$2n = \rho_0$

$$C_{j+1} = \left[\frac{2(j+l+1)-2n}{(j+1)(j+2l+2)}\right] C_j$$

S-wave $l=0$

$$C_{j+1} = \left[\frac{2(j+1)-2n}{(j+1)(j+2)}\right] C_j$$

1S $\quad j_{max}=0 \quad n=1 \quad v(\rho) = C_0$

2S $\quad j_{max}=1 \quad n=2 \quad v(\rho) = C_0(1-\rho) = C_0\left(1-\frac{1}{2}\frac{r}{a_0}\right)$

3S $\quad j_{max}=2 \quad n=3 \quad v(\rho) = C_0\left(1-2\rho+\frac{2}{3}\rho^2\right) = C_0\left(1-\frac{2}{3}\frac{r}{a_0}+\frac{2}{27}\left(\frac{r}{a_0}\right)^2\right) \qquad \rho = \frac{r}{3a_0}$

$$R_l(r) = \frac{1}{r}\rho^{l+1}e^{-\rho}v_l(\rho)$$

$R_{n=1,l=0} \sim C_0 e^{-\frac{r}{a_0}} \qquad R_{n=2\,l=0} \sim C_0 e^{-\frac{r}{2a_0}}\left(1-\frac{1}{2}\frac{r}{a_0}\right) \qquad R_{n=3\,l=0} \cdots$

$\downarrow$

$$\int_0^\infty r^2 dr\, |R_{nl}(r)|^2 = 1$$

$\downarrow$

$R_{n=1\,l=0} = \frac{2}{\sqrt{a_0^3}} e^{-\frac{r}{a_0}} \qquad R_{n=2\,l=0} = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{a_0^3}} e^{-\frac{r}{2a_0}}\left(1-\frac{r}{2a_0}\right) \qquad R_{n=3\,l=0} = \frac{2}{3\sqrt{3}}\frac{1}{\sqrt{a_0^3}} e^{-\frac{r}{3a_0}}\left(1-\frac{2r}{3a_0}+\frac{2}{27}\left(\frac{r}{a_0}\right)^2\right)$

$$\begin{cases} \phi_{n=1\,l=0\,m=0} = R_{n=1\,l=0} \cdot Y_{00}(\theta,\phi) \\ \phi_{n=2\,l=0\,m=0} = R_{n=2\,l=0}\, Y_{00}(\theta,\phi) \\ \phi_{n=3\,l=0\,m=0} = R_{n=3\,l=0}\, Y_{00}(\theta,\phi) \end{cases}$$

for any n.l

Laguerre polynomials

$v(\rho) = L_{n-l-1}^{2l+1}(2\rho)$

$$= \frac{1}{(n-l-1)!} \rho^{-2l-1} e^{2\rho} \left(\frac{d}{d\rho}\right)^{n-l-1} e^{-2\rho} \rho^{n+l}$$

$\phi_{nlm} =$ Lec 24 end.

$$\int r^2 dr\, d\Omega\, \phi^*_{n'l'm'}(\vec{r})\, \phi_{nlm}(\vec{r}) = \delta_{n'n}\delta_{l'l}\delta_{m'm}$$

Lec 25 Radiative transitions in hydrogen

$E_n = \frac{E_1}{n^2}$ $\quad E_1 = -13.6\,eV$

$$h\nu = E_{ni} - E_{nf} = -13.6\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) = 13.6\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Lyman $n_i^2 = 1^2$

Balmer $n_i^2 = 2^2$

Paschen $n_i^2 = 3^2$

Bracket $n_i^2 = 4^2$

Lec 26 把 uncoupled states → coupled states

$|l m_l\rangle |s m_s\rangle \to |l m_l\, s{=}\tfrac{1}{2}\, m_s\rangle$

即 $|l_1 m_1\rangle |l_2 m_2\rangle = |l_1 m_1 l_2 m_2\rangle \longleftrightarrow |(l_1 l_2) j\ m_j\rangle$

共 $(2l_1+1)(2l_2+1)$ 种组合方式

$j = |l_1 - l_2|, \dots, l_1+l_2 \qquad m_j \in [-j, j]$

$$|(l_1 l_2) j m_j\rangle = \sum_{m_1=-l_1}^{l_1} \sum_{m_2=-l_2}^{l_2} |l_1 m_1 l_2 m_2\rangle\langle l_1 m_1 l_2 m_2 | (l_1 l_2) j m_j\rangle$$

$2p:\ |n{=}2\ l{=}1\ m_l\rangle |\tfrac{1}{2} m_s\rangle \to \begin{cases} |n(l\tfrac{1}{2})\tfrac{3}{2} m_j\rangle & 4 个 m_j \\ |n(l\tfrac{1}{2})\tfrac{1}{2} m_j\rangle & 2 个 m_j \end{cases}$ 这些 coupled state 是大自然选择的基

$|lm\rangle$ 可以与 n 分开写，∵ 角向只与 $|lm\rangle$，径向只与 $|nl\rangle$ 有关

在 position representations $\langle \theta, \phi | l m\rangle = Y_{lm}(\theta, \phi)$ $\quad m$ 实际上是 j 投影

旋转 $\hat{U}(\hat{n}, \phi)|lm\rangle = \sum_{m'=-l}^{l} |lm'\rangle\langle lm'|\hat{U}|lm\rangle = \sum_{m'=-l}^{l} |lm'\rangle D^{l}_{m'm}(\hat{n}, \phi)$

$\hat{U}(\hat{n}, \phi) = \exp[-\frac{i}{\hbar}\phi\, \hat{n}\cdot \hat{L}] \to \exp[-\frac{i}{\hbar}\phi\, \hat{n}\cdot \hat{J}] \xrightarrow{当\hat{n}指向z\text{-axis}时} \exp[-\frac{i}{\hbar}\phi J_z]$ → $m\hbar$

now $D^{j}_{m'm}(\hat{n}, \phi) = \langle jm'|U|jm\rangle = \langle jm'|\exp[-\frac{i}{\hbar}\phi\, m\hbar]|jm\rangle = \langle jm'|\exp[-i\phi m]|jm\rangle = \delta_{m'm} \exp[-im\phi]$

$$\hat{U}(\alpha,\beta,\gamma) = \exp\left[-\frac{i}{\hbar}\alpha \hat{J}_z\right] \exp\left[-\frac{i}{\hbar}\beta \hat{J}_y\right] \exp\left[-\frac{i}{\hbar}\gamma \hat{J}_z\right]$$

$$D^{j}_{m'm}(\alpha,\beta,\gamma) = \langle j m' | \exp \exp \exp | j m \rangle$$

$$= \exp(-i\alpha m') \langle j m' | \exp[-\frac{i}{\hbar}\beta \hat{J}_z] | j m \rangle \exp[-i\gamma m]$$

$$= \exp(-i\alpha m') \cdot d^{j}_{m'm}(\beta) \exp(-i\gamma m)$$

Lec 27

$$[\hat{L}_i, \hat{L}_j] = i\hbar \varepsilon_{ijk} \hat{L}_k \qquad [\hat{L}^2, \hat{L}_i] = 0$$

$$\hat{L}_\pm = \hat{L}_x \pm i \hat{L}_y$$

$|lm\rangle$ 也一样

1. $\hat{L}^2 L_\pm |nlm\rangle = L_\pm \hat{L}^2 |nlm\rangle = \hbar^2 l(l+1) L_\pm |nlm\rangle$

2. $[\hat{L}_z, \hat{L}_\pm] = \pm\hbar \hat{L}_\pm$

$= \langle nlm' | \hat{L}_z \hat{L}_\pm - \hat{L}_\pm \hat{L}_z | nlm \rangle$

$= \langle nlm' | \hat{L}_z \hat{L}_\pm | nlm \rangle - \langle nlm' | \hat{L}_\pm \hat{L}_z | nlm \rangle$

$= m'\hbar \langle nlm' | \hat{L}_\pm | nlm \rangle - m\hbar \langle nlm' | \hat{L}_\pm | nlm \rangle$

$= (m'-m)\hbar \langle nlm' | \hat{L}_\pm | nlm \rangle = \langle nlm' | \pm\hbar \hat{L}_\pm | nlm \rangle$

$\hat{L}_\pm$ 决定 $m'-m = \pm 1$ $\Rightarrow$ $[\hat{L}_z, \hat{L}_\pm] = \pm\hbar \hat{L}_\pm$

