

# Lecture 3b: Finite State Machines, Sequential Logic and the Clock

CSCI11: Computer Architecture and Organization

Chapter 3.4-3.6 in Patt and Patel:  
Introduction to Computing Systems...

- Quick Combinational Circuits Review
- Finite State Machines
- Sequential Logic
- Memory
- The Clock

# 1 to 4 Demultiplexer

- data distributor
- one input to 1 of 4 outputs
- used to light multiple segments of an LED digit display
- not used in the LC3

FIGURE 13. 4-CHANNEL DEMULTIPLEXER CIRCUIT

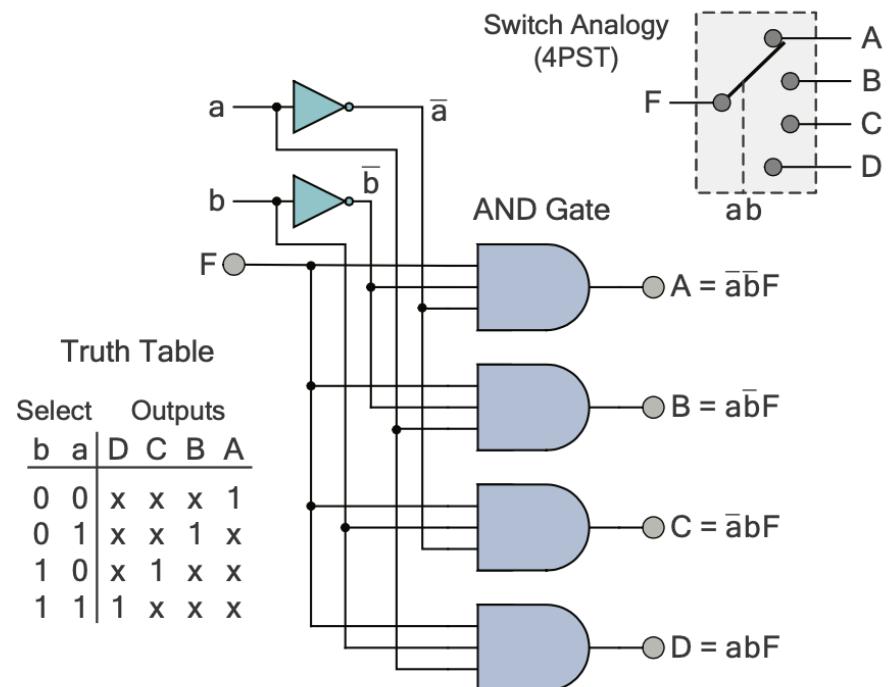
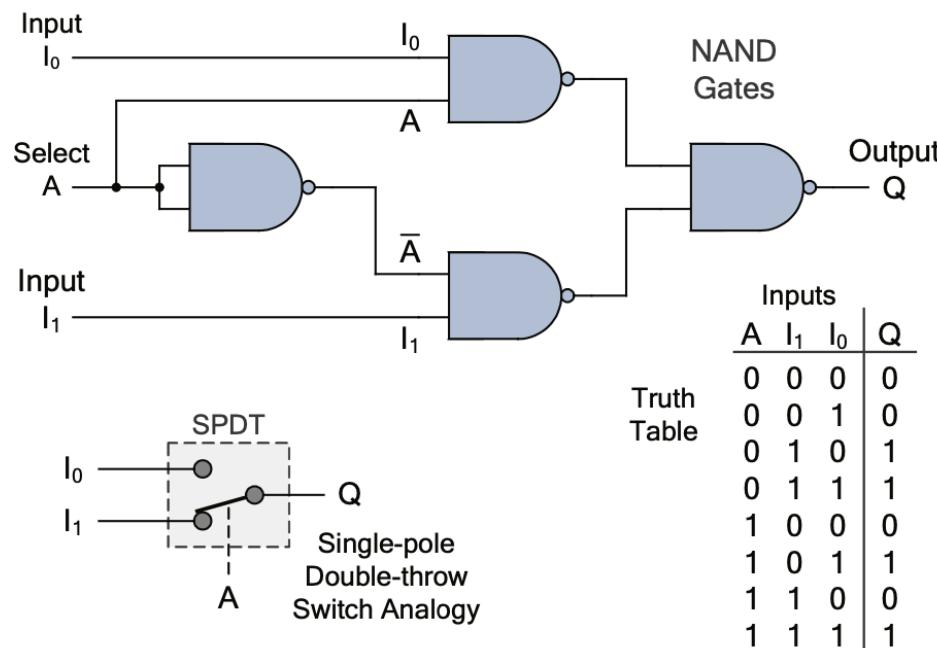


FIGURE 9. BASIC 2-INPUT MULTIPLEXER USING NAND GATES



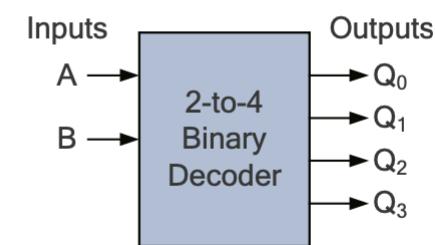
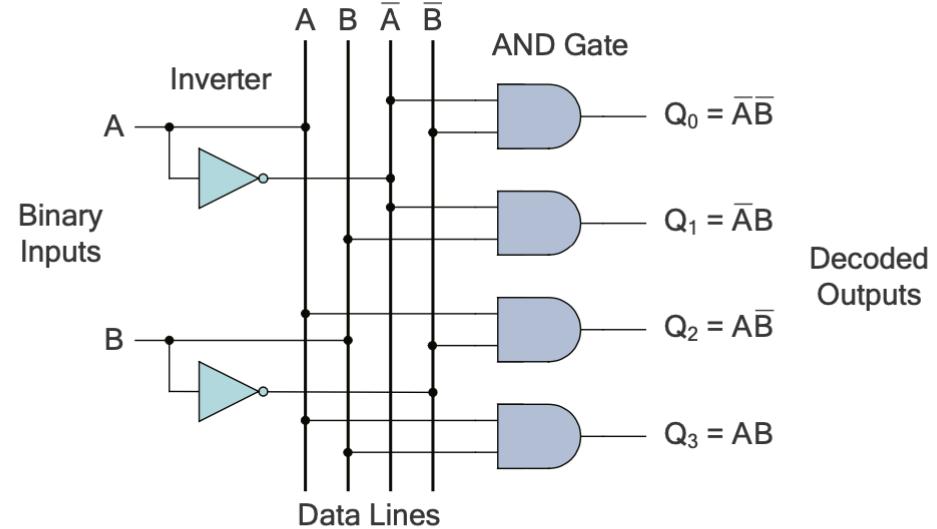
## 2-input Multiplexer

- **data selector**
- 1 of 2 inputs to one output
- use to select between multiple sets of address/data
- in the LC3, address lines pass through multiple muxes

## 2-4 Decoder

- data selector
- $2^n$  bits to select one of  $n$  lines
- use to select a register in a register file
- in the LC3, opcode's 3 bits select registers R0-R7

FIGURE 3. A 2-TO-4 BINARY DECODER



Truth Table

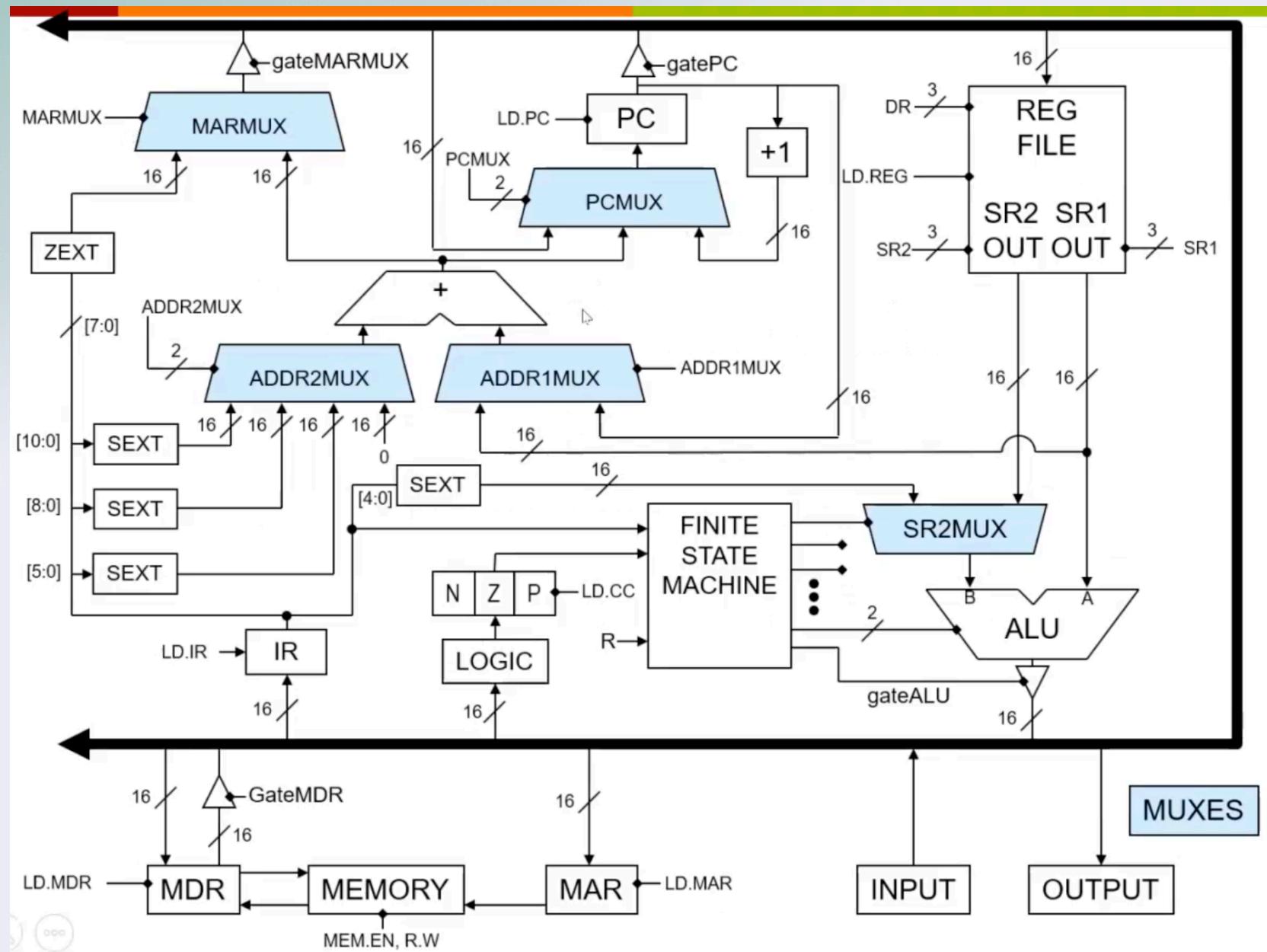
B	A	$Q_0$	$Q_1$	$Q_2$	$Q_3$
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

