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Mathematical models for ship path prediction in manoeuvring simulation systems

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Abstract

The problem of simulating the ship manoeuvring motion is studied mainly in connection with manoeuvring simulators. Several possible levels of solution to the problem with different degrees of complexity and accuracy are discussed. It is shown that the structure of the generic manoeuvring mathematical model leads naturally to two basic approaches based respectively on dynamic and purely kinematic prediction models. A simplified but fast dynamic manoeuvring model is proposed as well as two new advances in kinematic prediction methods: a prediction based on current values of velocities and accelerations and a method of anticipating the ship's trajectory in a course changing manoeuvre. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

The knowledge of the manoeuvring characteristics of a ship allows time simulations of her path as a function of her control settings. This feature has been explored in many training simulators, which incorporate the manoeuvring models of several ships and simulate realistic navigation situations, so as to train shipmasters.

Full mission simulators replicate the bridge of a ship and can provide full 360° vision of the outside environment as viewed from the ship. Some installations also simulate the motions that are felt in the bridge. These simulators aim at being as

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realistic as possible and thus they normally use the full manoeuvring equations for the simulation. Different approaches can be adopted to determine the coefficients in the full equations of motion of a ship on the horizontal plan (Crane et al., 1989).

There has been a significant effort recently to develop the Vessel Traffic Management Information Systems (VTMIS), which extend the capabilities of the present Vessel Traffic Systems (VTS) by incorporating more information that can be of interest to different actors involved in the operations of maritime transportation (Degré and Guedes Soares, 1998). These systems aim at keeping the traffic information over large areas of operation and also in maintaining a certain amount of information about each ship, which may be of interest to it or to any neighbouring ship.

Typically these systems follow the tracks of the ships and in congested areas, they detect potentially dangerous situations including ships in collision course and provide alerts for those situations. This will be a backup system for the onboard tracking of the neighbouring situation and thus provides an extra safety.

To decide on the potential for collision, in addition to the distances and courses of the ships it is useful to have information on their manoeuvring capabilities to assess how they can avoid collision. It is impractical in such a system to operate with detailed manoeuvring models of the many ships they will deal with and thus a simplified model like the one developed in this paper is useful.

Another application of a simplified manoeuvring model are on-board decision support systems that will show the master the estimated track of the ship for the choice of machinery and rudder commands made. This can be a training tool to test some manoeuvring strategies at approaches and entrances to ports that the ship may be aiming at.

These systems have a more stringent requirement on the response time and a less pronounced one on the accuracy. Therefore, with these types of applications in mind, this paper presents simplified manoeuvring models that provide a reasonable accuracy at the same time as they are computationally very fast.

The simplified mathematical models used for prediction purposes can be *dynamic* or *kinematic*. To clarify these concepts consider an abstract generic ship manoeuvring model. In the typical case of an unrestricted water area and in the absence of wind and current such a model can be represented in the following way:

$$\dot{\mathbf{X}}=\mathbf{F}_X(\mathbf{X},\mathbf{U},\mathbf{D}) \tag{1a}$$

$$\dot{\mathbf{Y}}=\mathbf{F}_Y(\mathbf{X},\mathbf{Y}), \tag{1b}$$

where $\mathbf{X}$ is the vector of velocities describing the instantaneous ship motion (more detailed definitions of all the vectors will be given later), $\mathbf{U}$ is the vector of controls which includes, first of all, the rudder order and the desired rpm, $\mathbf{D}$ is the vector of disturbances (usually neglected in dynamic prediction problems), $\mathbf{Y}$ is the vector of general coordinates describing the instantaneous ship's position and orientation, and $\mathbf{F}_x()$ and $\mathbf{F}_y()$ are functions defining respectively the dynamic and the kinematic part of the model.

The ship can also be treated as a closed-loop system when the control vector is continuously computed as a function of the coordinates and velocities:

$$\mathbf{U}=\mathbf{F}_U(\mathbf{X},\mathbf{Y},\mathbf{X}^*,\mathbf{Y}^*), \tag{2}$$

where $\mathbf{X}^*$ and $\mathbf{Y}^*$ are the vectors of ordered (desired) values of ship velocities and co-ordinates. The specific representation of the function $\mathbf{F}_U()$ depends on the particular human operator or auto-pilot.

The structure of the generic manoeuvring mathematical model suggests two sets of path prediction options. One is a prediction based on a relatively full dynamic mathematical model or only on its kinematic part, which is possible because the hydrodynamic forces do not depend on ship general coordinates. Another possibility is to make predictions at constant values of control actions. These actions are be unpredictable in principle but if the control law $\mathbf{U}=\mathbf{F}_U(\mathbf{X},\mathbf{Y},\mathbf{X}^*,\mathbf{Y}^*)$ is well defined, one can also predict a ship's trajectory before the actual manoeuvres starts, simulating also the order to execute.

Of course, using the same dynamic mathematical model for both actual simulation and path prediction is the most straightforward and accurate solution but it is also ineffective.

It is clear that the faster is the ship mathematical model used for simulations, the more powerful can be the ability of a simulation program to play manoeuvres faster than in real time scale. This is especially important for dynamic prediction purposes. However, generally speaking, the more accurate is a manoeuvring mathematical model, the more it is cumbersome and slower in computations. This means, that the dynamic prediction must be based on some specially designed mathematical models in which the accuracy is sacrificed in favour of speed.

That is why a simplified but realistic dynamic mathematical model aiming mainly at the fastest simulation was developed and is described below. Extensions of the usual kinematic prediction method improving its accuracy and flexibility are also presented.