③ 实际上与n无关

$$\hat{L}_+ |n l m\rangle = \hbar\sqrt{l(l+1)-m(m+1)}\ |n l m+1\rangle$$

$$\hat{L}_- |n l m\rangle = \hbar\sqrt{l(l+1)-m(m-1)}\ |n l m-1\rangle$$

④ $\hat{L}_+ |n l l\rangle = 0$, $\hat{L}_- |n l -l\rangle = 0$

⑤ $\hat{L}_\pm = \hat{L}_x \pm i\hat{L}_y$

$$\Rightarrow \hat{L}_x = \frac{1}{2}(\hat{L}_+ + \hat{L}_-) \Rightarrow \hat{L}_x |l m\rangle = \frac{\hbar}{2}\sqrt{l(l+1)-m(m+1)}\ |l m+1\rangle + \frac{\hbar}{2}\sqrt{l(l+1)-m(m-1)}\ |l m-1\rangle$$

$$\hat{L}_y = \frac{1}{2i}(\hat{L}_+ - \hat{L}_-) \Rightarrow \hat{L}_y |l m\rangle = \frac{\hbar}{2i}\sqrt{l(l+1)-m(m+1)}\ |l m+1\rangle - \frac{\hbar}{2i}\sqrt{l(l+1)-m(m-1)}\ |l m-1\rangle$$

e.g. $L=1$

$|l=\frac{1}{2}, m=\frac{1}{2}\rangle \to \begin{pmatrix}1\\0\end{pmatrix}$

$|l=\frac{1}{2}, m=-\frac{1}{2}\rangle \to \begin{pmatrix}0\\1\end{pmatrix}$

$|1,1\rangle \quad \begin{pmatrix}1\\0\\0\end{pmatrix}$ $\quad \hat{L}_+ |1,1\rangle = 0$

$|1,0\rangle \quad \begin{pmatrix}0\\1\\0\end{pmatrix}$ $\quad \hat{L}_+ |1,0\rangle = \hbar\sqrt{2}\ |1,1\rangle$

$|1,-1\rangle \quad \begin{pmatrix}0\\0\\1\end{pmatrix}$ $\quad \hat{L}_+ |1,-1\rangle = \hbar\sqrt{2}\ |1,0\rangle$

$$\hat{L}_+ \begin{pmatrix}1\\0\\0\end{pmatrix} = 0$$

$$\hat{L}_+ \begin{pmatrix}0\\1\\0\end{pmatrix} = \hbar\sqrt{2}\begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$\hat{L}_+ \begin{pmatrix}0\\0\\1\end{pmatrix} = \hbar\sqrt{2}\begin{pmatrix}0\\1\\0\end{pmatrix}$$

$$\hat{L}_- = \begin{bmatrix} 0 & 0 & 0 \\ \sqrt{2}\hbar & 0 & 0 \\ 0 & \sqrt{2}\hbar & 0 \end{bmatrix} \qquad \hat{L}_+ = \begin{bmatrix} 0 & \hbar\sqrt{2} & 0 \\ 0 & 0 & \hbar\sqrt{2} \\ 0 & 0 & 0 \end{bmatrix}$$

$$\hat{L}_x = \frac{1}{2}\left[\hat{L}_+ + \hat{L}_-\right] = \frac{\sqrt{2}}{2}\hbar \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\hat{L}_y = \frac{1}{2i}\left[\hat{L}_+ - \hat{L}_-\right] \quad \frac{\sqrt{2}\hbar}{2i} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

Lec. 28

$$[\hat{S}_i, \hat{S}_j] = i\hbar \varepsilon_{ijk} \hat{S}_k \Rightarrow [\hat{S}_x, \hat{S}_y] = i\hbar \hat{S}_z, \quad [\hat{S}_y, \hat{S}_z] = i\hbar \hat{S}_x, \quad [\hat{S}_z, \hat{S}_x] = i\hbar \hat{S}_y$$

$$S^2 |s\, m_s\rangle = \hbar^2 s(s+1) |s\, m_s\rangle$$

$$S_z |s\, m_s\rangle = \hbar m_s |s\, m_s\rangle$$

$$\hat{S}_\pm |s\, m_s\rangle = \hbar \sqrt{s(s+1) - m_s(m_s \pm 1)}\, |s\, m_s \pm 1\rangle$$

spin-$\frac{1}{2}$ $\quad |s\, m_s = \frac{1}{2}\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \equiv \chi_+ \quad |s\, m_s = -\frac{1}{2}\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \equiv \chi_-$ $\quad$ 2个基（解空间）

$$\Rightarrow \hat{S}^2 |s\, m_s\rangle = \hbar^2 s(s+1) |s\, m_s\rangle \Rightarrow \hat{S}^2 \chi_+ = \frac{3\hbar^2}{4}\chi_+ \qquad \hat{S}^2 \chi_- = \frac{3\hbar^2}{4}\chi_-$$

$$\hat{S}_z |s\, m_s\rangle = \hbar m_s |s\, m_s\rangle \Rightarrow \hat{S}_z \chi_+ = \frac{\hbar}{2}\chi_+ \qquad \hat{S}_z \chi_- = -\frac{\hbar}{2}\chi_-$$

$$\Rightarrow \hat{S}^2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{3\hbar^2}{4} \\ 0 \end{pmatrix} \Rightarrow \hat{S}^2 = \frac{3\hbar^2}{4} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\hat{S}^2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{3\hbar^2}{4} \end{pmatrix}$$

$$\Rightarrow \hat{S}_z \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{\hbar}{2} \\ 0 \end{pmatrix} \Rightarrow \hat{S}_z = \frac{\hbar}{2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\hat{S}_z \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{\hbar}{2} \end{pmatrix}$$

$$\hat{S}_+ \chi_+ = 0 \quad \hat{S}_+ \chi_- = \hbar \chi_+ \quad \hat{S}_- \chi_+ = \hbar \chi_- \quad \hat{S}_- \chi_- = 0$$

$$\Rightarrow \hat{S}_+ \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 0$$

$$\hat{S}_+ \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \hbar \begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow \hat{S}_+ = \hbar \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\hat{S}_\pm = \hat{S}_x \pm i \hat{S}_y$$

$$\Rightarrow \hat{S}_x = \frac{1}{2}(\hat{S}_+ + \hat{S}_-) = \frac{\hbar}{2} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\Rightarrow \hat{S}_- \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \hbar \begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow \hat{S}_- = \hbar \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$\hat{S}_- \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0$$

$$\hat{S}_y = \frac{1}{2i}(\hat{S}_+ - \hat{S}_-) = \frac{\hbar}{2} \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$\Rightarrow \hat{S} = \frac{\hbar}{2} \hat{\sigma} \qquad \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

general normalized state vector

$\chi_{\alpha\beta} = \alpha\chi_+ + \beta\chi_- = \begin{pmatrix}\alpha\\ \beta\end{pmatrix}$, $|\alpha|^2 + |\beta|^2 = 1$ $\quad \langle S_z\rangle = \frac{\hbar}{2}(\alpha^* \ \beta^*)\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \frac{\hbar}{2}(\alpha^*\alpha - \beta^*\beta)$

expectation $\quad = \frac{\hbar}{2}(|\alpha|^2 - |\beta|^2)$

$\langle \hat{S}_x\rangle = \frac{\hbar}{2}(\alpha^*, \beta^*)\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \frac{\hbar}{2}(\alpha^*\beta + \alpha\beta^*) = \hbar\, Re[\alpha\beta^*]$

$\alpha = 1,\ \beta = 0$

for $\alpha = \frac{1}{\sqrt{2}}$ and $\beta = \frac{1}{\sqrt{2}}$ $\quad \alpha\beta^* = \frac{1}{2} \Rightarrow \langle \hat{S}_x\rangle = \frac{\hbar}{2}$ $\quad \Rightarrow \langle \hat{S}_z\rangle = \frac{\hbar}{2}$

for $\alpha = -\beta = \frac{1}{\sqrt{2}} \Rightarrow \alpha\beta^* = -\frac{1}{2} \Rightarrow \langle \hat{S}_x\rangle = -\frac{\hbar}{2}$ $\quad P = |\langle \chi_+ | \chi_{\alpha=1,\beta=0}\rangle|^2 = 1$

$\chi_+^x = \begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix}$ $\quad \chi_-^x = \begin{pmatrix}\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}}\end{pmatrix}$ $\Rightarrow \chi_+ = \frac{1}{\sqrt{2}}(\chi_+^x + \chi_-^x)$, $\chi_- = \frac{1}{\sqrt{2}}(\chi_+^x - \chi_-^x)$ $\quad \alpha = 0 \quad \beta = 1$

$\Rightarrow \langle \hat{S}_z\rangle = -\frac{\hbar}{2}$

$\langle \hat{S}_y\rangle = \frac{\hbar}{2}(\alpha^*, \beta^*)\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \frac{\hbar}{2}(-i\alpha\beta^* + i\alpha\beta^*) = \hbar\, Re[i\alpha\beta^*]$ $\quad P = |\langle \chi_- | \chi_{\alpha=1,\beta=0}\rangle|^2 = 0$

$\uparrow$ initial state

for $\alpha = \frac{1}{\sqrt{2}}$, $\beta = \frac{i}{\sqrt{2}} \Rightarrow i\alpha\beta^* = \frac{1}{2} \Rightarrow \langle \hat{S}_y\rangle = \frac{\hbar}{2}$

for $\alpha = \frac{1}{\sqrt{2}}$, $\beta = -\frac{i}{\sqrt{2}} \Rightarrow i\alpha\beta^* = -\frac{1}{2} \Rightarrow \langle \hat{S}_y\rangle = -\frac{\hbar}{2}$

