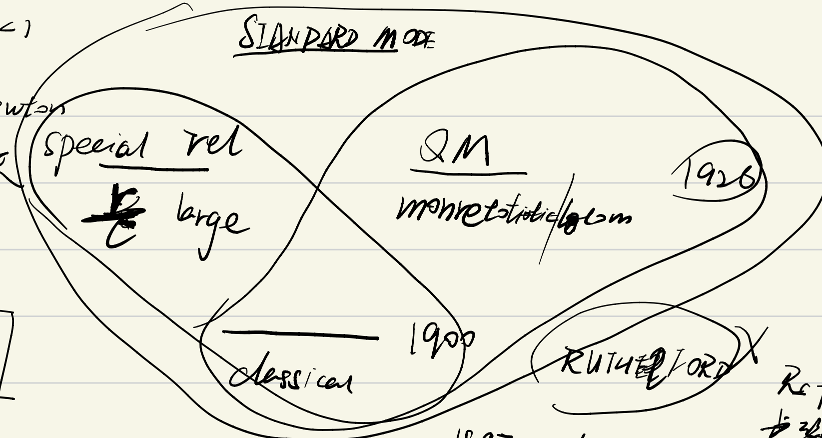
$\frac{\Delta x}{\Delta t} << c \Rightarrow v << c$ 有 $\frac{v u_x}{c^2} << 1$

$(x_1' - x_2') << c(t_1' - t_2')$ 可用Newton

~~速度变换~~

event simutanions

$c^2(t_1-t_2)^2 - (x_1-x_2)^2 \Rightarrow \boxed{\frac{v^2}{c^2} << 1}$

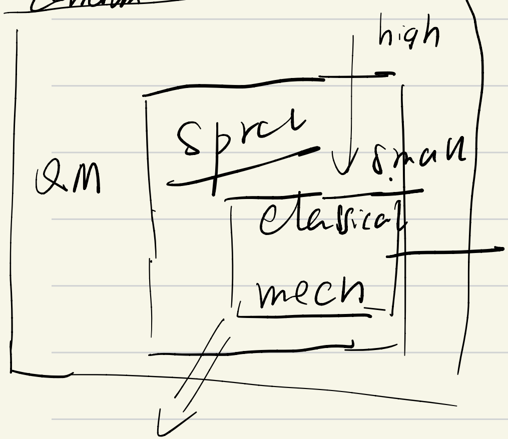


Ref
卢瑟福

1897 electron

atomic X

condition



conclusion ↓ match

$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \approx 1$

$\frac{v^2}{c^2} << 1$

$\Rightarrow$

$\frac{v}{c} << 1$

naturalines

$\Delta E \Delta t >> h$

$\Delta E \Delta x >> hc$ classical

$= 197\ eV \cdot nm$

质子 中

查德威克 Chad ⇒ proton - neu

$p \to e^-$

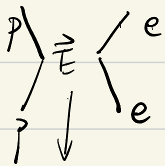
1960<前

1932 Pali

1935

$|1\rangle \quad e^{i\phi_1}$

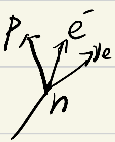
$|0\rangle \quad e^{i\phi_2}$



Ferman

Electric Force

Coulemb



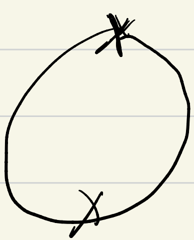
中微子

? V-A

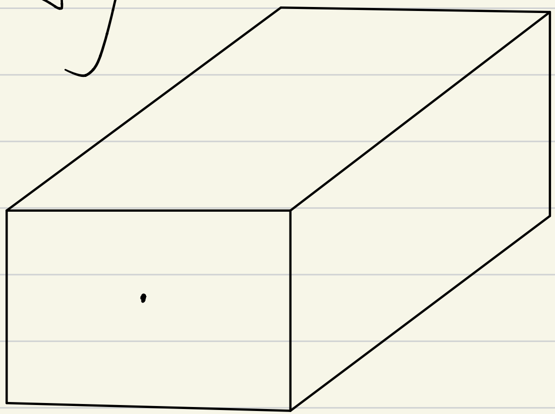
https://
baike.baidu.co
m/item/V-A理
论/22200002

$|1\rangle$

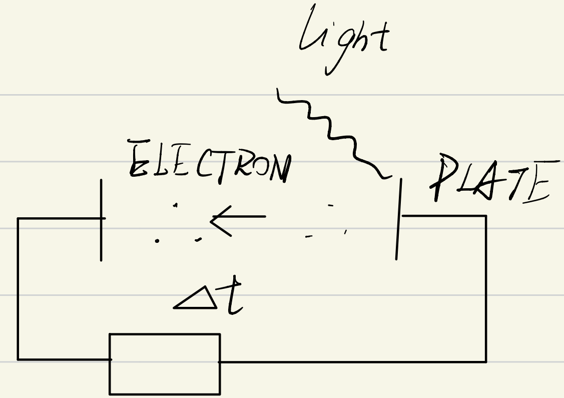
$|0\rangle$



27. Aug



BLACK BODY



proton 质

$$\frac{P}{A} = \sigma T^4$$

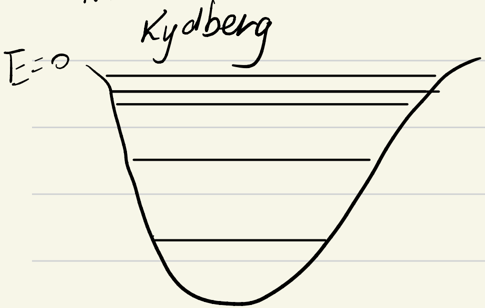
$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} + k^2 E = 0$$

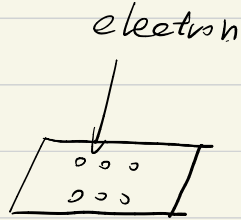
$$E(n_x, n_y, n_z) = N \sin\left(\frac{\pi n_x x}{L}\right) \sin\left(\frac{\pi n_y y}{L}\right) \sin\left(\frac{\pi n_z z}{L}\right)$$

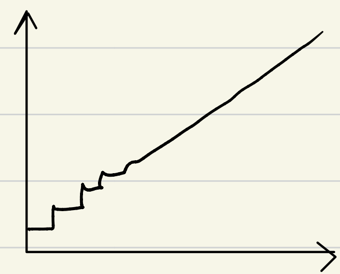
30 Aug.

QM $\supset$ classical phy.

① Bohr $\rightarrow$ large Q Num
$n \to \infty$





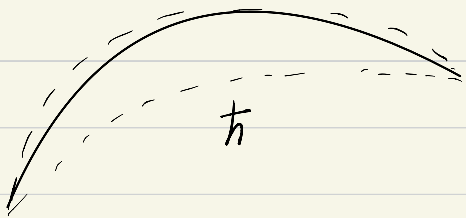


② $h \to 0$ $\Rightarrow$ classical phy.

matter is possibility waves

kenetic energy

$$S = \int_{E_x}^{E_y} [KE - V]\, dt$$



$$e^{\frac{iS}{\hbar}}$$

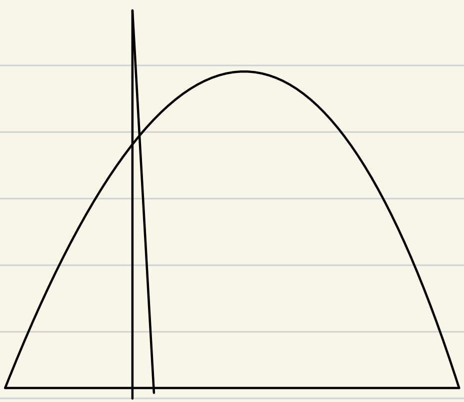
9.1 $\frac{d}{dt}\int_{-\infty}^{+\infty} |\psi|^2 dx$