2. Simplified dynamic mathematical model for fast simulation of ship manoeuvres

2.1. Preliminaries

Most modern manoeuvring mathematical models used in various simulators are built according to the so-called modular principle i.e. they include separate sub-models for the hull, propeller, rudder, engine and steering gear (e.g. Inoue et al., 1981a; Chislett, 1996). Every sub-model usually contains numerous regressions and empirical formulae for description of various components of forces and moments. All this adds some flexibility to the models and helps making them more accurate but at the same time reduces their computational speed.

The other end of the range of dynamic manoeuvring models are those based on the so-called Nomoto equations (Crane et al., 1989). The first-order Nomoto equation is especially suitable for predictors and collision–avoidance systems, and can also be used as reference model in adaptive auto-pilots. But to predict a ship's trajectory, the Nomoto equations must be supplemented with some additional ones including those for surge and drift motion. The resulting model loses its simplicity and becomes less attractive as its complexity becomes comparable to that of more consistent models.

So, in developing the fast dynamic model described here it was considered more reasonable to simplify the most sophisticated ones by eliminating all less important features and effects. The following issues were considered:

1. Although the accompanying roll motion can sensitively affect manoeuvring, this effect is in most cases weak and can be easily sacrificed. Thus, the equation of roll was omitted.
2. The engine-and-propeller inertia is much lower than that of the ship itself and it can be supposed that the propeller rotation rate n can be changed instantly. This simplification, although justified in the present context, wouldn't be appropriate for a simulator as it gives the operator a feeling of a wrong reaction.
3. An integrated polynomial regression model for the complex hull–rudder sway and yaw forces can be used instead of more complicated separate models. Historically, this approach was used first (Crane et al., 1989) and it can give a quite satisfactory description of hydrodynamic forces. The model can be further simplified by eliminating some secondary-importance terms.
4. A suitable mathematical model for longitudinal forces can be created from that (Inoue et al., 1981a) but with highly simplified representations for ship resistance and for the surge force induced by the rudder.
5. The steering gear dynamics can be also neglected in many cases but its presence is highly desirable when simulation of the automatic control is envisaged. However, this doesn't degrade sensibly the speed of computations.
6. The model is not supposed to reproduce precisely the manoeuvring properties of any vessels but some degree of flexibility and adaptability to ships with different turning ability and directional stability is required.
7. It is desirable to supply the model with control laws for automated execution of typical navigational manoeuvres (course change, lane change etc).

All the formulated requirements have affected substantially all the decisions taken during the development of the fast dynamic model tailored for dynamic prediction tasks.

2.2. Frames of reference and parameters definition

To introduce the frames and kinematic parameters that are adopted in this work reference is made to Fig. 1.

The ship origin's co-ordinates and her heading form the vector of general co-ordinates $\mathbf{Y}^{\mathrm{T}}=(\xi_C,\eta_C,\psi)$. The current ship velocity $\mathbf{V}$ can be decomposed in the body axes x and y with projections u and v respectively. These two velocities and the rate of yaw r are forming the minimum variant of the velocities vector $\mathbf{X}^{\mathrm{T}}=(u,v,r,\ldots)$ which can be extended and include the rudder angle, roll angle, rate of roll, propeller

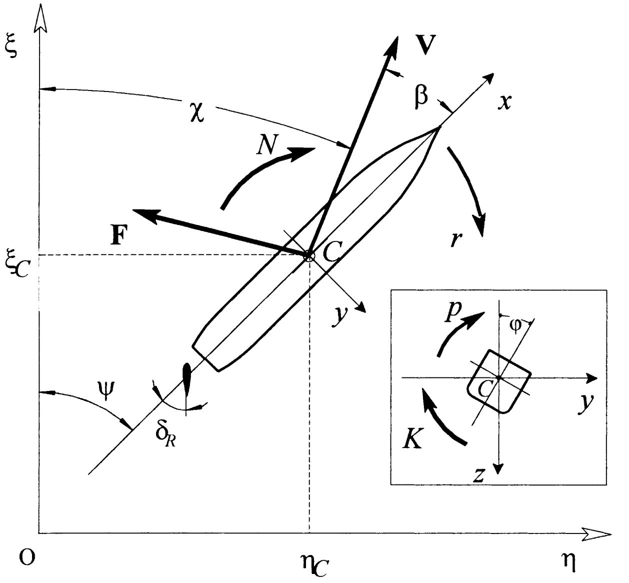

Fig. 1. Frames of reference and main parameters.

rotation rate etc. depending on the complexity of the used mathematical model. Of course, this vector becomes then the system state vector of more general meaning but we shall keep the term *velocities vector* regardless of this.

2.3. Description of the ship mathematical model

Assuming that the origin C is at the ship's centre of mass (G) and understanding that under the adopted assumptions the vector of ship velocities is $\mathbf{X}=(u,v,r,\delta_R)^T$ one can write the dynamic equations in the following form

$$m(\dot{u}-vr)=X \tag{3a}$$

$$m(\dot{v}+ur)=Y \tag{3b}$$

$$I_{zz}\dot{r}=N \tag{3c}$$

$$\dot{\delta}_R=F_\delta\left(\frac{1}{T_R}(\delta^*-\delta_R),\delta_m,\varepsilon_m\right) \tag{3d}$$

where m is the ship's mass, I_{zz} is the correspondent moment of inertia, X,Y,N are respectively the surge, sway, and yaw force/moment components, δ_m,ε_m are the maximum rudder deflection angle and the maximum rudder deflection rate, δ^* is the rudder order, and T_R is the steering gear time lag. $F_\delta()$ is a non-linear function accounting for limitations in the deflection angle and speed and for a possible dead zone in the steering gears's tracking system.