$\chi_+^y = \begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{i}{\sqrt{2}}\end{pmatrix}$ $\quad \chi_-^y = \begin{pmatrix}\frac{1}{\sqrt{2}}\\ -\frac{i}{\sqrt{2}}\end{pmatrix} \Rightarrow \chi_+ = \frac{1}{\sqrt{2}}(\chi_+^y + \chi_-^y)$, $\chi_- = \frac{i}{\sqrt{2}}[\chi_+^y - \chi_-^y]$

$$\chi_{\alpha,\beta} = \alpha\chi_+ + \beta\chi_- = \left(\frac{\alpha+\beta}{\sqrt{2}}\right)\chi_+^x + \left(\frac{\alpha-\beta}{\sqrt{2}}\right)\chi_-^x = \left(\frac{\alpha-i\beta}{\sqrt{2}}\right)\chi_+^y + \left(\frac{\alpha+i\beta}{\sqrt{2}}\right)\chi_-^y$$

性质 initial state $\alpha=1 \Rightarrow \chi_+$，通过 $\frac{\hbar}{2}$ 算出 (α,β)，待求概率态，后面的 χ_+ 是

① $|\langle\chi_+|\chi_+\rangle|^2 = 1$ $\hat{S}_z$ 测 $\frac{\hbar}{2}$ 的概率

$|\langle\chi_-|\chi_+\rangle|^2 = 0$ $\hat{S}_z$ 测 $-\frac{\hbar}{2}$ P

② $|\langle\chi_+^x|\chi_+\rangle|^2 = |\langle\frac{1}{\sqrt{2}}(\chi_+ + \chi_-)|\chi_+\rangle|^2 = \frac{1}{2}$ $\langle\hat{S}_x\rangle = \frac{\hbar}{2}$ 的 P

$|\langle\chi_-^x|\chi_+\rangle|^2 = |\langle\frac{1}{\sqrt{2}}(\chi_+ - \chi_-)|\chi_+\rangle|^2 = \frac{1}{2}$ $\langle\hat{S}_x\rangle = -\frac{\hbar}{2}$ 的 P

③ $|\langle\chi_+^y|\chi_{\alpha=1,\beta=0}\rangle|^2 = |\langle\frac{1}{\sqrt{2}}(\chi_+ + i\chi_-)|\chi_+\rangle|^2 = \frac{1}{2}$ $\langle\hat{S}_y\rangle = \frac{\hbar}{2}$ 的 P

$|\langle\chi_-^y|\chi_{\alpha=1,\beta=0}\rangle|^2 = |\langle\frac{1}{\sqrt{2}}(\chi_+ - i\chi_-)|\chi_+\rangle|^2 = \frac{1}{2}$ $\langle\hat{S}_y\rangle = -\frac{\hbar}{2}$ 的 P

④ if measurement ② 做过后，塌缩 $\chi_+ \Rightarrow \chi_+^x$

中第一个 collapse 到

此时若再测 这个 state if in state $\chi_+ \Rightarrow P = |\langle\chi_+|\chi_+^x\rangle|^2 = |\langle\chi_+^x|\chi_+\rangle|^2 = \frac{1}{2}$ $P \neq 1$ 了

initial state

先求 $\langle\hat{S}_i\rangle = \chi_{\alpha,\beta}^* \hat{S}_i \chi_{\alpha,\beta} \Rightarrow$ 与 α,β 有关的数

Electrons in a magnetic field:

$\vec{\mu} = \gamma \vec{S}$

$\hat{H} = -\vec{\mu}\cdot\vec{B} = -\gamma\vec{S}\cdot\vec{B} \qquad \vec{B} = B\hat{z}$

$\hat{H} = -\gamma B\hat{S}_z = -\frac{\gamma B\hbar}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ 对 $\hat{x},\hat{y}$ 同样

已知 $\chi_+ = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \chi_- = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ spin 其实 $s=\frac{1}{2}$ 时 就两个态 $|s\ m_s=\frac{1}{2}\rangle$ 及 $|s\ m_s=-\frac{1}{2}\rangle$

$\hat{H}\chi_+ = -\frac{\gamma B\hbar}{2}\chi_+ \Rightarrow E_+ = \frac{-\gamma B\hbar}{2}$ spin up E

$\hat{H}\chi_- = \frac{\gamma B\hbar}{2}\chi_- \Rightarrow E_- = \frac{\gamma B\hbar}{2}$ spin down E

Prime Directive

$\chi(0) = \alpha\chi_+ + \beta\chi_- = \cos\frac{\theta}{2}\chi_+ + \sin\frac{\theta}{2}\chi_-$

$\Rightarrow \chi(t) = \alpha\chi_+ e^{-i\frac{E_+t}{\hbar}} + \beta\chi_- e^{-i\frac{E_-t}{\hbar}}$

$= \cos\frac{\theta}{2}\chi_+ e^{i\frac{\gamma Bt}{2}} + \sin\frac{\theta}{2}\chi_- e^{-i\frac{\gamma Bt}{2}}$

$= \begin{bmatrix} \cos\frac{\theta}{2} e^{i\gamma Bt/2} \\ \sin\frac{\theta}{2} e^{-i\gamma Bt/2} \end{bmatrix}$

在这种波函数下（之前只是 $\chi^*_{\alpha,\beta}\hat{S}_x\chi_{\alpha\beta}$）

$\langle\hat{S}_x\rangle = \chi(t)^\dagger \frac{\hbar}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\chi(t) = \left(\cos\frac{\theta}{2}e^{-i\gamma Bt/2},\ \sin\frac{\theta}{2}e^{i\gamma Bt/2}\right)\frac{\hbar}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \cos\frac{\theta}{2}e^{i\gamma Bt/2} \\ \sin\frac{\theta}{2}e^{-i\gamma Bt/2} \end{bmatrix}$

$$= \left[\cos\frac{\theta}{2}\sin\frac{\theta}{2}\cdot e^{-i\gamma Bt} + \sin\frac{\theta}{2}\cos\frac{\theta}{2} e^{i\gamma Bt}\right]\frac{\hbar}{2}$$

$$= \frac{\hbar}{2}\cdot 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos(\gamma Bt) = \frac{\hbar}{2}\sin\theta\cos\gamma Bt$$

$$\langle\hat{S}_y\rangle = \chi^\dagger(t)\frac{\hbar}{2}\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}\chi(t) = \left(\cos\frac{\theta}{2}e^{-i\gamma Bt/2}, \sin\frac{\theta}{2}e^{i\gamma Bt/2}\right)\frac{\hbar}{2}\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}\begin{bmatrix}\cos\frac{\theta}{2}e^{i\gamma Bt/2}\\ \sin\frac{\theta}{2}e^{-i\gamma Bt/2}\end{bmatrix}$$

$$\left[-i\cos\frac{\theta}{2}\sin\frac{\theta}{2}e^{-i\gamma Bt/2} + i\sin\frac{\theta}{2}\cos\frac{\theta}{2}e^{i\gamma Bt/2}\right]\frac{\hbar}{2}$$

$$= \frac{\hbar i}{2}\left[i\,2\cos\frac{\theta}{2}\sin\frac{\theta}{2}\sin\gamma Bt/2\right] = -\frac{\hbar}{2}\sin\theta\sin\frac{\gamma Bt}{2}$$

$$\langle\hat{S}_z\rangle = \chi(t)^\dagger\frac{\hbar}{2}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\chi(t) = \left(\cos\frac{\theta}{2}e^{-i\gamma Bt/2}, \sin\frac{\theta}{2}e^{i\gamma Bt/2}\right)\frac{\hbar}{2}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}\cos\frac{\theta}{2}e^{i\gamma Bt/2}\\ \sin\frac{\theta}{2}e^{-i\gamma Bt/2}\end{bmatrix}$$

$$= \frac{\hbar}{2}\left(\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\right) = \frac{\hbar}{2}\cos\theta$$

29. Spin $\frac{1}{2}$

uncoupled representation

$$(\hat{S}^{(1)})^2|s_1 m_1 s_2 m_2\rangle = s_1(s_1+1)\hbar^2|s_1 m_1 s_2 m_2\rangle$$

$$(\hat{S}^{(2)})^2|s_1 m_1 s_2 m_2\rangle = s_2(s_2+1)\hbar^2|s_1 m_1 s_2 m_2\rangle$$

$$\hat{S}_z^{(1)}|s_1 m_1 s_2 m_2\rangle = m_1\hbar|s_1 m_1 s_2 m_2\rangle$$

$$\hat{S}_z^{(2)}|s_1 m_1 s_2 m_2\rangle = m_2\hbar|s_1 m_1 s_2 m_2\rangle$$

$\hat{S} = \hat{S}^{(1)} + \hat{S}^{(2)}$

$\hat{S}_z |s_1 m_1 s_2 m_2\rangle = (m_1+m_2)\hbar |s_1 m_1 s_2 m_2\rangle$

$\hat{S}^2$ 的作用不太清楚

$|\uparrow;\uparrow\rangle = |\frac{1}{2},\frac{1}{2};\frac{1}{2},\frac{1}{2}\rangle$ $s=1$ $m=1$

