# Lab 6

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## Naive Bayes

Column: Type A or B

Row: Characteristic traits of slugs

# Type A = 15

# Type B = 10

$X_1$:

| | 0 (No fluffy tail) | 1 (Fluffy tail) | MLE Estimates | |
|---|---|---|---|---|
| 0 (Type B) | 7 | 8 | 0.28 | 0.32 |
| 1 (Type A) | 3 | 7 | 0.12 | 0.28 |

$X_2$:

| | 0 (Black ears) | 1 (Green ears) | MLE Estimators | |
|---|---|---|---|---|
| 0 (Type B) | 13 | 2 | 0.69 | 0.08 |
| 1 (Type A) | 8 | 2 | 0.32 | 0.08 |

$X_3$ :

| | 0 (No spots) | 1 (Spots) | MLE Estimators | |
|---|---|---|---|---|
| 0 (Type B) | 10 | 5 | 0.40 | 0.20 |
| 1 (Type A) | 7 | 3 | 0.28 | 0.12 |

$X_4$ :

| | 0 (No extra tongue) | 1 (Extra tongue) | MLE Estimators | |
|---|---|---|---|---|
| 0 (Type B) | 10 | 5 | 0.40 | 0.20 |
| 1 (Type A) | 7 | 3 | 0.28 | 0.12 |

Testing on the last row:

$$\hat{Y} = \arg\max_y \hat{\mathbb{P}}(X|Y)\hat{\mathbb{P}}(Y) = \arg\max_y \hat{\mathbb{P}}(X_1|Y)\hat{\mathbb{P}}(X_2|Y)\hat{\mathbb{P}}(X_3|Y)\hat{\mathbb{P}}(X_4|Y)$$

$$\begin{aligned}
\mathbb{P}(X, Y=0) &= \hat{\mathbb{P}}(X_1=0|Y=0)\hat{\mathbb{P}}(X_2=0|Y=0)\hat{\mathbb{P}}(X_3=0|Y=0)\hat{\mathbb{P}}(X_4=1|Y=0)\hat{\mathbb{P}}(Y=0) \\
&= \frac{0.28}{0.60}\frac{0.69}{0.60}\frac{0.40}{0.60}\frac{0.20}{0.60}0.60 \\
&= 0.0715 \\
\mathbb{P}(X, Y=1) &= \hat{\mathbb{P}}(X_1=0|Y=1)\hat{\mathbb{P}}(X_2=0|Y=1)\hat{\mathbb{P}}(X_3=0|Y=1)\hat{\mathbb{P}}(X_4=1|Y=1)\hat{\mathbb{P}}(Y=1) \\
&= \frac{0.12}{0.40}\frac{0.32}{0.40}\frac{0.28}{0.40}\frac{0.12}{0.40}0.40 \\
&= 0.02016 \\
&\Rightarrow Y=0 \\
q & \quad SAME
\end{aligned}$$

$$\begin{aligned}
\mathbb{P}(X, Y=0) &= \hat{\mathbb{P}}(X_1=1|Y=0)\hat{\mathbb{P}}(X_2=0|Y=0)\hat{\mathbb{P}}(X_3=0|Y=0)\hat{\mathbb{P}}(X_4=0|Y=0)\hat{\mathbb{P}}(Y=0) \\
&= \frac{0.32}{0.60}\frac{0.69}{0.60}\frac{0.40}{0.60}\frac{0.40}{0.60}0.60 \\
&= 0.164 \\
\mathbb{P}(X, Y=1) &= \hat{\mathbb{P}}(X_1=1|Y=1)\hat{\mathbb{P}}(X_2=0|Y=1)\hat{\mathbb{P}}(X_3=0|Y=1)\hat{\mathbb{P}}(X_4=0|Y=1)\hat{\mathbb{P}}(Y=1) \\
&= \frac{0.28}{0.40}\frac{0.32}{0.40}\frac{0.28}{0.40}\frac{0.28}{0.40}0.40 \\
&= 0.10976 \\
&\Rightarrow Y=0 \\
& \quad DIFFERENT
\end{aligned}$$