$= \int (\frac{\partial}{\partial t}\psi^*)\psi + \psi^*(\frac{\partial \psi}{\partial t})\, dx$

$\frac{\partial}{\partial t}\psi = \frac{i\hbar}{2m}\frac{\partial^2 \psi}{\partial x^2} - \frac{i}{\hbar}V\psi$

$= \frac{i\hbar}{2m}\int_{-\infty}^{+\infty} \psi^* \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi^*}{\partial x^2}\psi$

$= \frac{i\hbar}{2m}$



$t \rightarrow t + \Delta t$

$x \rightarrow x + \frac{\delta x}{2}$

$$\frac{d}{dt}\int_{-\infty}^{+\infty} |\psi(x,t)|^2 \, dx = 0$$

$$E = \frac{p^2}{2m} + V$$

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi$$

$$\frac{\partial \psi}{\partial t} = \frac{i\hbar}{2m}\frac{\partial^2 \psi}{\partial x^2} + \frac{V}{i\hbar}\psi$$

$$\frac{\partial \psi^*}{\partial t} = \frac{-i\hbar}{2m}\frac{\partial^2 \psi^*}{\partial x^2} - \frac{V}{i\hbar}\psi^*$$

$$\psi = \Sigma c_i \phi_i e^{-i\frac{E_i t}{\hbar}}$$

$$\int_{-\infty}^{+\infty} \frac{\partial \psi^*}{\partial t}\psi + \psi^* \frac{\partial \psi}{\partial t} \, dx$$

$$= \int_{-\infty}^{+\infty} \left(\frac{-i\hbar}{2m}\frac{\partial^2 \psi^*}{\partial x^2} - \frac{V}{i\hbar}\psi^*\right)\psi + \psi^*\left(\frac{i\hbar}{2m}\frac{\partial^2 \psi}{\partial x^2} + \frac{V}{i\hbar}\psi\right) dx$$

$$= \int_{-\infty}^{+\infty} \frac{-i\hbar}{2m}\frac{\partial^2 \psi^*}{\partial x^2}\psi + \frac{i\hbar}{2m}\psi^*\frac{\partial^2 \psi}{\partial x^2} \, dx$$

$$= \frac{i\hbar}{2m}\int \psi^*\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi^*}{\partial x^2}\psi$$

$$= \underbrace{\psi^*\frac{\partial \psi}{\partial x}\Big|_{-\infty}^{+\infty}}_{=0} - \int \frac{\partial \psi^*}{\partial x}\frac{\partial \psi}{\partial x} dx - \frac{\partial \psi^*}{\partial x}\psi\Big|_{-\infty}^{+\infty} \int \frac{\partial^2 \psi^*}{\partial x^2}\psi \, dx = 0$$

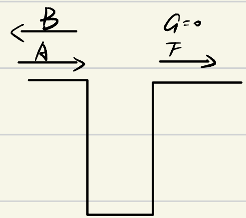
$$+ -$$

9.10 1. download lecture

2. listen, but not take notes

9.7

$$\frac{-\hbar^2}{2m}\phi'' = E\phi \qquad E>0.$$



$$\phi'' = -\frac{2mE}{\hbar^2}\phi = -k^2\phi$$

$$\begin{cases} \text{I} & Ae^{ikx} + Be^{-ikx} \\ \text{II} & Ce^{ilx} + De^{-ilx} \qquad l = \frac{\sqrt{2m(V_0+E)}}{\hbar} \\ \text{III} & Fe^{ikx} \end{cases}$$

boundary cd. ①

9.29

$$\int \psi^* \psi \, dr = \vec{v}_1 \cdot \vec{v}_2$$

$$\langle \psi_j | \hat{o} | \psi_i \rangle = (\quad)$$

bra (c) ket

$$\vec{v} = v_x \hat{x} + v_y \hat{y} + v_z \hat{z}$$

$\hat{x}, \hat{y}, \hat{z}$

$$(\hat{e}_{+1}, \hat{e}_0, \hat{e}_{-1})$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$-\frac{1}{\sqrt{2}}(\hat{x} + i\hat{y}) \quad \hat{e}_z \quad +\frac{1}{\sqrt{2}}(\hat{x} - i\hat{y})$$

$$\vec{v} = v_{+1}\hat{e}_{+1} + v_0 \hat{e}_0 + v_{-1}\hat{e}_{-1}$$

$$\{v_{+1}, v_0, v_{-1}\} = \left\{-\frac{1}{\sqrt{2}}(v_x - i v_y), v_z, \frac{1}{\sqrt{2}}(v_x + i v_y)\right\}$$

① $|\psi\rangle \to |\psi\rangle = \sum_i |i\rangle\langle i|\psi\rangle = \sum_i c_i |i\rangle$

$$\sum_i |i\rangle\langle i| = 1$$

② $\underset{\text{vector}}{|\psi\rangle} = \int dx \underset{\text{vector}}{|x\rangle} \underset{\text{coefficient}}{\langle x|\psi\rangle}$ Coordinate space $\hat{x}^2 \psi(x) \to x^2 \psi(x)$

$$\hat{x}^2 |\psi\rangle = \hat{x}^2 \int dx \, |x\rangle\langle x|\psi\rangle$$

$$\downarrow$$

$$x^2 \int dx \, |x\rangle \underbrace{\langle x|\psi\rangle}_{\text{wave func}}$$

$$\langle x | \hat{x}^2 | \psi\rangle = \langle x | E | \psi\rangle = E\psi(x) \Rightarrow \boxed{\langle x|\psi\rangle = \psi(x)}$$

$$\int dx' \langle x | \hat{x}^2 | x'\rangle \langle x'|\psi\rangle = x^2 \psi(x)$$

x'^2

$$x^2 \delta(x - x') \langle x'|\psi\rangle \qquad \boxed{\langle x|x'\rangle = \delta(x-x')}$$

$$\int dx' \, x^2 \delta(x-x') \langle x'|\psi\rangle = x^2\psi(x)$$

③ $|\psi\rangle = \int |p\rangle\langle p|\psi\rangle \, dp$

$$(\hat{p}^2 + \hat{x}^2)$$

$\sum |i\rangle\langle i| = 1 \rightarrow \int dx\, |x\rangle\langle x| = 1$

$\int dp\, |p\rangle\langle p| = 1$

orthonormality $\delta_{\alpha\beta} = \langle \phi_\alpha | \phi_\beta \rangle$

$= \int dx\, \langle \phi_\alpha | x \rangle \langle x | \phi_\beta \rangle$ $\quad \langle \phi_\alpha | x \rangle = \phi_\alpha^*(x) = \langle x | \phi_\alpha \rangle^*$

$= \int dx\, \phi_\alpha^*(x)\, \phi_\beta(x)$

10.1

Summary

① Dirac's ket $|\alpha\rangle$ $\quad \sum_i |i\rangle\langle i| = 1$ complete set

$|\alpha\rangle$ represented $\quad |\alpha\rangle = \sum_i |i\rangle\langle i|\alpha\rangle = \sum_i |i\rangle \alpha_i$

② $\sum_i |i\rangle\langle i| \Rightarrow \int dx\, |x\rangle\langle x| = 1$

Contintion $\quad |\alpha\rangle = \int dx\, |x\rangle\langle x|\alpha\rangle dx = \int |x\rangle \alpha_x\, dx$

③ adjoint 伴随 $\quad \langle\alpha| = \sum_{i=1}^{N} \langle\alpha|i\rangle\langle i| = \sum_{i=1}^{N} \alpha_i^* \langle i|$ $\quad \langle\alpha|i\rangle = \langle i|\alpha\rangle^*$