FIGURE 21. FULL-ADDER AND TRUTH TABLE

Symbol	Truth Table				
	Carry-In	B	A	SUM	Carry-Out
	0	0	0	0	0
	0	0	1	1	0
	0	1	0	1	0
	0	1	1	0	1
	1	0	0	1	0
	1	0	1	0	1
	1	1	0	0	1
	1	1	1	1	1

## 2 bit Full Adder

- n bit adder
- counts up to  $2^n$  bits or divides by  $2^n$  bits on Cout
- use as a program counter (PC) or a clock divider
- LC3 uses a 16bit full adder as a program counter

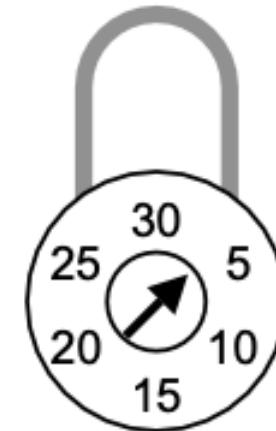
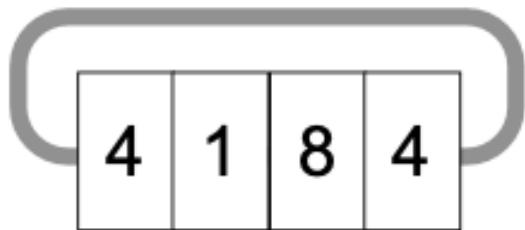


# Combinational Logic Circuits

*By definition, combinational logic circuits depend on their inputs and have no knowledge of previous events. That is, they are memory-less.*

# Combinational vs. Sequential

Two types of “combination” locks



## Combinational

Success depends only on the **values**, not the order in which they are set.

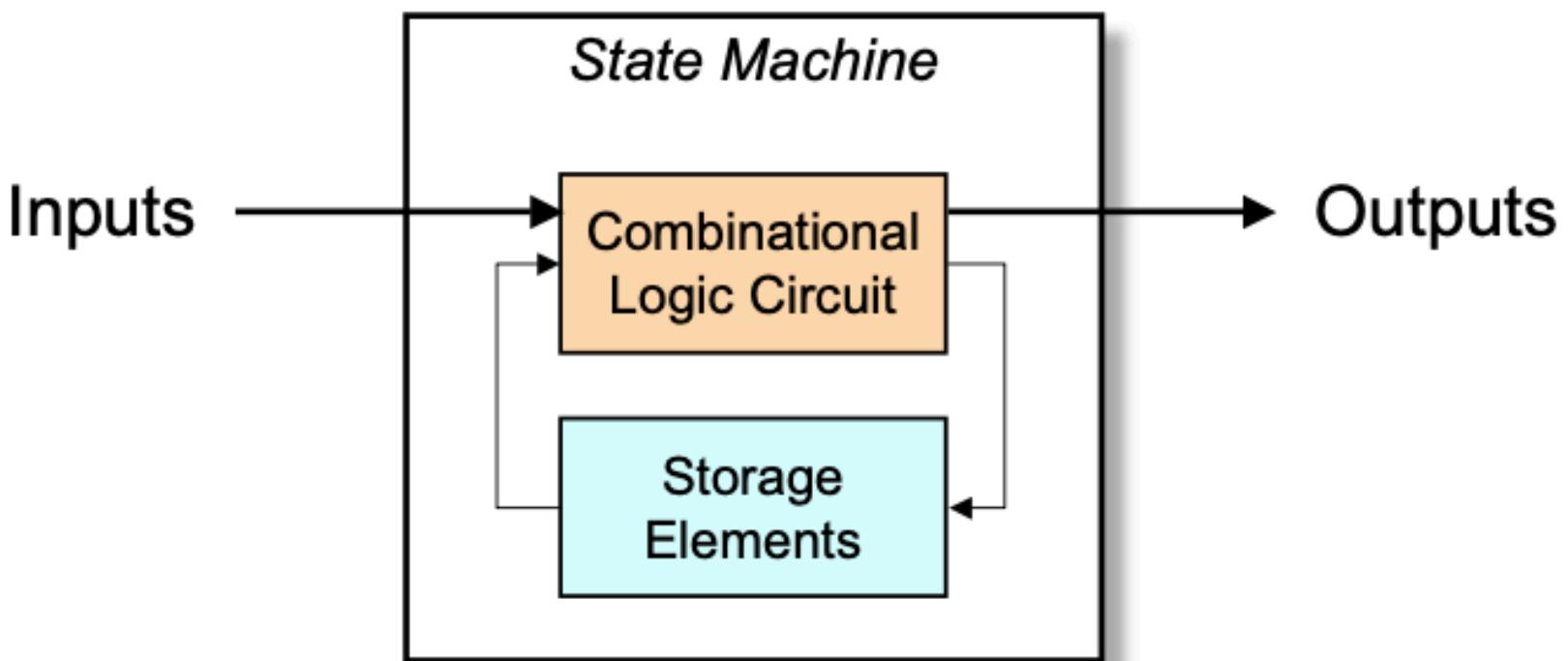
## Sequential

Success depends on the **sequence** of values (e.g., R-13, L-22, R-3).

# State Machine

Another type of sequential circuit

- Combines combinational logic with storage
- “Remembers” state, and changes output (and state) based on **inputs** and **current state**



## State

The **state** of system is **snapshot** of **all relevant elements** of system at moment snapshot is taken

## Examples

- The state of a basketball game can be represented by the scoreboard
  - Number of points, time remaining, possession, etc.
- The state of a tic-tac-toe game can be represented by the placement of X's and O's on the board (and turn)

## **Finite State Machine**

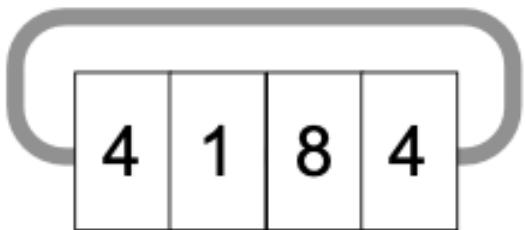
**A description of a system with the following components:**

- 1. A finite number of states**
- 2. A finite number of external inputs**
- 3. A finite number of external outputs**
- 4. An explicit specification of all state transitions**
- 5. An explicit specification of what determines each external output value**

**Often described by a state diagram**

- Inputs trigger state transitions**
- Outputs are associated with each state (or with each transition)**

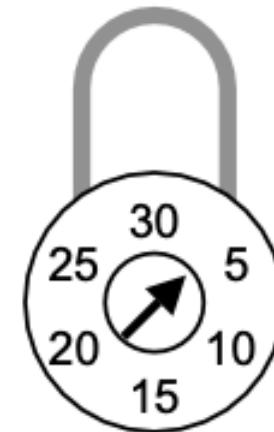
## Two State Lock



### Combinational

Success depends only on the **values**, not the order in which they are set.

## Finite State Machine



### Sequential

Success depends on the **sequence** of values (e.g., R-13, L-22, R-3).

