

Hands On Continuous Time Modeling With ctsem

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Outline

- Recap
- Introduction to ctsem
- Walk-through
- Exercise
- Solution

Recap I

What are the assumptions about time in the general cross lagged panel model (GCLPM)?

1. time progresses in discrete steps
2. equally spaced measurement occasions
3. subjects are measured at the same time points

Recap II

What are limitations of treating time as a discrete variable when modeling longitudinal data?

- biased parameter estimates due to:
 - unequally spaced measurement waves in the panel data
 - measurement of individual subjects at each wave spread out over time
- uncomparable parameter estimates across studies

Recap II

Multivariate stochastic differential equation: *[Math Processing Error]*

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Multivariate stochastic equation: *[Math Processing Error]*
modelling change in latent variables $\boldsymbol{\eta}_i(t)$ as a function of
time over infinitesimal small time steps dt

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Multivariate stochastic differential equation: *[Math Processing Error]* modelling change as a function of time over infinitesimal small time steps dt

A: drift matrix with auto effects on the diagonal and cross effects on the off-diagonals ($\mathbf{A} \in \mathbb{R}^{v \times v}$).

Recap II

Multivariate stochastic differential equation: *[Math Processing Error]* modelling change as a function of time over infinitesimal small time steps dt

\mathbf{A} : drift matrix with auto effects on the diagonal and cross effects on the off-diagonals ($\mathbf{A} \in \mathbb{R}^{v \times v}$).

\mathbf{z}_i : time independent predictors

$\chi_i(t)$: time dependent predictors

$\mathbf{W}_i(t)$: stochastic error term (Wiener process)

\mathbf{G} : diffusion matrix

Recap II

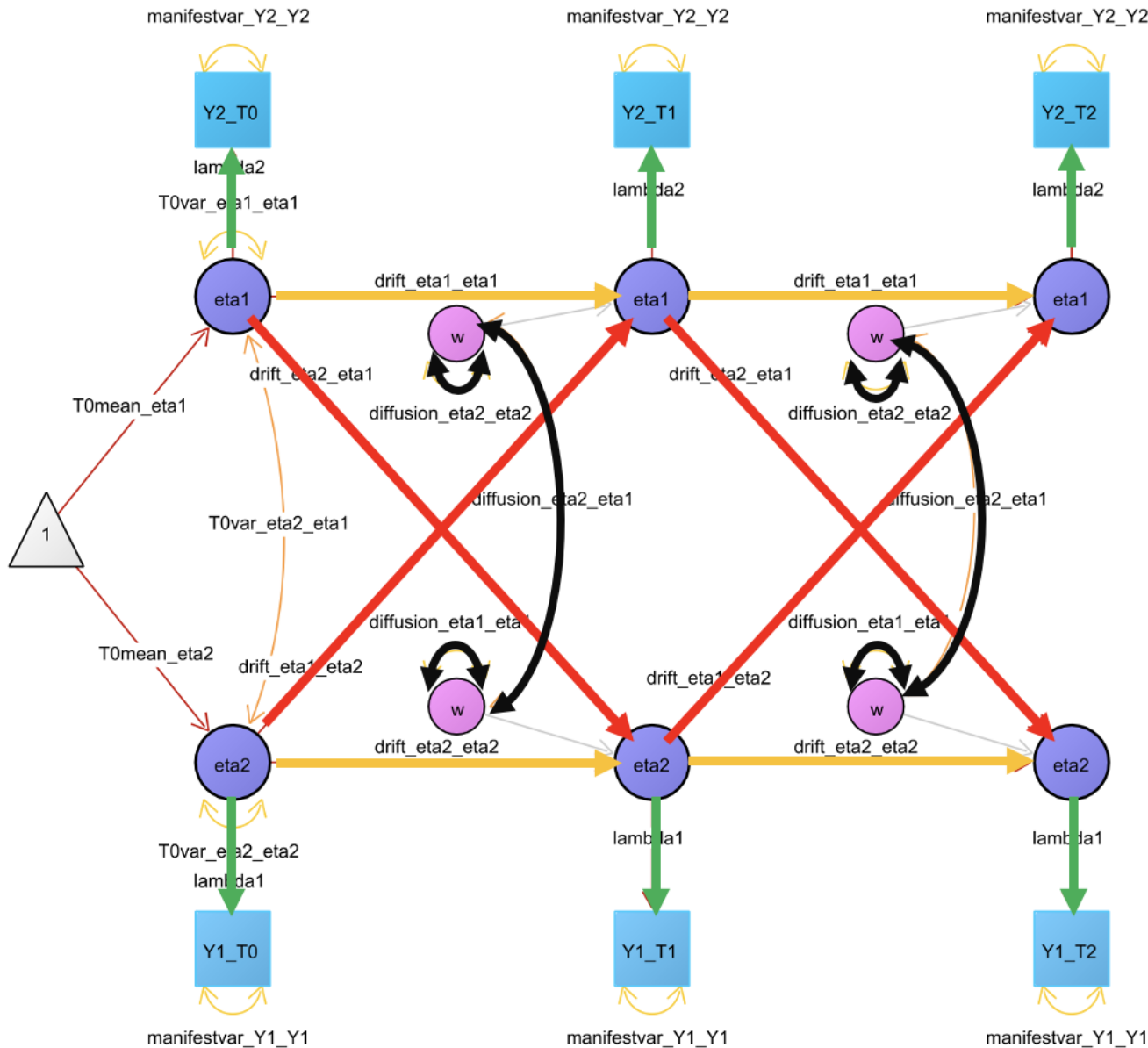
Multivariate stochastic differential equation: *[Math Processing Error]* measurement part: *[Math Processing Error]*