Continuous $\quad \langle\alpha| \Rightarrow \int dx\, \langle a|x\rangle\langle x| = \int dx\, \langle x|\alpha\rangle^* \langle x| = \int dx\, \phi_\alpha^*(x)\, \langle x|$

④ inner product $\quad \langle\beta|\alpha\rangle = \sum_{i=1}^{N} \langle\beta|i\rangle\langle i|\alpha\rangle = \sum_{i=1}^{N} \langle i|\beta\rangle^* \langle i|\alpha\rangle = \sum_{i=1}^{N} b_i^* a_i$

$\langle\beta|\alpha\rangle \Rightarrow \int dx\, \langle\beta|x\rangle\langle x|\alpha\rangle = \int dx\, \langle x|\beta\rangle^* \langle x|\alpha\rangle = \int \psi_\beta^*(x)\, \psi_\alpha(x)\, dx$

$|\alpha\rangle, |\beta\rangle$, orthonormal set (e.g. stationary state)

$\delta_{\alpha\beta} = \langle\beta|\alpha\rangle = \sum_{i=1}^{N} \langle\beta|i\rangle\langle i|\alpha\rangle = \sum_{i=1}^{N} b_i^* a_i$

$\delta_{\alpha\beta} \to \int dx\ \phi_\beta^*(x)\ \phi_\alpha(x)$

Hilbert space properties

① basis states are normalizable

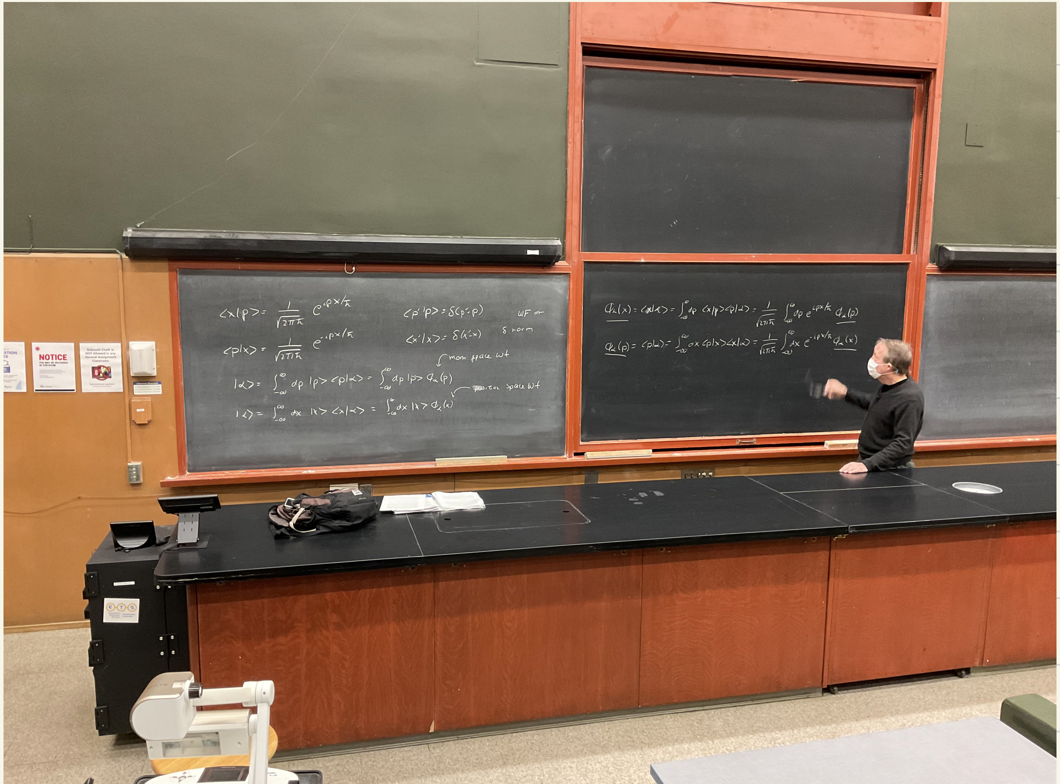
$\langle\psi|\psi\rangle = 1 = \int dx\ \psi^*(x)\psi(x) = 1$

②

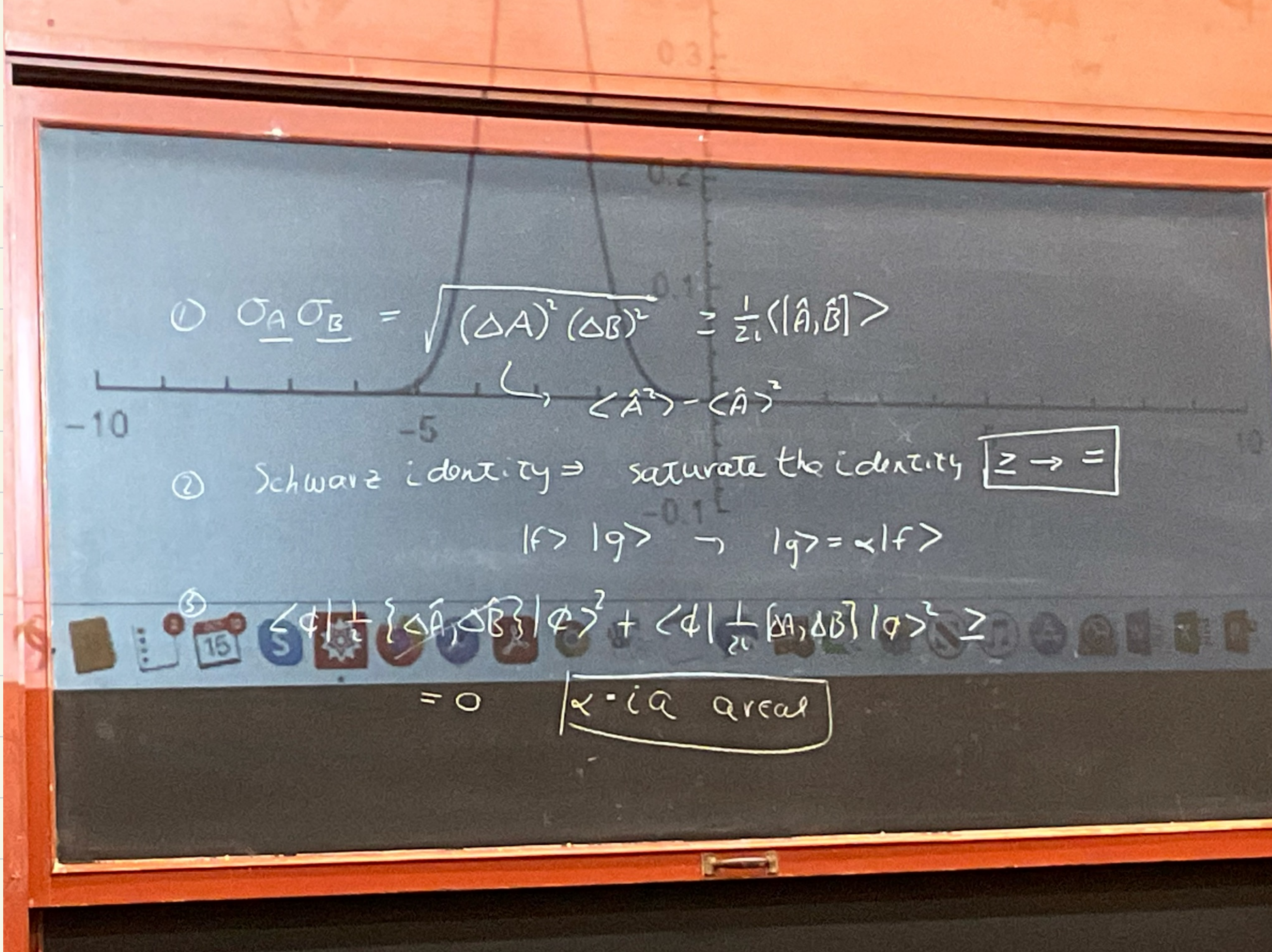
$\langle\phi_j|\phi_i\rangle = \langle\phi_i|\phi_j\rangle^* =$

