,)\,) \\
&= \tfrac{1}{2}(|0\rangle\langle 0|0\rangle + |0\rangle\langle 0|1\rangle) \otimes |\psi\rangle + i\,(|0\rangle\langle 0|0\rangle - |0\rangle\langle 0|1\rangle) \otimes U|\psi\rangle\,) \\
&= \tfrac{1}{2}(|0\rangle \otimes |\psi\rangle + i|0\rangle \otimes U|\psi\rangle)
\end{aligned}
$$

测量的概率：

$$
\begin{aligned}
\mathrm{Prob}(0) &= ||\,\tfrac{1}{2}(|0\rangle \otimes |\psi\rangle + i|0\rangle \otimes U|\psi\rangle)\,||^2 \\
&= \tfrac{1}{4}(|0\rangle \otimes |\psi\rangle + i|0\rangle \otimes U|\psi\rangle)^\dagger\,(|0\rangle \otimes |\psi\rangle + i|0\rangle \otimes U|\psi\rangle) \\
&= \tfrac{1}{4}(\langle 0| \otimes \langle\psi| - i\,\langle 0| \otimes \langle\psi|U^\dagger)\,(|0\rangle \otimes |\psi\rangle + i|0\rangle \otimes U|\psi\rangle) \\
&= \tfrac{1}{4}(\,(\langle 0| \otimes \langle\psi|)(|0\rangle \otimes |\psi\rangle) + (\langle 0| \otimes \langle\psi|)(i|0\rangle \otimes U|\psi\rangle) + (-i\,\langle 0| \otimes \langle\psi|U^\dagger)(|0\rangle \otimes |\psi\rangle\,) + (-i\,\langle 0| \otimes \langle\psi|U^\dagger)(i|0\rangle \otimes U|\psi\rangle)) \\
&= \tfrac{1}{4}(\,\langle 0|0\rangle \otimes \langle\psi|\psi\rangle + i\,\langle 0|0\rangle \otimes \langle\psi|U|\psi\rangle - i\,\langle 0|0\rangle \otimes \langle\psi|U^\dagger|\psi\rangle + \langle 0|0\rangle \otimes \langle\psi|U^\dagger U|\psi\rangle\,) \\
&= \tfrac{1}{4}(\,1 + i\,\langle\psi|U|\psi\rangle - i\,\langle\psi|U^\dagger|\psi\rangle + 1) \\
&= \tfrac{1}{4}(\,2 + (\,\langle\psi|U|\psi\rangle - \langle\psi|U|\psi\rangle^*\,)\,i\,) \\
&= \tfrac{1}{4}(\,2 - 2\mathrm{b}\,) \\
&= \frac{1 - \mathrm{Im}(\langle\psi|U|\psi\rangle)}{2}
\end{aligned}
$$

$\mathrm{Im}(\langle\psi|U|\psi\rangle)$

幺正算符 $U$ 在量子态 $\psi$ 上投影期望的虚部

公式：

- $\langle 0|1\rangle = 1 \quad \langle 0|1\rangle = 0$
- $(A \otimes B)\,(C \otimes D) = (AC) \otimes (BD)$
- $|\,|\psi\rangle\,|^2 = |\psi\rangle^\dagger|\psi\rangle = \langle\psi|\psi\rangle$
- $(A \otimes B)^\dagger = A^\dagger \otimes B^\dagger$
- $(cA)^\dagger = c^* A^\dagger$
- $(A + B)^\dagger = A^\dagger + B^\dagger$
- $(AB)^\dagger = B^\dagger A^\dagger$
- $\langle e_j|A|e_k\rangle = \langle e_k|A^\dagger|e_j\rangle^*$
- $\langle u|A|v\rangle = \langle A^\dagger u|v\rangle = \langle v|A^\dagger|u\rangle^*$

令：

$\langle\psi|U|\psi\rangle$ = a + b $i$

则：

b = $Im(\langle\psi|U|\psi\rangle)$

$\langle\psi|U|\psi\rangle^*$ = a - b $i$

$(\langle\psi|U|\psi\rangle - \langle\psi|U|\psi\rangle^*)\,i$ = 2b $i * i$ = − 2b