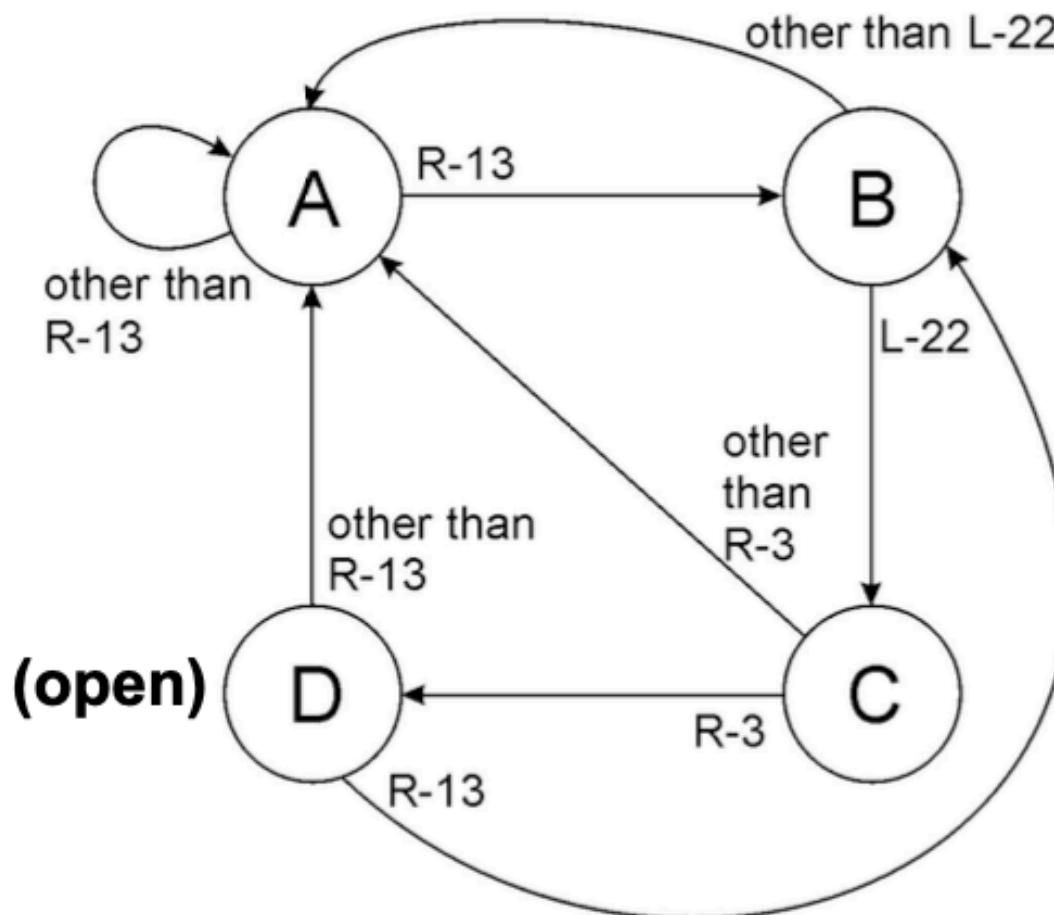
## State of Sequential Lock

Our lock example has four different states, labeled A-D:

- A: The lock is **not open**,  
and no relevant operations have been performed
- B: The lock is **not open**,  
and the user has completed the **R-13** operation
- C: The lock is **not open**,  
and the user has completed **R-13**, followed by **L-22**
- D: The lock is **open**

## Sequential Lock State Diagram

Shows **states** and **actions** that cause a **transition between states**



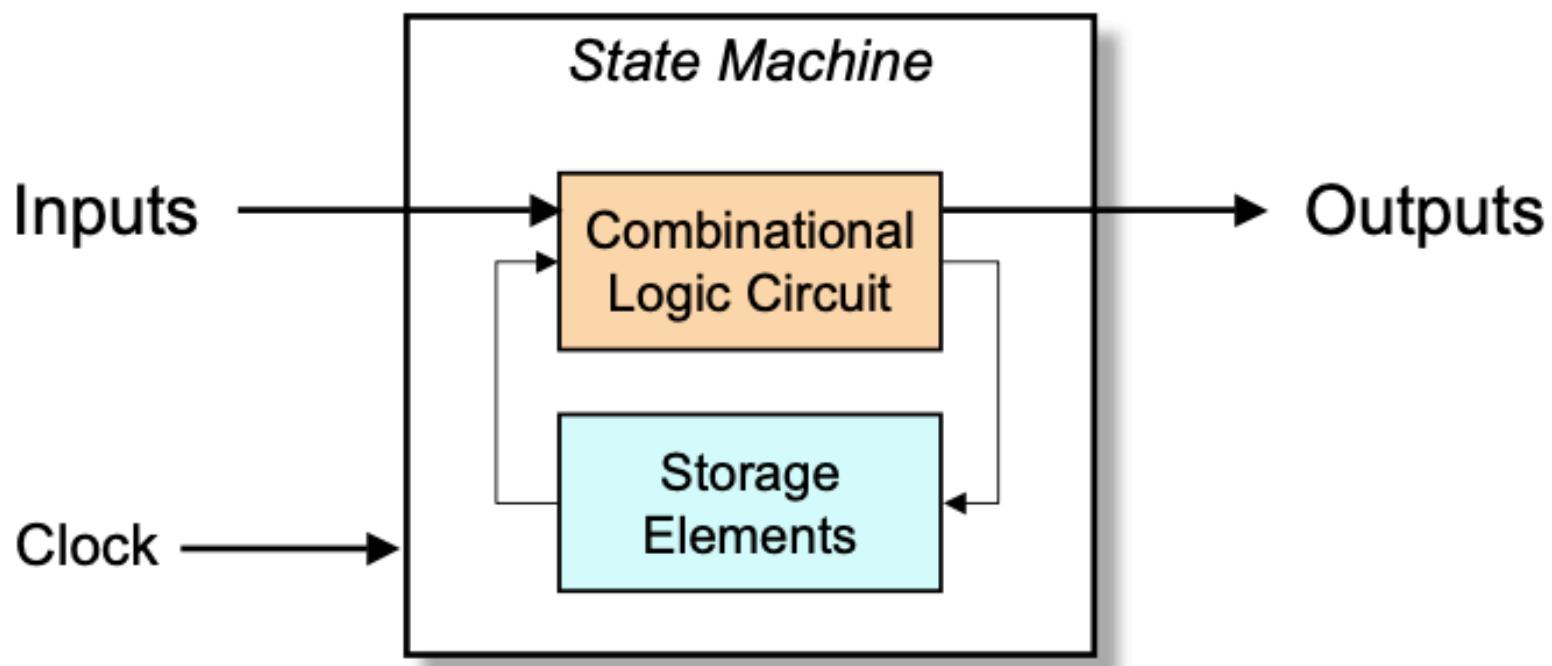
# Implementing a Finite State Machine

## Combinational logic

- Determine outputs and next state.

## Storage elements

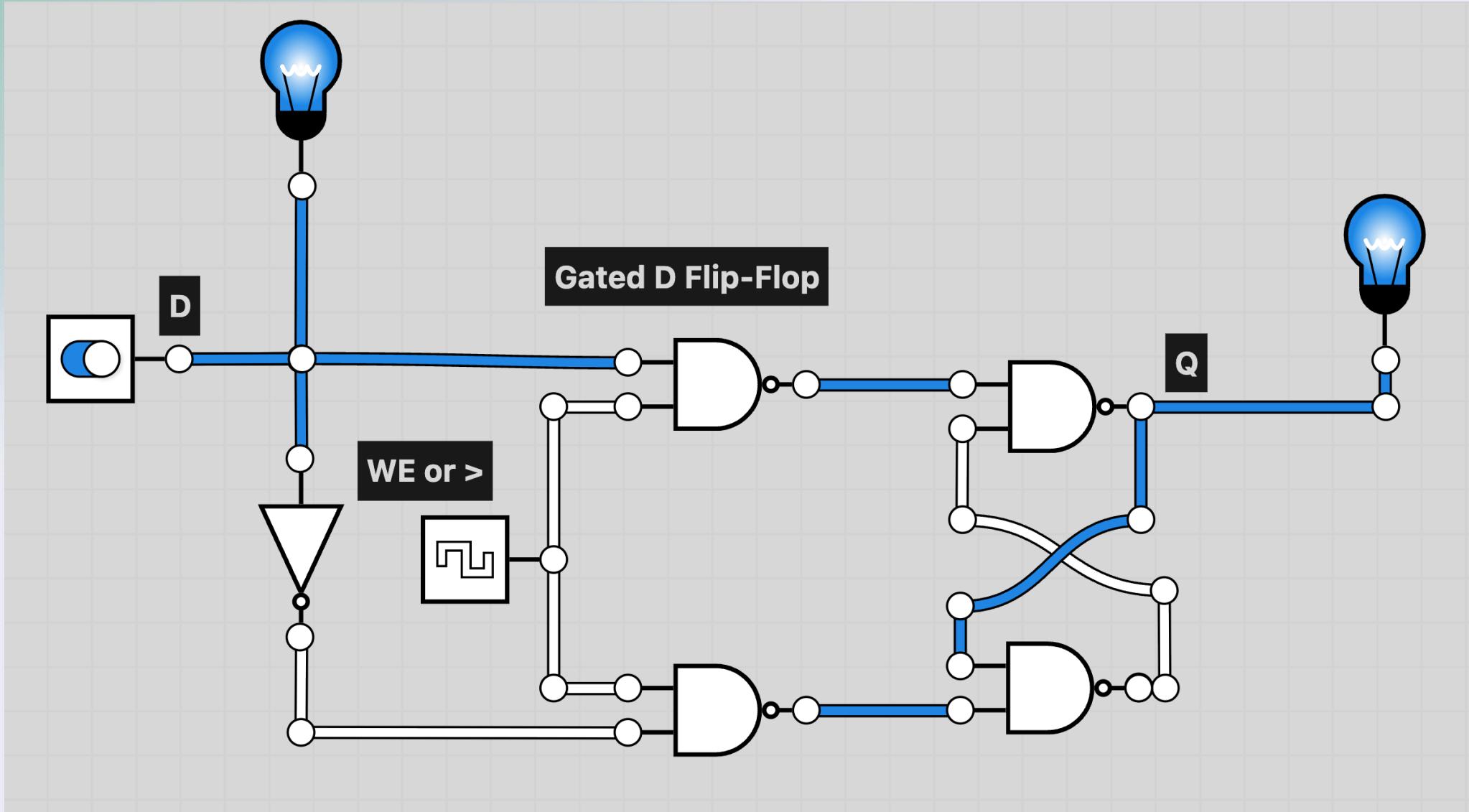
- Maintain state representation.