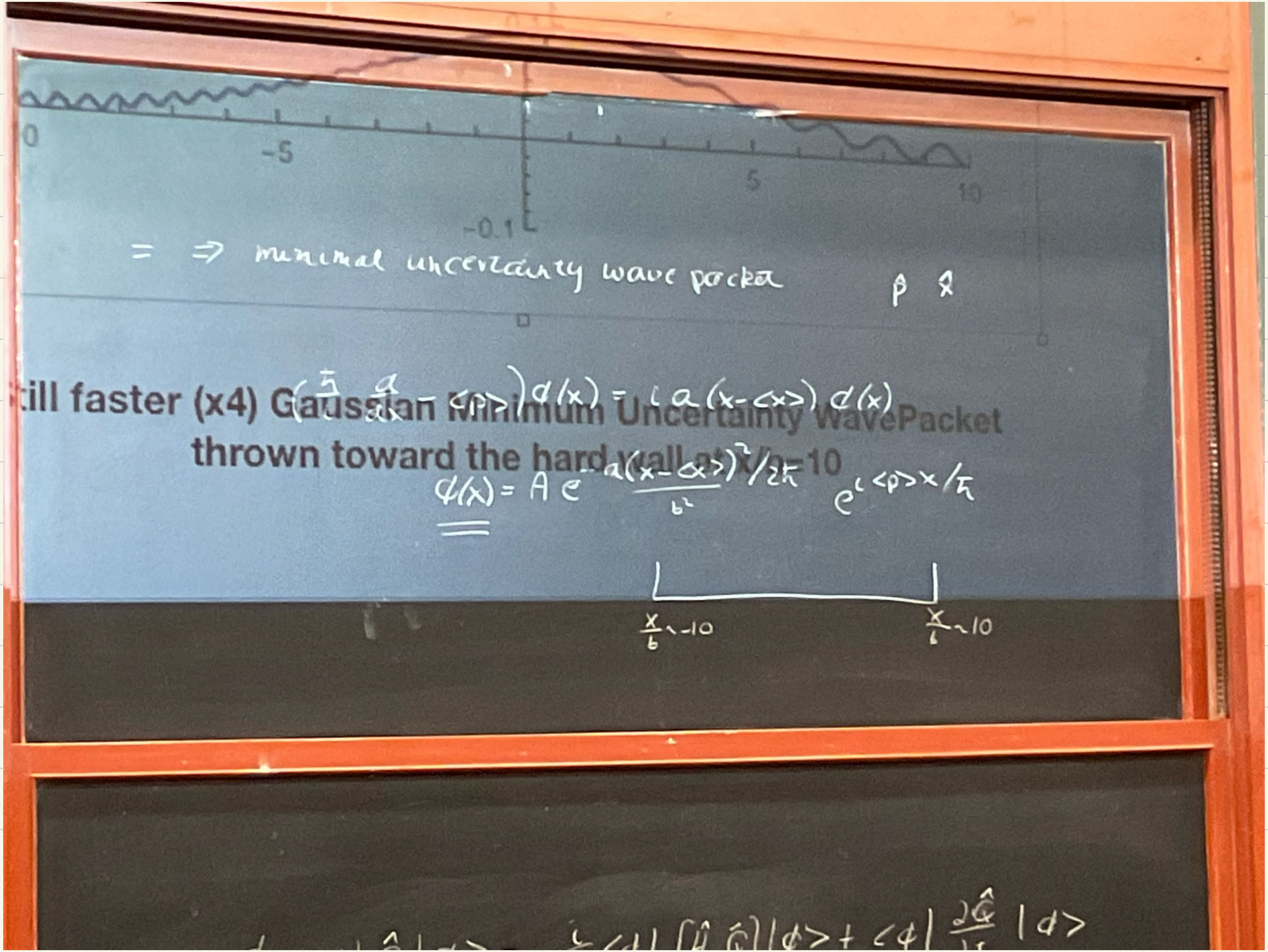
③ etc...

10.4


$\langle x|p\rangle = \frac{1}{\sqrt{2\pi\hbar}} e^{ipx/\hbar}$
$\langle p|x\rangle = \frac{1}{\sqrt{2\pi\hbar}} e^{-ipx/\hbar}$
$\langle p'|p\rangle = \delta(p'-p)$
$\langle x'|x\rangle = \delta(x'-x)$
WF or
δ norm
mom space wf
$|\alpha\rangle = \int_{-\infty}^{\infty} dp\, |p\rangle\langle p|\alpha\rangle = \int_{-\infty}^{\infty} dp\, |p\rangle\, \phi_\alpha(p)$
position space wf
$|\alpha\rangle = \int_{-\infty}^{\infty} dx\, |x\rangle\langle x|\alpha\rangle = \int_{-\infty}^{\infty} dx\, |x\rangle\, \phi_\alpha(x)$
$\phi_\alpha(x) = \langle x|\alpha\rangle = \int_{-\infty}^{\infty} dp\, \langle x|p\rangle\langle p|\alpha\rangle = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} dp\, e^{ipx/\hbar}\, \phi_\alpha(p)$
$\phi_\alpha(p) = \langle p|\alpha\rangle = \int_{-\infty}^{\infty} dx\, \langle p|x\rangle\langle x|\alpha\rangle = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} dx\, e^{-ipx/\hbar}\, \phi_\alpha(x)$
NOTICE

10.15


① $\sigma_A \sigma_B = \sqrt{(\Delta A)^2 (\Delta B)^2} \geq \frac{1}{2i}\langle [\hat{A}, \hat{B}] \rangle$
$\hookrightarrow \langle \hat{A}^2 \rangle - \langle \hat{A} \rangle^2$
② Schwarz identity ⇒ saturate the identity $\boxed{\geq \to =}$
$|f\rangle \; |g\rangle \to |g\rangle = \alpha |f\rangle$
③ $\langle \phi | \frac{1}{2}\{\Delta\hat{A}, \Delta\hat{B}\} | \phi \rangle^2 + \langle \phi | \frac{1}{2i}[\Delta A, \Delta B] | \phi \rangle^2 \geq$
$= 0$
$\boxed{\alpha = ia \quad a \text{ real}}$


-5
5
10
-0.1
= ⇒ minimal uncertainty wave packet
p x
till faster (x4) Gaussian Minimum Uncertainty WavePacket
thrown toward the hard wall at X/b=10
$\left(\frac{\hbar}{i}\frac{d}{dx} - \langle p\rangle\right)\phi(x) = i a (x-\langle x\rangle)\phi(x)$
$\phi(x) = A e^{-a(x-\langle x\rangle)^2/2\hbar b^2} e^{i\langle p\rangle x/\hbar}$
$\frac{x}{b} \sim -10$
$\frac{x}{b} \sim 10$

