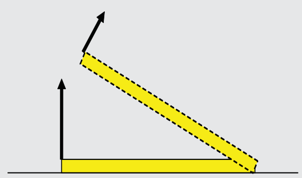
Physics Challenge for Teachers and Students

***Boris Korsunsky,* Column Editor**
Weston High School, Weston, MA 02493
korsunbo@post.harvard.edu

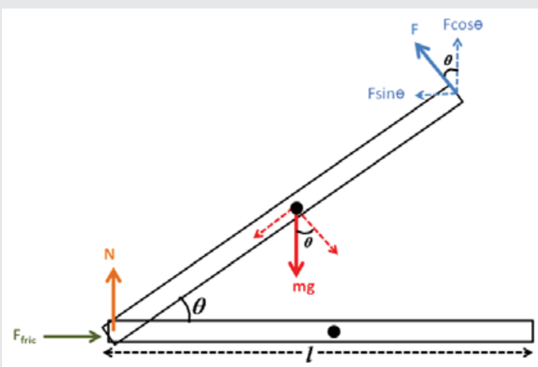
Solution to January 2010 Challenge

Rise and shine

A long thin uniform rod lies flat on the table as shown. One end of the rod is slowly pulled up by a force that remains perpendicular to the rod at all times. What minimum coefficient of static friction is required so that the rod can be brought to the vertical position without any slipping of the bottom end?



Solution:



Assume that $0 \leq \theta \leq \frac{\pi}{2}$.

Since the rod is raised slowly (quasi-statically), the entire system remains in equilibrium at any moment in time. Thus, the torque about the point of rotation of the rod is zero about any axis, and the net external forces are 0.

$F_{fric} = F \sin\theta$

$$\tau = -Fl + \frac{mgl\cos\theta}{2} = 0 \Rightarrow F = \frac{mg\cos\theta}{2}$$

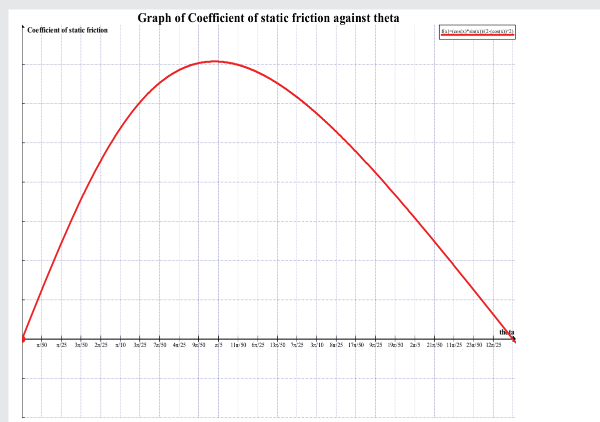
$N = mg - F\cos\theta$

The $F\sin\theta$ force causes the rod's bottom end to slip in one direction. The friction force counters this force to stop the slipping when $F_{fric} = \mu_s N \geq F\sin\theta$.
Hence,

$$\mu_s \geq \frac{F\sin\theta}{mg - F\cos\theta} = \frac{\sin\theta\cos\theta}{2 - \cos^2\theta}$$

The graph of μ_s shows that the values of μ_s above the graph will prevent the rod from slipping. Thus, we need to find the μ_s at its highest point within the range of θ given.

From the graph below, there is a value of θ that corresponds to the maximum value of μ_s needed for the entire cycle of raising the rod to a vertical position. This is the minimum μ_s needed as if we use a smaller value for the minimum.



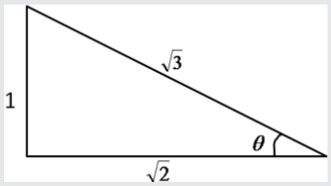
Maximum of μ_s can be found by setting its derivative with respect to theta equal to zero:

$$\mu'_s = \frac{(2-\cos^2\theta)(2\cos^2\theta - 1) - 2\cos^2\theta\,(1-\cos^2\theta)}{(2-\cos^2\theta)^2} = 0$$

The denominator is always greater than 0, so all Θ values are possible.
Let $\cos^2\theta = x$.
Hence, $(2-x)(2x-1) - 2x(1-x) = 0$

$$x = \cos^2\theta = \frac{2}{3} \Rightarrow \cos\theta = \sqrt{\frac{2}{3}};\sin\theta = \frac{1}{\sqrt{3}}$$

$$\mu_{s,\min} = \frac{\left(\frac{\sqrt{2}}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)}{2 - \frac{2}{3}} = \frac{\sqrt{2}}{4}.$$



(Contributed by Loh Lay Kuan, student, NUS High School of Mathematics and Science, Singapore)

We would also like to recognize the following contributors:

André Bellemans (Université Libre de Bruxelles, Belgium)

Hubert N. Biezeveld (Zwaag, The Netherlands)

Dave Bittel (Bristol Eastern High School, Bristol, CT)

R. R. Bukrey (retired, Loyola University, Evanston, IL)

Phil Cahill (Lockheed Martin Corporation, North Yorkshire, United Kingdom)

Daniel Cartin (Naval Academy Preparatory School, Middetown, RI)

Don Easton (Lacombe, Alberta, Canada)

Hasan Fakhruddin (The Indiana Academy for Science, Mathematics, and Humanities, Ball State University, Muncie, IN)

Fernando Ferreira (Universidade da Beira Interior, Covilhã, Portugal)

Fredrick P. Gram (Cuyahoga Community College, Cleveland, OH)

Gerald E. Hite (TAMUG, Galveston, TX)

Art Hovey (Milford, CT)

J. Iñiguez (Universidad de Salamanca, Salamanca, Spain)

Per-Olof Jansson (Stockholm, Sweden)

David Jones (Florida International University, Miami, FL)

Jack Kingston (Weston High School, Weston, CT)

Jeff Melmed (Eastern Maine Community College, Bangor, ME)

Matthew W. Milligan (Farragut High School, Knoxville, TN)

Daniel Mixson (Naval Academy Preparatory School, Newport, RI)

Eugene P. Mosca (U.S. Naval Academy, Annapolis, MD)

Clark M. Neily, Jr. (Hermon's of Alaska Christian School, Allston, MA)

Bill Nettles (Union University, Jackson, TN)

Pascal Renault (John Tyler Community College, Chester, VA)

Daniel Schumayer (University of Otago, Dunedin, New Zealand)

Jason L. Smith (Richland Community College, Decatur, IL)

Cássio dos Santos Sousa, student (Colégio Objetivo, São Paulo, Brazil)

Clint Sprott (University of Wisconsin – Madison, WI)

John B. Zacharias (Edgewood Jr./Sr. High School, Merritt Island, FL)

Many thanks to all contributors and we hope to hear from you in the future!