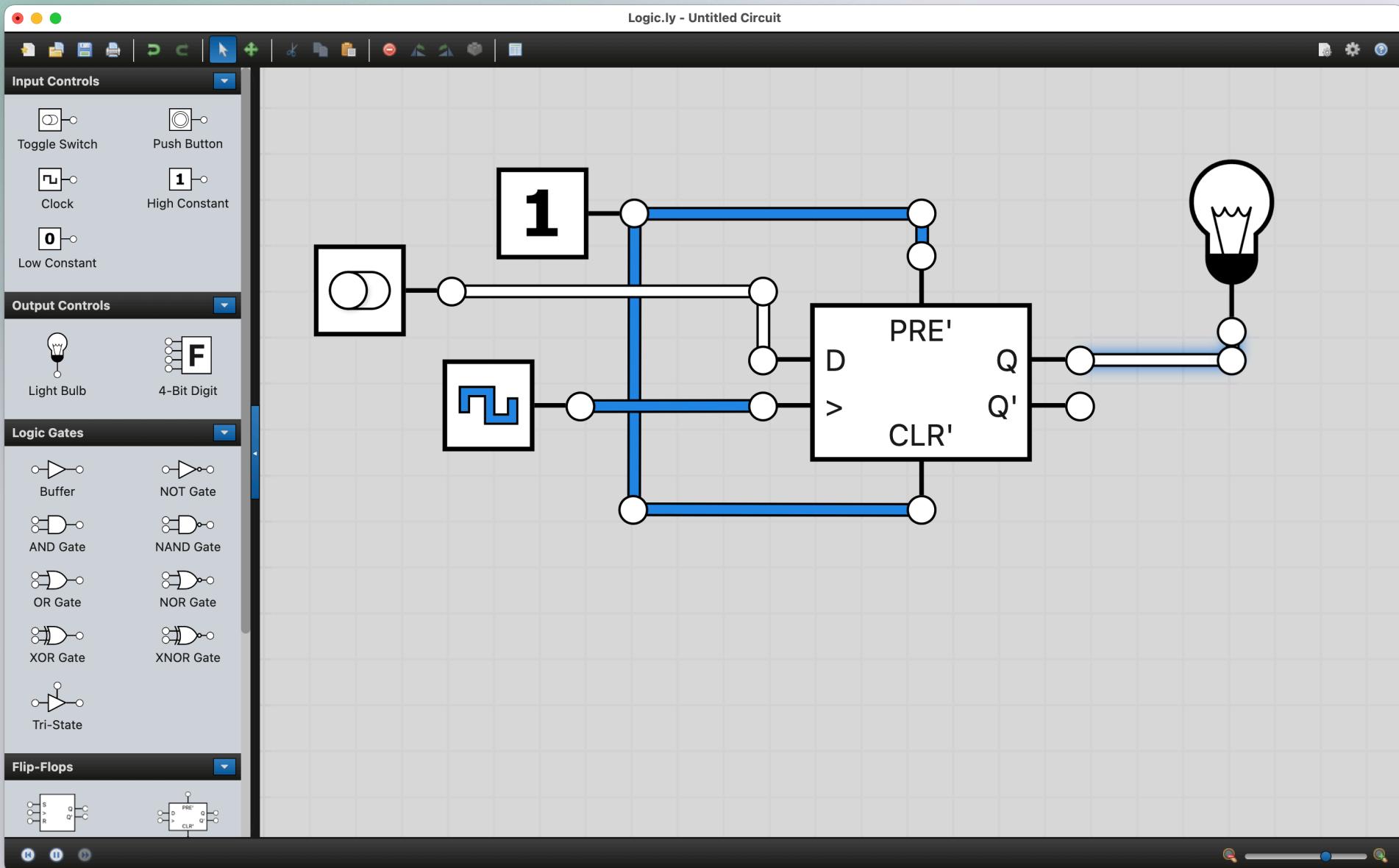


## Why is a Clock Needed?

1. To ensure there is a time element to the circuit
2. To ensure things are synchronized

See *D Latch as an example*





# Clocks

## Clock Characteristics

- Usually generated by an oscillating crystal
- Fixed period
- Frequency = 1/period
- $1\text{Mhz} = 1/10^6$  seconds or 1 microsecond or 1us
- $1\text{GHz} = 1/10^9$  seconds or 1 nanosecond or 1ns

# Clock Disciplines

- **Level sensitive:** State changes when clock is high (or low).
- **Edge triggered:** State changes at clock edge.
  - Positive edge-triggered
  - Negative edge-triggered

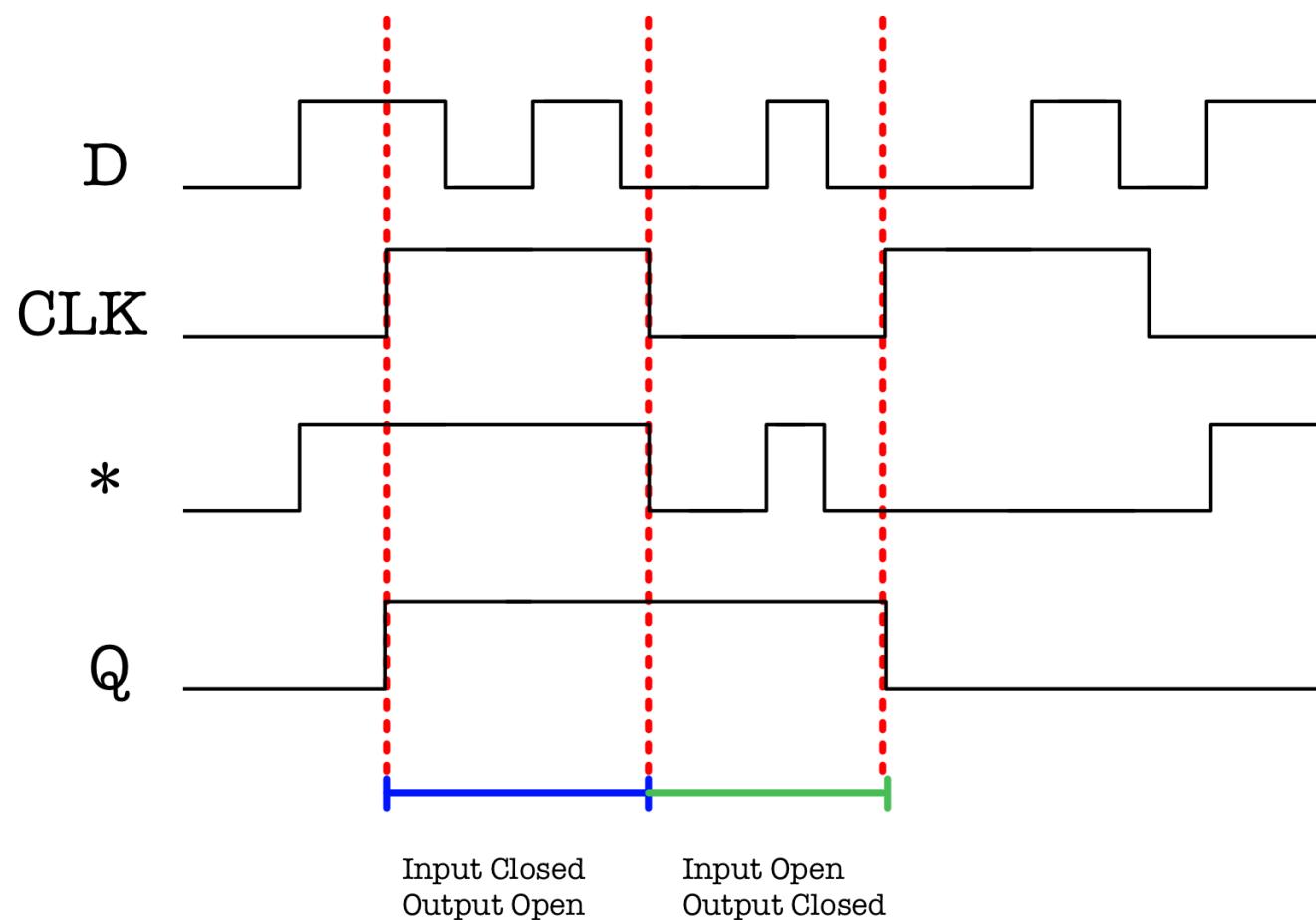
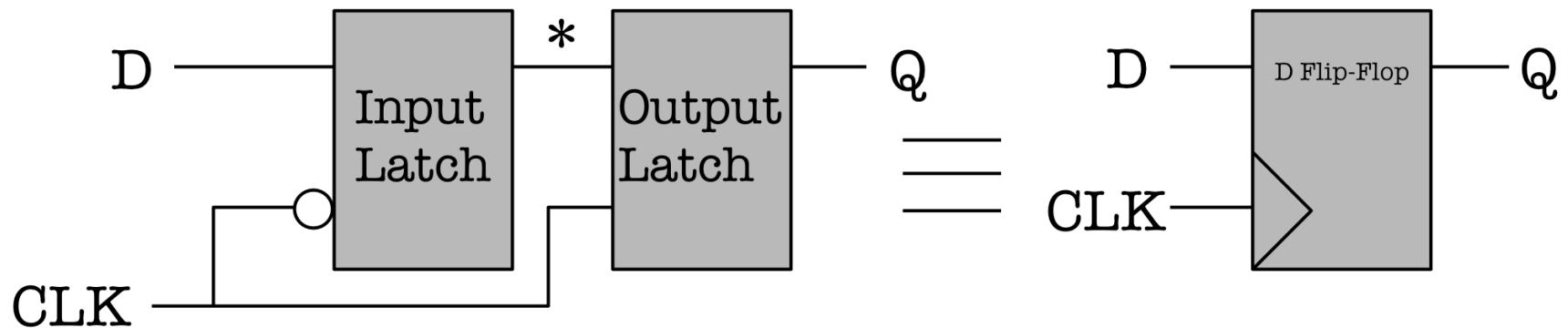
# **Edge-Triggering**