The kinematic equations are

$$\dot{\xi}_C = u\cos\psi - v\sin\psi, \tag{4a}$$

$$\dot{\eta}_C = u\sin\psi + v\cos\psi, \tag{4b}$$

$$\dot{\psi} = r. \tag{4c}$$

The set Eqs. (4a, 4b, 4c) and (5) can be complemented with a control law representing an autopilot or a human operator. For instance, in the case of the course change manoeuvring the control law can be represented in the general form

$$\delta^* = f(\psi^*, \psi, u, v, r, \dot{u}, \dot{v}, \dot{r}, \delta_R, \ldots), \tag{5}$$

where some additional parameters can be introduced as arguments for specific law realisations and ψ^* is the heading order.

The hydrodynamic forces are represented as follows

$$X = 0.625\mu_{22}vr + T_E(u,n) + C_R u^2 + \frac{\rho}{2} LTX'_{\delta\delta} u^2 \delta_R^2, \tag{6a}$$

$$Y = -\mu_{11}\dot{u} + \frac{\rho}{2} LTV^2 (Y'_v v' + Y'_r r' + Y'_{vvv} v'^3 + Y'_{vvr} v'^2 r' + Y'_\delta \delta_R + Y'_{vv\delta} v'^2 \delta_R), \tag{6b}$$

$$N = -\mu_{66}\dot{r} + \frac{\rho}{2} L^2 TV^2 (N'_v v' + N'_r r' + N'_{vvv} v'^3 + N'_{vvr} v'^2 r' + N'_\delta \delta_R + N'_{vv\delta} v'^2 \delta_R) \tag{6c}$$

where $\mu_{11}, \mu_{22}, \mu_{66}$ are the added masses, T_E is the effective thrust of the propeller, C_R is the resistance coefficient, ρ is the water density, L, T are respectively the ship's length and draught, $X'_{\delta\delta}, Y'_v, \ldots, N'_{vv\delta}$ are the dimensionless hydrodynamic derivatives, $V = \sqrt{u^2 + v^2}$ is the ship's speed and $v' = v/V$ and $r' = rL/V$ are the dimensionless velocities.

Four linear hydrodynamic derivatives are supposed to depend on the relative trim $d' = d/T$, where the trim d is positive by the stern:

$$Y'_v = (1 + b_1 d') Y'_{v0}, \tag{7a}$$

$$Y'_r = (1 + b_2 d') Y'_{r0}, \tag{7b}$$

$$N'_v = (1 + b_3 d') N'_{v0}, \tag{7c}$$

$$N'_r = (1 + b_4 d') N'_{r0}, \tag{7d}$$

where an additional subscript "0" stands for even-keel values and the parameters $b_1, \ldots, b_4$ are constants (Inoue et al., 1981b).

The added masses can be linked to the ship's mass and moment of inertia with the help of dimensionless inertial coefficients k_{11}, k_{22}, k_{66}:

$$\mu_{11} = k_{11} m,\ \mu_{22} = k_{22} m,\ \mu_{66} = k_{66} I_{zz}, \tag{8}$$

All dimensionless coefficients and derivatives are constant for a given ship. Moreover, some averaged or standard values can be taken for most of them if the accuracy requirements are moderate. For instance, it can be recommended to use data for a *Mariner* ship, which are considered to be enough reliable (Eda et al., 1989).

Some adjustment is, however, highly desirable. The dynamic properties of the Mariner ship are typical for merchant vessels and correspond to a directionally stable ship with moderate turning ability. However, introducing some imaginary trim can make the ship mathematical model even more directionally stable with poorer turning ability ("trim" by the stern) or it can generate a directionally unstable easily turning object. There is also an additional possibility to tune the turning ability without affecting the stability through changing values of the derivatives Y'_δ and N'_δ.

The effective thrust is calculated with a relatively simple 4-quadrant mathematical model of the propeller proposed by Oltmann and Sharma (1984) which in the present implementation is described with the following sequence of formulae:

$$n = k_n k_{\mathrm{BACK}} n_0, \tag{9a}$$

$$V_{CP} = 0.7\pi D_P n, \tag{9b}$$

$$V_B = \sqrt{u^2 + V_{CP}^2}, \tag{9c}$$

$$\sin\beta_B = \frac{u}{V_B}, \tag{9d}$$

$$\cos\beta_B = \frac{V_{CP}}{V_B}, \tag{9e}$$

$$C_T = \begin{cases} C_{T0} + C_T^c \cos\beta_B + C_T^s \sin\beta_B & \text{at } \cos\beta_B \geq 0.9336 \\ C_T^{cc} |\cos\beta_B| \cos\beta_B + C_T^{ss} |\sin\beta_B| \sin\beta_B & \text{otherwise,} \end{cases} \tag{9f}$$

$$T_E = \frac{\rho}{2} A_d k_{\mathrm{COR}} C_T(\beta_B) V_B^2, \tag{9g}$$

where k_n is the control order varying from -1 for FULL ASTERN to +1 for FULL SEA AHEAD, k_{BACK} is the astern thrust degradation coefficient (usually equal to 0.7 when the propeller is reversing), n_0 is the effective propeller rotation rate, D_P is the propeller diameter, k_{COR} is the correction factor and $C_{T0}, \ldots, C_T^{ss}$ are constant coefficients.

