

量子计算

—数学基础

Quantum Computer

网址: www.qubits.top

Calvin Tang

179209347@qq.com

介绍

教程简介：

- 面向对象：量子计算初学者
- 依赖课程：线性代数，解析几何，量子力学（非必需）

知乎专栏：

https://www.zhihu.com/column/c_1501138176371011584

Github & Gitee 地址：

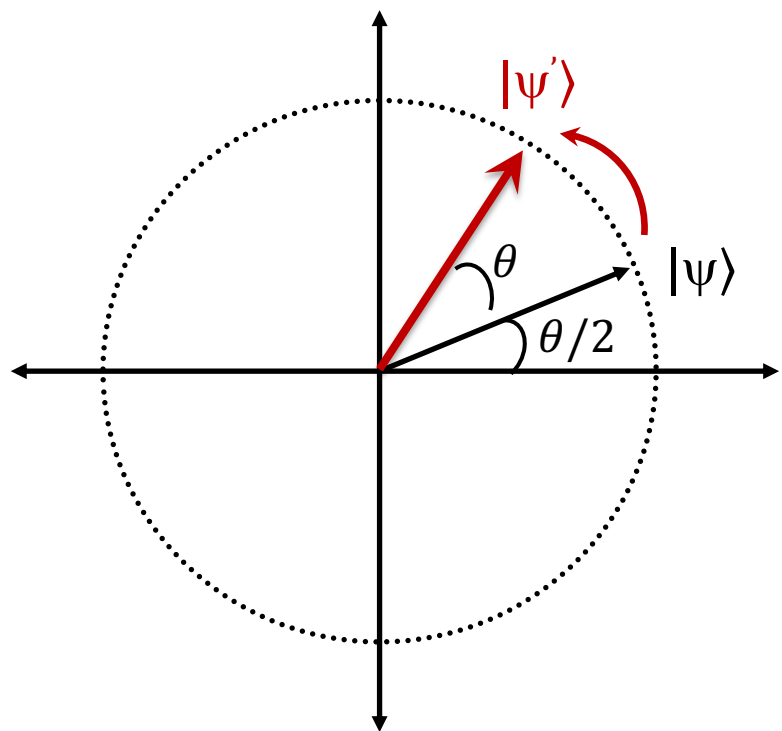
<https://github.com/mymagicpower/qubits>

<https://gitee.com/mymagicpower/qubits>

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常用几何变换 - 逆时针旋转 θ



$$|\psi\rangle = \cos(\theta/2) |0\rangle + \sin(\theta/2) |1\rangle = \begin{bmatrix} \cos(\theta/2) \\ \sin(\theta/2) \end{bmatrix}$$

$$Q = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

* 每次作用于向量，相当于逆时针旋转 θ

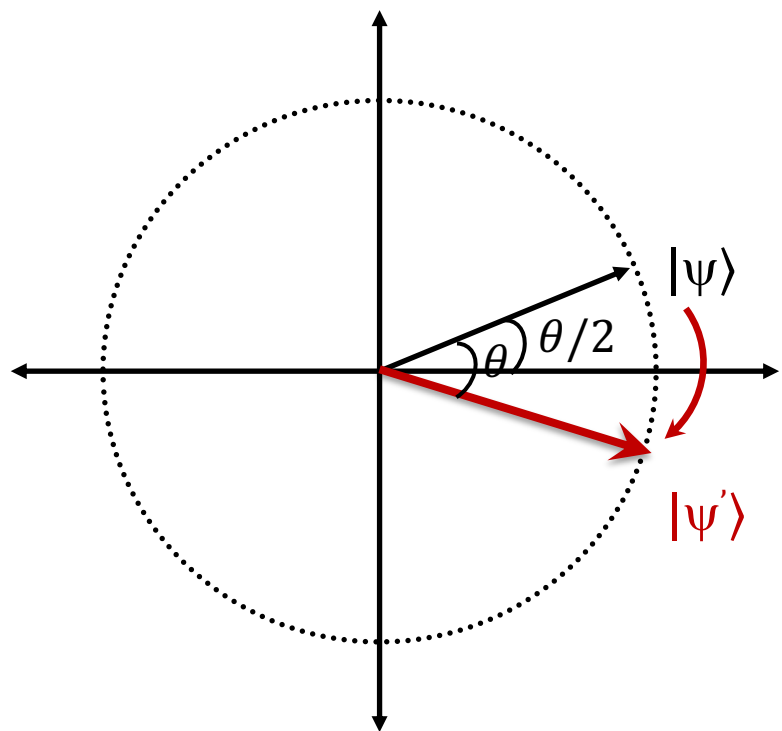
证明：

两角和与差的三角函数公式：

$$\begin{aligned} \sin(\alpha \pm \beta) &= \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \\ \cos(\alpha \pm \beta) &= \cos \alpha \cos \beta \mp \sin \alpha \sin \beta \end{aligned}$$