$$\boldsymbol{\varepsilon}(t) \sim N(\mathbf{0}, \boldsymbol{\Theta})$$

$\boldsymbol{\Lambda}$: factor loadings

$\boldsymbol{\tau}$: intercepts

$\boldsymbol{\varepsilon}(t)$: measurement error



Bivariate Process (no Factor Structure)

stochastic differential equation:

$$d\boldsymbol{\eta}_i(t) = (\mathbf{A}\boldsymbol{\eta}_i(t) + \mathbf{B}\mathbf{z}_i + \mathbf{M}\boldsymbol{\chi}_i(t)) dt + \mathbf{G}d\mathbf{W}_i(t)$$

Measurement:

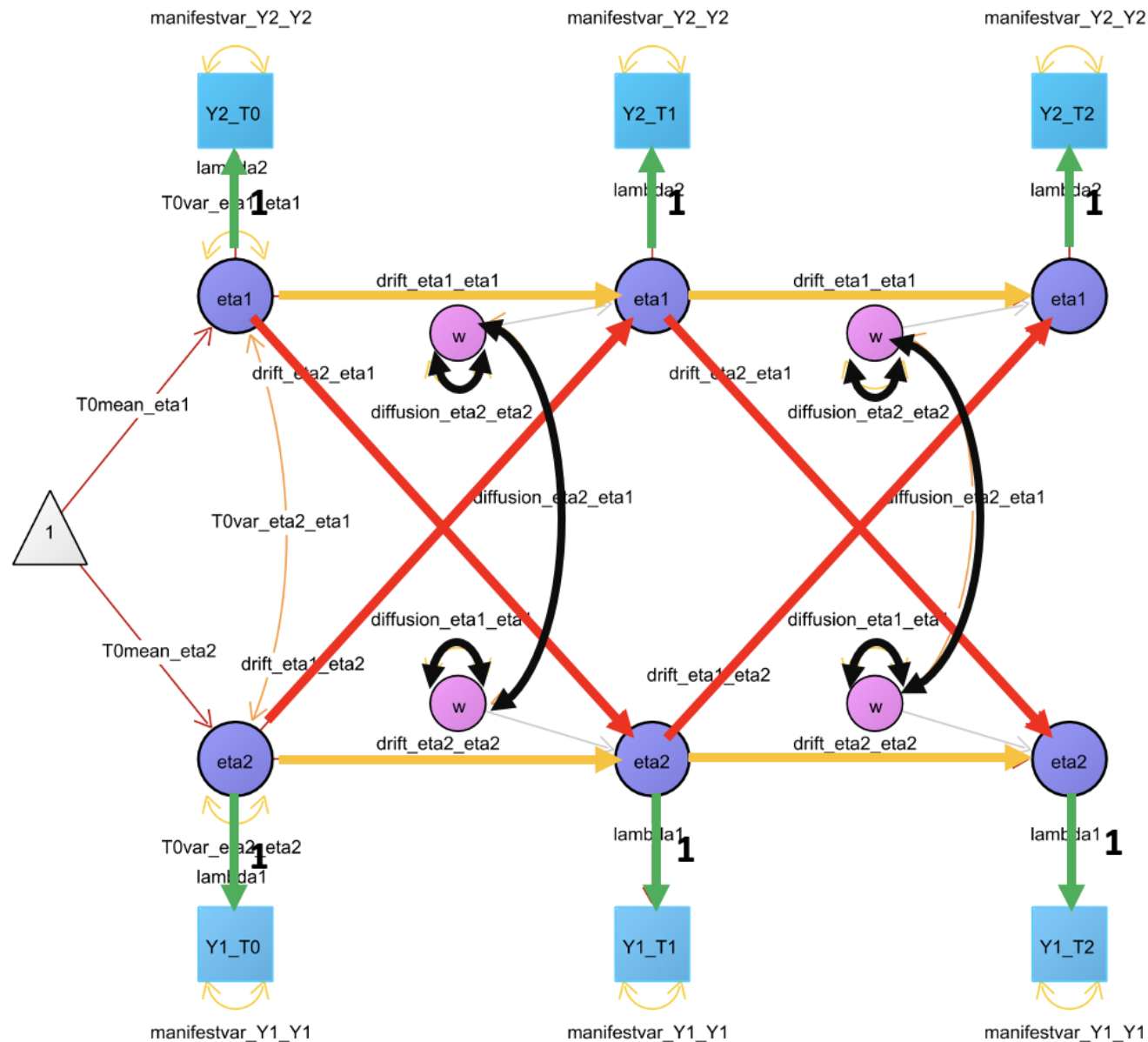
$$\mathbf{y}(t) = \mathbf{\Lambda}\boldsymbol{\eta}(t) + \boldsymbol{\tau} + \boldsymbol{\varepsilon}(t) \quad \boldsymbol{\varepsilon}(t) \sim N(\mathbf{0}, \boldsymbol{\Theta})$$

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad \text{Drift Matrix}$$

$$\mathbf{\Lambda} = \begin{pmatrix} \lambda_{11} & 0 \\ 0 & \lambda_{22} \end{pmatrix} \quad \text{Loading Matrix}$$

$$\mathbf{G} = \begin{pmatrix} g_{11} & 0 \\ g_{21} & g_{22} \end{pmatrix} \quad \text{Diffusion Matrix}$$

Driver, C. C., Oud, J. H., & Voelkle, M. C. (2017). Continuous time structural equation modeling with R package *ctsem*. *Journal of Statistical Software*, 77, 1-35.



Bivariate Process (no Factor Structure)