In general, the latter coefficients must be estimated individually for any ship screw propeller but it appeared that because of approximate similarity of propellers' dynamic characteristics one can use the same values for all the cases adjusting only the parameters n_0 and k_{COR}. The former is defined in such a way that the propeller work at maximum efficiency at the ship's design speed (it is clear that n_0 can become rather different from the actual design rotation rate) while the latter must equalise the magnitudes of the ship's resistance and the propeller's effective thrust. It is clear that with this approach there is no need in using the wake fraction and thrust deduction factors as they are accounted for implicitly.

The described mathematical model seems to be as simplified as possible since any further simplifications can deteriorate realism of the resulting modelled motion.

Mathematically, the resulting model represents a set of ordinary differential equations together with a number of auxiliary relations and any prediction or simulation of a ship's motion is equivalent to the solution of a Cauchy initial-value problem. Whatever simple the mathematical model is, there is no way to obtain any analytic solution and the motion must be simulated numerically. The simplest Euler's method of numerical integration appeared to be the most suitable in this case as it is the most robust with respect to possible discontinuities at the right-hand sides.

The model can, through an appropriate adjustment of the "trim" describe ships with different degree of directional stability/instability as is illustrated with Fig. 2, where the relative stability parameter $S=2\underline{d}$ was introduced with $S=0$ corresponding to the *Mariner* case.

While the spiral curves are difficult to obtain with full-scale trials, the zigzag manoeuvre results can be achieved much easier and such tests are now considered as mandatory by the Interim Standards for Ship Manoeuvrability (1993).

Three zigzag examples are presented on Fig. 3 while Fig. 4 presents a dependence of the second overshoot angle (this angle is, however, practically identical to all the subsequent overshoots and the authors can recommend strongly to take an average of several measured overshoots, say, from 2nd to 5th to reduce the influence of initial conditions and possible asymmetry of a ship) on the value of the stability parameter.

2.4. Time-suboptimal steering in course changing manoeuvre

As stated earlier, a suitable control law would be a useful augmentation of the proposed mathematical model making possible automated simulation of, say, course alterations. The time-suboptimal (i.e. nearly optimal) control law briefly presented below approximately models careful but efficient control performed by an experi-

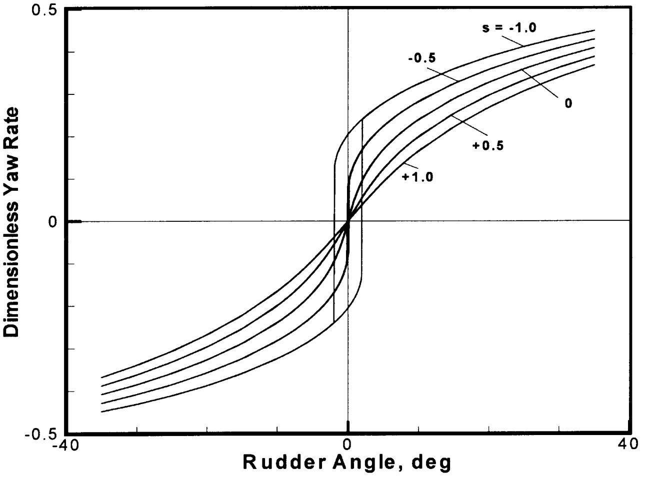

Fig. 2. Spiral curves for different degrees of directional stability.

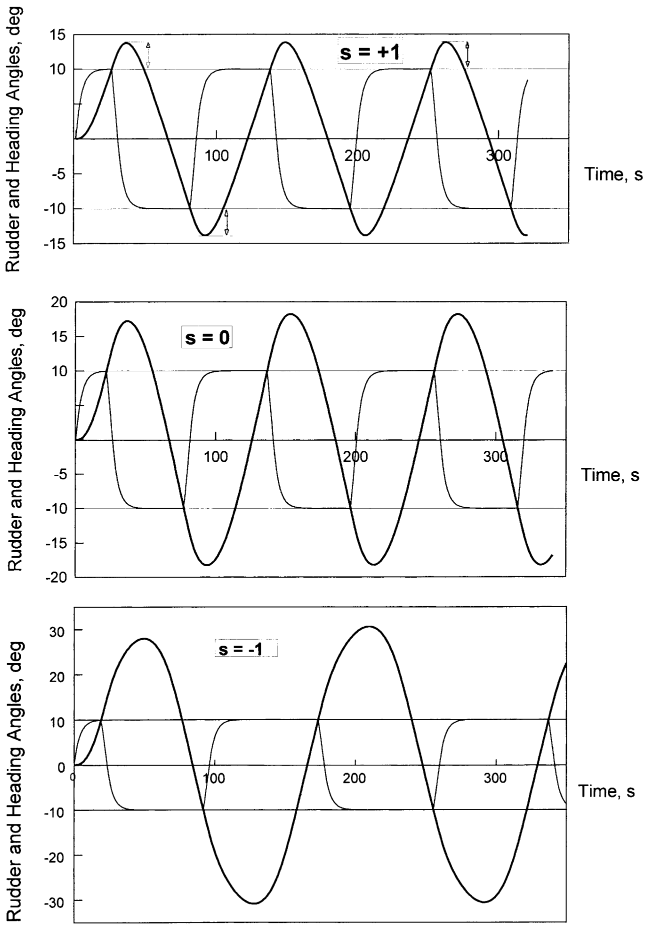

Fig. 3. Zigzag 10–10 time histories.

enced helmsmen in course-changing manoeuvres. More detailed information can be found in Sutulo (1997, 1998).

The control law was first devised by Oldenburger and in this particular case its use implies executing the following steps.