$|\uparrow;\downarrow\rangle = |\frac{1}{2},\frac{1}{2};\frac{1}{2},-\frac{1}{2}\rangle$ $s=?$ $m=0$ Why ? s

$|\downarrow;\uparrow\rangle = |\frac{1}{2},-\frac{1}{2};\frac{1}{2},\frac{1}{2}\rangle$ $s=?$ $m=0$

$|\downarrow;\downarrow\rangle = |\frac{1}{2},-\frac{1}{2},\frac{1}{2},-\frac{1}{2}\rangle$ $s=1$ $m=-1$ 总 s 和 m 都 clear 的可以写成耦合形式

用其它方法: $\hbar\sqrt{s(s+1)-m(m-1)}$ $\frac{3}{4}+\frac{1}{4}$

$\hat{S}_-|\uparrow;\uparrow\rangle = (\hat{S}_-^{(1)}+\hat{S}_-^{(2)})|\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2}\rangle = \hbar\,(|\frac{1}{2}-\frac{1}{2};\frac{1}{2}\frac{1}{2}\rangle + |\frac{1}{2}\frac{1}{2};\frac{1}{2}-\frac{1}{2}\rangle$

$\hat{S}_-|(\frac{1}{2}\frac{1}{2})\,s=1\;m=1\rangle = \hbar\sqrt{2}\,|(\frac{1}{2}\frac{1}{2})\,s=1\;m=0\rangle$

$\Rightarrow \frac{1}{\sqrt{2}}(|\frac{1}{2}-\frac{1}{2};\frac{1}{2}\frac{1}{2}\rangle + |\frac{1}{2}\frac{1}{2};\frac{1}{2}-\frac{1}{2}\rangle = |(\frac{1}{2}\frac{1}{2})\,s=1\;m=0\rangle$

$\hat{S}_-|(\frac{1}{2}\frac{1}{2})\,s=1\;m=0\rangle = \hbar\sqrt{2}\,|(\frac{1}{2}\frac{1}{2})\,s=1\;m=-1\rangle$

$= (\hat{S}_-^{(1)}+\hat{S}_-^{(2)})\left[\frac{1}{\sqrt{2}}(|\frac{1}{2}-\frac{1}{2};\frac{1}{2}\frac{1}{2}\rangle + |\frac{1}{2}\frac{1}{2};\frac{1}{2}-\frac{1}{2}\rangle)\right]$

$\hat{S}_-|\quad\rangle$

若此态是coupled形式, $\hat{S}_-$正常

若 uncoupled, $\hat{S}_- = \hat{S}_-^{(1)} + \hat{S}_-^{(2)}$

$= \hbar \frac{1}{\sqrt{2}} \left(|\tfrac12 -\tfrac12; \tfrac12 -\tfrac12\rangle + |\tfrac12 -\tfrac12; \tfrac12 -\tfrac12\rangle \right)$

$= \hbar\sqrt{2}\, |\tfrac12 -\tfrac12; \tfrac12 -\tfrac12\rangle$

$|\uparrow\uparrow\rangle = [\tfrac12 \otimes \tfrac12]_{11}$

$\frac{1}{\sqrt{2}}\left(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle\right) = [\tfrac12 \otimes \tfrac12]_{10}$

$|\downarrow\downarrow\rangle = [\tfrac12 \otimes \tfrac12]_{1-1}$

$\Rightarrow |\tfrac12 -\tfrac12; \tfrac12 -\tfrac12\rangle = |(\tfrac12 \tfrac12)\, S=1 \ m=-1\rangle = |\downarrow;\downarrow\rangle$

$\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) = [\tfrac12 \otimes \tfrac12]_{00}$

Results

$|\uparrow;\uparrow\rangle = |\tfrac12 \tfrac12; \tfrac12 \tfrac12\rangle = |(\tfrac12 \tfrac12)\, S=1\ m=1\rangle \quad S=1 \quad m=1$

lower

$\frac{1}{\sqrt{2}}\left(|\uparrow;\downarrow\rangle + |\downarrow;\uparrow\rangle\right) = \frac{1}{\sqrt{2}}\left(|\tfrac12 \tfrac12; \tfrac12 -\tfrac12\rangle + |\tfrac12 -\tfrac12; \tfrac12 \tfrac12\rangle\right) \equiv |(\tfrac12 \tfrac12), S=1\ m=0\rangle \quad S=1 \quad m=0$

lower

$|\downarrow;\downarrow\rangle = |\tfrac12 -\tfrac12; \tfrac12 -\tfrac12\rangle = |(\tfrac12 \tfrac12)\, S=1\ m=-1\rangle \quad S=1 \quad m=-1$

对 $S=0$ $m=0$ 用所有 $m=0$ 的单态 linear combination，并 anti symmetric + normalization + orthogonal

$\Rightarrow |(\tfrac12 \tfrac12) S=0\ m=0\rangle = \frac{1}{\sqrt{2}}\left(|\tfrac12 \tfrac12; \tfrac12 -\tfrac12\rangle - |\tfrac12 -\tfrac12; \tfrac12 \tfrac12\rangle\right)$

$|s_1 m_1 s_2 m_2\rangle \Rightarrow |(s_1 s_2)\, S M\rangle$

$|l\, m_l\, s\, m_s\rangle \Rightarrow |(ls)\, j\ m_j\rangle$

$\Rightarrow |(ls)\, j=l+\tfrac12\ m_j\rangle \quad 2l+2$ states spin aligned

$|(ls)\, j=l-\tfrac12\ m_j\rangle \quad 2l$ states spin-antialigned

$\Rightarrow 2(2l+1)$ states

Lec 30: Clebsch - Gordan Coefficients

$$|(s_1 s_2) s m\rangle = \sum_{\substack{m_1, m_2 \\ m_1+m_2=m}} |s_1 m_1 s_2 m_2\rangle\langle s_1 m_1 s_2 m_2|(s_1 s_2) s m\rangle$$

$$\begin{bmatrix} |(\frac{1}{2}\frac{1}{2})\,1\,1\rangle \\ |(\frac{1}{2}\frac{1}{2})\,1\,0\rangle \\ |(\frac{1}{2}\frac{1}{2})\,0\,0\rangle \\ |(\frac{1}{2}\frac{1}{2})\,1\,{-1}\rangle \end{bmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & 0 \\ 0 & -\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} |\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2}\rangle \\ |\frac{1}{2}\frac{1}{2};\frac{1}{2}-\frac{1}{2}\rangle \\ |\frac{1}{2}-\frac{1}{2};\frac{1}{2}\frac{1}{2}\rangle \\ |\frac{1}{2}-\frac{1}{2};\frac{1}{2}-\frac{1}{2}\rangle \end{pmatrix}$$

$\downarrow$

$$\begin{bmatrix} \langle\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2}|(\frac{1}{2}\frac{1}{2})\,1\,1\rangle & 0 & 0 & 0 \\ 0 & - - & - - & 0 \\ 0 & - - & - - & 0 \\ 0 & 0 & 0 & - - \end{bmatrix}$$

Lec 31. Fine structure of hydrogen atom

$$\hat{H} = -\hat{\mu}\cdot\vec{B} = -\gamma\hat{S}\cdot\vec{B} = -\frac{e}{m}\hat{S}\cdot\vec{B}$$ $\Rightarrow \mu \Rightarrow \hat{S}$

$$\vec{B} = -\frac{\vec{v}\times\vec{E}}{c^2} = -\frac{\vec{p}\times\vec{r}}{mc^2}\frac{E}{r} = \frac{\vec{l}\,E}{mc^2 r}$$ $\vec{p}$ is electron momentum $\Rightarrow$ 轨道 $(\vec{B}) \Rightarrow \hat{l}$

$$\Rightarrow \hat{H} = \frac{\hat{S}\cdot\hat{l}}{m^2c^2}\frac{eE}{r}$$

$$e\vec{E} = -e\nabla V = \nabla -\frac{\alpha\hbar c}{r} = \frac{\alpha\hbar c}{r^2}\hat{r} \Rightarrow \frac{eE}{r} = \frac{\alpha\hbar c}{r^3}$$

电势 xe = 势能 见 Lec 23

$$\Rightarrow \hat{H}' = \frac{1}{m^2c^2}\frac{\alpha\hbar c}{r^3}\hat{S}\cdot\hat{l} = \frac{(\hbar c)^3}{(mc^2)^2}\frac{\alpha}{r^3}\frac{\hat{S}\cdot\hat{l}}{\hbar^2}$$

$$\hat{H}^0 = \frac{-\alpha\hbar c}{r}$$

$$\hat{H} = \hat{H}^0 + \hat{H}' \Rightarrow \hat{H} = \hat{H}^0 + \lambda\hat{H}'$$

$$\boxed{\psi_n = \psi_n^0 + \lambda\psi_n^1 + \lambda^2\psi_n^2 + \cdots} \qquad E_n = E_n^0 + \lambda E_n^1 + \lambda^2 E_n^2 + \cdots$$ ②