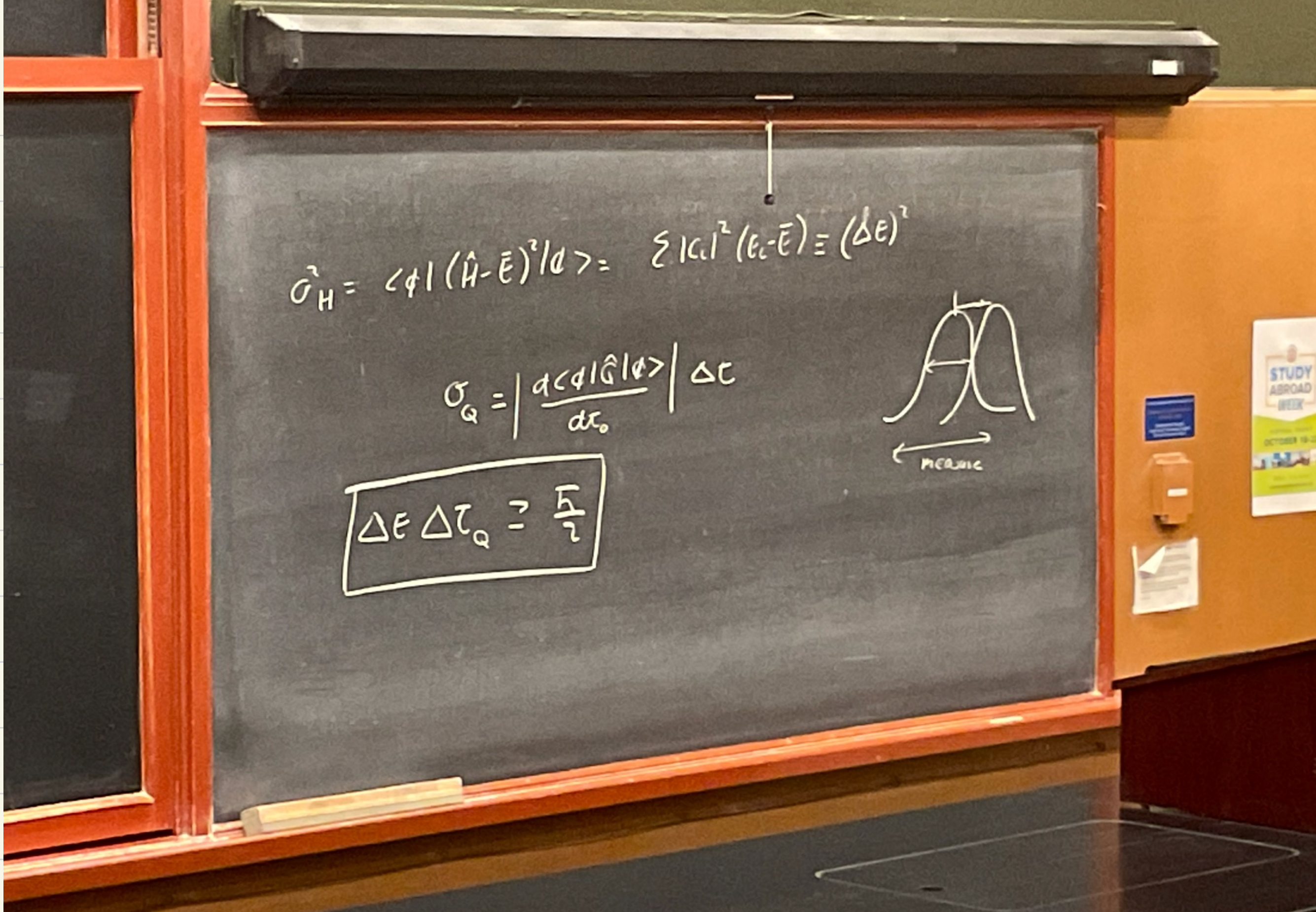
$\frac{x}{b} \sim -10$ $\qquad$ $\frac{x}{b} \sim 10$

$$\frac{d}{dt}\langle\psi|\hat{Q}|\psi\rangle = \underbrace{\frac{i}{\hbar}\langle\psi|[\hat{H},\hat{Q}]|\psi\rangle} + \cancel{\langle\psi|\frac{\partial\hat{Q}}{\partial t}|\psi\rangle}$$

$$\sigma_H^2\,\sigma_Q^2 \geq \langle\psi|\frac{1}{2i}[H,Q]|\psi\rangle^2$$

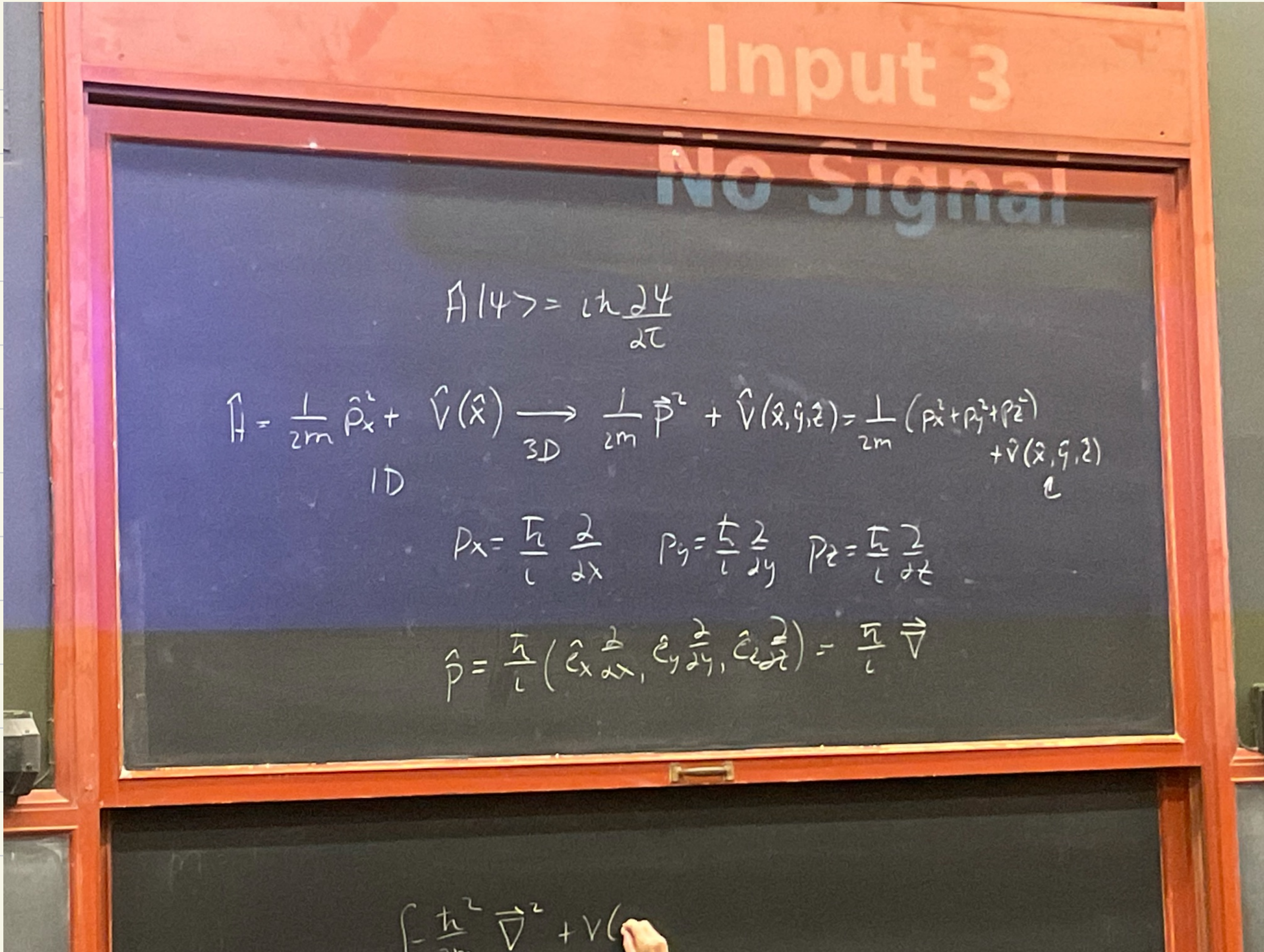
$$\sigma_H\,\sigma_Q \geq \frac{\hbar}{2}\left|\frac{d}{dt}\langle\psi|\hat{Q}|\psi\rangle\right|^2$$