1. The non-linear dynamic Eqs. (3a, 3b, 3c) and (3d) are linearized in the least-square sense over a region of anticipated variation of the dimensionless velocities of sway and yaw.
2. The equation of surge is dropped and the remaining equations of sway and yaw are non-dimensionalized for convenience. The steering gear is supposed to be ideal which means that $\delta_R \equiv \delta^*$.
3. The linear sway-yaw equations are transformed into the Nomoto equations of second and first order:

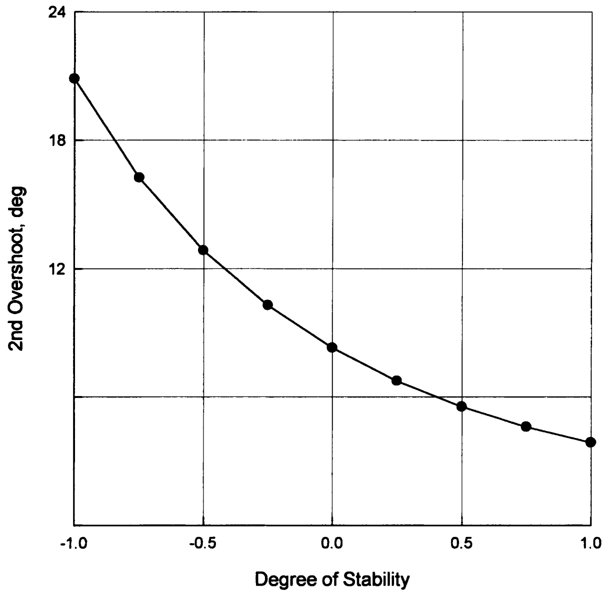

Fig. 4. Zigzag 10–10: the second overshoot angle versus the stability index.

$$T_1'T_2'\ddot{r}'+(T_1'+T_2')\dot{r}'+r'=K'(\delta_R+T_3'\dot{\delta}_R) \tag{10}$$

$$T'\dot{r}'+r'=K'\delta_R \tag{11}$$

where the dimensionless time lags $T'_1,T'_2,T'_3,T'=T'_1+T'_2\text{-}T'_3$ and the gain K' are directly linked to the coefficients of the linearized sway-yaw equations.

4. The control law is then expressed as

$$\delta^*=-\frac{V\delta_m}{L\varepsilon_m}\mathrm{sign}\left(\frac{T'r'+\psi-\psi*}{K'}-\frac{V}{2L\varepsilon_m}\delta_R|\delta_R|\right). \tag{12}$$

This control law is of bang–bang (discontinuous, relay) type but being applied to initial non-linear mathematical models it works a significant part of time in the sliding mode assuring very smooth transients. Very extensive testing did not reveal any cases of overshoots though during course changes by small angles (5–10 degrees) the process is somewhat delayed compared to the truly time-optimal one but the absolute time loss remains negligible.

However, a more efficient though much more complicated time-suboptimal control law also exists (Sutulo, 1998) and it can be used if it is necessary to model a faster steering performed accurately by an experienced operator. Also, a control law had been obtained (Sutulo, 1994) allowing overshoot-free simulation of the lane change (the co-ordinate manoeuvre) or of a complete overtaking manoeuvre.

3. Kinematic mathematical models for ship path prediction

The fastest ship mathematical models are purely kinematic i.e. based exclusively on the kinematic Eqs. (4a, 4b) and (4c). Such models can be effectively used only for short-term path predictions and, as a rule, cannot account for any control change.

As the set Eqs. (4a, 4b) and (4c) contains only three equations for six state variables, certain assumptions must be made to reduce the number of unknowns. A natural assumption is some hypothesis concerning the behaviour of the velocities $u(t)$,$v(t)$,$r(t)$ on the time interval $t \in [t_P, t_p+T_P]$. The most evident is the hypothesis of constant values of the mentioned velocities kept during the prediction interval i.e.

$$u(t) \equiv u(t_P) \equiv u_P, \tag{13a}$$

$$v(t) \equiv v(t_P) \equiv v_P, \tag{13b}$$

$$r(t) \equiv r(t_P) \equiv r_P. \tag{13c}$$

The Eqs. (4a, 4b) and (4c) can then easily be integrated (the subscript P always corresponds to the prediction time moment):

$$\psi(t) = \psi_P + r_P(t - t_P), \tag{14a}$$

$$\xi_C(t) = \xi_{CP} + u_P A(t) - v_P B(t), \tag{14b}$$

$$\eta_C(t) = \eta_{CP} + u_P B(t) + v_P A(t), \tag{14c}$$

where

$$A(t) = \int_{t_P}^{t} \cos[\psi_P + r_P(t - t_P)]dt = \frac{1}{r_P}\{\sin[\psi_P + r_P(t - t_P)] - \sin\psi\}, \tag{15a}$$

$$B(t) = \int_{t_P}^{t} \sin[\psi_P + r_P(t - t_P)]dt = -\frac{1}{r_P}\{\cos[\psi_P + r_P(t - t_P)] - \cos\psi\}. \tag{15b}$$

The previous formulae have a removable singularity at small absolute values of r_P and it is then preferable to use their asymptotic forms at $r_P \to 0$

$$A(t) = (t - t_P)\cos\psi_P - \frac{r_P}{2}(t - t_P)^2 \sin\psi_P, \tag{16a}$$

$$B(t)=(t-t_P)\sin\psi_P+\frac{r_P}{2}(t-t_P)^2\cos\psi_P, \tag{16b}$$

where the truncation error is equal to $1/6(t-t_P)^3 r_P^2$ in both cases.

This simplest kinematic prediction model provides extremely effective and fast prediction along both rectilinear and curvilinear paths but it leads to sensible errors in accelerating-decelerating manoeuvres where a non-constant speed is the essence of the motion.