$\hat{H}^0|\psi_n^0\rangle = E_n^0|\psi_n^0\rangle$ unperturbed eigenstates

$$(\hat{H}^0 + \lambda\hat{H}')[\,|\psi_n^0\rangle + \lambda|\psi_n^1\rangle + \lambda^2|\psi_n^2\rangle + \cdots] = (E_n^0 + \lambda E_n^1 + \lambda^2 E_n^2 + \cdots)[\,|\psi_n^0\rangle + \lambda|\psi_n^1\rangle + \lambda^2|\psi_n^2\rangle + \cdots]$$

$$\hat{H}^0|\psi_n^0\rangle = E_n^0|\psi_n^0\rangle \quad ①$$

$$\lambda\hat{H}^0|\psi_n^1\rangle + \lambda\hat{H}'|\psi_n^0\rangle = \lambda E_n^0|\psi_n^1\rangle + \lambda E_n^1|\psi_n^0\rangle \quad ②$$

②'

$$\langle \psi_n^0 | \hat{H}^0 | \psi_n^1 \rangle + \langle \psi_n^0 | \hat{H}' | \psi_n^0 \rangle = E_n^0 \cancel{\langle \psi_n^0 | \psi_n^1 \rangle} + E_n^1 \langle \psi_n^0 | \psi_n^0 \rangle = E_n^0 \langle \psi_n^0 | \psi_n^1 \rangle + E_n^1$$

$\downarrow$

$\hat{H}^0$ is Hermitian $\Rightarrow \langle \psi_n^0 | \hat{H}^0 | \psi_n^1 \rangle = \langle \hat{H}^0 \psi_n^0 | \psi_n^1 \rangle = E_n^0 \cancel{\langle \psi_n^0 | \psi_n^1 \rangle}$

$$\Rightarrow \langle \psi_n^0 | \hat{H}' | \psi_n^0 \rangle = E_n^1$$

由 ③ $\Rightarrow$ $\boxed{E_n \sim E_n^0 + \langle \psi_n^0 | \hat{H}' | \psi_n^0 \rangle + o(\lambda^2)}$

$\downarrow$

the first order energy shift

The fine-structure interaction:

$\hat{H}_{rel.s.o}$ $\quad \Delta T \quad$ $\hat{H}_{Darwin}$

$$\hat{s} \cdot \hat{\ell} = \frac{1}{2} [\hat{j}^2 - \hat{\ell}^2 - \hat{s}^2] = \frac{\hbar^2}{2} [j(j+1) - \ell(\ell+1) - s(s+1)]$$

$$\langle n(\ell s) j m | \hat{H}_{rel\ s.o.} | n(\ell s) j m \rangle = \frac{|E_n|^2}{mc^2} n \frac{j(j+1) - \ell(\ell+1) - \frac{3}{4}}{\ell(\ell+\frac{1}{2})(\ell+1)}$$

$$\langle n(\ell s) j m | \Delta T | n(\ell s) j m \rangle = -\frac{|E_n|^2}{2mc^2} \left[\frac{4n}{\ell+\frac{1}{2}} - 3 \right]$$

$$\langle n(\ell s) j m | \hat{H}_{Darwin} | n(\ell s) j m \rangle = 2n \frac{|E_n|^2}{mc^2}$$

For $l>0$ $\quad \langle \hat{H}_{\text{fine structure}} \rangle = \langle \hat{H}_{rel.so} + \hat{\delta T} \rangle = \dfrac{|E_n|^2}{2mc^2}\left[3 - \dfrac{4n}{j+\frac{1}{2}}\right]$

For $l=0$ $\quad \langle \hat{H}_{\text{fine structure}} \rangle = \langle \hat{\delta T} + \hat{H}_{Darwin} \rangle = \dfrac{|E_n|^2}{2mc^2}\left[3 - \dfrac{4n}{j+\frac{1}{2}}\right]$ $\quad j=\frac{1}{2} = S$

ΔE_n

$$\frac{\Delta E_n}{|E_n|} = \frac{|E_n|}{2mc^2}\left[3 - \frac{4n}{j+\frac{1}{2}}\right] = \frac{|E_1|}{2n^2mc^2}\left[3 - \frac{4n}{j+\frac{1}{2}}\right] \sim \frac{1.33\times 10^{-5}}{n^2}\left[3 - \frac{4n}{j+\frac{1}{2}}\right]$$

Lec. 32

$$H = \frac{1}{2m}\left[\hat{p} - q\hat{A}(x,t)\right]^2 + q\Phi(x,t)$$

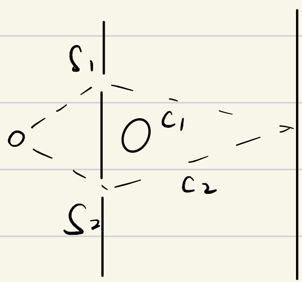
$$\Phi \to \Phi' \equiv \Phi - \frac{\partial \Lambda}{\partial t} \qquad \vec{A} \to \vec{A}' = \vec{A} + \nabla\Lambda$$

$$\Phi = \pi a^2 B = \int \vec{B}\cdot d\vec{S} = \int \nabla\times\vec{A}\cdot d\vec{S} = \oint \vec{A}\cdot d\vec{l}$$

$$\Rightarrow A(r) = \frac{\Phi}{2\pi r}\hat{\phi} \quad r>a$$

$$\pi r^2 B = \frac{r^2}{a^2}\Phi = 2\pi r A(r)$$

$$\Rightarrow A(r) = \frac{\Phi r}{2\pi a^2}\hat{\phi} \quad r<a$$



$\hat{H}$ $\quad q = -e$

$$\frac{1}{2m}[\hat{p} + e\vec{A}(\vec{x},t)]^2 \psi = i\hbar \frac{\partial \psi}{\partial t}$$

$$\Rightarrow \psi(x,t) = \psi_0(x,t) \exp\left[-\frac{ie}{\hbar}\int_{x_0}^{x} \vec{A}\cdot d\vec{l}\right]$$

$$\oint_{x_0}^{x} \vec{A}\cdot d\vec{l}\Big|_{c_2} - \oint_{x_0}^{x} \vec{A}\cdot d\vec{l}\Big|_{c_1} = \Phi$$

$$\psi_1(\vec{x},t) + \psi_2(\vec{x},t) = \psi_0(\vec{x},t)\left[\exp\left[-\frac{ie}{\hbar}\int_{x_0}^{x} \vec{A}\cdot d\vec{l}\Big|_{c_1}\right] + \exp\left[-\frac{ie}{\hbar}\int_{x_0}^{x} \vec{A}\cdot d\vec{l}\Big|_{c_2}\right]\right]$$

$$= \psi_0(\vec{x},t) \exp\left[-\frac{ie}{\hbar}\int_{x_0}^{x} \vec{A}\cdot d\vec{l}\Big|_{c_1}\right]\left[1 + \exp\left[-\frac{ie}{\hbar}\int_{x_0}^{x} \vec{A}\cdot d\vec{l}\Big|_{c_2-c_1}\right]\right]$$

$$= \psi_0(\vec{x},t) \exp\left[-\frac{ie}{\hbar}\int_{x_0}^{x} \vec{A}\cdot d\vec{l}\Big|_{c_1}\right]\left[1 + \exp\left[-\frac{ie}{\hbar}\Phi\right]\right]$$

Lec. 33

$$\hat{H} = -\frac{\hbar^2}{2m_1}\nabla_1^2 - \frac{\hbar}{2m_2}\nabla_2^2 + V(\vec{r}_1,\vec{r}_2,t) \qquad \hat{H}\psi = i\hbar\frac{\partial \psi}{\partial t} \Rightarrow \psi(\vec{r}_1, \vec{r}_2, t)$$

if $V(\vec{r}_1,\vec{r}_2,t) \Rightarrow V(\vec{r}_1,\vec{r}_2) \Rightarrow \psi(\vec{r}_1,\vec{r}_2) = \langle \vec{r}_1 \vec{r}_2 | \psi \rangle$

$$\psi(\vec{r}_1,\vec{r}_2,t) = \sum_{n_1 n_2} C_{n_1 n_2} \psi_{n_1 n_2}(\vec{r}_1,\vec{r}_2)\, e^{-i\frac{E_{n_1 n_2} t}{\hbar}}$$

$$C_{n_1 n_2} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \psi_{n_1 n_2}(\vec{r}_1,\vec{r}_2)\, \psi(\vec{r}_1,\vec{r}_2,t=0)\, d\vec{r}_1\, d\vec{r}_2$$

$$\sum |C_{n_1 n_2}|^2 = 1$$

1. Noninteracting particles

$$V(\vec{r}_1, \vec{r}_2) = V_1(\vec{r}_1) + V_2(\vec{r}_2)$$

$$\hat{H} = \hat{H}_1 + \hat{H}_2 = \left[-\frac{\hbar^2}{2m_1}\nabla_1^2 + V_1(\vec{r}_1)\right] + \left[-\frac{\hbar^2}{2m_2}\nabla_2^2 + V_2(\vec{r}_2)\right]$$

$$\hat{H}_1 \phi_n(\vec{r}_1) = E_n \phi_n(\vec{r}_1) \qquad \hat{H}_2 \phi_m'(\vec{r}_2) = E_m' \phi_m'(\vec{r}_2)$$

$$\Rightarrow \hat{H} \phi_n(\vec{r}_1) \phi_m'(\vec{r}_2) = (\hat{H}_1 + \hat{H}_2) \phi_n(\vec{r}_1) \phi_m'(\vec{r}_2) = (E_n + E_m') \phi_n(\vec{r}_1) \phi_m'(\vec{r}_2)$$

$$\Psi(\vec{r}_1, \vec{r}_2, t) = \sum_{nm} C_{nm} \phi_n(\vec{r}_1) \phi_m'(\vec{r}_2) e^{-i\frac{(E_n + E_m')t}{\hbar}}$$

