

Lecture 3b: Finite State Machines, Sequential Logic and the Clock

CSCI11: Computer Architecture and Organization

Chapter 3.4-3.6 in [Patt and Patel: Introduction to Computing Systems...](#)

- Quick Combinational Circuits Review
- Finite State Machines
- Sequential Logic
- Memory
- The Clock

1 to 4 Demultiplexer

- data distributor
- one input to 1 of 4 outputs
- used to light multiple segments of an LED digit display
- not used in the LC3

FIGURE 13. 4-CHANNEL DEMULTIPLEXER CIRCUIT

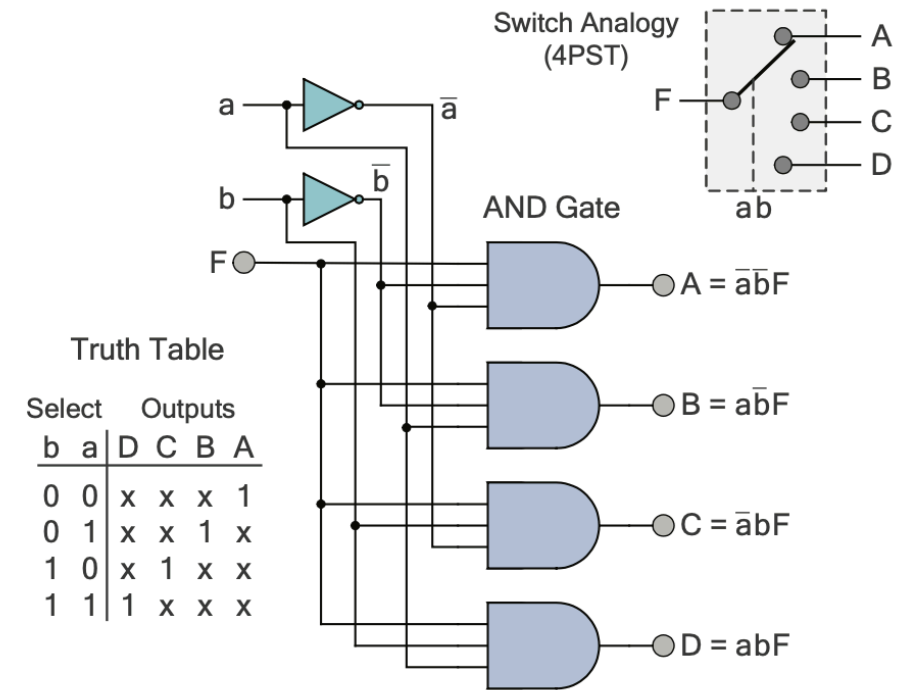
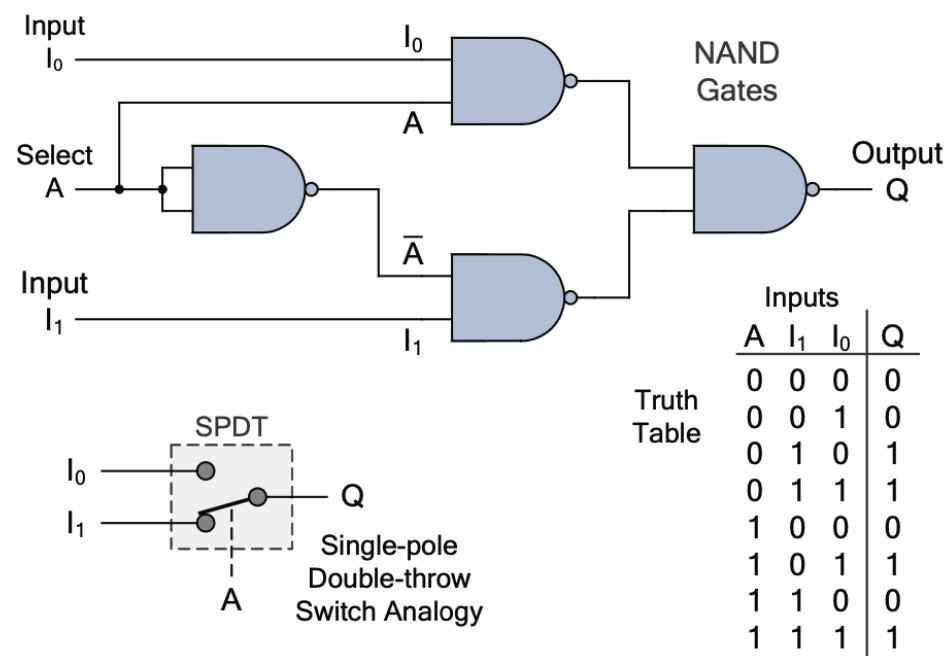


FIGURE 9. BASIC 2-INPUT MULTIPLEXER USING NAND GATES



2-input Multiplexer

- data selector
- 1 of 2 inputs to one output
- use to select between multiple sets of address/data
- in the LC3, address lines pass through multiple muxes

2-4 Decoder

- **data selector**
- 2^n bits to select one of n lines
- use to select a register in a register file
- in the LC3, opcode's 3 bits select registers R0-R7

FIGURE 3. A 2-TO-4 BINARY DECODER

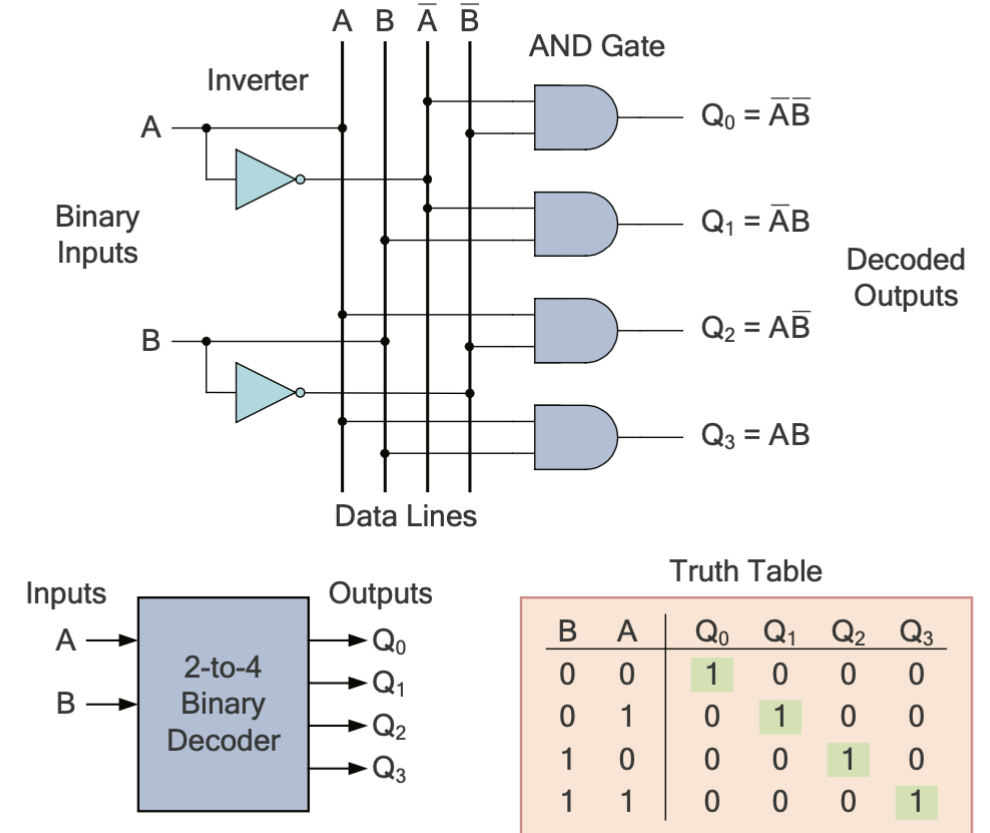
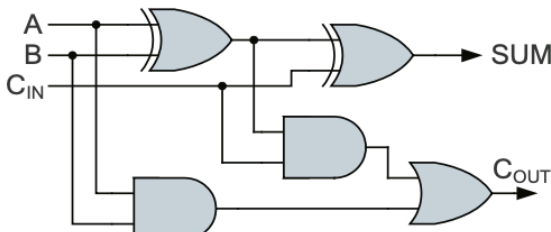
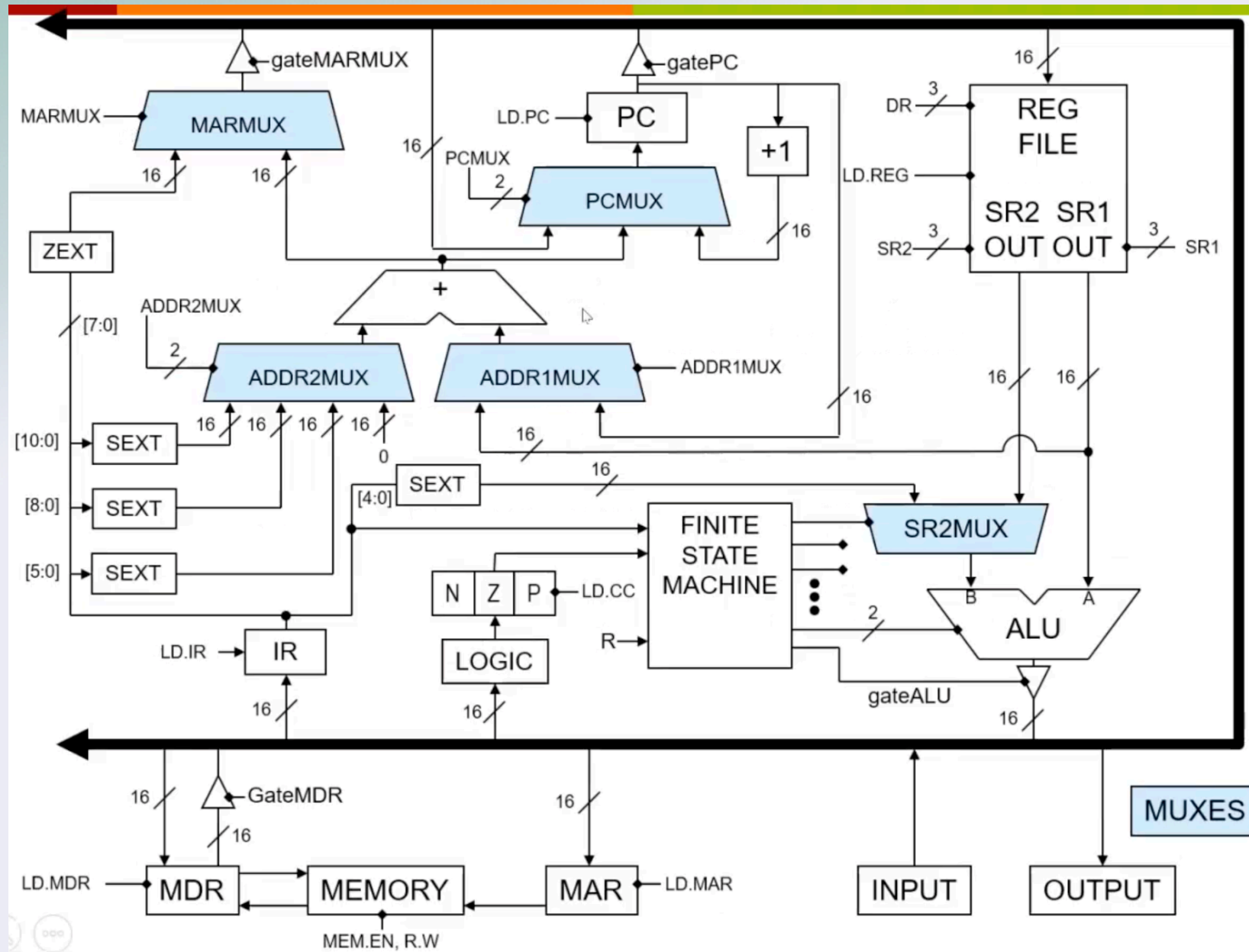