Then, the natural extension is to consider the effect of constant accelerations $a_u(t)\equiv\dot{u}(t)\equiv a_{uP}$, $a_v(t)\equiv\dot{v}(t)\equiv a_{vP}$ and $a_r(t)\equiv\dot{r}(t)\equiv a_{rP}$. This yields the following representations for the velocities on the prediction interval:

$$u(t)=u_P+a_{uP}(t-t_P), \tag{17a}$$

$$v(t)=v_P+a_{vP}(t-t_P), \tag{17b}$$

$$r(t)=r_P+a_{rP}(t-t_P). \tag{17c}$$

A formula for the heading angle prediction is derived immediately:

$$\psi(t)=\psi_P+r_P(t-t_P)+\frac{a_{rP}(t-t_P)^2}{2} \tag{18}$$

but the expressions for the predicted advance and transfer become significantly more complicated:

$$\xi_C(t)=\xi_{CP}+\int_{t_P}^{t}u(t)\cos\psi(t)dt-\int_{t_P}^{t}v(t)\sin\psi(t)dt, \tag{19a}$$

$$\eta_C(t)=\eta_{CP}+\int_{t_P}^{t}u(t)\sin\psi(t)dt+\int_{t_P}^{t}v(t)\cos\psi(t)dt, \tag{19b}$$

where the time functions at the right-hand sides are defined by Eqs. (17a, 17b, 17c) and (18). However, the resulting integrals still can be evaluated analytically:

$$\xi_C(t)=\xi_{CP}+\frac{1}{a_{rP}}[a_{uP}(\sin a_1-\sin\psi_P)+a_{vP}(\cos a_1-\cos\psi_P)]+\frac{\sqrt{p}}{|a_{rP}|^{3/2}}\{[S(a_2) \tag{20a}$$
$$-S(a_3)][c_1\sin a_4-c_2\cos a_4]\operatorname{sign}a_{rP}+[C(a_2)-C(a_3)][c_1\cos a_4+c_2\sin a_4]\},$$

$$\eta_C(t)=\eta_{CP}+\frac{1}{a_{rP}}[-a_{uP}(\cos a_1-\cos\psi_P)+a_{vP}(\sin a_1-\sin\psi_P)]$$

$$+\frac{\sqrt{\pi}}{|a_{rP}|^{3/2}}\{[S(a_2)-S(a_3)][c_1\cos a_4+c_2\sin a_4]\operatorname{sign}a_{rP}-[C(a_2) \tag{20b}$$

$$-C(a_3)][c_1 \sin a_4 - c_2 \cos a_4]\},$$

where $C(x)$, $S(x)$ are the Fresnel integrals and the auxiliary arguments and coefficients are:

$$a_1 = \psi_P + r_P(t-t_P) + \frac{a_{rP}(t-t_P)^2}{2}, \tag{21a}$$

$$a_2 = \frac{r_P}{\sqrt{\pi|a_{rP}|}}, \tag{21b}$$

$$a_3 = \frac{r_P + a_{rP}(t-t_P)}{\sqrt{\pi|a_{rP}|}}, \tag{21c}$$

$$a_4 = \frac{r_P^2}{2a_{rP}} - \psi_P, \tag{21d}$$

$$c_1 = r_P a_{uP} - u_P a_{rP}, \tag{21e}$$

$$c_2 = r_P a_{vP} - v_P a_{rP}. \tag{21f}$$

An example of prediction made with the help of Eqs. (20a, 20b, 21a, 21b, 21b, 21c, 21d, 21e) and (21f) is shown on Fig. 5 for the case of zero initial velocities of sway and yaw, and of the initial velocity of surge being equal to 10 m/s, with nonzero initial accelerations $a_{uP}=a_{vP}=0.01$ m/s^2 and $a_{rP}=0.001$ rad/s^2. Of course, the spiral, which is clearly seen on the figure, is never observed in reality because these acceler-

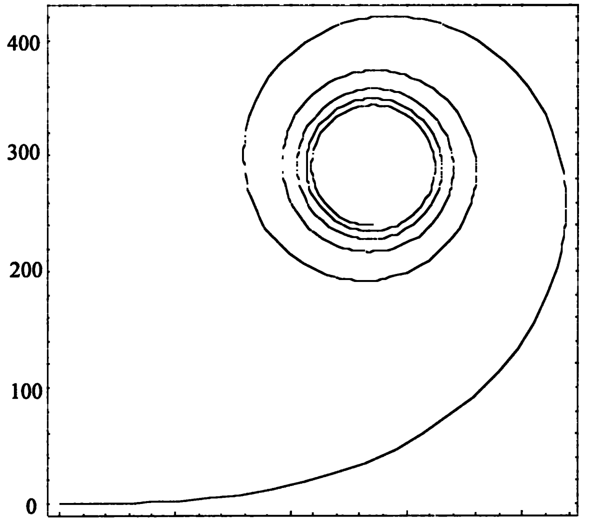

Fig. 5. An example of the predicted trajectory (horizontal axis — advance, vertical axis — transfer).

ations cannot be kept constant all the time but the initial part of the trajectory is predicted very realistically.

The Eqs. (21a, 21b, 21c, 21d, 21e) and (21f) contain an apparent singularity at a_{rP}=0. The calculations show this singularity doesn't really exist but the corresponding asymptotic formulae appeared to be too bulky and complicated. However, it is possible to indicate a threshold value of the yaw acceleration a_{rP}^T below which it is possible to neglect it completely:

$$a_{rP}^T = \frac{\Delta r}{T_P^{\max}}, \tag{22}$$

where Δr is the maximum acceptable rate-of-yaw error at the end of the greatest anticipated prediction interval with lead time $T_P^{\max}$.

