University of Toronto – Faculty of Arts & Science – MAT235Y1: Multivariable Calculus

Term Test 1 – Fall 2022/Winter 2023

Family Name (PRINT): _____
(As on your Student ID)

Given Name(s) (PRINT): _____
(As on your Student ID)

U of T Email: _____

Student Number: _____

Signature: _____

This exam contains **10** pages (including this cover page) and **7** problems. The last page includes a formula sheet. Once the exam begins, check to see if any pages are missing. There are **50** possible points to be earned in this exam.

- Duration: **90 minutes**
- **No aids or calculators are permitted on the exam.**
- **Do not tear any pages off this exam.**
- **One scrap page is provided at the end.** This page will not be graded unless specifically indicated. Please enter all of your answers in the space provided on the answer sheets.
- Do not write in the page margins. Make sure that your writing is dark enough to be readable.
- **Unsupported answers to short answer questions will not receive full credit**. A correct answer without explanation will receive no credit unless otherwise noted; an incorrect answer supported by substantially correct calculations and explanations may receive partial credit.
  - **Organize your work** in a reasonably neat and coherent way.
  - You must use the methods learned in this course to solve all of the problems.

1. (8 points) For each part, write your final answer on the indicated line. Only your final answer will be graded for this question. Each part is worth 2 marks.

   1. Let $r^2 = \sin\theta$ be a polar equation. Find a point (r, θ) other than $(0, 0)$ which lies on the polar curve.

      Answer _____

   2. Write the Cartesian coordinates $(-3, -\sqrt{3})$ as polar coordinates.

      Answer _____

   3. Let $\vec{u} = \vec{i} + \vec{j}$, and let $\vec{v} = \vec{j} - \vec{i}$. Then $\vec{u} \times \vec{v}$ is:

      Answer _____

   4. Let $\vec{r}(t) = \langle\ t^2,\ e^{-3t},\ \cos^2 t\ \rangle$. Calculate $\vec{r}\,''(t)$.

      Answer _____

2. (4 points) Match each equation with its graph. Write **A, B, C** or **D** for each equation. Only your final answer will be graded. Each part is worth 1 mark.

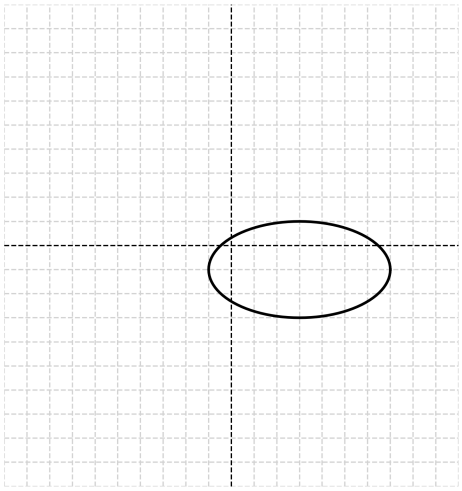
   1. $x^2 + 4y^2 - 6x + 8y = 3$ Answer _____