$$\begin{aligned} |\psi'\rangle &= Q |\psi\rangle \\ &= \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} \cos(\theta/2) \\ \sin(\theta/2) \end{bmatrix} \\ &= \begin{bmatrix} \cos(\theta) \cos(\theta/2) - \sin(\theta) \sin(\theta/2) \\ \sin(\theta) \cos(\theta/2) + \cos(\theta) \sin(\theta/2) \end{bmatrix} \\ &= \begin{bmatrix} \cos(\theta/2 + \theta) \\ \sin(\theta/2 + \theta) \end{bmatrix} \end{aligned}$$

常用几何变换 - 顺时针旋转 θ



$$|\psi\rangle = \cos(\theta/2) |0\rangle + \sin(\theta/2) |1\rangle = \begin{bmatrix} \cos(\theta/2) \\ \sin(\theta/2) \end{bmatrix}$$

$$Q = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$$

* 每次作用于向量，相当于顺时针旋转 θ

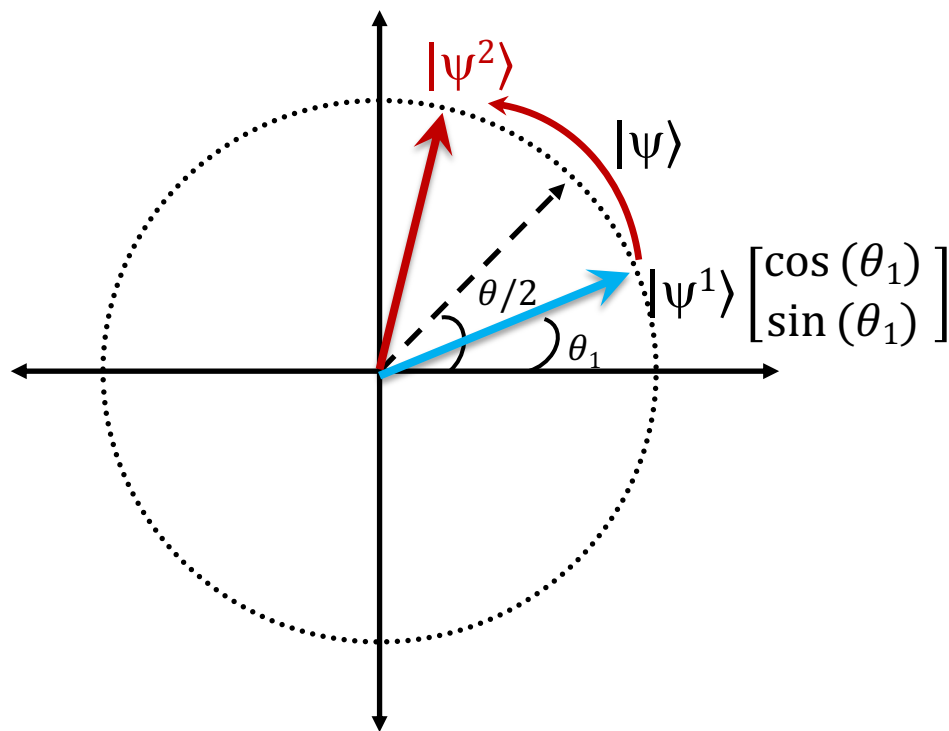
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两角和与差的三角函数公式：

$$\begin{aligned} \sin(\alpha \pm \beta) &= \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \\ \cos(\alpha \pm \beta) &= \cos \alpha \cos \beta \mp \sin \alpha \sin \beta \end{aligned}$$

$$\begin{aligned} |\psi'\rangle &= Q |\psi\rangle \\ &= \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} \cos(\theta/2) \\ \sin(\theta/2) \end{bmatrix} \\ &= \begin{bmatrix} \cos(\theta) \cos(\theta/2) + \sin(\theta) \sin(\theta/2) \\ -\sin(\theta) \cos(\theta/2) + \cos(\theta) \sin(\theta/2) \end{bmatrix} \\ &= \begin{bmatrix} \cos(\theta/2 - \theta) \\ \sin(\theta/2 - \theta) \end{bmatrix} \end{aligned}$$