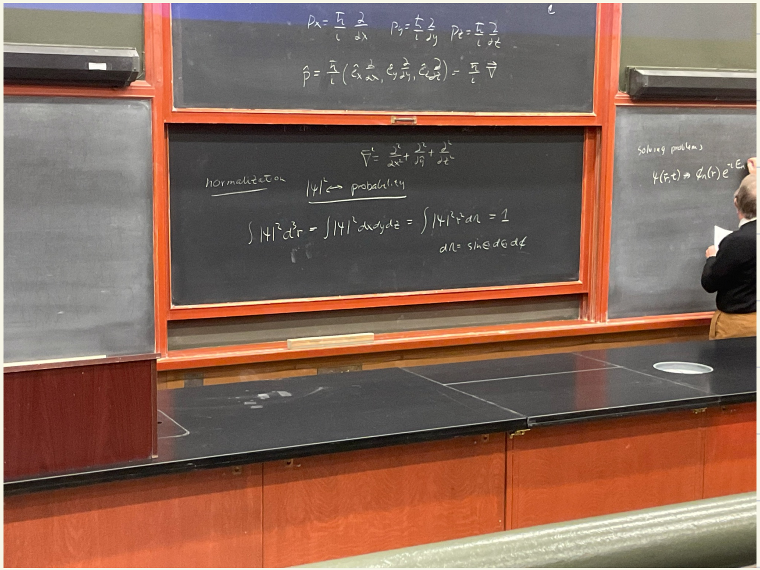
$$\sigma_H\left(\sigma_Q\,\frac{1}{\left|\frac{d}{dt}\langle\psi|\hat{Q}|\psi\rangle\right|}\right) \geq \frac{\hbar}{2}$$

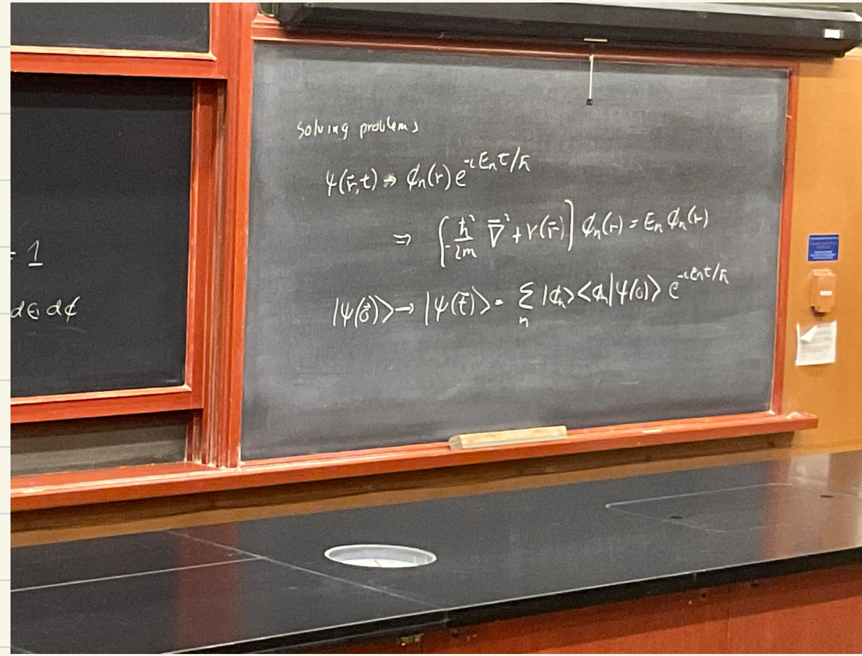


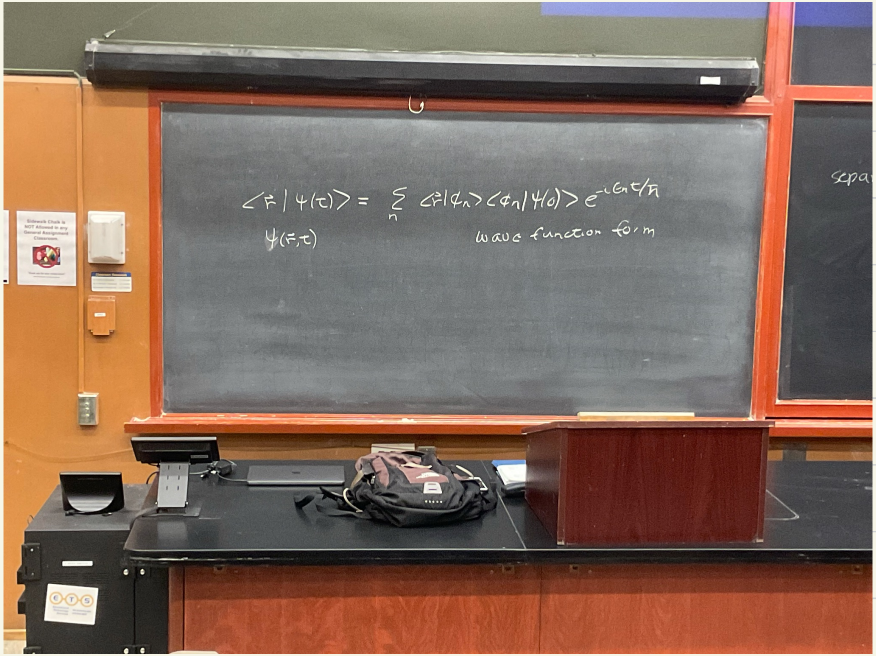
$\sigma_H^2 = \langle \phi | (\hat{H} - \bar{E})^2 | \phi \rangle = \sum |c_i|^2 (E_i - \bar{E}) \equiv (\Delta E)^2$
$\sigma_Q = \left| \frac{d\langle \phi | \hat{Q} | \phi \rangle}{dt_0} \right| \Delta t$
$\boxed{\Delta E \, \Delta \tau_Q \geq \frac{\hbar}{2}}$