FIGURE 21. FULL-ADDER AND TRUTH TABLE

Symbol	Truth Table				
	Carry-In	B	A	SUM	Carry-Out
	0	0	0	0	0
	0	0	1	1	0
	0	1	0	1	0
	0	1	1	0	1
	1	0	0	1	0
	1	0	1	0	1
	1	1	0	0	1
	1	1	1	1	1

2 bit Full Adder

- **n bit adder**
- counts up to 2^n bits or divides by 2^n bits on Cout
- use as a program counter (**PC**) or a clock divider
- LC3 uses a 16bit full adder as a program counter

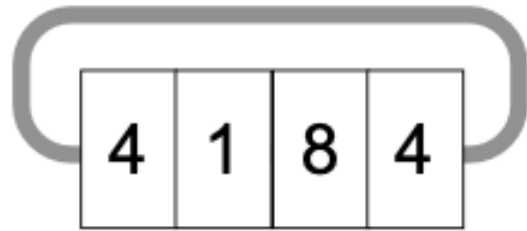


Combinational Logic Circuits

By definition, combinational logic circuits depend on their inputs and have no knowledge of previous events. That is, they are memory-less.

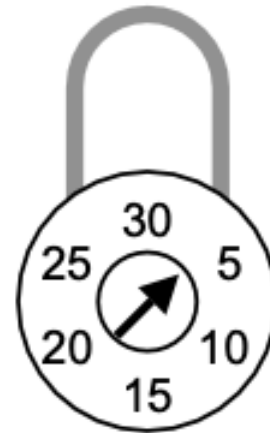
Combinational vs. Sequential

Two types of “combination” locks



Combinational

Success depends only on the **values**, not the order in which they are set.



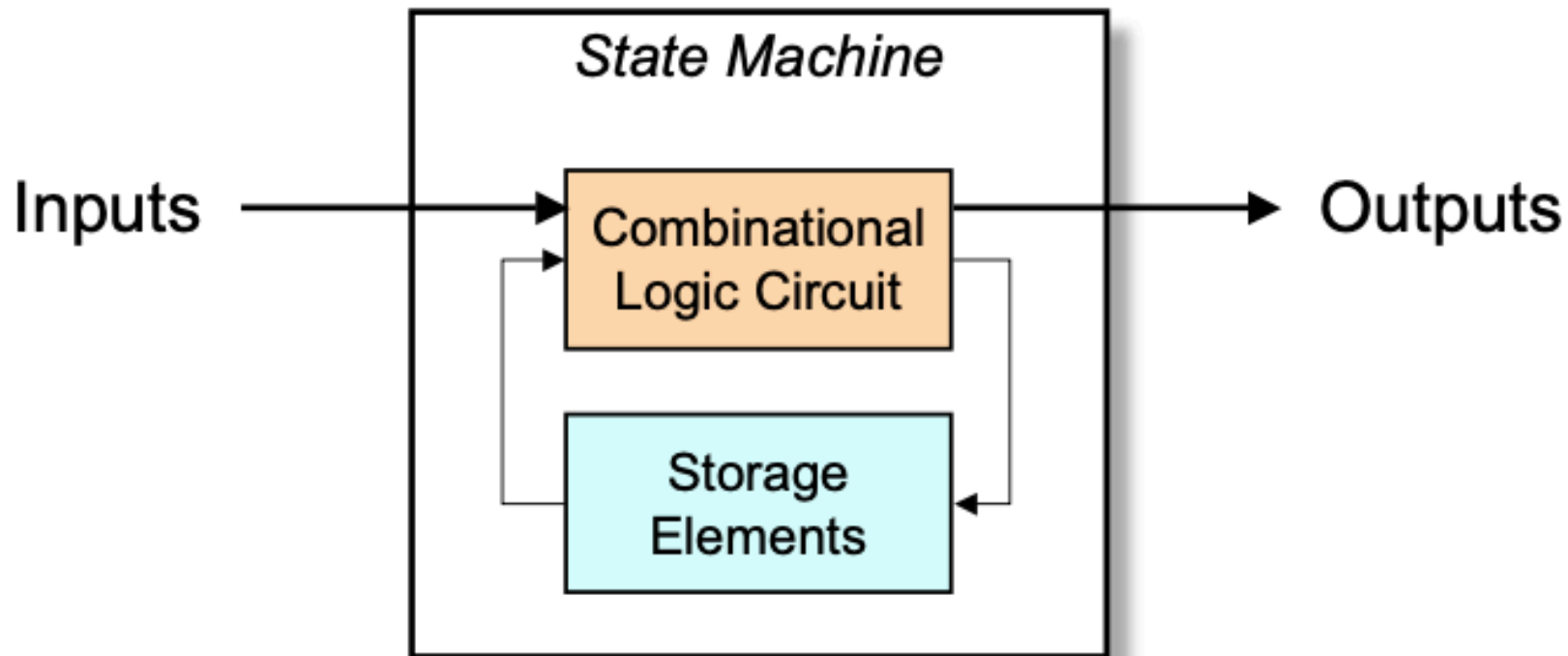
Sequential

Success depends on the **sequence** of values (e.g, R-13, L-22, R-3).

State Machine

Another type of sequential circuit

- Combines combinational logic with storage
- “Remembers” state, and changes output (and state) based on **inputs** and **current state**



State

The **state** of system is **snapshot** of **all relevant elements** of system at moment snapshot is taken

Examples

- The state of a basketball game can be represented by the scoreboard
 - Number of points, time remaining, possession, *etc.*
- The state of a tic-tac-toe game can be represented by the placement of X's and O's on the board (and turn)

Finite State Machine

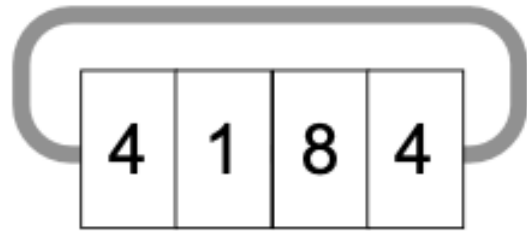
A description of a system with the following components:

- 1. A finite number of **states****
- 2. A finite number of external **inputs****
- 3. A finite number of external **outputs****
- 4. An explicit specification of all **state transitions****
- 5. An explicit specification of what determines each external **output value****

Often described by a state diagram

- Inputs trigger state transitions**
- Outputs are associated with each state (or with each transition)**

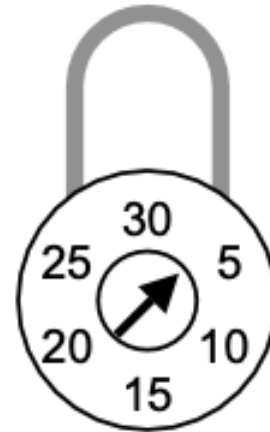
Two State Lock



Combinational

Success depends only on the **values**, not the order in which they are set.

Finite State Machine



Sequential

Success depends on the **sequence** of values (e.g, R-13, L-22, R-3).