常用几何变换 – 镜像



$$|\psi\rangle = \cos(\theta/2) |0\rangle + \sin(\theta/2) |1\rangle = \begin{bmatrix} \cos(\theta/2) \\ \sin(\theta/2) \end{bmatrix}$$

$$Q = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ \sin(\theta) & -\cos(\theta) \end{bmatrix}$$

* 关于通过原点、方向和水平轴夹角为 $\theta/2$ 直线镜像;

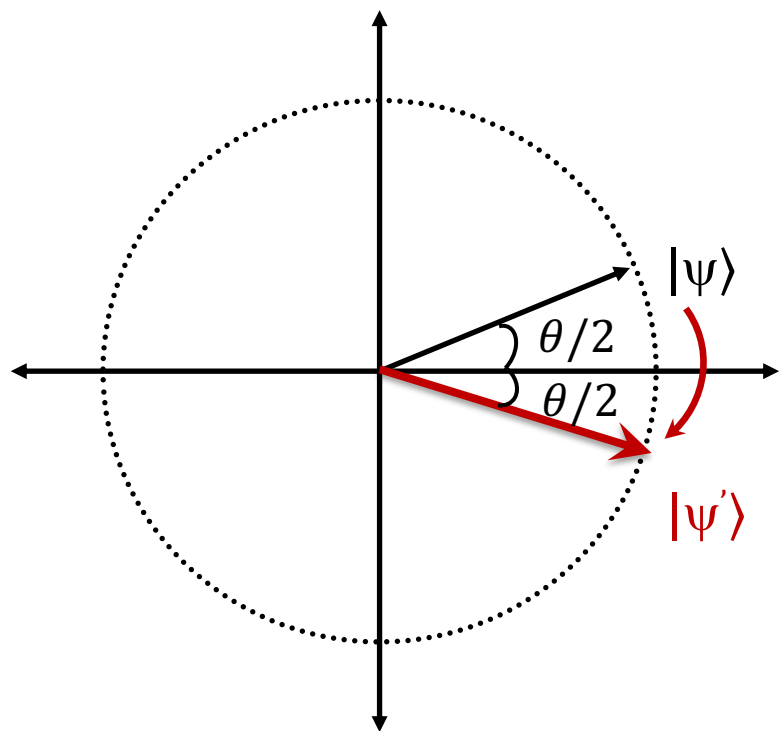
证明：

$$\begin{aligned} |\psi^2\rangle &= Q |\psi^1\rangle \\ &= \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ \sin(\theta) & -\cos(\theta) \end{bmatrix} \begin{bmatrix} \cos(\theta_1) \\ \sin(\theta_1) \end{bmatrix} \\ &= \begin{bmatrix} \cos(\theta) \cos(\theta_1) + \sin(\theta) \sin(\theta_1) \\ \sin(\theta) \cos(\theta_1) - \cos(\theta) \sin(\theta_1) \end{bmatrix} \\ &= \begin{bmatrix} \cos(\theta - \theta_1) \\ \sin(\theta - \theta_1) \end{bmatrix} \end{aligned}$$

关于通过原点、方向和水平轴夹角为 $\theta/2$ 直线镜像，
 可以理解为逆时针旋转 $2(\frac{\theta}{2} - \theta_1)$ ，则：

$$|\psi^2\rangle = \begin{bmatrix} \cos(\theta_1 + 2(\frac{\theta}{2} - \theta_1)) \\ \sin(\theta_1 + 2(\frac{\theta}{2} - \theta_1)) \end{bmatrix} = \begin{bmatrix} \cos(\theta - \theta_1) \\ \sin(\theta - \theta_1) \end{bmatrix}$$

