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## Proof By Contradictions Quiz

Q: There are no rational solutions to  $x^2 - 2 = 0$ . Prove by contradiction  
(not no solns)  
↓

Soln: We assume there IS a rational soln to  $x^2 - 2 = 0$

Assume:  $x = \frac{p}{q}$ ,  $p$  and  $q$  have no common factors,  $q \neq 0$  if rational

$$\text{Therefore } \left(\frac{p}{q}\right)^2 - 2 = 0 \rightarrow \frac{p^2}{q^2} = 2q^2 \rightarrow p^2 = 2q^2$$

- This means  $p^2$  is even, therefore  $p$  is also even
- However,  $q$  cannot be even, bc  $p$  and  $q$  have no common factors
- This means  $q$  is odd
- Since  $p$  is even, we can write  $p = 2k$ . Therefore  
 $p^2 = 4k^2 = 2q^2$
- We can conclude that  $q^2 = 2k^2$ . In other words,  
 $q^2$  is even, hence  $q$  is even
- This is a contradiction because  $q$  cannot be even and odd at the same time

NOTE: I followed the procedure in the First slide set, slide 28, to complete this assignment. I worked through the problem following the format and logic presented there.

The assumption that there is a rational solution to  $x^2 - 2 = 0$  leads to contradictions in the proof, therefore, there must be no rational soln to  $x^2 - 2 = 0$ .