## **Triggering Mechanisms**

- Rising edge: Positive edge-triggered
- Falling edge: Negative edge-triggered

## **Requirements**

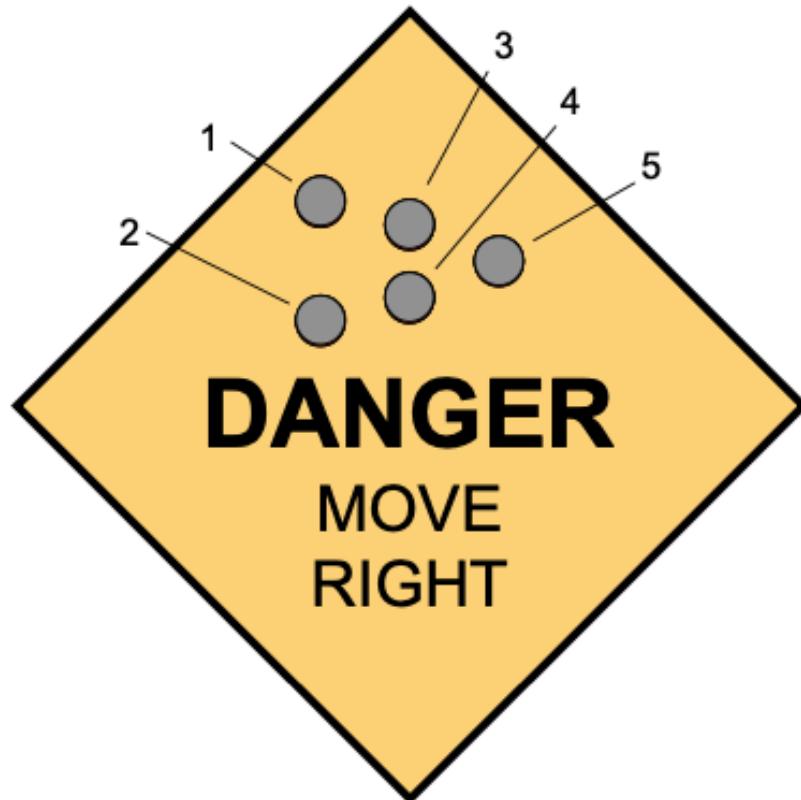
- Inputs must be stable just before the triggering edge.



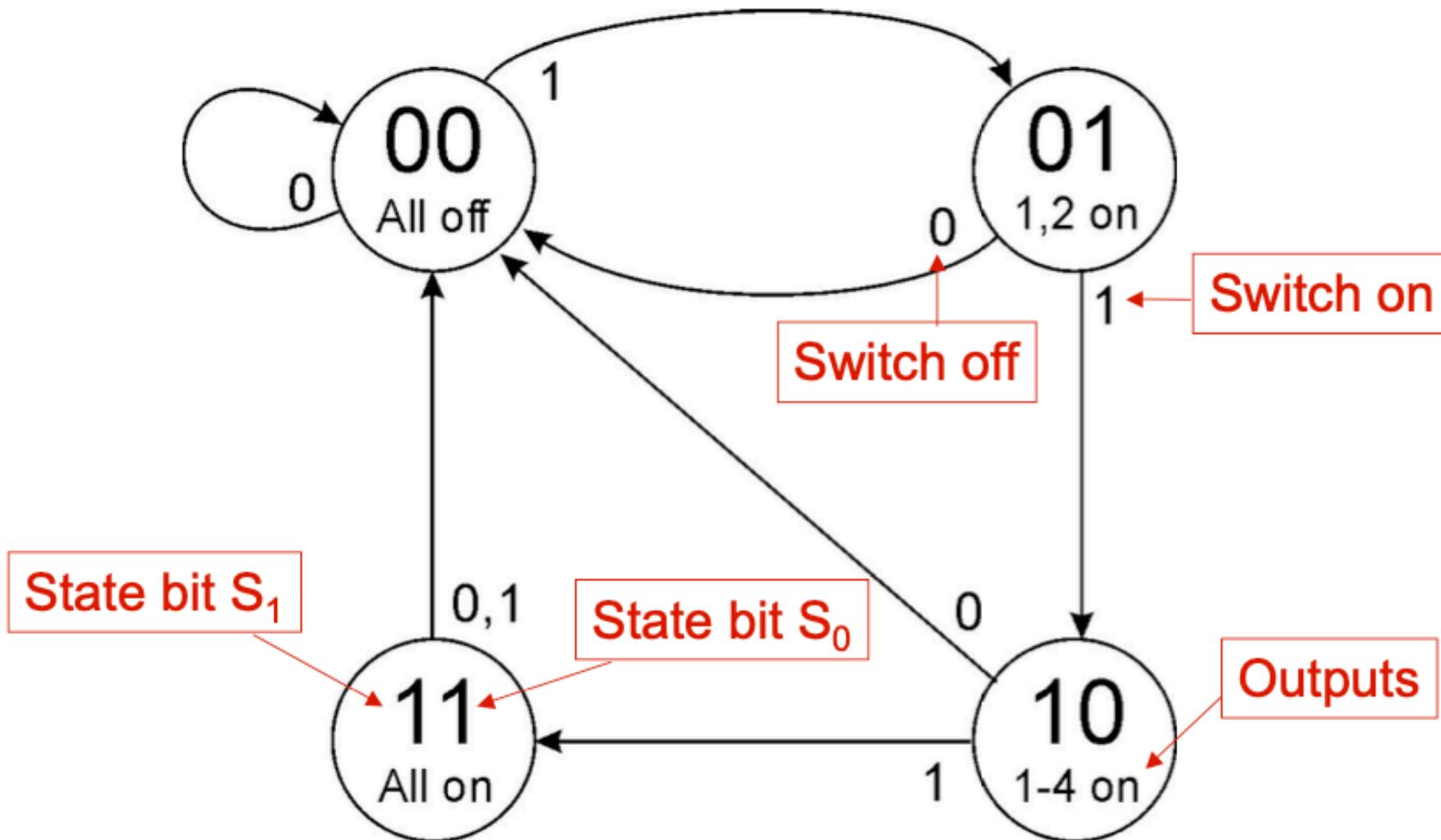
## Complete Example

### A blinking traffic sign

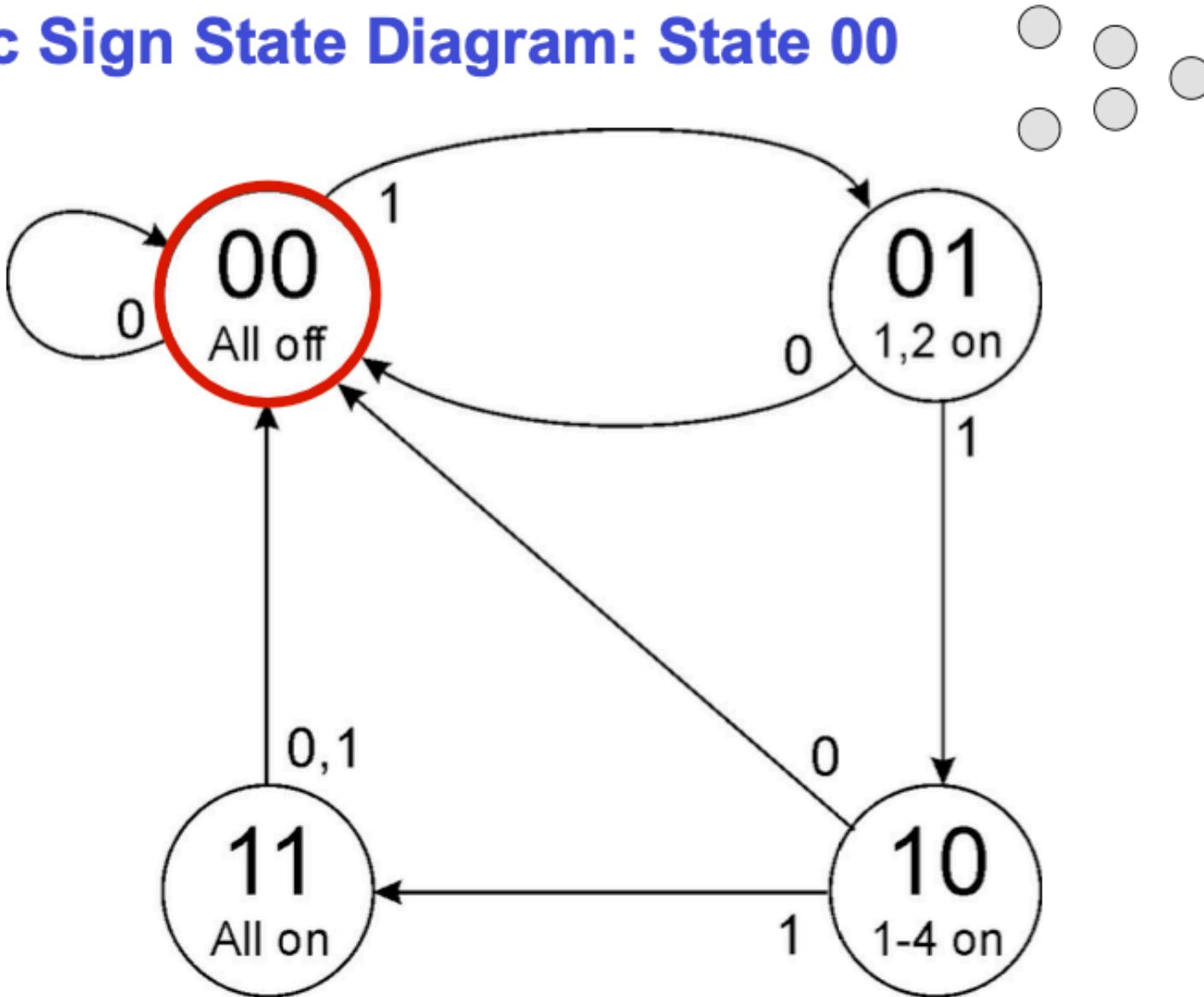
- No lights on
- 1 & 2 on
- 1, 2, 3, & 4 on
- 1, 2, 3, 4, & 5 on
- (repeat as long as switch is turned on)