2. Central potential:

$$\left[-\frac{\hbar^2}{2m_1}\nabla_1^2 + \frac{\hbar^2}{2m_2}\nabla_2^2 + V(|\vec{r}_1 - \vec{r}_2|)\right] \phi(\vec{r}_1, \vec{r}_2) = E \phi(\vec{r}_1, \vec{r}_2)$$

$$\vec{r} \equiv \vec{r}_1 - \vec{r}_2 \qquad M\vec{R} = m_1 \vec{r}_1 + m_2 \vec{r}_2 \qquad M \equiv m_1 + m_2$$

$$\Rightarrow \left[-\frac{\hbar^2}{2\mu}\nabla_r^2 - \frac{\hbar^2}{2M}\nabla_R^2 + V(r)\right] \phi(\vec{r}, \vec{R}) \qquad \mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$\hat{H} = \hat{H}^{rel} + \hat{H}^{cm} = \left[-\frac{\hbar^2}{2\mu}\nabla_r^2 + V(r)\right] + \left[-\frac{\hbar^2}{2M}\nabla_R^2\right]$$

$$\hat{H} \phi(\vec{r}, \vec{R}) = \hat{H} \phi_n^{rel}(\vec{r}) \phi_k^{cm}(\vec{R}) = (\hat{H}^{rel} + \hat{H}^{cm}) \phi_n^{rel}(\vec{r}) \phi_k^{cm}(\vec{R}) = (E_n^{rel} + E_k^{cm}) \phi_n^{rel}(\vec{r}) \phi_k^{cm}(\vec{R})$$

$$\psi(\vec{r},\vec{R},t)=\sum_{nk} C_{nk}\,\phi_n^{rel}(\vec{r})\,\phi_{\vec{k}}^{cm}(\vec{R})\,e^{-i\frac{(E_n^{rel}+E_k^{cm})t}{\hbar}}$$

3、Other cases He atom

$$V(\vec{r}_1,\vec{r}_2)=-\alpha\hbar c\left(-\frac{2}{|\vec{r}_1|}-\frac{2}{|\vec{r}_2|}+\frac{1}{|\vec{r}_1-\vec{r}_2|}\right)$$

Indistinguishable particles:

$\psi_+(\vec{r}_1,\vec{r}_2)=\psi_+(\vec{r}_2,\vec{r}_1)$ bosons 整数

$\psi_-(\vec{r}_1,\vec{r}_2)=-\psi_-(\vec{r}_2,\vec{r}_1)$ fermions 半整数

fermions

$\phi_n(\vec{r}_1)\phi_{m}'(\vec{r}_2) \Rightarrow \psi(\vec{r}_1,\vec{r}_2)=\sum_{nm}\phi_n(\vec{r}_1)\phi_m(\vec{r}_2)$ & $\sum_{nm}|C_{nm}|^2=1$

$\psi(\vec{r}_1,\vec{r}_2)=-\psi(\vec{r}_2,\vec{r}_1)$

$$\psi(\vec{r}_1,\vec{r}_2)=\sum_{n>m} C_{nm}\frac{1}{\sqrt{2}}\left(\phi_n(\vec{r}_1)\phi_m(\vec{r}_2)-\phi_m(\vec{r}_1)\phi_n(\vec{r}_2)\right)$$

$\langle n'm'|nm\rangle=\iint \psi^*(\vec{r}_1,\vec{r}_2)\,\psi(\vec{r}_1,\vec{r}_2)\,d\vec{r}_1\,d\vec{r}_2=\delta_{n'n}\delta_{m'm}$ fermionic / ortho normal

$=\frac{1}{\sqrt{2}}\begin{vmatrix}\phi_n(\vec{r}_1) & \phi_n(\vec{r}_2)\\ \phi_m(\vec{r}_1) & \phi_m(\vec{r}_2)\end{vmatrix}$ $n>m$ Slater determinant $\Rightarrow$ anti-symmetric

three fermions

$$\psi(\vec r_1,\vec r_2,\vec r_3) = \frac{1}{\sqrt{3!}}\begin{vmatrix} \phi_n(\vec r_1) & \phi_n(\vec r_2) & \phi_n(\vec r_3) \\ \phi_m(\vec r_1) & \phi_m(\vec r_2) & \phi_m(\vec r_3) \\ \phi_l(\vec r_1) & \phi_l(\vec r_2) & \phi_l(\vec r_3) \end{vmatrix} \qquad n>m>l$$

Lec 34

The Pauli exclusion principle

fermion

if $n=m \Rightarrow$ Slater vanished $\Rightarrow$ if one electron occupies state n, all other electrons are excluded from that state

Boson and Bose-Einstein condensation

: there is no restriction on the occupation number of a given state

$\phi_n(\vec r_1)\,\phi_n(\vec r_2)\cdots\phi_n(\vec r_N) \Rightarrow$ cool $\to$ almost to absolute zero

Particle seperation and statistics

(space)

Dirac representation — state exist independent of their representations.

$|a\rangle = \int d\vec r\ |\vec r\rangle\langle\vec r|a\rangle = \int d\vec r\ |\vec r\rangle\,\phi_a(r)$ $\quad\phi_a(r)$ is just an expansion coefficient.

$|a;b\rangle_{fermions} = \frac{1}{\sqrt2}(|a;b\rangle - |b;a\rangle)$ $\qquad |a;b\rangle_{bosons} = \frac{1}{\sqrt2}(|a;b\rangle + |b;a\rangle)$

we are exchanging quantum labels.

$$\Rightarrow |a;b\rangle_{fermions} = \int d\vec{r}_1 d\vec{r}_2 \, |\vec{r}_1 \vec{r}_2\rangle\langle \vec{r}_1 \vec{r}_2 | a;b\rangle_{fermions} = \int d\vec{r}_1 d\vec{r}_2 \, |\vec{r}_1 \vec{r}_2\rangle \frac{1}{\sqrt{2}} \left(\phi_a(\vec{r}_1)\phi_b(\vec{r}_2) - \phi_b(\vec{r}_1)\phi_a(\vec{r}_2) \right)$$

$$|a;b\rangle_{bosons} = \int d\vec{r}_1 d\vec{r}_2 \, |\vec{r}_1 \vec{r}_2\rangle\langle \vec{r}_1 \vec{r}_2 | a;b\rangle_{bosons} = \int d\vec{r}_1 d\vec{r}_2 \, |\vec{r}_1 \vec{r}_2\rangle \frac{1}{\sqrt{2}} \left(\phi_a(\vec{r}_1)\phi_b(\vec{r}_2) + \phi_a(\vec{r}_1)\phi_b(\vec{r}_2) \right)$$

Space and spin - Hilbert space of the two-electron two-state problem

$|\alpha_1\rangle \equiv |a \ \tfrac{1}{2} \ \tfrac{1}{2}\rangle$ $\quad |\alpha_2\rangle \equiv |a \ \tfrac{1}{2} \ -\tfrac{1}{2}\rangle$ $\quad |\alpha_3\rangle \equiv |b \ \tfrac{1}{2} \ \tfrac{1}{2}\rangle$ $\quad |\alpha_4\rangle \equiv |b \ \tfrac{1}{2} \ -\tfrac{1}{2}\rangle$

$s \ m_s$

anti-symmetric

$$|\alpha_i;\alpha_j\rangle_{fermion} = \frac{1}{\sqrt{2}} \left(|\alpha_i;\alpha_j\rangle - |\alpha_j;\alpha_i\rangle \right) \qquad i<j$$

$|\alpha_1;\alpha_2\rangle_{fermion}$ ...... six antisymmetric basis states.