It is not likely to predict more than 1000 s ahead and the error of 0.001 rad/s sounds acceptable. Then, a_{rP}^T=1×10^{-6} but smaller values can also be used. If the current value of the angular acceleration a_{rP} is less than a_{rP}^T then one must use the formulae derived from Eqs. (19a) and (19b) at $a_{rP}\equiv 0$:

$$\xi_C(t) = \xi_{CP} = \frac{1}{r_P^2}[d_1(\cos a_{10} - \cos\psi_P) - d_2(\sin a_{10} - \sin\psi_P)] + \frac{1}{r_P}(a_{uP}\sin a_{10} + a_{vP}\cos a_{10})(t - t_P), \tag{23a}$$

$$\eta_C(t) = \eta_{CP} + \frac{1}{r_P^2}[d_1(\sin a_{10} - \sin\psi_P) + d_2(\cos a_{10} - \cos\psi_P)] + \frac{1}{r_P}(a_{vP}\sin a_{10} - a_{uP}\cos a_{10})(t - t_P), \tag{23b}$$

where

$$a_{10} = \psi_P + r_P(t - t_P), \tag{24a}$$

$$d_1 = a_{uP} + v_P r_P, \tag{24b}$$

$$d_2 = a_{vP} - u_P r_P. \tag{24c}$$

These equations are a generalisation of the Eqs. (16a, 16b, 17a, 17b) and (17c) and similarly have an apparent singularity as $r_P \to 0$. Their asymptotic form valid at small r_P is

$$\xi(t) = \xi_P + (u_P\cos\psi_P - v_P\sin\psi_P)(t - t_P) + \tfrac{1}{2}(a_{uP}\cos\psi_P - a_{vP}\sin\psi_P)(t - t_P)^2 + [-\tfrac{1}{2}(u_P\sin\psi_P + v_P\cos\psi_P)(t - t_P)^2 + \tfrac{1}{3}(a_{uP}\sin\psi_P + a_{vP}\cos\psi_P)(t - t_P)^3]r_P \tag{25a}$$

$$\eta(t) = \eta_P + (u_P\sin\psi_P + v_P\cos\psi_P)(t - t_P) + \tfrac{1}{2}(a_{uP}\sin\psi_P + a_{vP}\cos\psi_P)(t - t_P)^2 = [\tfrac{1}{2}(u_P\cos\psi_P - v_P\sin\psi_P)(t - t_P)^2 + \tfrac{1}{3}(a_{uP}\cos\psi_P - a_{vP}\sin\psi_P)(t - t_P)^3]r_P. \tag{25b}$$

with the truncation error not exceeding

$$\left[\frac{a_P}{8}(t - t_P)^4 + \frac{V_P}{6}(t - t_P)^3\right]r_P^2,$$

where

$$a_P = \sqrt{a_{uP}^2 = a_{vP}^2} \text{ and } V_P = \sqrt{u_P^2 + v_P^2}.$$

It seems that the most appropriate would be implementing both prediction schemes as displaying two not identical predicted positions will give the operator an assessment of the prediction uncertainty. It is evident that in the case of uniform motion both predicted positions will converge. On the other hand, it is clear that at fixed control actions the accelerations will decrease in course of time as the motion will tend to a steady one. This means that the accelerations-based prediction will always overestimate the ship's displacement (in the case of positive accelerations) while the velocities-based prediction will underestimate it.

It is also clear that even more sophisticated prediction schemes based on an idea of Taylor expansions of kinematic parameters' time histories can be developed. However, this hardly makes some sense as the assumption on the analytical properties of these histories while control action applied to the ship can be altered at any moment seems to be too strong and artificial.

4. Course changing manoeuvre prediction: time lag influence

A ship can in general follow any curvilinear path but the most typical is tracking some segmented trajectory formed by a number of rectilinear pieces. The ship is connecting these pieces together by means of the course-changing manoeuvre, which seems to be the most routine one in ship handling practice. It is then likely that the short-term prediction of these manoeuvres is of major importance. It is highly desirable to perform the prediction before the course changing begins which eliminates using methods described and developed in previous sections of the report.

On the other hand, as the final result of the manoeuvre i.e. a new straight path is known in advance, the simplest prediction method is simply to draw a new straight course rotated by the ordered angle with respect to the previous one and intersecting it at the point corresponding to the start of the rudder deflection. However, a substantial error can be introduced when following this oversimplified approach as the ship, due to her inertia, can follow but a smooth trajectory and some delay in terms of time or advance is inevitable. In the case of a complicated navigational situation, this error can become dangerous.

The most accurate prediction can be obtained with the help of some ship dynamic mathematical model completed with an appropriate control law. The latter issue is of primary importance because actually any course change can be executed in different ways and one can create an infinite quantity of control signal reaching the same goal.

To remove the ambiguity, some additional criterion is necessary. Observations on helmsmen's behaviour show that there is a tendency to complete the manoeuvre in more or less the shortest time possible but at the same time with special care to avoid overshoots.

The time optimality criterion seems to guide the ship handling primarily at hard course changes (by 30 degrees or more) while at small course corrections the manoeuvre is executed more gently with most attention paid to avoiding overshoots. Fortunately, a time sub-optimal (i.e. approximately optimal) control law with these properties does really exist (Sutulo 1997, 1998) and can be effectively used in simulation programs. An example of the predicted trajectory is shown on Fig. 6 (smooth solid line).