$$
\begin{aligned}
\mathbb{P}(X, Y=0) &= \hat{\mathbb{P}}(X_1=0|Y=0)\hat{\mathbb{P}}(X_2=0|Y=0)\hat{\mathbb{P}}(X_3=1|Y=0)\hat{\mathbb{P}}(X_4=1|Y=0)\hat{\mathbb{P}}(Y=0) \\
&= \frac{0.28}{0.60}\frac{0.69}{0.60}\frac{0.20}{0.60}\frac{0.20}{0.60}0.60 \\
&= 0.0358 \\
\mathbb{P}(X, Y=1) &= \hat{\mathbb{P}}(X_1=0|Y=1)\hat{\mathbb{P}}(X_2=0|Y=1)\hat{\mathbb{P}}(X_3=1|Y=1)\hat{\mathbb{P}}(X_4=1|Y=1)\hat{\mathbb{P}}(Y=1) \\
&= \frac{0.12}{0.40}\frac{0.32}{0.40}\frac{0.28}{0.12}\frac{0.12}{0.40}0.40 \\
&= 0.02016 \\
&\Rightarrow Y=0 \\
&\quad SAME
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{P}(X, Y=0) &= \hat{\mathbb{P}}(X_1=1|Y=0)\hat{\mathbb{P}}(X_2=1|Y=0)\hat{\mathbb{P}}(X_3=1|Y=0)\hat{\mathbb{P}}(X_4=1|Y=0)\hat{\mathbb{P}}(Y=0) \\
&= \frac{0.32}{0.60}\frac{0.08}{0.60}\frac{0.20}{0.60}\frac{0.20}{0.60}0.60 \\
&= 0.004741 \\
\mathbb{P}(X, Y=1) &= \hat{\mathbb{P}}(X_1=1|Y=1)\hat{\mathbb{P}}(X_2=1|Y=1)\hat{\mathbb{P}}(X_3=1|Y=1)\hat{\mathbb{P}}(X_4=1|Y=1)\hat{\mathbb{P}}(Y=1) \\
&= \frac{0.28}{0.40}\frac{0.08}{0.40}\frac{0.12}{0.40}\frac{0.12}{0.40}0.40 \\
&= 0.00504 \\
&\Rightarrow Y=1 \\
&\quad SAME
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{P}(X, Y=0) &= \hat{\mathbb{P}}(X_1=1|Y=0)\hat{\mathbb{P}}(X_2=0|Y=0)\hat{\mathbb{P}}(X_3=0|Y=0)\hat{\mathbb{P}}(X_4=1|Y=0)\hat{\mathbb{P}}(Y=0) \\
&= \frac{0.32}{0.60}\frac{0.69}{0.60}\frac{0.40}{0.60}\frac{0.20}{0.60}0.60 \\
&= 0.082 \\
\mathbb{P}(X, Y=1) &= \hat{\mathbb{P}}(X_1=1|Y=1)\hat{\mathbb{P}}(X_2=0|Y=1)\hat{\mathbb{P}}(X_3=0|Y=1)\hat{\mathbb{P}}(X_4=1|Y=1)\hat{\mathbb{P}}(Y=1) \\
&= \frac{0.28}{0.40}\frac{0.32}{0.40}\frac{0.28}{0.40}\frac{0.12}{0.40}0.40 \\
&= 0.047 \\
&\Rightarrow Y=0 \\
&\quad SAME
\end{aligned}
$$

Success Rate: 80%

## Laplace Smoothing

Column: Type A or B

Row: Characteristic traits of slugs

# Type A = 17

# Type B = 12

$X_1$:

| | 0 (No fluffy tail) | 1 (Fluffy tail) |
|---|---|---|
| 0 (Type B) | 8 | 9 |
| 1 (Type A) | 4 | 8 |

$X_2$:

| | 0 (Black ears) | 1 (Green ears) |
|---|---|---|
| 0 (Type B) | 14 | 3 |
| 1 (Type A) | 9 | 3 |

$X_3$ :

| | 0 (No spots) | 1 (Spots) |
|---|---|---|
| 0 (Type B) | 11 | 6 |
| 1 (Type A) | 8 | 4 |

$X_4$ :

| | 0 (No extra tongue) | 1 (Extra tongue) |
|---|---|---|
| 0 (Type B) | 11 | 6 |
| 1 (Type A) | 8 | 4 |

Testing on the last row:

$$\hat{Y} = \arg\max_y \hat{\mathbb{P}}(X|Y)\hat{\mathbb{P}}(Y) = \arg\max_y \hat{\mathbb{P}}(X_1|Y)\hat{\mathbb{P}}(X_2|Y)\hat{\mathbb{P}}(X_3|Y)\hat{\mathbb{P}}(X_4|Y)$$