State of Sequential Lock

Our lock example has four different states, labeled A-D:

A: The lock is **not open**,
and no relevant operations have been performed

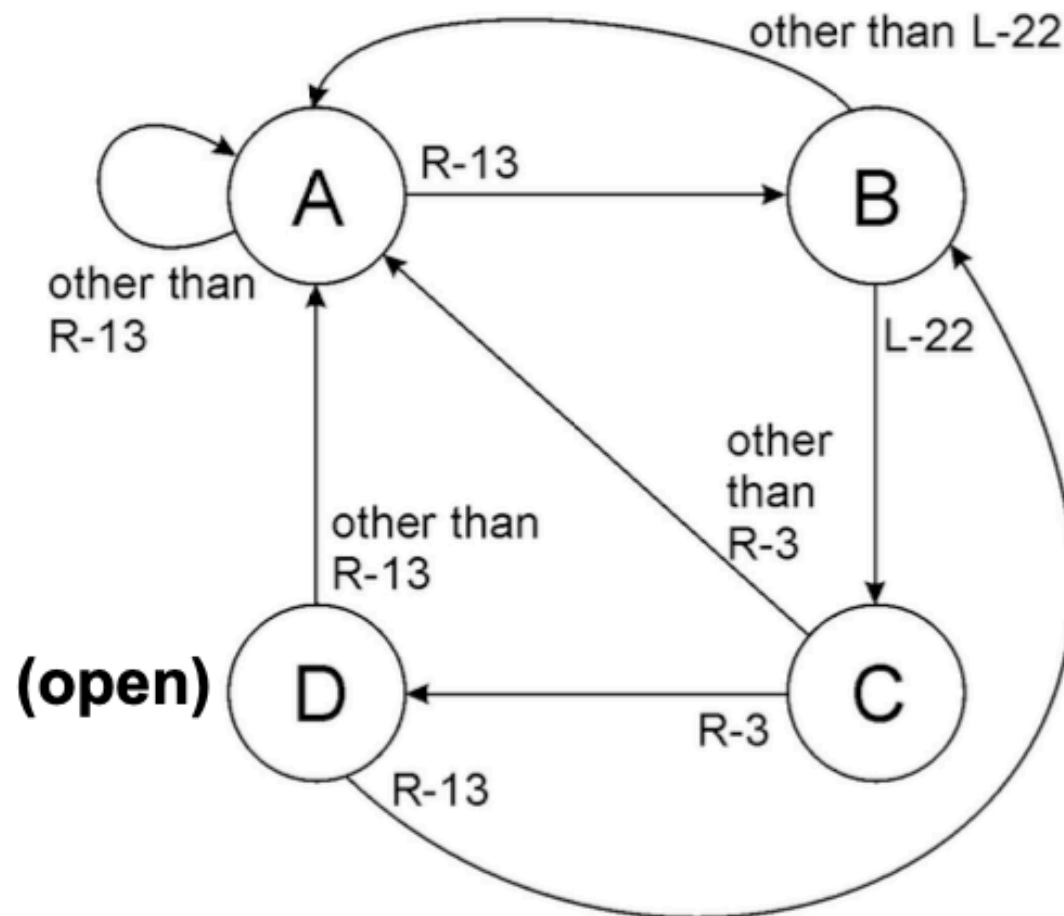
B: The lock is **not open**,
and the user has completed the **R-13** operation

C: The lock is **not open**,
and the user has completed **R-13**, followed by **L-22**

D: The lock is **open**

Sequential Lock State Diagram

Shows **states** and **actions** that cause a **transition** between states



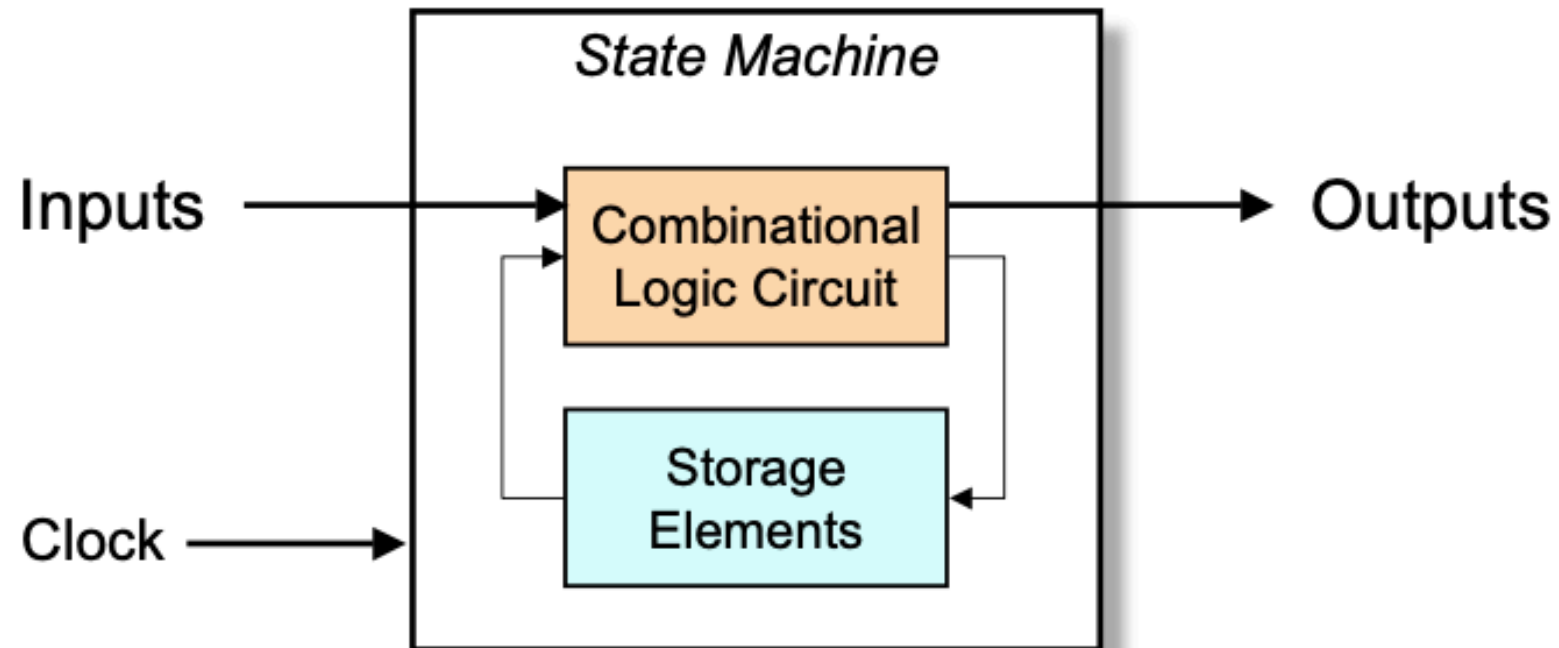
Implementing a Finite State Machine

Combinational logic

- Determine outputs and next state.

Storage elements

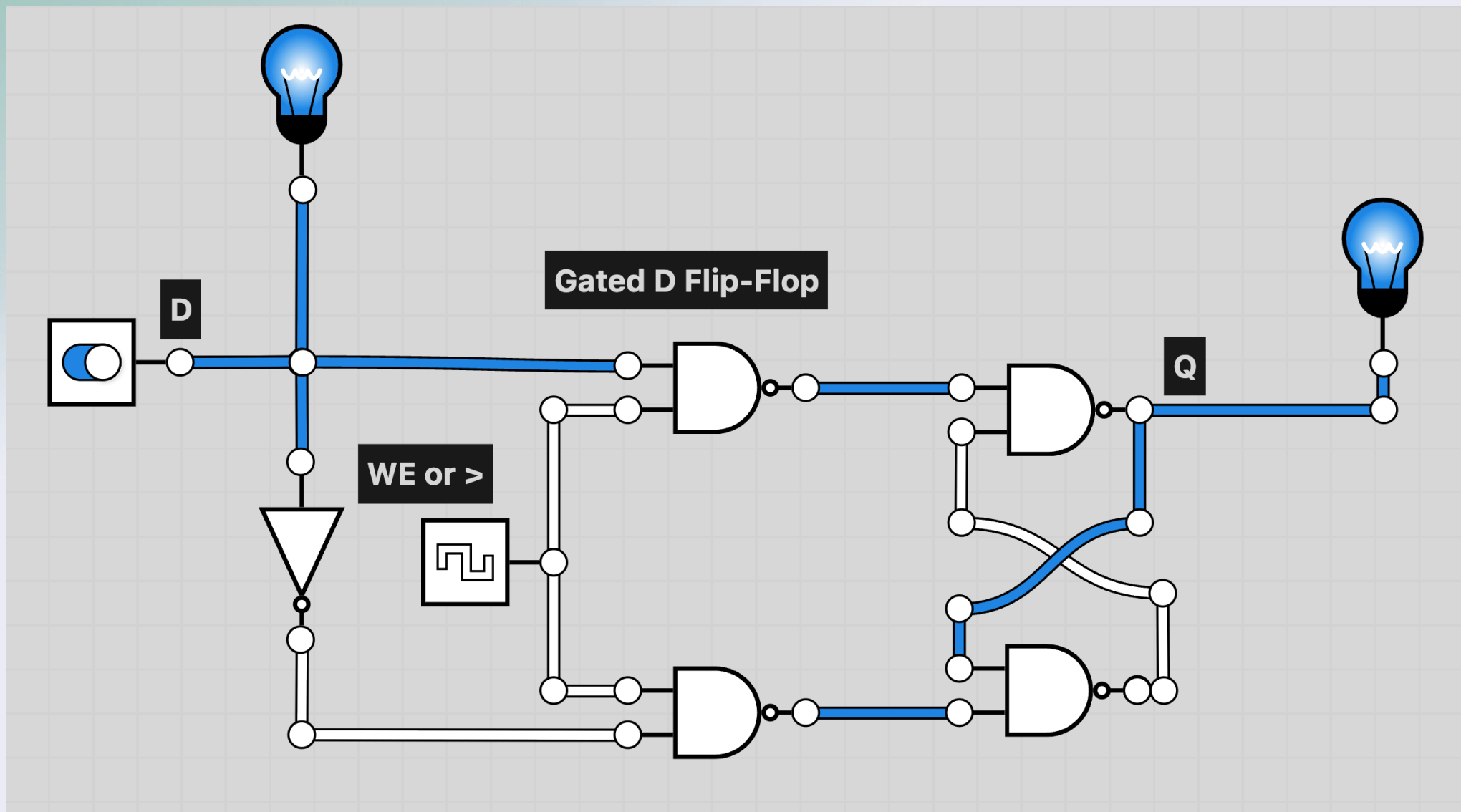
- Maintain state representation.

