Lab 5 Probability

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Yesterday's Puzzle

Parity
n different colors of hats

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Question 1

(a) Royal Flush: Dealt the 5 highest cards from a single suit.

$|E| = 4$ as there are 4 suits, each with five highest cards

$\mathbb{P} = \dfrac{4}{\binom{52}{5}}$

(b) Four of a kind: Four cards of the same rank, one of another kind

$|E| = 13 \cdot \binom{4}{4} \cdot 48$

We have 13 possible ranks to choose for those four cards, and there are 48 remaining cards to choose the other one from.
$\mathbb{P} = \dfrac{13 \cdot 48}{\binom{52}{5}}$

(c) Two Pairs: two pairs of same rank, one different

$|E| = 13\binom{4}{2} \cdot 12\binom{4}{2} \cdot 11\binom{4}{1}$

(d) Flush:

All five cards of same suit, not in sequence

$|S| = 4\binom{13}{5}$

(e) Straight flush:

Five cards in sequence, all of the same suit.

$|E| = 4 \cdot 10$

10 possible consecutive sequence.

Question 2

$\frac{\binom{n}{k}}{\binom{n}{k}!}$

Question 3

- First r bits have exactly k zeros, there other $M+N-r$ bits have exactly $M-k$ zeros.
- $\frac{\binom{r}{k}\binom{M+N-r}{r-k}}{\binom{M}{k}}$

Question 4

- $J_I = \#$ jurors voting the defendant innocent.

  $J_G = \#$ jurors voting the defendant guilty
- $A_I =$ Event the defendant is really innocent

  $A_G =$ Event the defendant is really guilty
- $F_G =$ Event a juror votes guilty

  $\mathbb{P}(F_G|A_I) = 0.1$

  $\mathbb{P}(F_G|A_G) = 0.8$
- $F_I =$ Event a juror votes innocent

  $\mathbb{P}(F_I|A_I) = 0.9$

  $\mathbb{P}(F_I|A_G) = 0.2$
- $C =$ Event the jury makes the correct decision.

$$\begin{aligned} C &= \mathbb{P}(J_G \geq 9|A_G)\mathbb{P}(A_G) + \mathbb{P}(J_I \geq 9|A_I)\mathbb{P}(A_I) \\ &= (0.65)\sum_{i=9}^{12}\binom{12}{i}0.8^i 0.2^{12-i} + (0.35)\sum_{i=9}^{12}\binom{12}{i}0.9^i 0.1^{12-i} \\ &= 0.857 \end{aligned}$$

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$$\begin{aligned} &\text{Percentage of defendants found guilty by the jury} \\ &= \mathbb{P}(J_G \geq 9) \\ &= \mathbb{P}(J_G \geq 9|A_G)\mathbb{P}(A_G) + \mathbb{P}(J_G \geq 9|A_I)\mathbb{P}(A_I) \\ &= (0.65)\sum_{i=9}^{12} 0.8^i 0.2^{12-i} + (0.35)\sum_{i=9}^{12} 0.9^i 0.1^{12-i} \end{aligned}$$

Question 5

(a) William's parents must be Bb since otherwise his sister cannot be bb. Therefore William can be Bb, Bb, or BB. So he has probability $\frac{2}{3}$ to have a blue-eyed gene.

(b) If William is Bb and wife is bb, his first child has blue eyes with probability $\frac{1}{2}$.

If William is BB and wife is bb, no chance his first child has blue eyes.

Therefore $\mathbb{P}(\text{first child has blue eyes is}) = \frac{2}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot 0 = \frac{1}{3}$

(c) W-Bb: William has Bb genotype

W-BB: William has BB genotype

CBrown: William's first child is brown

$$\begin{aligned}
\mathbb{P}(W - Bb|CBrown) &= \frac{\mathbb{P}(CBrown \cap W - Bb)}{\mathbb{P}(CBrown)} \\
&= \frac{\mathbb{P}(CBrown|W - Bb)\mathbb{P}(W - Bb)}{\mathbb{P}(CBrown|W - Bb)\mathbb{P}(W - Bb) + \mathbb{P}(CBrown|W - BB))\mathbb{P}(W - BB)} \\
&= \frac{\frac{1}{2} \cdot \frac{2}{3}}{\frac{1}{2} \cdot \frac{2}{3} + 1 \cdot \frac{1}{3}} \\
&= \frac{1}{2}
\end{aligned}$$

$\mathbb{P}(SCBrown) = \mathbb{P}(SCBrown|W - Bb) + \mathbb{P}(SCBrown|W - BB) = \frac{1}{2} \cdot \frac{1}{2} + 1\frac{1}{2} = \frac{3}{4}$