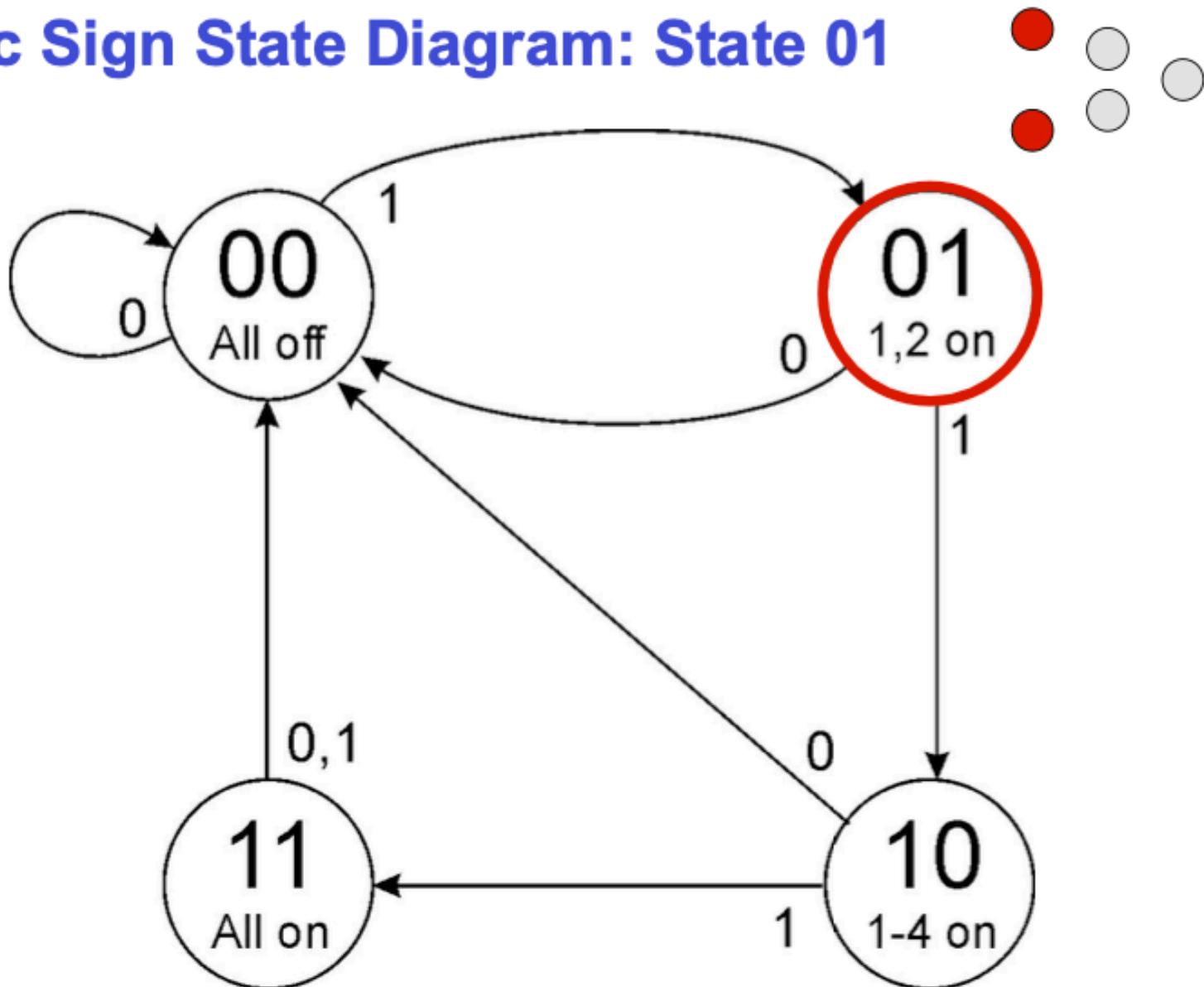
# Traffic Sign State Diagram



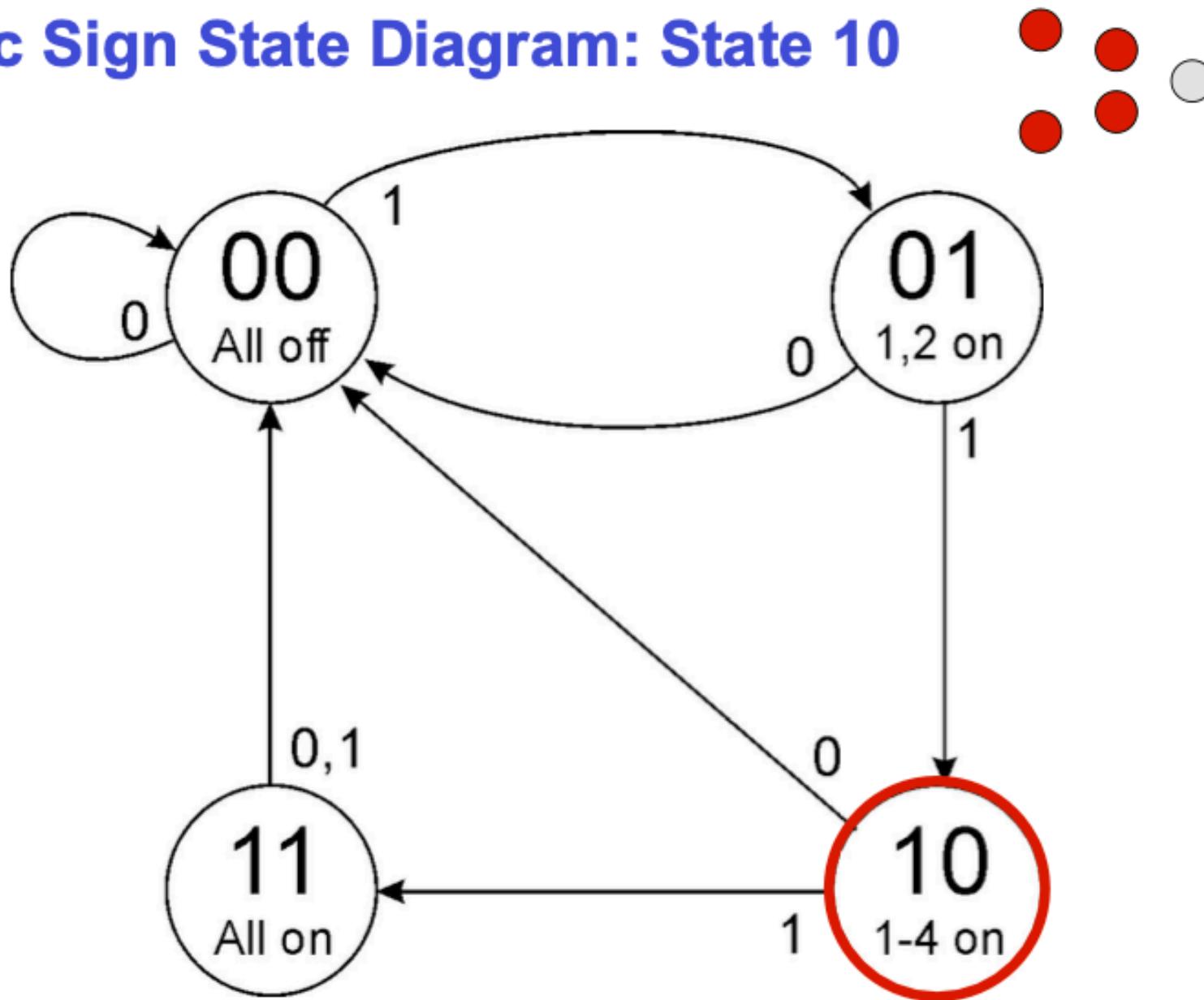
## Traffic Sign State Diagram: State 00



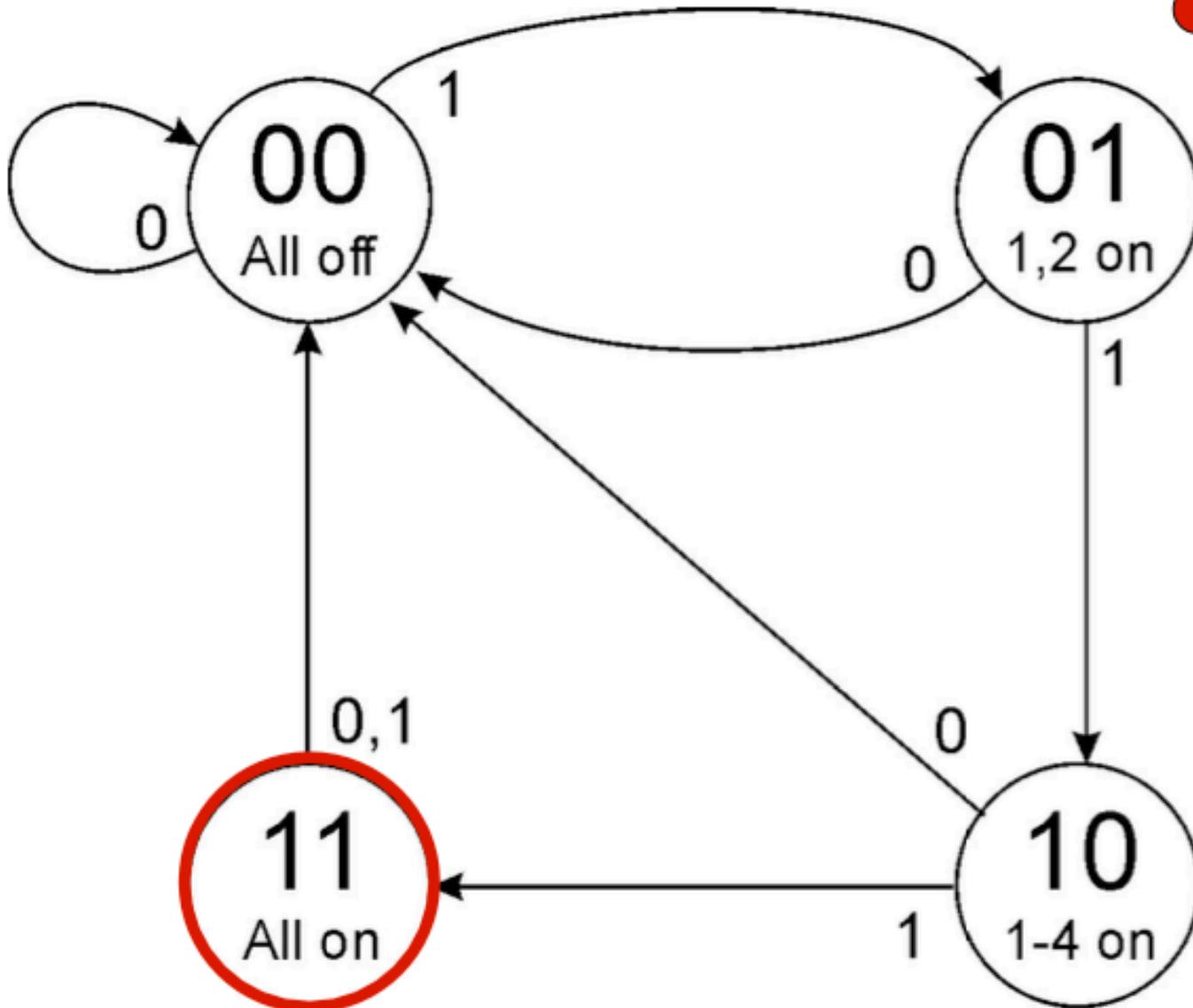
## Traffic Sign State Diagram: State 01



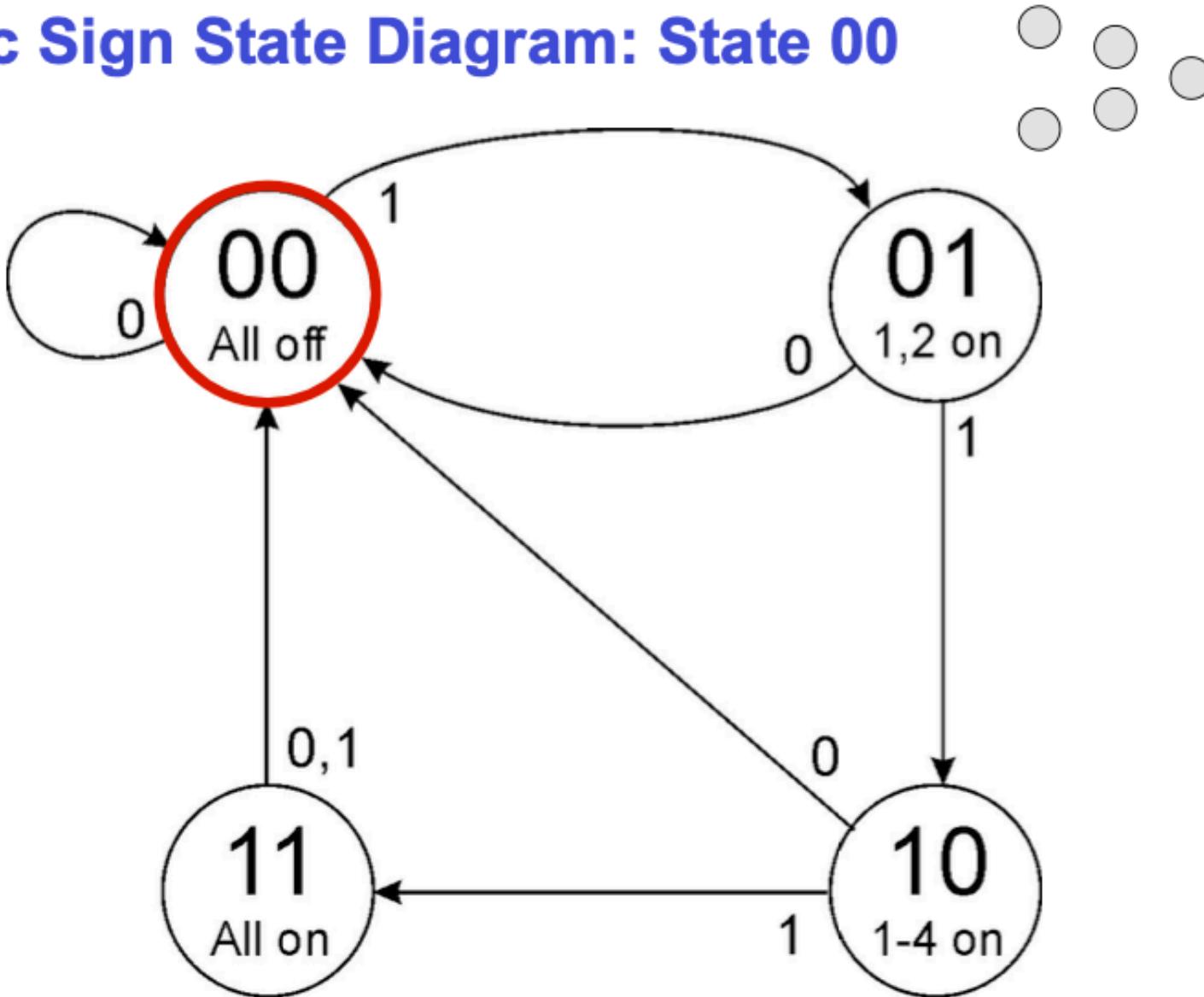
## Traffic Sign State Diagram: State 10



# Traffic Sign State Diagram: State 11



## Traffic Sign State Diagram: State 00



# Traffic Sign Truth Tables

Outputs  
(depend only on state:  $S_1 S_0$ )

$S_1$	$S_0$	Z	Y	X
0	0	0	0	0
0	1	1	0	0
1	0	1	1	0
1	1	1	1	1

Lights 1 and 2  
Lights 3 and 4  
Light 5

Don't care  
(1 or 0)

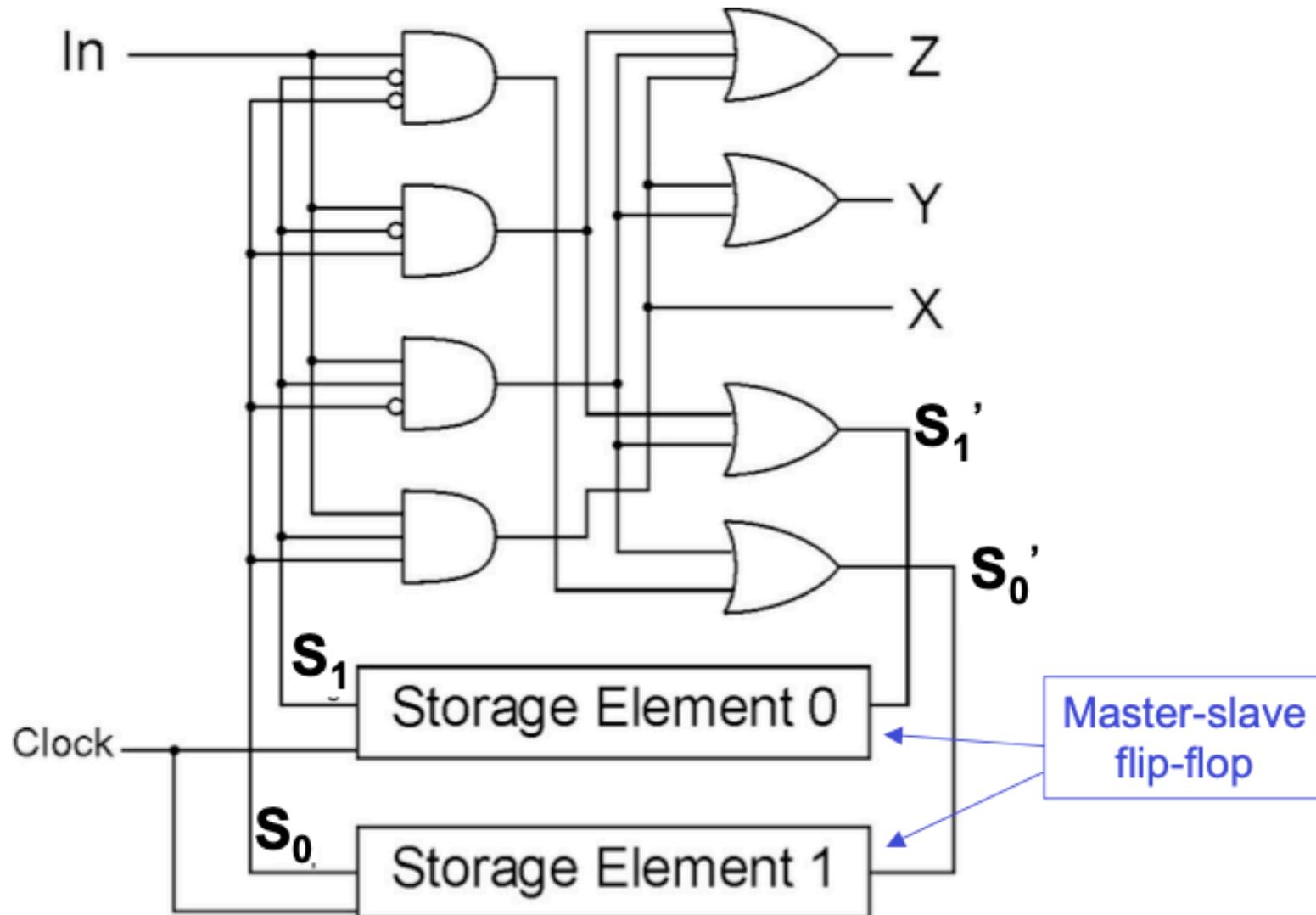
Next State:  $S'_1 S'_0$   
(depend on state and input)