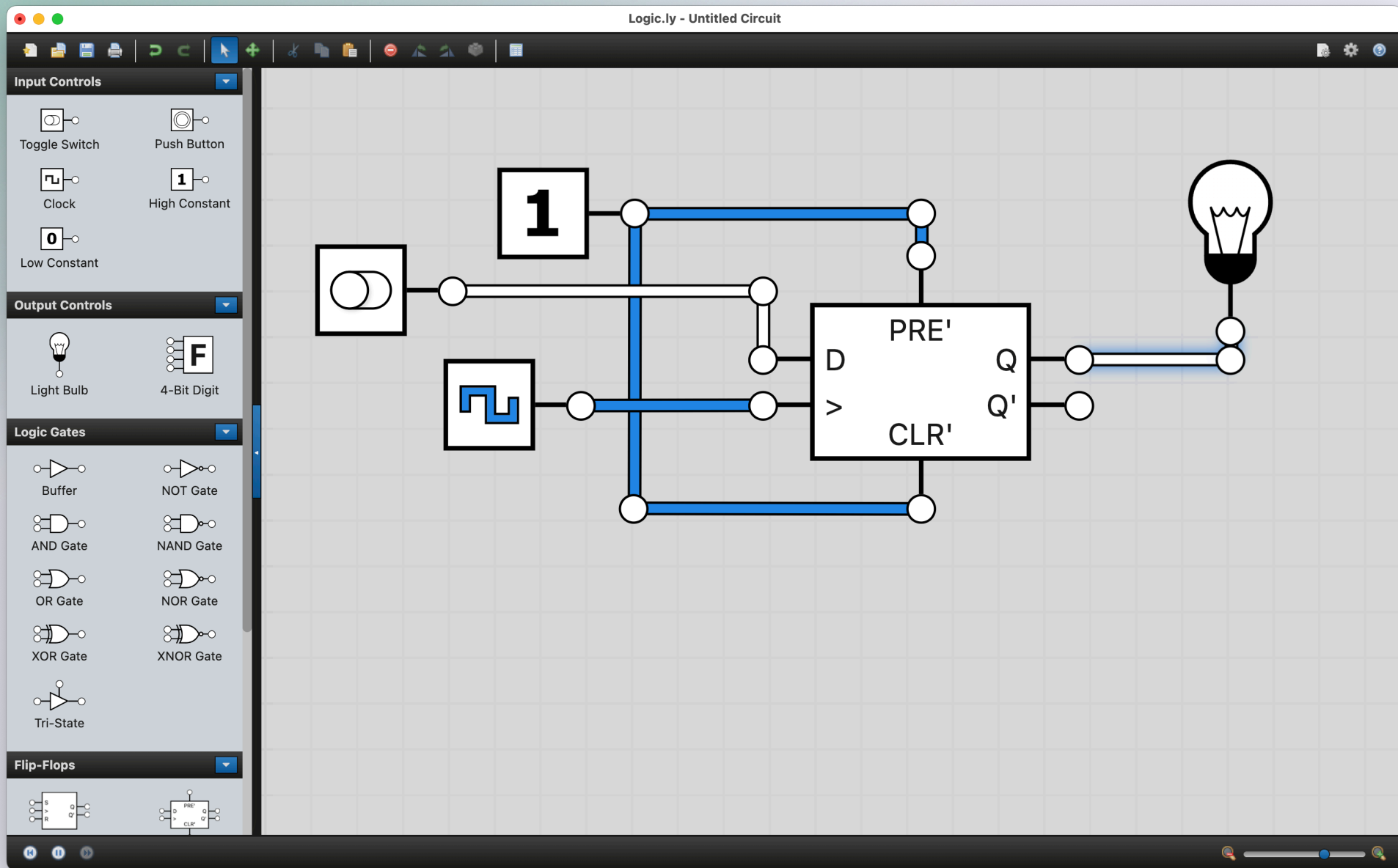


Why is a Clock Needed?

1. To ensure there is a time element to the circuit
2. To ensure things are synchronized

See D Latch as an example





Clocks

Clock Characteristics

- Usually generated by an oscillating crystal
- Fixed period
- Frequency = $1/\text{period}$
- 1Mhz = $1/10^6$ seconds or 1 microsecond or 1us
- 1GHz = $1/10^9$ seconds or 1 nanosecond or 1ns

Clock Disciplines

- **Level sensitive:** State changes when clock is high (or low).
- **Edge triggered:** State changes at clock edge.
 - Positive edge-triggered
 - Negative edge-triggered

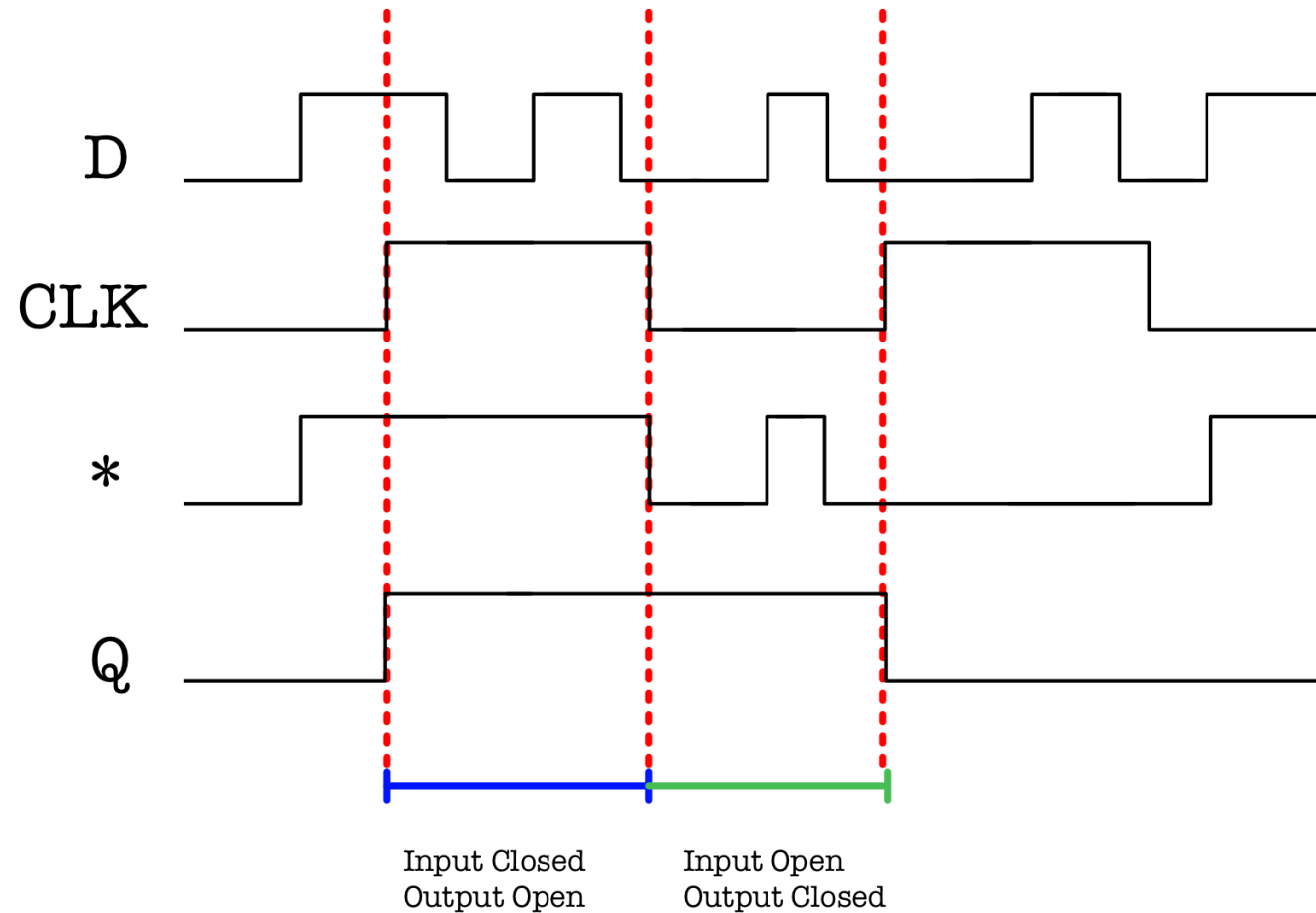
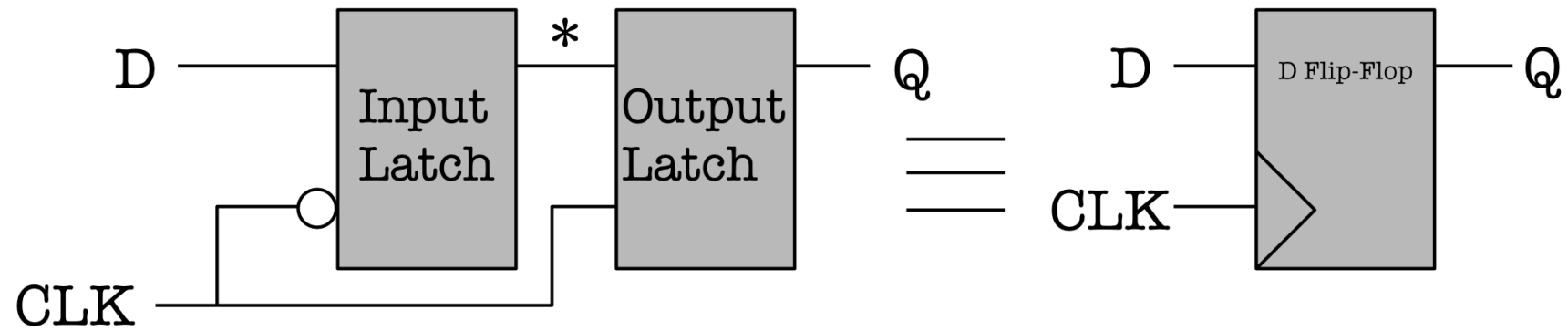
Edge-Triggering