As always, using a relatively sophisticated dynamic model can become too time-consuming and it can be desirable to suggest simpler approximate solutions. An evident proposal stems immediately from the mentioned plot: it is possible to keep using straight-line approximation but with a certain advance lag A introduced. The latter is expected to depend on ship's dynamic properties and on the magnitude of the course change.

Serial calculations carried out by means of the same computer code for ships with different degree of directional stability and for different course alterations resulted in the plot presented on Fig. 7. Values of the quantity A are given there in meters but as the ship was supposed to be 100 meters long, one may and should interpret these data in terms of the ship length percentage. These numerical results can be believed to be approximately valid for any surface displacement ship and used in form of a database inside the core of computerised kinematical predictors.

Moreover, it was found that the linear polynomial regression

$$A+C_o+C_s s+C_\psi \psi^*+C_{s\psi} s\psi^*+C_{ss}s^2+C_{\psi\psi}\psi^{*2}+C_{sss}s^3+C_{\psi\psi\psi}\psi^{*3} \qquad (26)$$
$$+C_{ss\psi}s^2\psi^*+C_{s\psi\psi}s\psi^{*2}$$

approximates the numerical data on Fig. 7 fairly well when the coefficients are estimated in the least square sense (their numerical values are given in Table 1). The

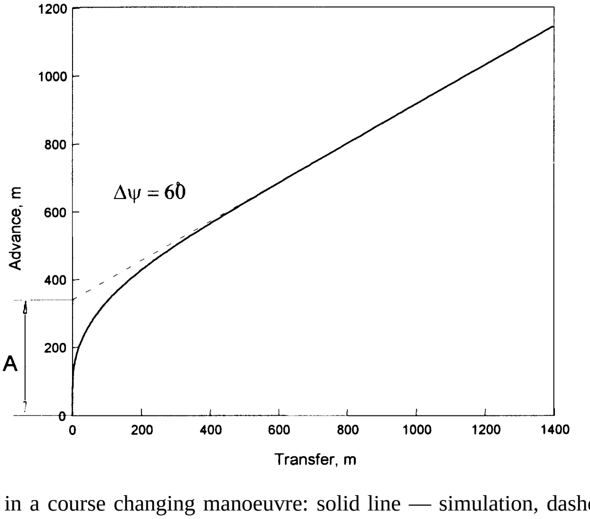

Fig. 6. Trajectories in a course changing manoeuvre: solid line — simulation, dashed line — approximation.

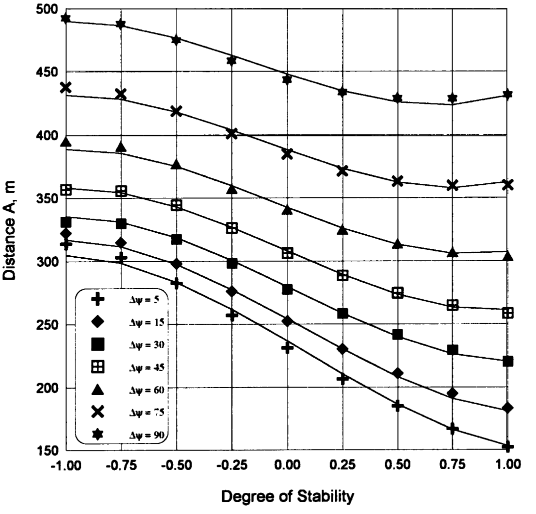

Fig. 7. Advance lag A as function of the stability index and course change magnitude: symbols — computed values, lines — regression response.

Table 1
The estimated values of regression coefficients

Term	Coeff.	Term	Coeff.
1	227.1921	ψ^{*2}	−0.011883
s	−108.309	s^3	28.25132
ψ^*	1.926033	ψ^{*3}	0.000197
$s\psi^*$	0.843768	$s^2\psi^*$	0.238110
s^2	−8.82075	$s\psi^{*2}$	−0.003156

time lag A shown on Fig. 7 and defining the predicted trajectory was restored from the regression but the error appeared to be practically undetectable in this case. However, it can be somewhat greater if the dynamic properties of a ship are substantially different from those described with the used mathematical model.

A more accurate and sophisticated approximation is possible if one more parameter is assumed to vary: the relative rudder efficiency described by the relevant derivatives. It is possible then to vary the ship turning ability leaving the directional stability intact and vice versa. But in this case it is impossible to find a simple and immediate connection between the adjusted parameters and the observed parameters of the manoeuvres: with two parameters the task becomes already nontrivial and requires using in full the parameter identification methods.

5. Conclusions

An analysis of the overall IMO vessel traffic system guidelines resulted in the conclusion that due to the variety of ship traffic control problems one cannot stick on some unique method of ship's path tracking and prediction: the available methods should be used depending on the available information on ship's manoeuvring properties, on the current accuracy requirements and on the available hardware performance.

The simplified dynamic ship mathematical model presented here makes possible a reasonably accurate, fast and realistic simulation of any moderate manoeuvre. Additional advantages of the model are the high computational speed achieved through elimination of a number of secondary effects and a very small number of necessary input data.

The model has a certain flexibility in that the degree of directional stability can be adjusted according to the properties of any given ship. It is enough to know main manoeuvring performance measures required by the IMO Interim Standards for Ship Manoeuvring, which are in effect since 1994.

An advanced analytical scheme for the short-term kinematic prediction accounting for current values of accelerations is proposed.

A method of fast trajectory prediction in course changing manoeuvre is developed. The salient feature of the method is that it accounts for a time lag due to ship's inertia. Approximate values of this lag in terms of ship's length were computed with the help of the simplified dynamic model as function of ship's stability parameter and of the course change magnitude.

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