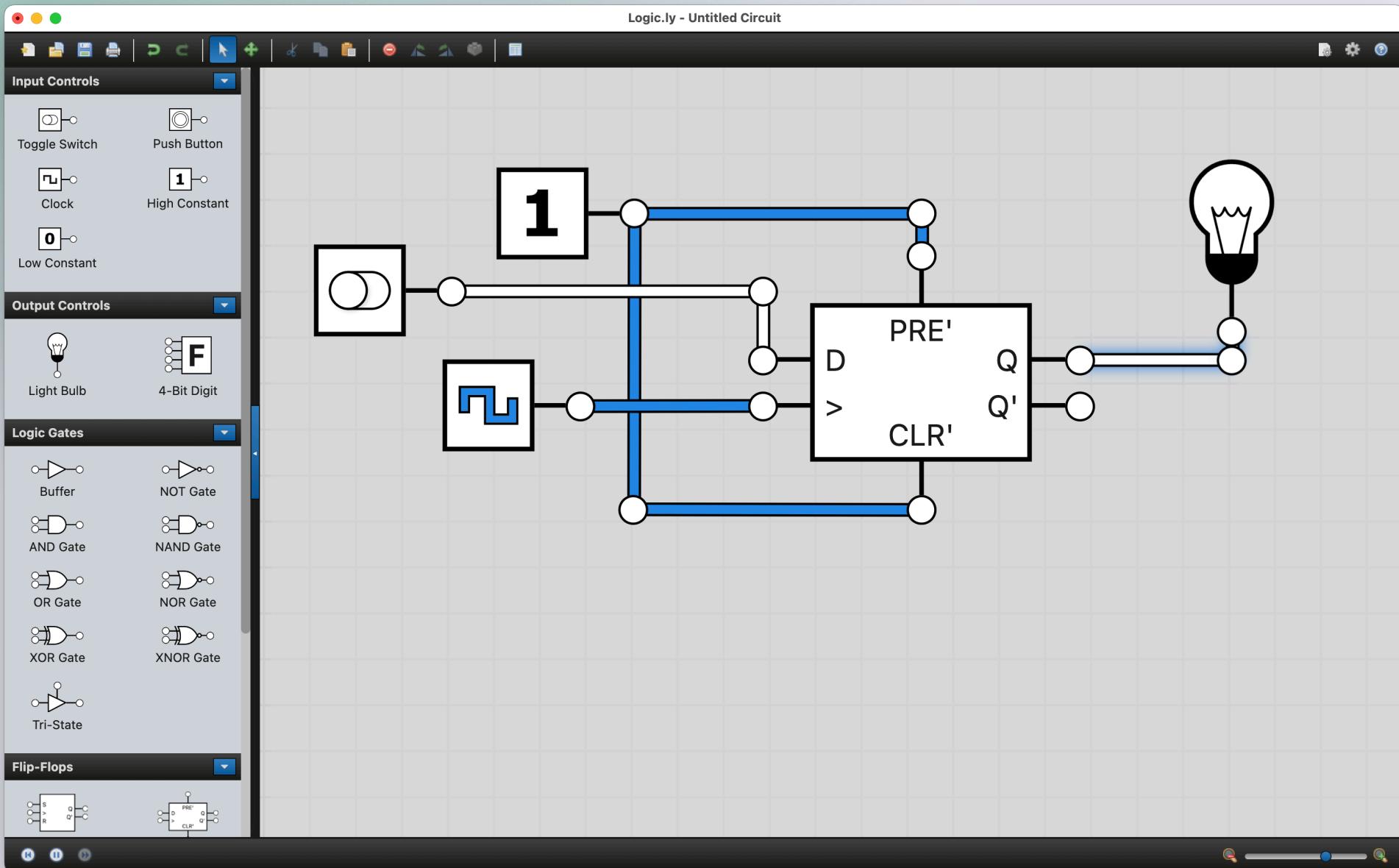
In	$S_1$	$S_0$	$S'_1$	$S'_0$
0	x	x	0	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	1
1	1	1	0	0

Switch

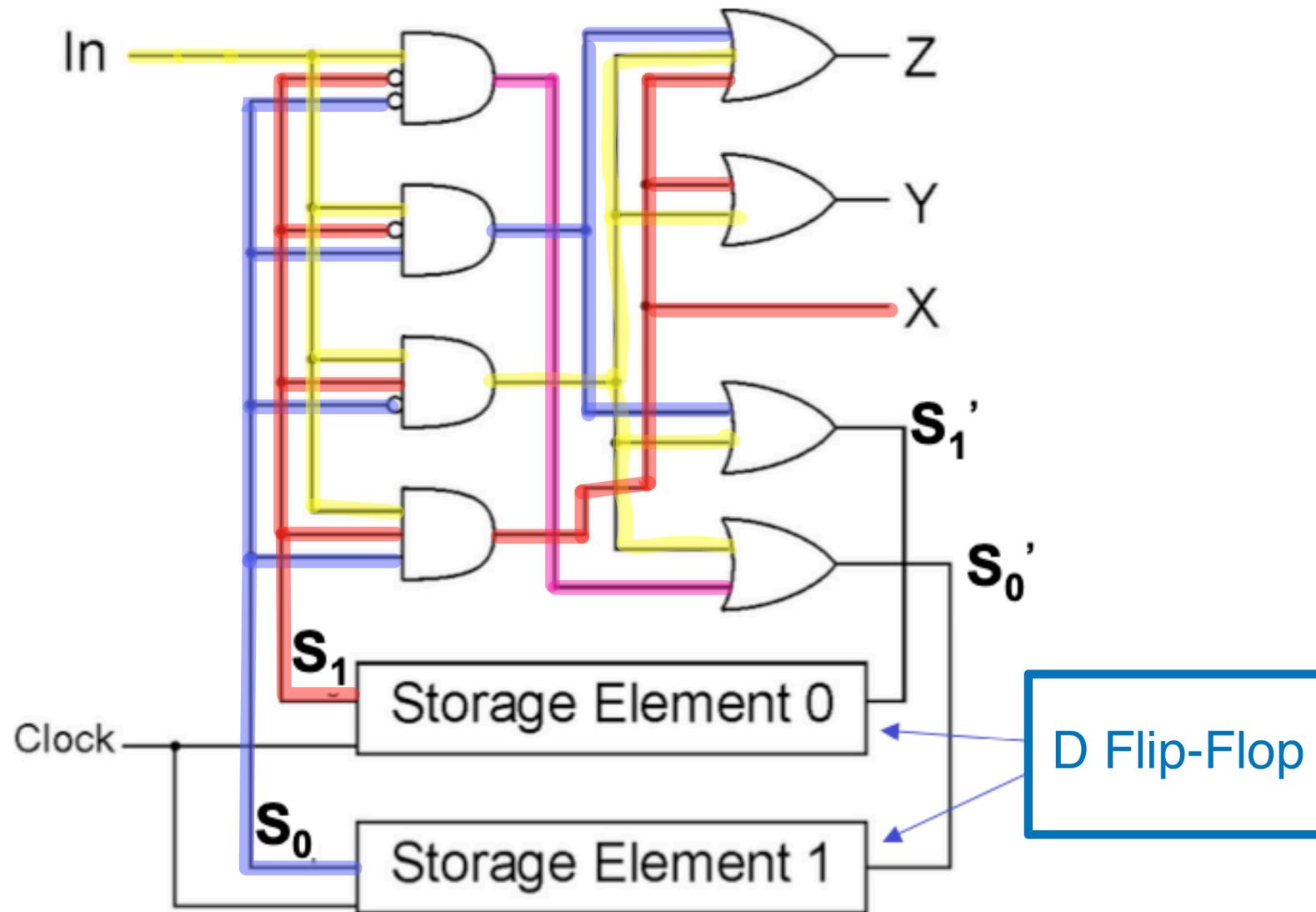
Whenever In=0, next state is 00.

# Traffic Sign Logic





# Traffic Sign Logic



## Credits:

Patt and Patel: "*Introduction to Computing Systems*"

AspenCore: "*Sequential Logic*"

UPenn CSE240 Fall 2006 Fassell: "*Chapter 3 Lecture*"

Sequential Logic and Synchronization [CS 50.002](#)