Triggering Mechanisms

- Rising edge: Positive edge-triggered
- Falling edge: Negative edge-triggered

Requirements

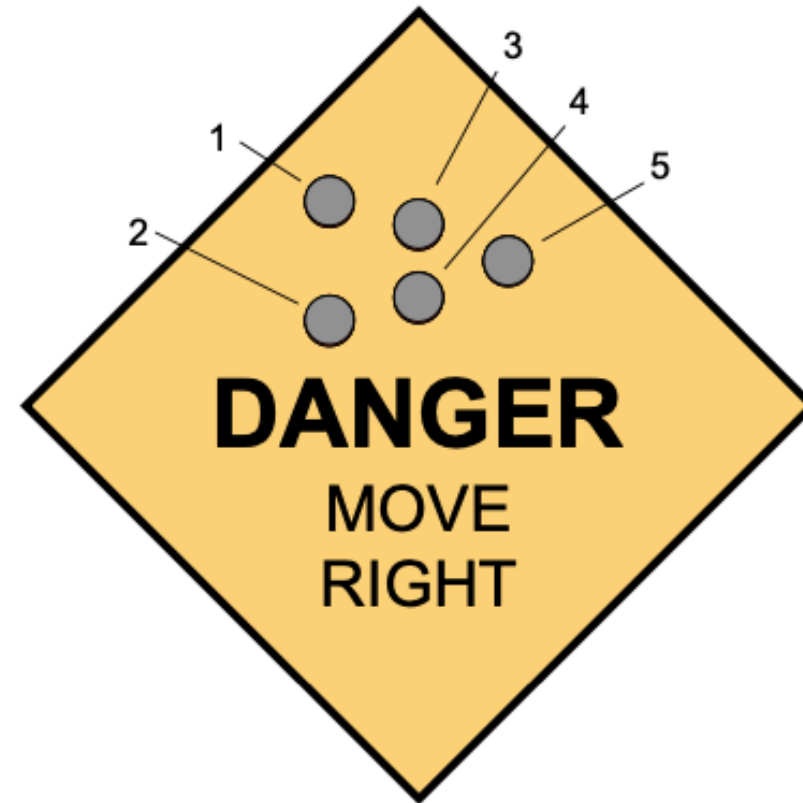
- Inputs must be stable just before the triggering edge.



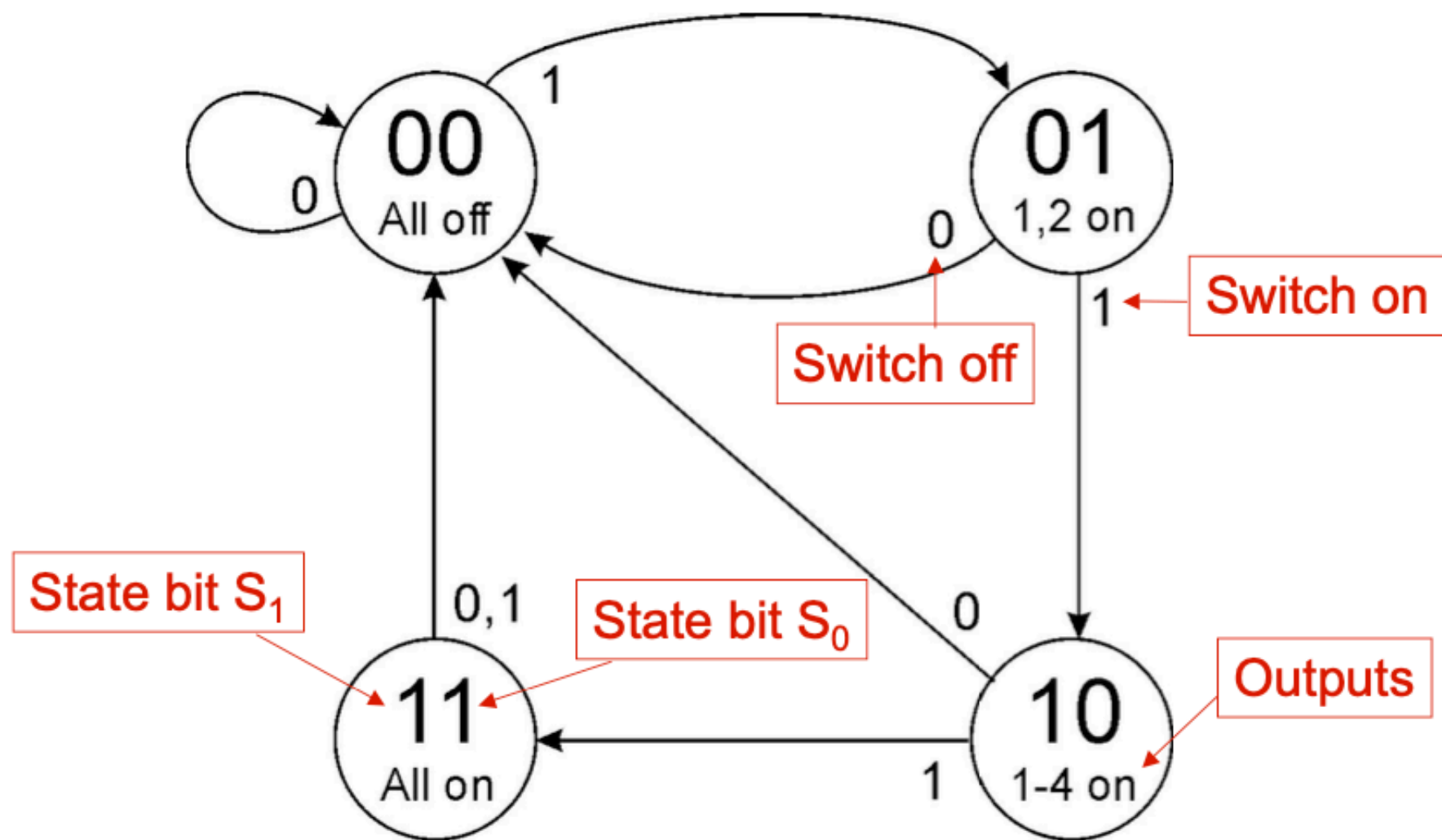
Complete Example

A blinking traffic sign

- No lights on
- 1 & 2 on
- 1, 2, 3, & 4 on
- 1, 2, 3, 4, & 5 on
- (repeat as long as switch is turned on)