不加也行

#

1 $|\alpha_1 \alpha_2\rangle = \frac{1}{\sqrt{2}}(|a\uparrow;a\downarrow\rangle - |a\downarrow;a\uparrow\rangle) = |aa\rangle \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) = |aa\rangle [\tfrac{1}{2}\otimes\tfrac{1}{2}]_{00}$

2 $|\alpha_1 \alpha_3\rangle = \frac{1}{\sqrt{2}}(|a\uparrow;b\uparrow\rangle - |b\uparrow;a\uparrow\rangle) = \frac{1}{\sqrt{2}}(|ab\rangle - |ba\rangle)|\uparrow\uparrow\rangle = \frac{1}{\sqrt{2}}(|ab\rangle - |ba\rangle)[\tfrac{1}{2}\otimes\tfrac{1}{2}]_{11}$

3 $|\alpha_2 \alpha_4\rangle = \frac{1}{\sqrt{2}}(|a\downarrow;b\downarrow\rangle - |b\downarrow;a\downarrow\rangle) = \frac{1}{\sqrt{2}}(|ab\rangle - |ba\rangle)|\downarrow\downarrow\rangle = \frac{1}{\sqrt{2}}(|ab\rangle - |ba\rangle)[\tfrac{1}{2}\otimes\tfrac{1}{2}]_{1-1}$

4 $|\alpha_3 \alpha_4\rangle = \frac{1}{\sqrt{2}}(|b\uparrow;b\downarrow\rangle - |b\downarrow;b\uparrow\rangle) = |bb\rangle \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) = |bb\rangle [\tfrac{1}{2}\otimes\tfrac{1}{2}]_{00}$

#5 $\frac{1}{\sqrt{2}}\left(|\alpha_1\alpha_4\rangle + |\alpha_2\alpha_3\rangle\right) = \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\left(|a\uparrow; b\downarrow\rangle - |b\downarrow; a\uparrow\rangle + |a\downarrow; b\uparrow\rangle - |b\uparrow; a\downarrow\rangle\right)$

$= \frac{1}{\sqrt{2}}\left(|ab\rangle - |ba\rangle\right)\underbrace{\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)}_{顺序无所谓} = \frac{1}{\sqrt{2}}\left(|ab\rangle - |ba\rangle\right)\left[\tfrac{1}{2}\otimes\tfrac{1}{2}\right]_{10}$

Sym

$|\uparrow\uparrow\rangle = \left[\tfrac{1}{2}\otimes\tfrac{1}{2}\right]_{11}$

$\frac{1}{\sqrt{2}}\left(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle\right) = \left[\tfrac{1}{2}\otimes\tfrac{1}{2}\right]_{10}$

$|\downarrow\downarrow\rangle = \left[\tfrac{1}{2}\otimes\tfrac{1}{2}\right]_{1-1}$

#6 $\frac{1}{\sqrt{2}}\left(|\alpha_1\alpha_4\rangle - |\alpha_2\alpha_3\rangle\right) = \frac{1}{2}\left(|a\uparrow; b\downarrow\rangle - |b\downarrow; a\uparrow\rangle - |a\downarrow; b\uparrow\rangle + |b\uparrow; a\downarrow\rangle\right)$

$= \frac{1}{\sqrt{2}}\left(|ab\rangle + |ba\rangle\right)\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) = \frac{1}{\sqrt{2}}\left(|ab\rangle + |ba\rangle\right)\left[\tfrac{1}{2}\otimes\tfrac{1}{2}\right]_{00}$

anti sym

$\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) = \left[\tfrac{1}{2}\otimes\tfrac{1}{2}\right]_{00}$

Hilbert space of the two-level, two-electron problems

$|\alpha_1;\alpha_4\rangle$ 本身会蹦出一个 $\frac{1}{\sqrt{2}}$

three are space sym, spin antisym
$$\left\{\begin{array}{l} |aa\rangle \\ \frac{1}{\sqrt{2}}\left(|ab\rangle + |ba\rangle\right) \quad \text{with} \quad \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) = \left[\tfrac{1}{2}\otimes\tfrac{1}{2}\right]_{00} \\ |bb\rangle \end{array}\right.$$

three are spatially antisym, spin sym $\frac{1}{\sqrt{2}}|ab - ba\rangle$ with
$$\left\{\begin{array}{l} |\uparrow\uparrow\rangle = \left[\tfrac{1}{2}\otimes\tfrac{1}{2}\right]_{11} \\ \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right) = \left[\tfrac{1}{2}\otimes\tfrac{1}{2}\right]_{10} \\ |\downarrow\downarrow\rangle = \left[\tfrac{1}{2}\otimes\tfrac{1}{2}\right]_{1-1} \end{array}\right.$$

for two level (a, b), two electron system

$S=0 \Rightarrow$ two ground
two excited

one in each with spatical symmetry

$S=1 \Rightarrow$ one in each with spatical antisymmetry
+ spin sym wave functions.

Lec 35

He atom

$$\hat{H} = \frac{\vec{p}_1^{\,2}}{2m} + \frac{\vec{p}_2^{\,2}}{2m} + \hbar c\alpha\left[-\frac{2}{r_1} - \frac{2}{r_2} + \frac{1}{r_{12}}\right] \qquad r_{12} = |\vec{r}_1 - \vec{r}_2|$$

more descriptive quantum numbers:

$|\alpha\beta\rangle \to |n_1 l_1 m_1 m_{s_1};\ n_2 l_2 m_2 m_{s_2}\rangle$ 　 s都是$\frac{1}{2}$就不写了

$n\,l\,m$ 掌控 space 　 m_s control spin

$\hat{E}_{12}^{space} \, | n_1 l_1 m_1 m_{s_1} ; n_2 l_2 m_2 m_{s_2} \rangle = | n_2 l_2 m_2 m_{s_1} ; n_1 l_1 m_1 m_{s_2} \rangle$

$\hat{E}_{12}^{spin} \, | n_1 l_1 m_1 m_{s_1} ; n_2 l_2 m_2 m_{s_2} \rangle = | n_1 l_1 m_1 m_{s_2} ; n_2 l_2 m_2 m_{s_1} \rangle$

$\hat{E}_{12} = \hat{E}_{12}^{space} \, E_{12}^{spin}$

$\hat{E}_{12} \, | n_1 l_1 m_1 m_{s_1} ; n_2 l_2 m_2 m_{s_2} \rangle = | n_2 l_2 m_2 m_{s_2} ; n_1 l_1 m_1 m_{s_1} \rangle$

$[\hat{E}_{12}^{space}, \hat{E}_{12}^{spin}] = 0$ Commute $\hat{H}$ also commute $\hat{H} \, | \alpha\beta \rangle = E_{n_1 l_1 m_1 ; n_2 l_2 m_2}$

换不换无所谓

He involves two fermions

the eigenvalue $\varepsilon_{12} = \varepsilon_{12}^{space} \, \varepsilon_{12}^{spin}$ must be odd

$\hat{E}_{12}^{spin} \, | (\tfrac{1}{2} \tfrac{1}{2}) S=1 \; M_S \rangle = \hat{E}_{12}^{spin} \begin{cases} | \tfrac{1}{2} ; \tfrac{1}{2} \rangle & M_S = 1 \\ \tfrac{1}{\sqrt{2}} (| \tfrac{1}{2} ; -\tfrac{1}{2} \rangle + | -\tfrac{1}{2} ; \tfrac{1}{2} \rangle) & M_S = 0 \\ | -\tfrac{1}{2} ; -\tfrac{1}{2} \rangle & M_S = -1 \end{cases} = + | (\tfrac{1}{2} \tfrac{1}{2}) S=1 \; M_S \rangle$ $\quad \varepsilon_{12}^{spin} = 1$ $\quad S=1 \Rightarrow$ 需 space odd

$\hat{E}_{12}^{spin} \, | (\tfrac{1}{2} \tfrac{1}{2}) S=0 \; M_S=0 \rangle = \hat{E}_{12}^{spin} \, \tfrac{1}{\sqrt{2}} (| \tfrac{1}{2} ; -\tfrac{1}{2} \rangle - | -\tfrac{1}{2} ; \tfrac{1}{2} \rangle) = - | (\tfrac{1}{2} \tfrac{1}{2}) S=0 \; M_S=0 \rangle$ $\quad \varepsilon_{12}^{spin} = -1$ $\quad S=0 \Rightarrow$ 须 space even

$S = S^{(1)} + S^{(2)}$ Commute with H

$L = L^{(1)} + L^{(2)}$ Commute with H

$$[L^{(1)}, H] = \hbar c \alpha \left[\vec{r}_1 \times \vec{p}_1 , \frac{1}{|\vec{r}_1 - \vec{r}_2|} \right] = i \hbar c \alpha \, \vec{r}_1 \times \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|^3} = - i \hbar c \alpha \frac{\vec{r}_1 \times \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|^3}$$

$[L^{(2)}, H] = -i\hbar c\alpha \dfrac{\vec{r}_2\times\vec{r}_1}{|\vec{r}_1-\vec{r}_2|^3} \Rightarrow [\hat{L}, H] = 0$