$$
\begin{aligned}
\mathbb{P}(X, Y=0) &= \hat{\mathbb{P}}(X_1=0|Y=0)\hat{\mathbb{P}}(X_2=0|Y=0)\hat{\mathbb{P}}(X_3=0|Y=0)\hat{\mathbb{P}}(X_4=1|Y=0)\hat{\mathbb{P}}(Y=0) \\
&= \frac{8}{17}\frac{14}{17}\frac{11}{17}\frac{6}{17}\frac{17}{29} \\
&= 0.0519 \\
\mathbb{P}(X, Y=1) &= \hat{\mathbb{P}}(X_1=0|Y=1)\hat{\mathbb{P}}(X_2=0|Y=1)\hat{\mathbb{P}}(X_3=0|Y=1)\hat{\mathbb{P}}(X_4=1|Y=1)\hat{\mathbb{P}}(Y=1) \\
&= \frac{4}{12}\frac{9}{12}\frac{8}{12}\frac{4}{12}\frac{12}{29} \\
&= 0.023 \\
&\Rightarrow Y=1 \\
q & \quad DIFFERENT
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{P}(X, Y=0) &= \hat{\mathbb{P}}(X_1=1|Y=0)\hat{\mathbb{P}}(X_2=0|Y=0)\hat{\mathbb{P}}(X_3=0|Y=0)\hat{\mathbb{P}}(X_4=0|Y=0)\hat{\mathbb{P}}(Y=0) \\
&= \frac{9}{17}\frac{14}{17}\frac{11}{17}\frac{11}{17}\frac{17}{29} \\
&= 0.107 \\
\mathbb{P}(X, Y=1) &= \hat{\mathbb{P}}(X_1=1|Y=1)\hat{\mathbb{P}}(X_2=0|Y=1)\hat{\mathbb{P}}(X_3=0|Y=1)\hat{\mathbb{P}}(X_4=0|Y=1)\hat{\mathbb{P}}(Y=1) \\
&= \frac{8}{12}\frac{9}{12}\frac{8}{12}\frac{8}{12}\frac{12}{29} \\
&= 0.092 \\
&\Rightarrow Y=0 \\
& \quad DIFFERENT
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{P}(X, Y=0) &= \hat{\mathbb{P}}(X_1=0|Y=0)\hat{\mathbb{P}}(X_2=0|Y=0)\hat{\mathbb{P}}(X_3=1|Y=0)\hat{\mathbb{P}}(X_4=1|Y=0)\hat{\mathbb{P}}(Y=0) \\
&= \frac{8}{17}\frac{14}{17}\frac{6}{17}\frac{6}{17}\frac{17}{29} \\
&= 0.48 \\
\mathbb{P}(X, Y=1) &= \hat{\mathbb{P}}(X_1=0|Y=1)\hat{\mathbb{P}}(X_2=0|Y=1)\hat{\mathbb{P}}(X_3=1|Y=1)\hat{\mathbb{P}}(X_4=1|Y=1)\hat{\mathbb{P}}(Y=1) \\
&= \frac{4}{12}\frac{9}{12}\frac{4}{12}\frac{4}{12}\frac{12}{29} \\
&= 0.011 \\
&\Rightarrow Y=0 \\
& \quad SAME
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{P}(X, Y=0) &= \hat{\mathbb{P}}(X_1=1|Y=0)\hat{\mathbb{P}}(X_2=1|Y=0)\hat{\mathbb{P}}(X_3=1|Y=0)\hat{\mathbb{P}}(X_4=1|Y=0)\hat{\mathbb{P}}(Y=0) \\
&= \frac{9}{17}\frac{3}{17}\frac{6}{17}\frac{6}{17}\frac{17}{29} \\
&= 0.0068 \\
\mathbb{P}(X, Y=1) &= \hat{\mathbb{P}}(X_1=1|Y=1)\hat{\mathbb{P}}(X_2=1|Y=1)\hat{\mathbb{P}}(X_3=1|Y=1)\hat{\mathbb{P}}(X_4=1|Y=1)\hat{\mathbb{P}}(Y=1) \\
&= \frac{8}{12}\frac{3}{12}\frac{4}{12}\frac{4}{12}\frac{12}{29} \\
&= 0.0077 \\
&\Rightarrow Y=1 \\
&\quad SAME
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{P}(X, Y=0) &= \hat{\mathbb{P}}(X_1=1|Y=0)\hat{\mathbb{P}}(X_2=0|Y=0)\hat{\mathbb{P}}(X_3=0|Y=0)\hat{\mathbb{P}}(X_4=1|Y=0)\hat{\mathbb{P}}(Y=0) \\
&= \frac{9}{17}\frac{14}{17}\frac{11}{17}\frac{6}{17}\frac{17}{29} \\
&= 0.0583 \\
\mathbb{P}(X, Y=1) &= \hat{\mathbb{P}}(X_1=1|Y=1)\hat{\mathbb{P}}(X_2=0|Y=1)\hat{\mathbb{P}}(X_3=0|Y=1)\hat{\mathbb{P}}(X_4=1|Y=1)\hat{\mathbb{P}}(Y=1) \\
&= \frac{8}{12}\frac{9}{12}\frac{8}{12}\frac{4}{12}\frac{12}{29} \\
&= 0.046 \\
&\Rightarrow Y=0 \\
&\quad DIFFERENT
\end{aligned}
$$

Success Rate: 40%

Success rate decreased.

Laplace smoothing is helpful when something always has the change to happen. Harmful if the case where the data really does not exist, but you pretended to see it.

In this case, since we have data for all possible observations, not really useful.