   2. $4x^2 + 8x + (y-3)^2 = 12$ Answer _____

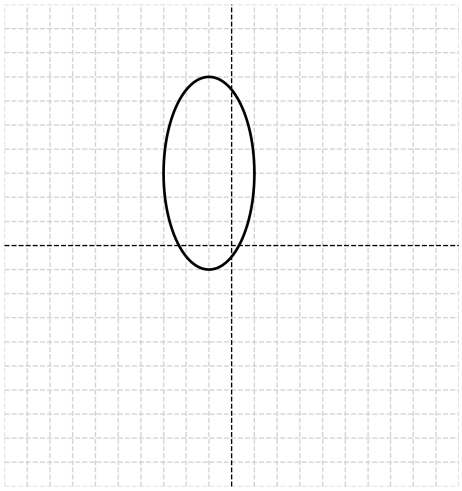
   3. $2y + x^2 = 2x + 5$ Answer _____

   4. $2x + 7 = y^2 + 2y$ Answer _____

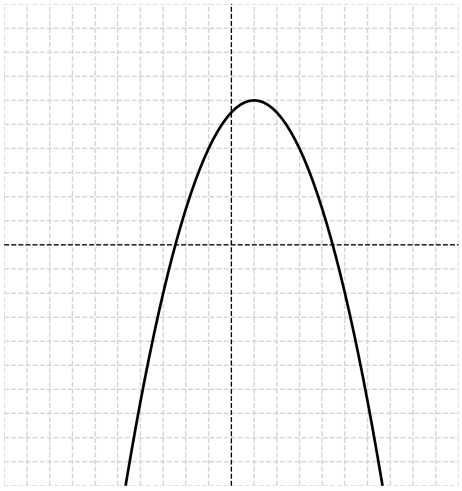
A



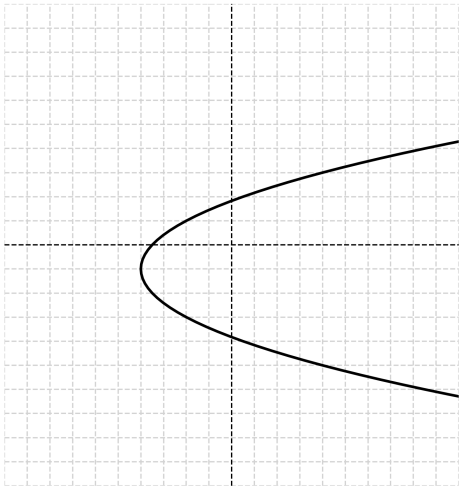
B



C



D



3. (9 points) A curve is parametrized by $x(t) = t^2 - 3, y(t) = t^3 - 4t$ for $-2 \leq t \leq 2$.

(a) (3 points) Give an expression for $\frac{dy}{dx}$ in terms of t.

(b) (6 points) Find all points (x, y) at which the curve has a horizontal or vertical tangent line.

4. (6 points) Find the equation of the plane that passes through the point $(3, 1, 4)$ and contains the line $x(t) = 4 - t, y(t) = 2t - 1, z(t) = -3t$.

5. (6 points) Find the area of the region bounded by one loop of the polar curve $r = 1 - \cos(4\theta)$.

6. (7 points) Let C be the curve of intersection of the following surfaces:

$$x^2 + y^2 + z^2 = 2 \text{ and } z = \sqrt{x^2 + y^2}.$$

(a) (4 points) Find a vector function $\vec{r}(t)$ that represents C.

(b) (3 points) Using your answer to part (a), find the direction vector of the tangent line to C when $t = \pi$.

7. (10 points) (a) (2 points) Sketch the points (r, θ) which lie on the polar curve $r = 1/2$ for $\pi \leq \theta \leq 2\pi$.

(b) (2 points) Sketch the polar curve $r = \sin(2\theta)$ for $0 \leq \theta \leq \pi$.

(c) (6 points) Set up an integral which represents the area enclosed inside both of the above polar curves. (You may assume the area is symmetric. You do not need to fully solve the integral.)

Do not tear this page off. This page is a formula sheet and will not be graded under any circumstances. It can be used for rough work only.

Trigonometry	Differentiation	Integration
Identities	Rules	Rules
$\sin^2 x + \cos^2 x = 1$	$(cf(x) \pm g(x))' = cf'(x) \pm g'(x)$	$\int cf(x) \pm g(x)\ dx = c\int f(x)\ dx + \int g(x)\ dx$
$\cos^2 x - \sin^2 x = \cos 2x$	$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$	$\int_{u(a)}^{u(b)} f(u)\ du = \int_a^b f(u(x))u'(x)\ dx$
$\sin 2x = 2\sin x \cos x$	$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$	$\int f(x)g'(x)\ dx = f(x)g(x) - \int g(x)f'(x)\ dx$
$2\cos^2 x = 1 + \cos 2x$	$(f(g(x)))' = f'(g(x))g'(x)$	$\int_a^b F'(x)\ dx = F(b) - F(a)$
$\tan^2 x + 1 = \sec^2 x$	$\left(f^{-1}(x)\right)' = \frac{1}{f'(f^{-1}(x))}$	
Special angles	Derivatives	Integrals (Constants omitted)
$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$	$\frac{d}{dx}(c) = 0$	$\int \frac{1}{x}\ dx = \ln x$
$\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$	$\frac{d}{dx}(x^n) = nx^{n-1}, \quad n \neq 0$	$\int x^n\ dx = \frac{1}{n+1}x^{n+1} \quad n \neq -1$
$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$	$\frac{d}{dx}(e^x) = e^x$	$\int e^x\ dx = e^x$
$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$	$\frac{d}{dx}(\ln x) = \frac{1}{x}$	$\int \ln x\ dx = x(\ln x - 1)$
$\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$	$\frac{d}{dx}(\sin x) = \cos x$	$\int \sin x\ dx = -\cos x$
$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$	$\frac{d}{dx}(\cos x) = -\sin x$	$\int \cos x\ dx = \sin x$

Do not tear this page off. This page is for additional work and will not be graded, unless you clearly indicate it on the original question page.