Traffic Sign State Diagram

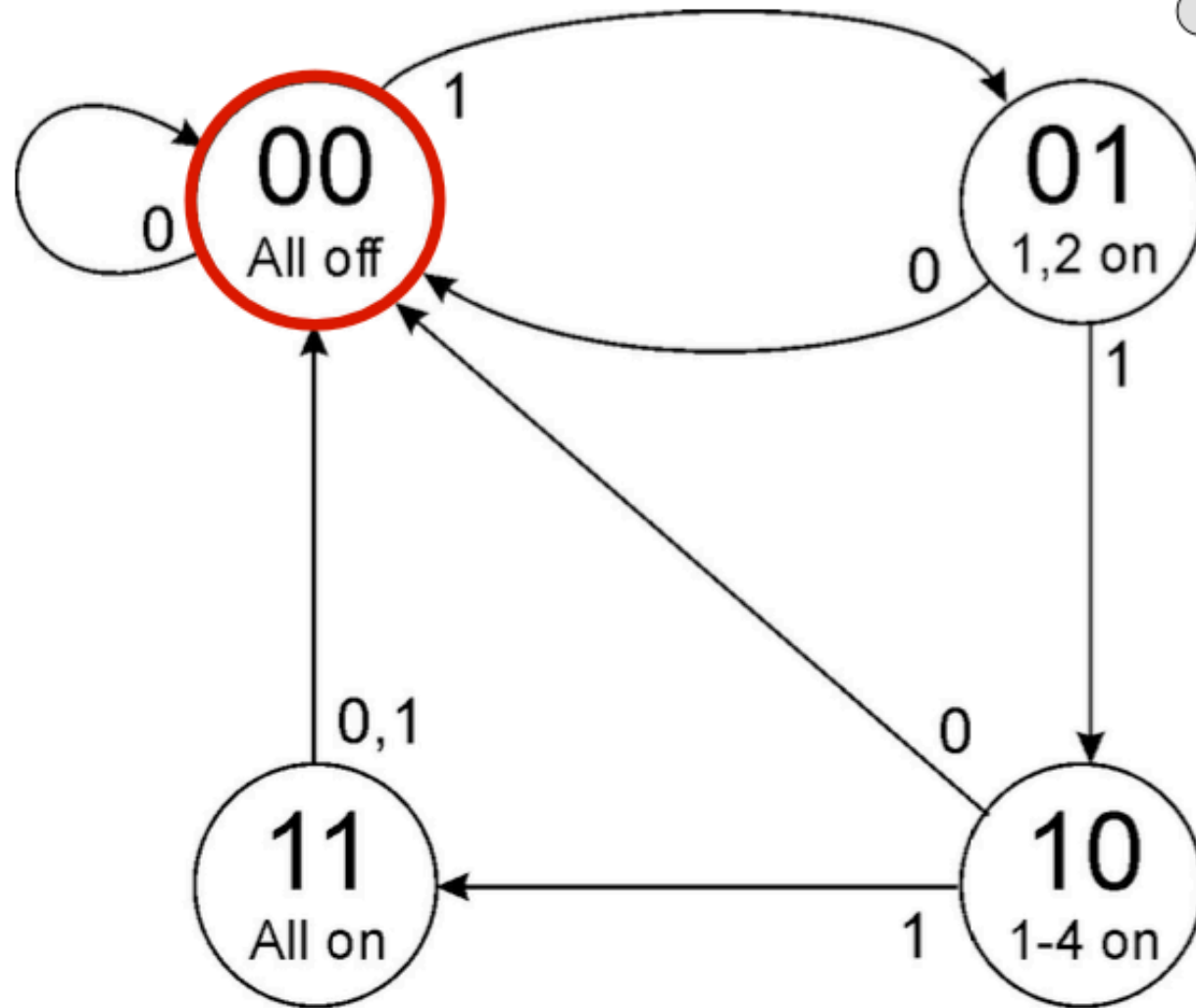


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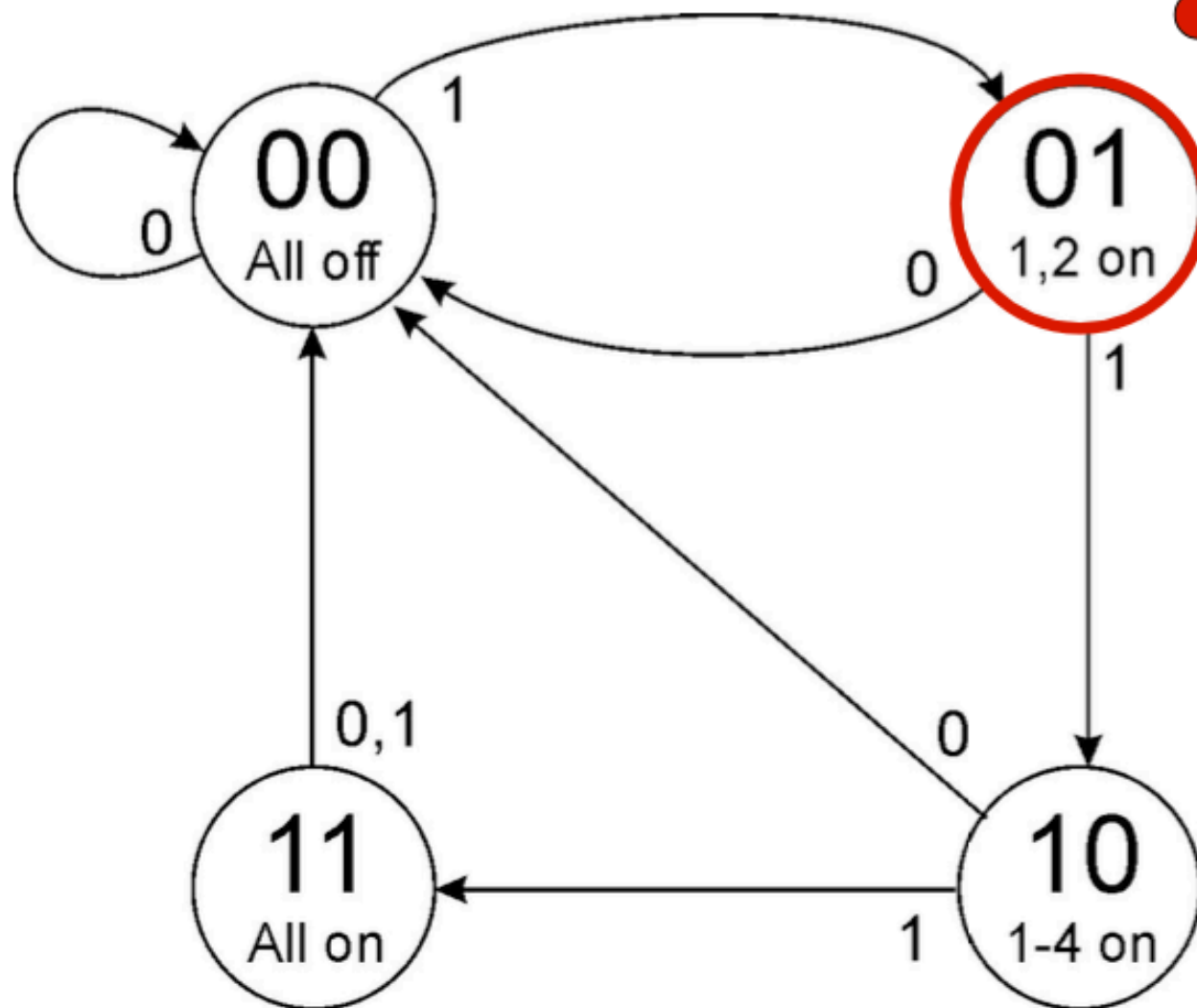
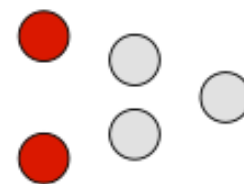
Transition on each clock cycle.