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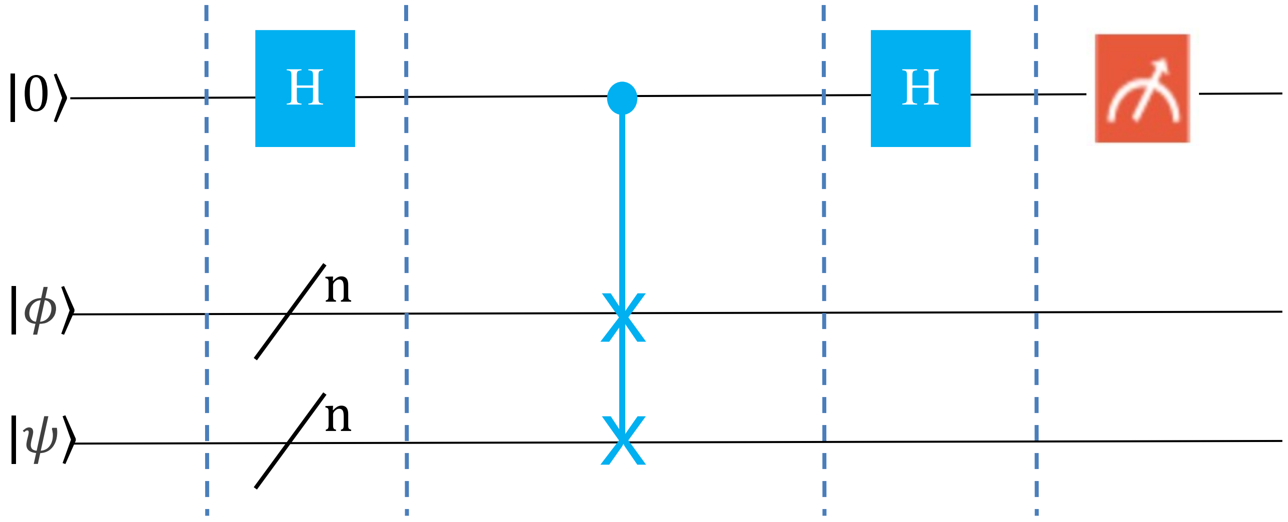
# SWAP Test

Hadamard Test有着多种形式和广泛用途，其中一种特殊形式是基本量子线路SWAP Test。
任给两个维数相同的量子态，通过SWAP Test线路，可以得到两个量子态的保真度，反应了它们的重叠情况。
两个量子态 $|\phi\rangle, |\psi\rangle$ 的保真度是指量子态内积范数的平方 $|\langle\phi|\psi\rangle|^2$

SWAP Test的量子线路图结构：



# SWAP Test – 测量前的状态

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$

$$H|0\rangle = \frac{1}{\sqrt{2}}\left(\,|0\rangle + |1\rangle\,\right)$$
$$H|1\rangle = \frac{1}{\sqrt{2}}\left(\,|0\rangle - |1\rangle\,\right)$$