常用几何变换 - 关于横轴镜像对称



$$|\psi\rangle = \cos(\theta/2) |0\rangle + \sin(\theta/2) |1\rangle = \begin{bmatrix} \cos(\theta/2) \\ \sin(\theta/2) \end{bmatrix}$$

$$Q = \begin{bmatrix} \cos(0) & \sin(0) \\ \sin(0) & -\cos(0) \end{bmatrix}$$

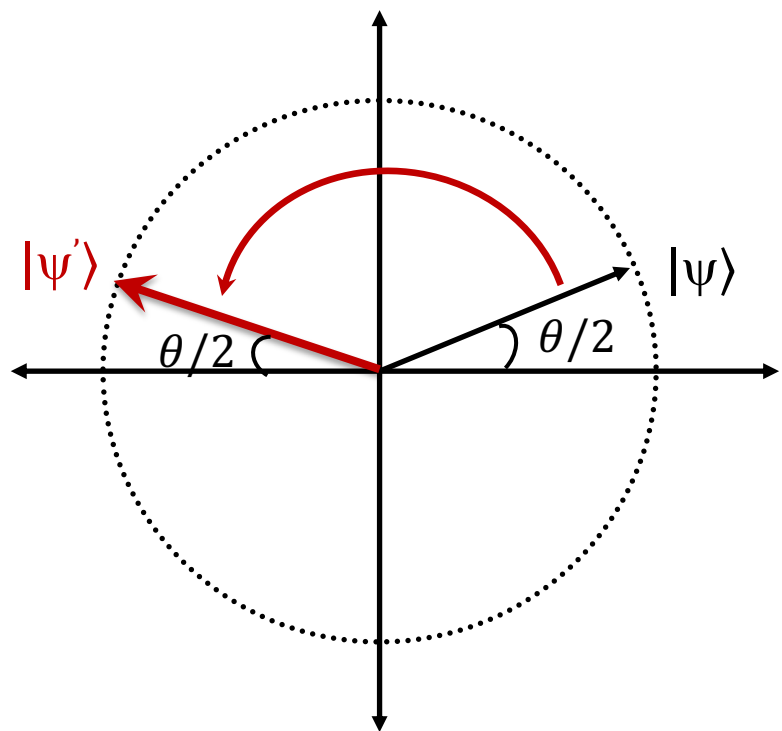
$$Q = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

* 作用于向量，相当于关于横轴镜像

证明：

$$\begin{aligned} |\psi'\rangle &= Q |\psi\rangle \\ &= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \cos(\theta/2) \\ \sin(\theta/2) \end{bmatrix} \\ &= \begin{bmatrix} \cos(\theta/2) \\ -\sin(\theta/2) \end{bmatrix} \end{aligned}$$

常用几何变换 - 关于纵轴镜像对称



$$|\psi\rangle = \cos(\theta/2) |0\rangle + \sin(\theta/2) |1\rangle = \begin{bmatrix} \cos(\theta/2) \\ \sin(\theta/2) \end{bmatrix}$$

$$Q = \begin{bmatrix} \cos(2\frac{\pi}{2}) & \sin(2\frac{\pi}{2}) \\ \sin(2\frac{\pi}{2}) & -\cos(2\frac{\pi}{2}) \end{bmatrix}$$

$$Q = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

* 作用于向量，相当于关于纵轴镜像

证明：

$$\begin{aligned} |\psi'\rangle &= Q |\psi\rangle \\ &= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta/2) \\ \sin(\theta/2) \end{bmatrix} \\ &= \begin{bmatrix} -\cos(\theta/2) \\ \sin(\theta/2) \end{bmatrix} \end{aligned}$$

常用几何变换 – 镜像

$$\text{令 } |\psi\rangle = \begin{bmatrix} \cos(\theta/2) \\ \sin(\theta/2) \end{bmatrix}$$

$$\text{则 } |\psi\rangle\langle\psi| = \begin{bmatrix} \cos(\theta/2) \\ \sin(\theta/2) \end{bmatrix} \begin{bmatrix} \cos(\theta/2) & \sin(\theta/2) \end{bmatrix} = \begin{bmatrix} \cos^2(\theta/2) & \cos(\theta/2)\sin(\theta/2) \\ \cos(\theta/2)\sin(\theta/2) & \sin^2(\theta/2) \end{bmatrix}$$

$$\text{则 } 2|\psi\rangle\langle\psi| - I = \begin{bmatrix} 2\cos^2(\theta/2) - 1 & 2\cos(\theta/2)\sin(\theta/2) \\ 2\cos(\theta/2)\sin(\theta/2) & 2\sin^2(\theta/2) - 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ \sin(\theta) & -\cos(\theta) \end{bmatrix}$$

- $2|\psi\rangle\langle\psi| - I$, 相当于关于 $|\psi\rangle$ 镜像
- $|\psi\rangle$ 为镜像轴



Thank

You