3-98

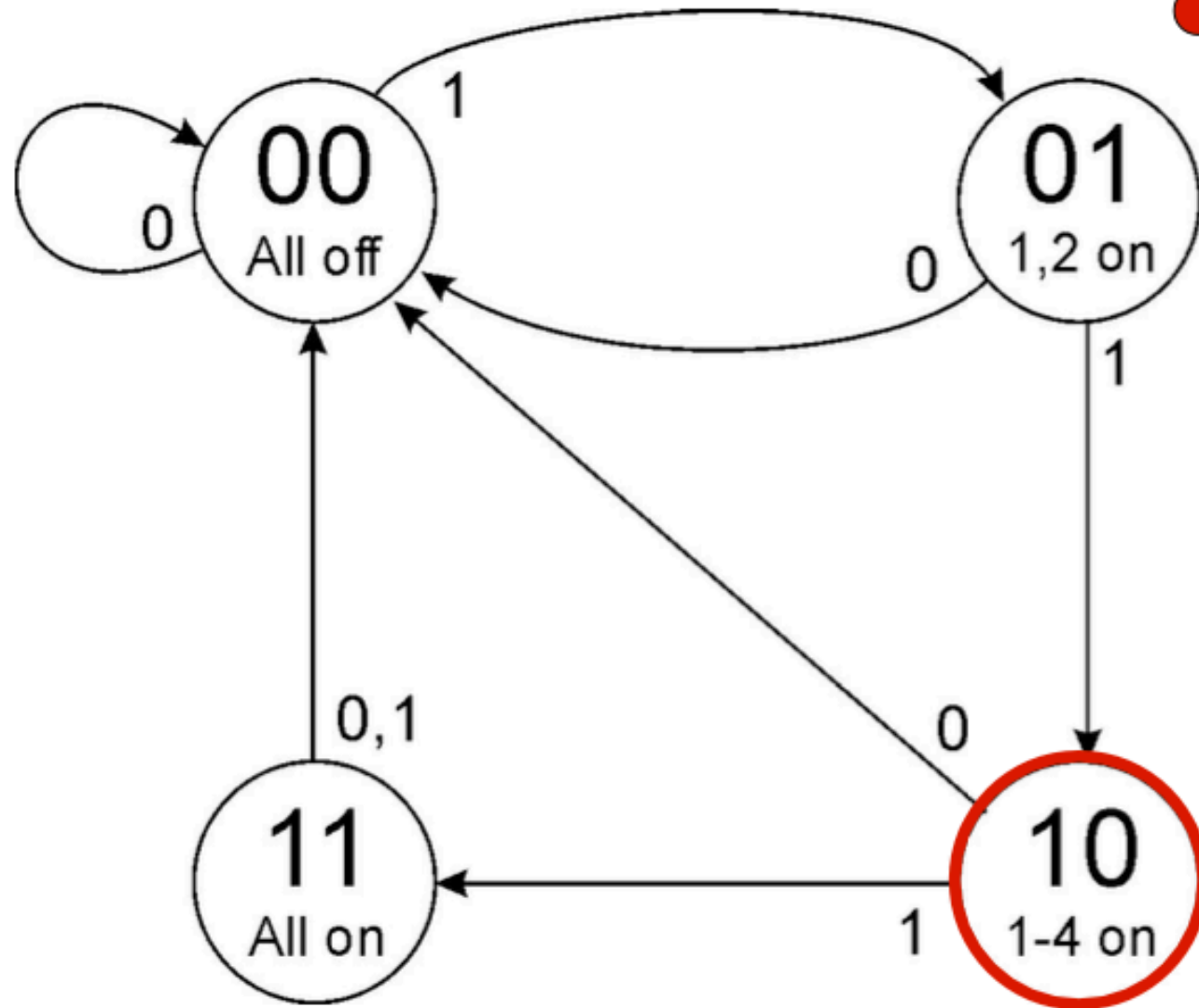
Traffic Sign State Diagram: State 00



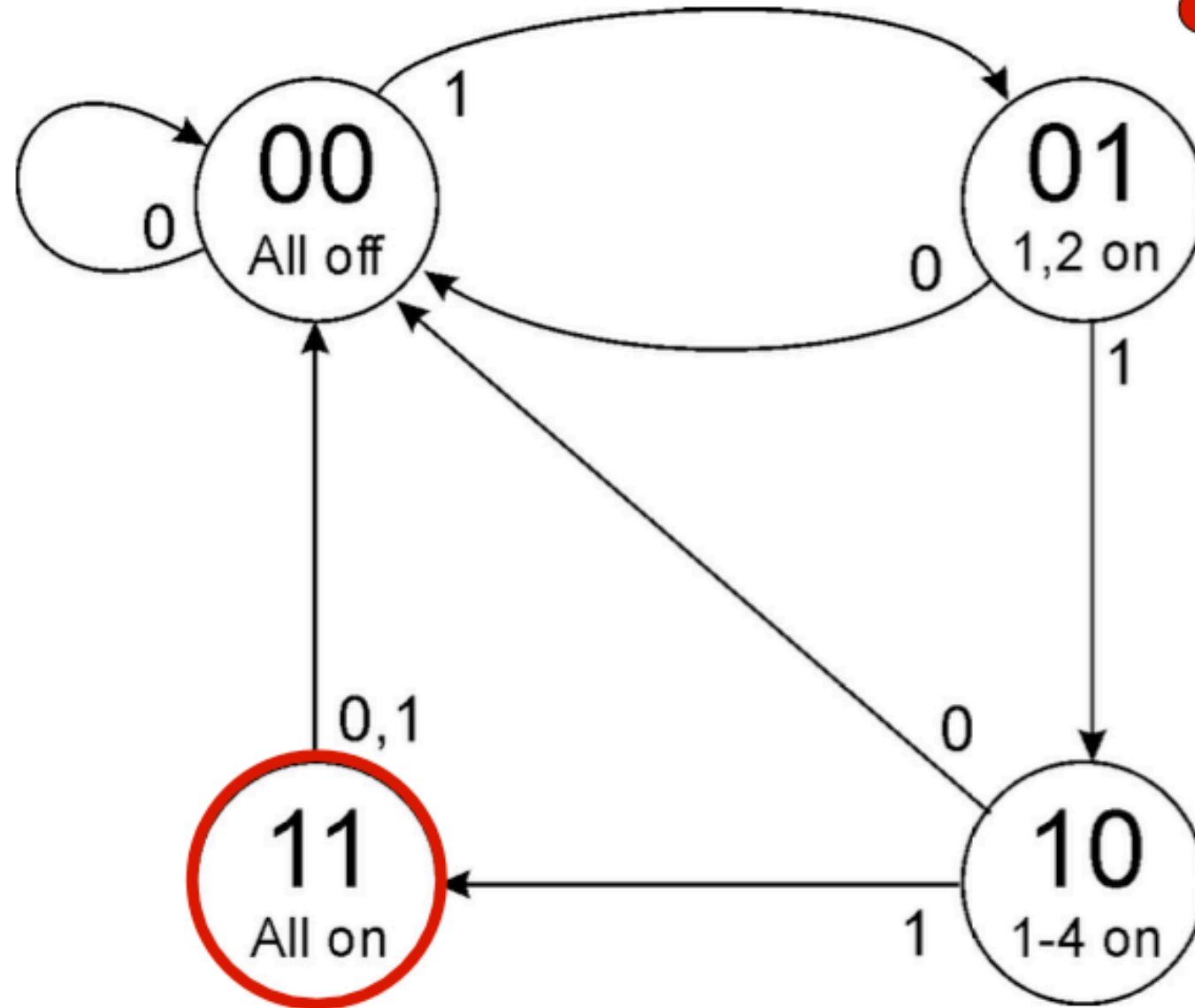
Traffic Sign State Diagram: State 01



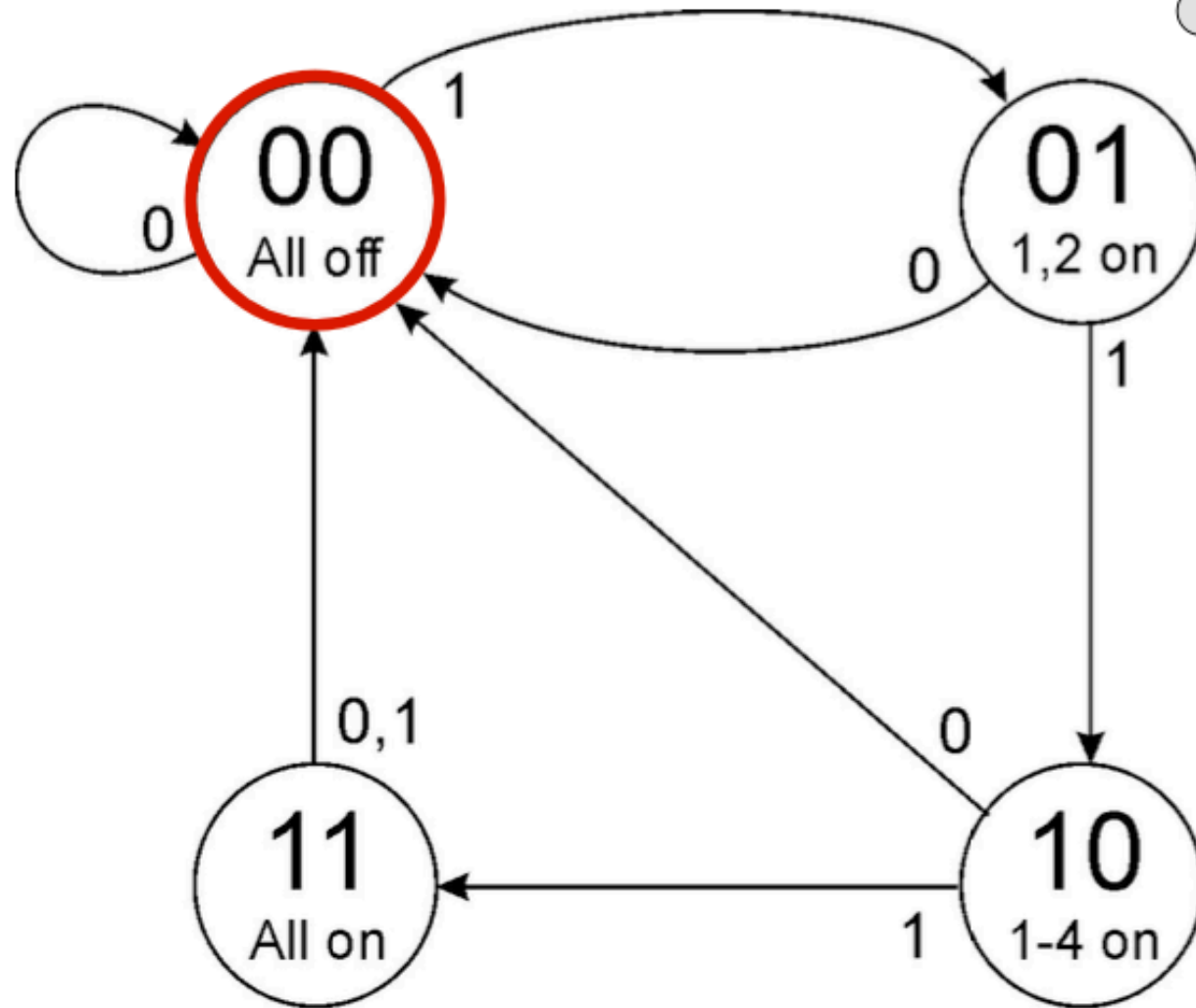
Traffic Sign State Diagram: State 10



Traffic Sign State Diagram: State 11



Traffic Sign State Diagram: State 00



Traffic Sign Truth Tables

Outputs
(depend only on state: S_1S_0)

S_1	S_0	Z	Y	X
0	0	0	0	0
0	1	1	0	0
1	0	1	1	0
1	1	1	1	1

Lights 1 and 2 → Z
 Lights 3 and 4 → Y
 Light 5 → X

Next State: $S_1'S_0'$
(depend on state and input)