1. 输入态

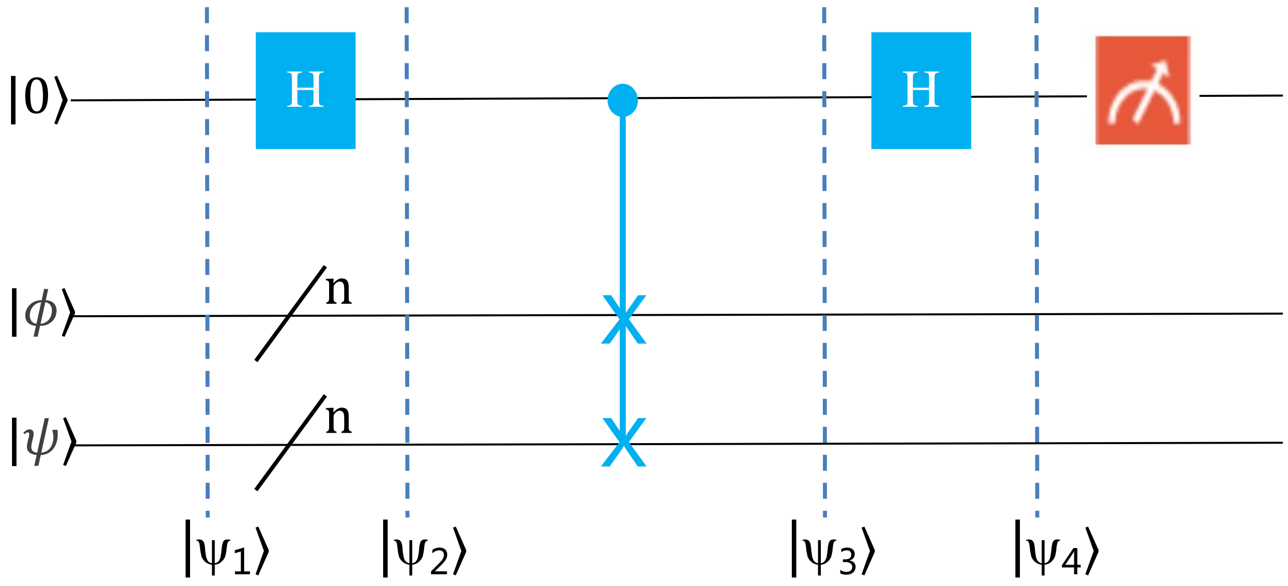
$$|\psi_1\rangle = |0\rangle \otimes |\phi\rangle \otimes |\psi\rangle$$

2. 输入态经过第一个 H 门

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\,|\phi\rangle|\psi\rangle$$
$$= \frac{1}{\sqrt{2}}(|0\rangle|\phi\rangle|\psi\rangle + |1\rangle|\phi\rangle|\psi\rangle)$$

3. 再经过一个 swap 门（ $|1\rangle$ 时交换 ）

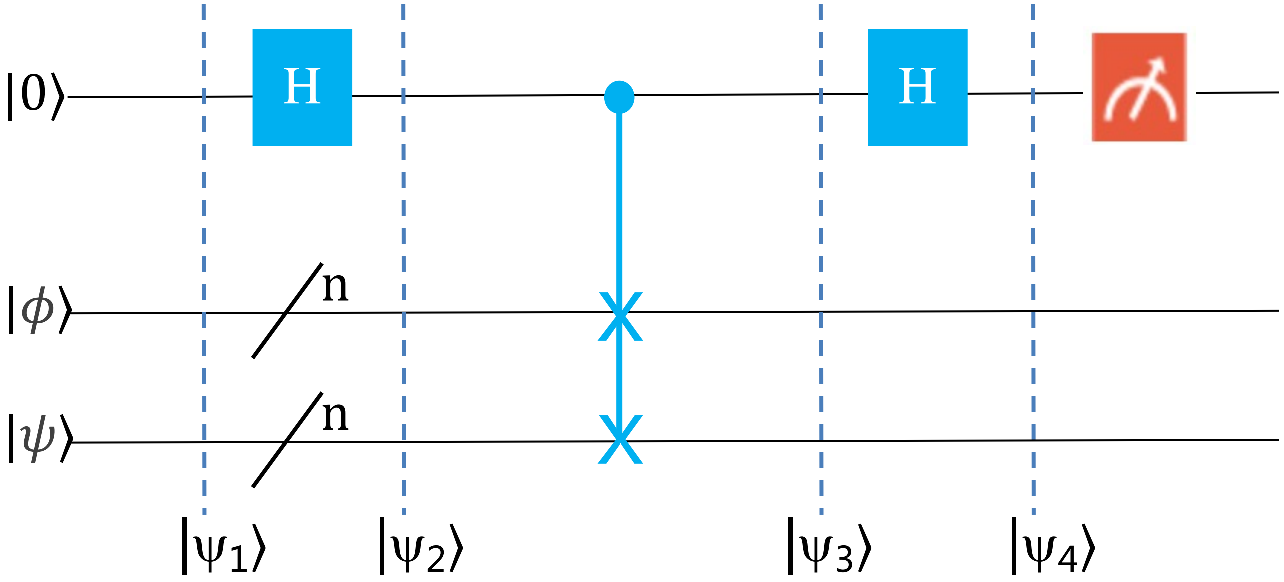
$$|\psi_3\rangle = \frac{1}{\sqrt{2}}(|0\rangle|\phi\rangle|\psi\rangle + |1\rangle|\psi\rangle|\phi\rangle)$$