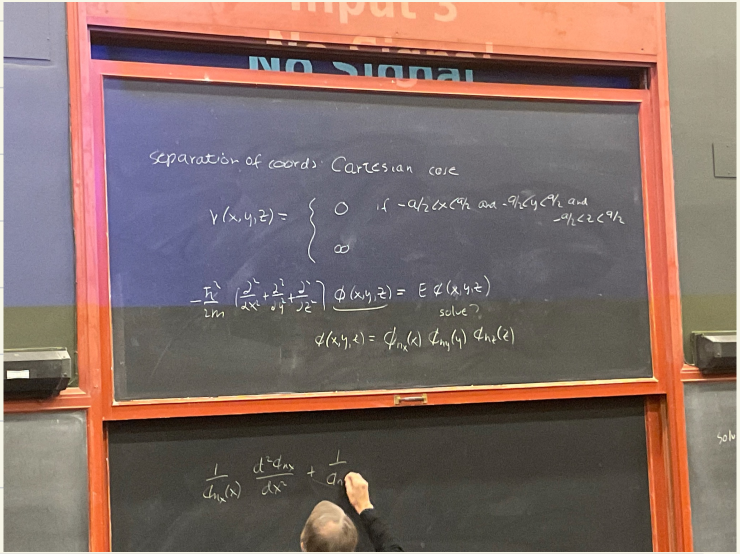
10.18

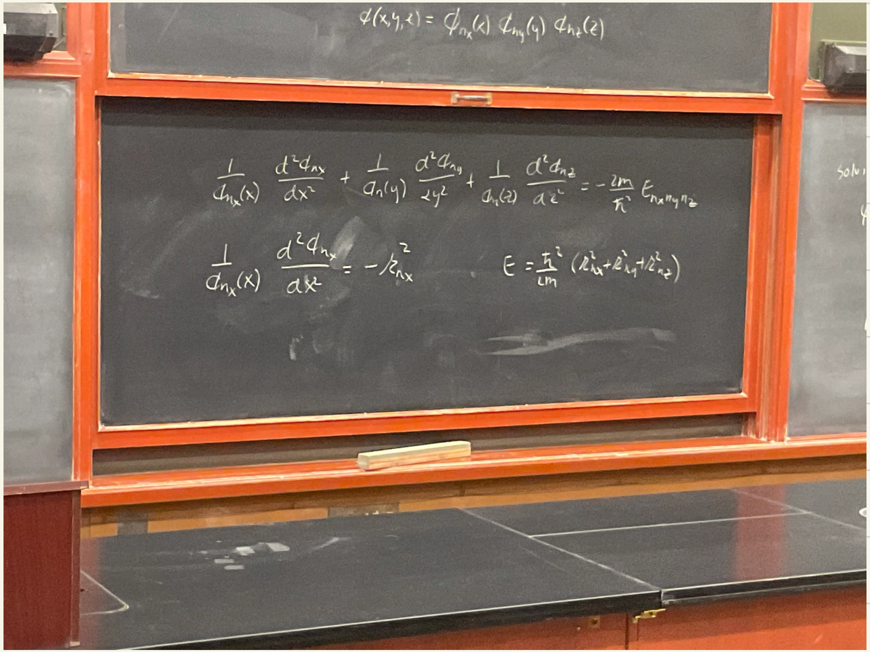

Input 3
No Signal
$\hat{H}|\psi\rangle = i\hbar \frac{\partial \psi}{\partial t}$
$\hat{H} = \frac{1}{2m}\hat{p}_x^2 + \hat{V}(\hat{x}) \xrightarrow[3D]{} \frac{1}{2m}\vec{p}^2 + \hat{V}(\hat{x},\hat{y},\hat{z}) = \frac{1}{2m}(p_x^2+p_y^2+p_z^2) + \hat{V}(\hat{x},\hat{y},\hat{z},t)$
1D
$p_x = \frac{\hbar}{i}\frac{\partial}{\partial x}$ $p_y = \frac{\hbar}{i}\frac{\partial}{\partial y}$ $p_z = \frac{\hbar}{i}\frac{\partial}{\partial z}$
$\hat{p} = \frac{\hbar}{i}\left(\hat{e}_x \frac{\partial}{\partial x}, \hat{e}_y \frac{\partial}{\partial y}, \hat{e}_z \frac{\partial}{\partial z}\right) = \frac{\hbar}{i}\vec{\nabla}$
$[-\frac{\hbar^2}{2m}\vec{\nabla}^2 + V($


$p_x = \frac{\hbar}{i}\frac{\partial}{\partial x}$ $p_y = \frac{\hbar}{i}\frac{\partial}{\partial y}$ $p_z = \frac{\hbar}{i}\frac{\partial}{\partial z}$
$\hat{p} = \frac{\hbar}{i}\left(\hat{e}_x\frac{\partial}{\partial x}, \hat{e}_y\frac{\partial}{\partial y}, \hat{e}_z\frac{\partial}{\partial z}\right) = \frac{\hbar}{i}\vec{\nabla}$
$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$
normalization
$|\psi|^2 \leftrightarrow$ probability
$\int |\psi|^2 d^3r = \int |\psi|^2 dx\,dy\,dz = \int |\psi|^2 r^2 dr\, d\Omega = 1$
$d\Omega = \sin\theta\, d\theta\, d\phi$
Solving problems
$\psi(\vec{r},t) \Rightarrow \phi_n(r)e^{-iE_n}$

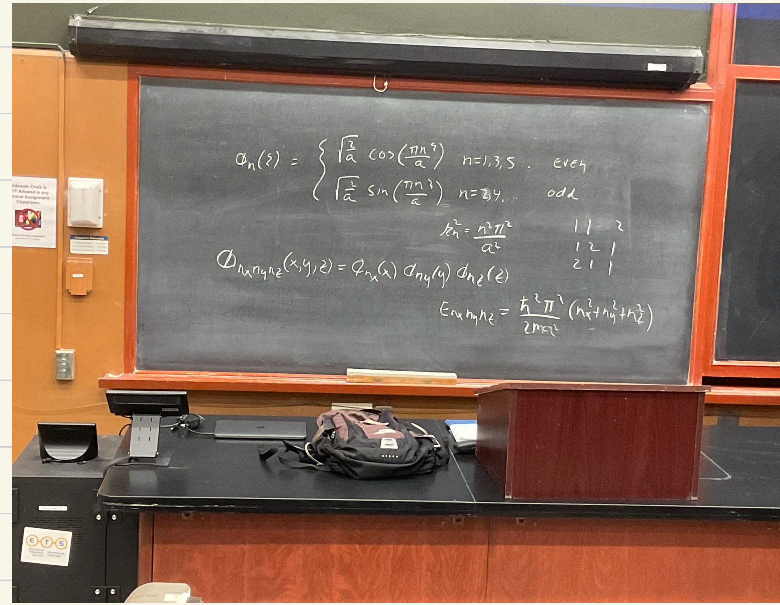

Solving problems
$\psi(\vec{r},t) \Rightarrow \phi_n(r)e^{-iE_nt/\hbar}$
$\Rightarrow \left[-\frac{\hbar^2}{2m}\vec{\nabla}^2 + V(\vec{r})\right]\phi_n(r) = E_n\phi_n(r)$
$|\psi(0)\rangle \rightarrow |\psi(t)\rangle = \sum_n |\phi_n\rangle\langle\phi_n|\psi(0)\rangle e^{-iE_nt/\hbar}$


$\langle \vec{r} | \psi(t) \rangle = \sum_n \langle \vec{r} | \phi_n \rangle \langle \phi_n | \psi(0) \rangle e^{-iE_n t/\hbar}$
$\psi(\vec{r},t)$
wave function form

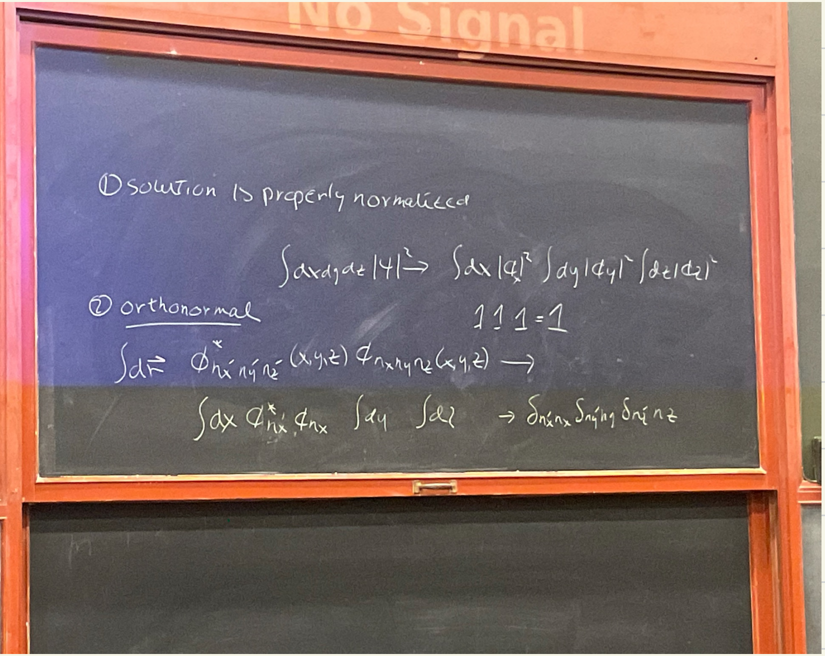

separation of coords: Cartesian case
$V(x,y,z) = \begin{cases} 0 & \text{if } -a/2<x<a/2 \text{ and } -a/2<y<a/2 \text{ and } -a/2<z<a/2 \\ \infty & \end{cases}$
$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\phi(x,y,z) = E\,\phi(x,y,z)$
solve?
$\phi(x,y,z) = \phi_{n_x}(x)\,\phi_{n_y}(y)\,\phi_{n_z}(z)$
$\frac{1}{\phi_{n_x}(x)}\frac{d^2\phi_{n_x}}{dx^2} + \frac{1}{\phi_n}$