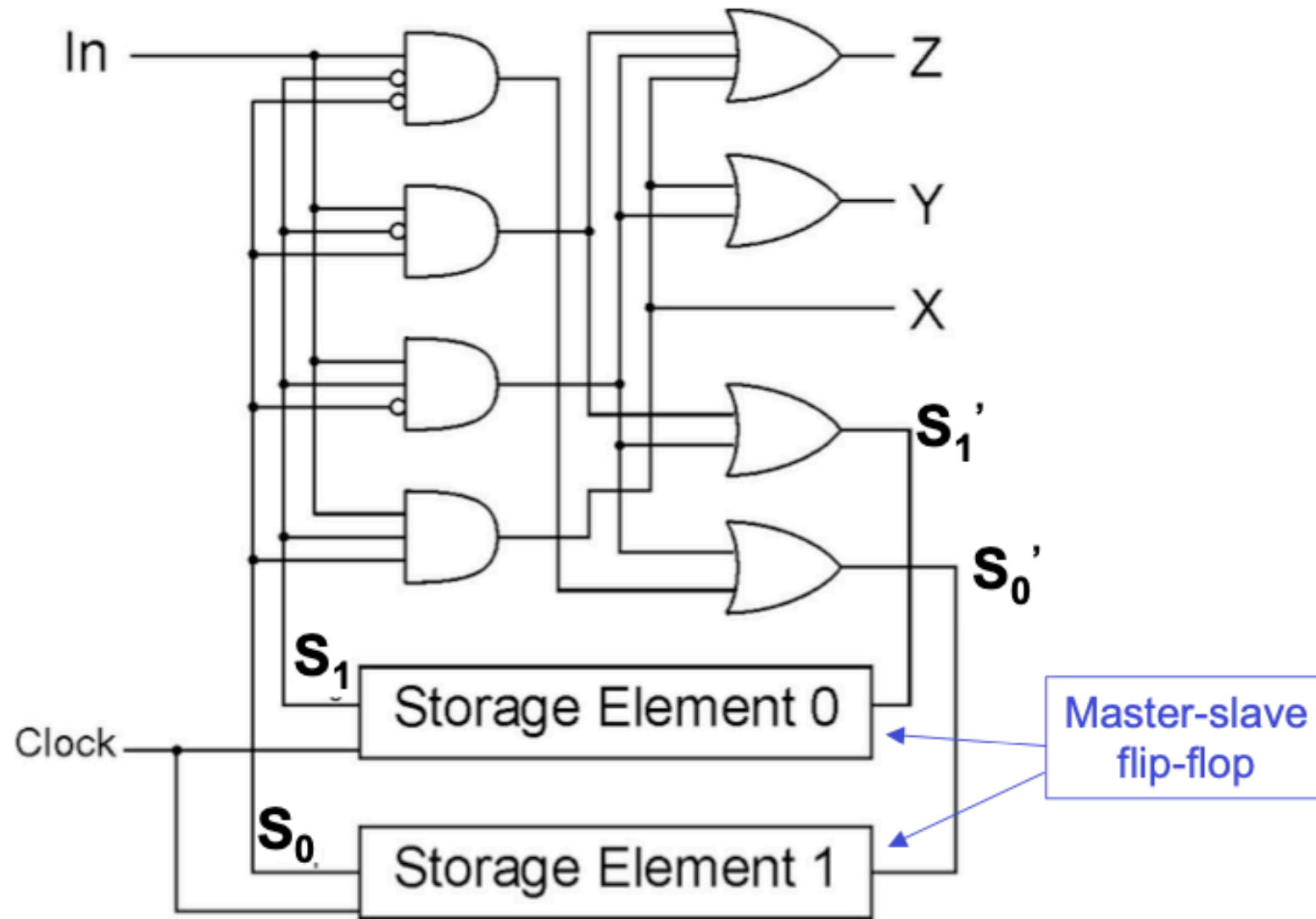
Switch

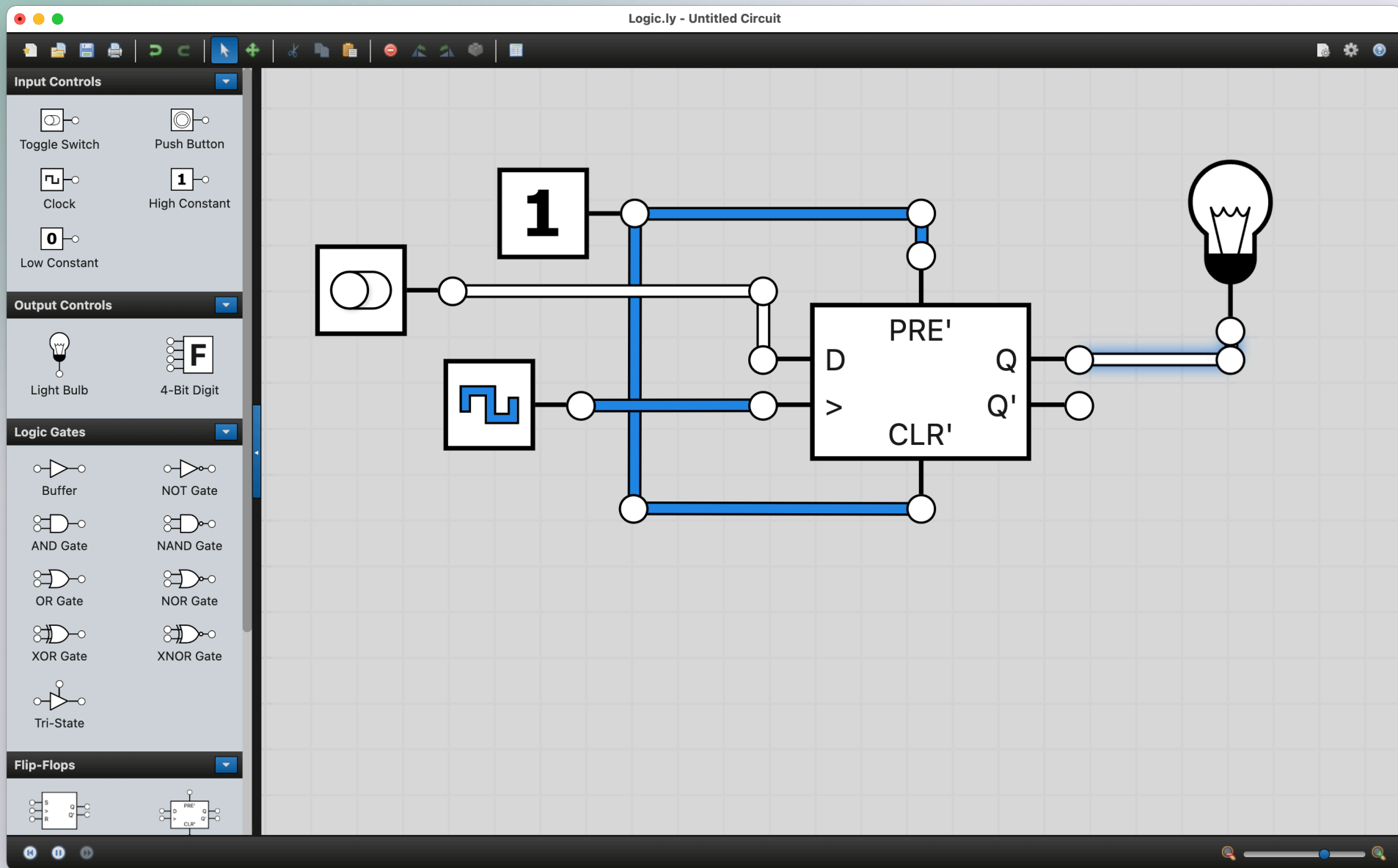
In	S_1	S_0	S_1'	S_0'
0	X	X	0	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	1
1	1	1	0	0

Don't care (1 or 0)

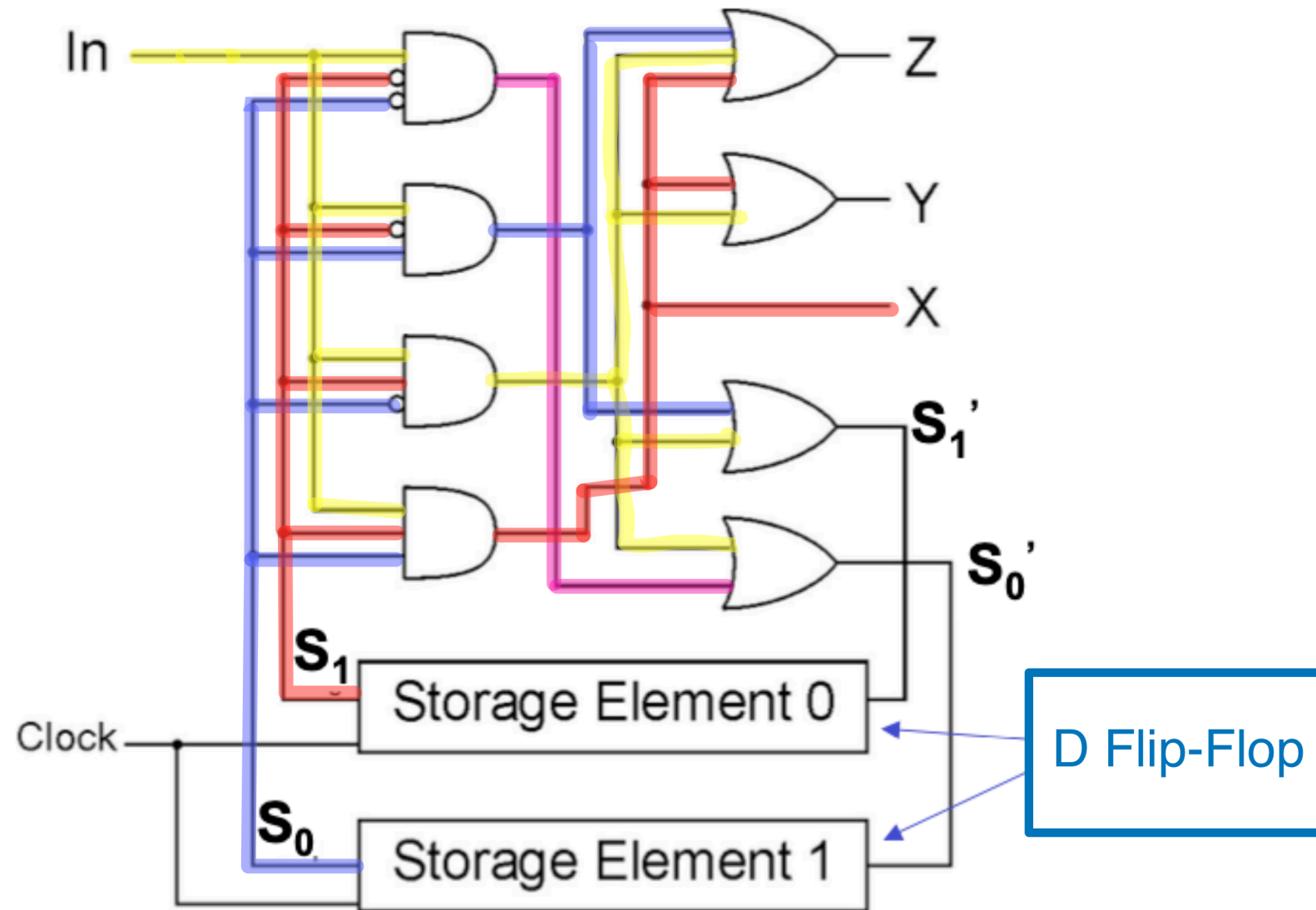
Whenever In=0, next state is 00.

Traffic Sign Logic





Traffic Sign Logic



Credits:

Patt and Patel: *"Introduction to Computing Systems"*

AspenCore: *"Sequential Logic"*

UPenn CSE240 Fall 2006 Fassell: *"Chapter 3 Lecture"*

Sequential Logic and Synchronization [CS 50.002](#)