# SWAP Test – 测量前的状态

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$H|0\rangle = \frac{1}{\sqrt{2}} \left( |0\rangle + |1\rangle \right)$$
$$H|1\rangle = \frac{1}{\sqrt{2}} \left( |0\rangle - |1\rangle \right)$$




4. 再经过最后一个 H 门

$$|\psi_4\rangle = H|\psi_3\rangle$$
$$= \frac{1}{\sqrt{2}}(H|0\rangle|\phi\rangle|\psi\rangle + H|1\rangle|\psi\rangle|\phi\rangle)$$
$$= \frac{1}{\sqrt{2}}(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|\phi\rangle|\psi\rangle + \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)|\psi\rangle|\phi\rangle)$$
$$= \frac{1}{2}[|0\rangle (|\phi\rangle|\psi\rangle + |\psi\rangle|\phi\rangle) + |1\rangle (|\phi\rangle|\psi\rangle - |\psi\rangle|\phi\rangle)]$$

# SWAP Test – 测量

$$|\psi_4\rangle = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|\phi\rangle|\psi\rangle + \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)|\psi\rangle|\phi\rangle\right)$$
$$= \frac{1}{2}[|0\rangle\,(|\phi\rangle|\psi\rangle + |\psi\rangle|\phi\rangle) + |1\rangle\,(|\phi\rangle|\psi\rangle - |\psi\rangle|\phi\rangle)]$$

测量 $|0\rangle$ ：

$$(\,|0\rangle\langle 0| \otimes I\,)\,|\psi_4\rangle = (\,|0\rangle\langle 0| \otimes I\,)\,\frac{1}{2}[|0\rangle\,(|\phi\rangle|\psi\rangle + |\psi\rangle|\phi\rangle) + |1\rangle\,(|\phi\rangle|\psi\rangle - |\psi\rangle|\phi\rangle)]$$
$$= \frac{1}{2}[|0\rangle\langle 0|0\rangle\,(|\phi\rangle|\psi\rangle + |\psi\rangle|\phi\rangle) + |0\rangle\langle 0|1\rangle\,(|\phi\rangle|\psi\rangle - |\psi\rangle|\phi\rangle)]$$
$$= \frac{1}{2}[|0\rangle \otimes (|\phi\rangle|\psi\rangle + |\psi\rangle|\phi\rangle)]$$

那么测量的概率：

$$\mathrm{Prob}(0) = ||\,\frac{1}{2}[|0\rangle \otimes (|\phi\rangle|\psi\rangle + |\psi\rangle|\phi\rangle)]\,||^2 = \frac{1}{4}||\,|0\rangle||^2\,||\,(|\phi\rangle|\psi\rangle + |\psi\rangle|\phi\rangle)\,||^2$$
$$= \frac{1}{4}(|\phi\rangle|\psi\rangle + |\psi\rangle|\phi\rangle)^\dagger\,(|\phi\rangle|\psi\rangle + |\psi\rangle|\phi\rangle)$$
$$= \frac{1}{4}(\,\langle\psi|\langle\phi| + \langle\phi|\langle\psi|\,)(|\phi\rangle|\psi\rangle + |\psi\rangle|\phi\rangle)$$
$$= \frac{1}{4}(\,\langle\psi|\langle\phi|\phi\rangle|\psi\rangle + \langle\phi|\langle\psi|\phi\rangle|\psi\rangle + \langle\phi|\langle\psi|\phi\rangle|\psi\rangle + \langle\phi|\langle\psi|\psi\rangle|\phi\rangle)$$
$$= \frac{1}{4}(\,1 + \langle\phi|\langle\psi|\phi\rangle|\psi\rangle + \langle\phi|\langle\psi|\phi\rangle|\psi\rangle + 1)$$
$$= \frac{1}{2}\,(1 + ||\langle\psi|\phi\rangle||^2)$$