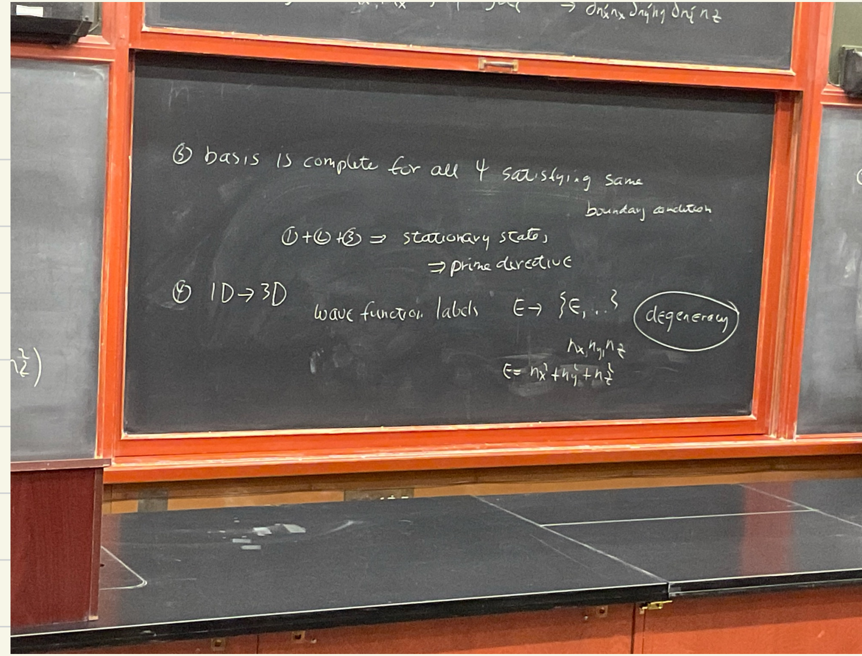


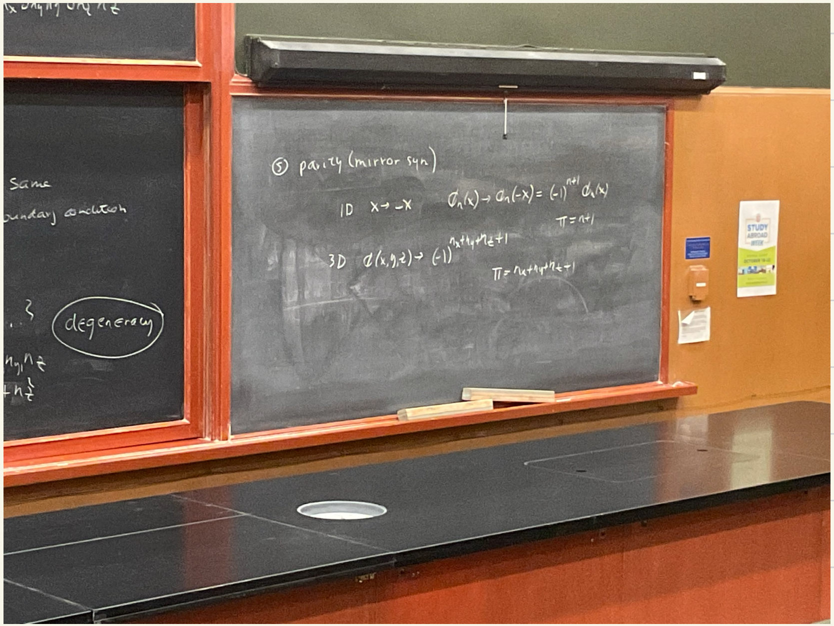
$\phi(x,y,z) = \phi_{n_x}(x)\, \phi_{n_y}(y)\, \phi_{n_z}(z)$
$\frac{1}{\phi_{n_x}(x)} \frac{d^2\phi_{n_x}}{dx^2} + \frac{1}{\phi_n(y)} \frac{d^2\phi_{n_y}}{dy^2} + \frac{1}{\phi_n(z)} \frac{d^2\phi_{n_z}}{dz^2} = -\frac{2m}{\hbar^2} E_{n_x n_y n_z}$
$\frac{1}{\phi_{n_x}(x)} \frac{d^2\phi_{n_x}}{dx^2} = -k_{n_x}^2$
$E = \frac{\hbar^2}{2m}(k_{n_x}^2 + k_{n_y}^2 + k_{n_z}^2)$



$\phi_n(\xi) = \begin{cases} \sqrt{\frac{2}{a}} \cos(\frac{\pi n \xi}{a}) & n=1,3,5.. \text{ even} \\ \sqrt{\frac{2}{a}} \sin(\frac{\pi n \xi}{a}) & n=2,4,.. \text{ odd} \end{cases}$
$k_n^2 = \frac{n^2\pi^2}{a^2}$
1 1 2
1 2 1
2 1 1
$\Phi_{n_x n_y n_z}(x,y,z) = \phi_{n_x}(x)\, \phi_{n_y}(y)\, \phi_{n_z}(z)$
$E_{n_x n_y n_z} = \frac{\hbar^2\pi^2}{2ma^2}(n_x^2 + n_y^2 + n_z^2)$


No Signal
① solution is properly normalized
$\int dx\,dy\,dz\,|\psi|^2 \rightarrow \int dx\,|\phi_x|^2 \int dy\,|\phi_y|^2 \int dz\,|\phi_z|^2$
② orthonormal
$1\ 1\ 1 = 1$
$\int d\vec{r}\ \phi^*_{n_x' n_y' n_z'}(x,y,z)\ \phi_{n_x n_y n_z}(x,y,z) \longrightarrow$
$\int dx\ \phi^*_{n_x'}\phi_{n_x}\ \int dy\ \int dz\ \rightarrow \delta_{n_x' n_x}\delta_{n_y' n_y}\delta_{n_z' n_z}$


$\rightarrow \delta_{n_x' n_x}\delta_{n_y' n_y}\delta_{n_z' n_z}$
③ basis is complete for all ψ satisfying same
boundary condition
①+②+③ ⇒ stationary states
⇒ prime directive
④ 1D→3D
wave function labels
$E \rightarrow \{E, \ldots\}$
n_x, n_y, n_z
$E = n_x^2 + n_y^2 + n_z^2$
degeneracy
⑤



same
oundary condition
degeneracy
⑤ parity (mirror sym)
1D $x \to -x$ $\phi_n(x) \to \phi_n(-x) = (-1)^{n+1}\phi_n(x)$
$\pi = n+1$
3D $\phi(x,y,z) \to (-1)^{n_x+n_y+n_z+1}$
$\pi = n_x+n_y+n_z+1$
STUDY ABROAD WEEK

10.20

R part = $\ell(\ell+1)$

Y part = $-\ell(\ell+1)$
$$\begin{cases} \theta \text{ part} = m^2 \\ \phi \text{ part} = -m^2 \end{cases}$$

$\frac{1}{\phi}\phi'' = -m^2 \Rightarrow \phi(\phi) = e^{im\phi}$

$\Phi(0) = \phi(2\pi) \Rightarrow \quad e^{im0} = 1 \quad e^{im \cdot 2\pi} = 1$

sphere ~~ϕ~~

$\Rightarrow m \cdot 2\pi = 2k\pi$

$\Rightarrow m$ intorger $\quad m = 0, \pm 1, \pm 2$

$\epsilon_{xyz} = 1$

$\epsilon_{yxz} = -1$

$\epsilon_{xxz} = 0$

10.22

Commuting operators