S、L 都可作 quantum numbers

$|L M_L S M_S\rangle \Rightarrow H|N L M_L S M_S\rangle = E_{NLS}|N L M_L S M_S\rangle$

degenerate $(2L+1)(2S+1)$ $[M_L \times M_S]$

$N\,{}^{2S+1}L_J$

NLS 空间

if $S=0$ we need $|1s\,2p\rangle + |2p\,1s\rangle$

antisym sym

if $S=1$ we need $|1s\,2p\rangle - |2p\,1s\rangle$

H atom

$\psi_{1s}(\vec{r}) = \dfrac{1}{\sqrt{4\pi}} \dfrac{2}{\sqrt{a_0^3}} e^{-\frac{r}{a_0}}$

Y_{00} R_{00}

He atom

$\psi^{Z_{eff}}_{1s^2}(\vec{r}_1,\vec{r}_2) = \langle \vec{r}_1\vec{r}_2|1s^2 Z_{eff}\rangle = \sqrt{\dfrac{Z_{eff}^3}{\pi a_0^3}}\, e^{-\frac{Z_{eff} r_1}{a_0}} \cdot \sqrt{\dfrac{Z_{eff}^3}{\pi a_0^3}}\, e^{-\frac{Z_{eff} r_2}{a_0}}$

$= \dfrac{Z_{eff}^3}{\pi a_0^3} e^{-\frac{Z_{eff}}{a_0}(r_1+r_2)}$ symmetric

$\Updownarrow$ + 上

antisym spin singlet state $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) = [\frac{1}{2}\otimes\frac{1}{2}]_{00}$

normalized

$$\int d\vec{r}_1 d\vec{r}_2 \left| \psi^{Z_{eff}}_{1s^2}(\vec{r}_1, \vec{r}_2) \right|^2 = 1$$

expectation of H

$$\langle 1s^2 Z_{eff} | H | 1s^2 Z_{eff} \rangle = \langle 1s^2 Z_{eff} | \frac{\vec{p}_1^{\,2}}{2m} + \frac{\vec{p}_2^{\,2}}{2m} + \hbar c \alpha \left[-\frac{2}{r_1} - \frac{2}{r_2} + \frac{1}{r_{12}} \right] | 1s^2 Z_{eff} \rangle$$

$Z_{eff} = \frac{27}{16}$ $\quad |E_0| = (\alpha Z_{eff})^2 m_e c^2 = 77.52 eV$

Hydrogen atom $\quad E_{gs} = \frac{1}{2} (\alpha Z_{eff})^2 m_e c^2 = 13.6 eV$

A simple example: the lowest two-electron s-wave states in He

at lease 1 stay in 1s

$$^1S_0 : \begin{cases} |1s\,1s\rangle \\ \frac{1}{\sqrt{2}}(|1s\,2s\rangle + |2s\,1s\rangle) \end{cases} \quad \text{with } S=0 \quad \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) = [\tfrac{1}{2} \otimes \tfrac{1}{2}]_{00}$$

$$^3S_1 : \frac{1}{\sqrt{2}}(|1s2s\rangle - |2s\,1s\rangle) \quad \text{with } S=1 \begin{cases} |\uparrow\uparrow\rangle = [\tfrac{1}{2} \otimes \tfrac{1}{2}]_{11} \\ \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) = [\tfrac{1}{2} \otimes \tfrac{1}{2}]_{10} \\ |\downarrow\downarrow\rangle = [\tfrac{1}{2} \otimes \tfrac{1}{2}]_{1-1} \end{cases}$$

Spatial

Lec. two neutrons

$|p\rangle = |\tau = \frac{1}{2}, m_\tau = \frac{1}{2}\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\quad |n\rangle = |\tau = \frac{1}{2}, m_\tau = -\frac{1}{2}\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

proton isospin-up $\qquad$ neutron isospin-down

$\hat{T}_i = \sum_{j=1}^{A} \hat{\tau}_i(j) \qquad [\hat{T}_i, \hat{T}_j] = i\varepsilon_{ijk}\hat{T}_k \qquad M_T = \sum_{j=1}^{A} m_\tau(j)$

$[\hat{H}_{strong}, \hat{T}_i] = 0$ strong interaction Hamiltonian commute with T_i

$\hat{T}^2 = \hat{T}_1^2 + \hat{T}_2^2 + \hat{T}_3^2$

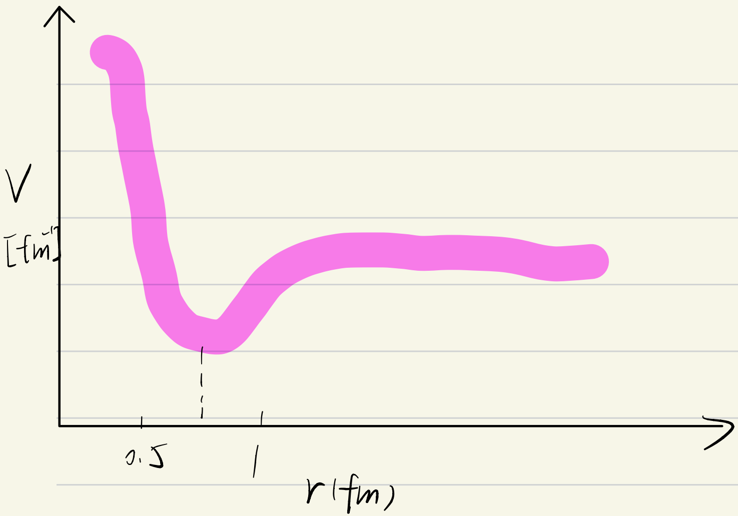
$\hat{T}^2 |\alpha T M_T\rangle = T(T+1)|\alpha T M_T\rangle \qquad \hat{T}_3 |\alpha T M_T\rangle = M_T |\alpha T M_T\rangle \qquad \hat{H}|\alpha T M_T\rangle = E_{\alpha T M_T}|\alpha T M_T\rangle$

$T_\pm = T_x \pm i T_y$

$\hat{T}_+ H_{strong} |\alpha T M_T\rangle = H_{strong} \hat{T}_+ |\alpha T M_T\rangle$

$E_{\alpha T M_T} \sqrt{(T - M_T)(T + M_T + 1)}\, |\alpha T M_T\rangle = E_{\alpha T M_{T+1}} \sqrt{(T - M_T)(T + M_T + 1)}\, |\alpha T M_{T+1}\rangle$

$E_{\alpha T M_T}$ is independent of M_T



four nucleons can reside in the lowest s-state.

Antisymmetric two-nucleon states: two fermions

Isospin sym — $T=1$ — Spin antisym — $S=0$

$$^1S_0 \begin{cases} |pp\rangle \equiv |\uparrow\uparrow\rangle = [\tfrac{1}{2}\otimes\tfrac{1}{2}]_{11} & \\ |pn\rangle_{sym} \equiv \tfrac{1}{\sqrt{2}}|\uparrow\downarrow+\downarrow\uparrow\rangle = [\tfrac{1}{2}\otimes\tfrac{1}{2}]_{10} & \tfrac{1}{\sqrt{2}}|\uparrow\downarrow-\downarrow\uparrow\rangle = [\tfrac{1}{2}\otimes\tfrac{1}{2}]_{00} \\ |nn\rangle \equiv |\downarrow\downarrow\rangle = [\tfrac{1}{2}\otimes\tfrac{1}{2}]_{1-1} & \end{cases}$$

Isospin antisym 核子 $T=0$

Spin sym 电子 $S=1$

$^3S_1$ $|pn\rangle_{antisym} = \frac{1}{\sqrt{2}}|\uparrow\downarrow - \downarrow\uparrow\rangle$

$$\begin{cases} |\uparrow\uparrow\rangle = [\frac{1}{2}\otimes\frac{1}{2}]_{11} \\ \frac{1}{\sqrt{2}}|\uparrow\downarrow + \downarrow\uparrow\rangle = [\frac{1}{2}\otimes\frac{1}{2}]_{10} \\ |\downarrow\downarrow\rangle = [\frac{1}{2}\otimes\frac{1}{2}]_{1-1} \end{cases}$$

$S+T$ must be odd to produce a fermion state that is antisymmetric under exchange.

$Y_{LM} = (-1)^L Y_{LM}$ spatial $\vec{r}_{12} \to -\vec{r}_{12}$

$|spin\rangle = (-1)^{S+1}|spin\rangle$ spin $S=0 \Rightarrow$ odd, $S=1 \Rightarrow$ even

$|Isospin\rangle = (-1)^{T+1}|Isospin\rangle$ Isospin $T=0 \Rightarrow$ odd, $T=1 \Rightarrow$ even

$(-1)^{L+S+T} \Rightarrow = -1 \Rightarrow L+S+T = \text{odd}$ antisymmetry

Two-nucleon states

$|nLM_L; (\frac{1}{2}\frac{1}{2})SM_S; (\frac{1}{2}\frac{1}{2})TM_T\rangle$