公式：

- $\langle 0|1\rangle = 1 \quad \langle 0|1\rangle = 0$
- $(A \otimes B)\,(C \otimes D) = (AC) \otimes (BD)$
- $|\,|\psi\rangle\,|^2 = |\psi\rangle^\dagger|\psi\rangle = \langle\psi|\psi\rangle$
- $(A \otimes B)^\dagger = A^\dagger \otimes B^\dagger$
- $(cA)^\dagger = c^* A^\dagger$
- $(A + B)^\dagger = A^\dagger + B^\dagger$
- $(AB)^\dagger = B^\dagger A^\dagger$
- $\langle e_j|A|e_k\rangle = \langle e_k|A^\dagger|e_j\rangle^*$
- $\langle u|A|v\rangle = \langle A^\dagger u|v\rangle = \langle v|A^\dagger|u\rangle^*$

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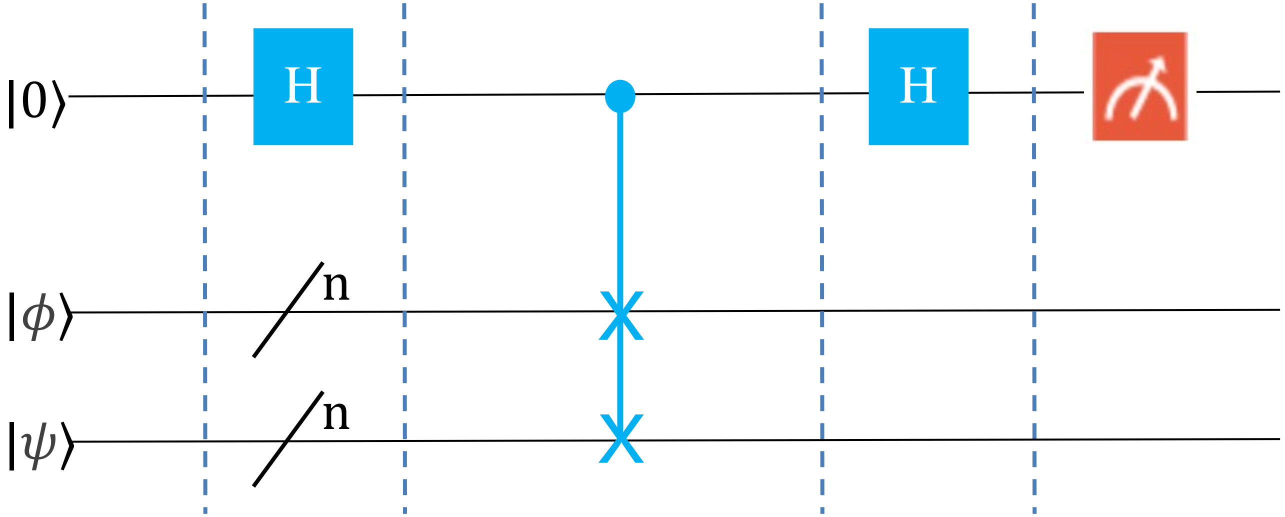
# SWAP Test

对SWAP Test的公式推导验证过程完全类似于Hadamard Test，结果量子态的第一个寄存器测量得到$|0\rangle, |1\rangle$的概率均与给定的两个量子态的保真度相关。也就是可以多次测量，判断两个量子态 $|\phi\rangle, |\psi\rangle$ 具体区别有多大。

$$P_0 = \frac{1+|\langle\psi|\phi\rangle|^2}{2}, \ P_1 = 1 - P_0$$

SWAP Test作为Hadamard的一种特殊形式，它对两个给定量子态给出了其保真度相关的测量结果，具有重要应用意义。在量子态的内积相关研究中有着重要作用。

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# Thank You