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Measurement:

$$\mathbf{y}(t) = \mathbf{\Lambda}\boldsymbol{\eta}(t) + \boldsymbol{\tau} + \boldsymbol{\varepsilon}(t) \quad \boldsymbol{\varepsilon}(t) \sim N(\mathbf{0}, \boldsymbol{\Theta})$$

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad \text{Drift Matrix}$$

$$\mathbf{\Lambda} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{Loading Matrix}$$

$$\mathbf{G} = \begin{pmatrix} g_{11} & 0 \\ g_{21} & g_{22} \end{pmatrix} \quad \text{Diffusion Matrix}$$

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Introducing **ctsem** & **ctsemOMX**

- R-packages **ctsem** & **ctsemOMX** to handle the math
- **ctsem** interfaces with Stan
- **ctsemOMX** interfaces with OpenMx
- today we will use **ctsemOMX**

Data Structure

`ctsemOMX` uses wide data with time intervals specified in columns:

	Quality_T0	Quality_T1	Quality_T2	dT1	dT2	dT3
1	-8.016139	-8.940450	-8.243518	1	1	1
2	-6.367648	-6.762183	-6.731700	1	1	1
3	-7.927406	-8.518258	-8.907190	1	1	1
4	-5.887097	-7.304229	-5.552340	1	1	1
5	-5.415876	-10.365073	-8.733621	1	1	1
6	-6.408571	-7.434263	-4.577385	1	1	1

(time points can also be interindividually varying)

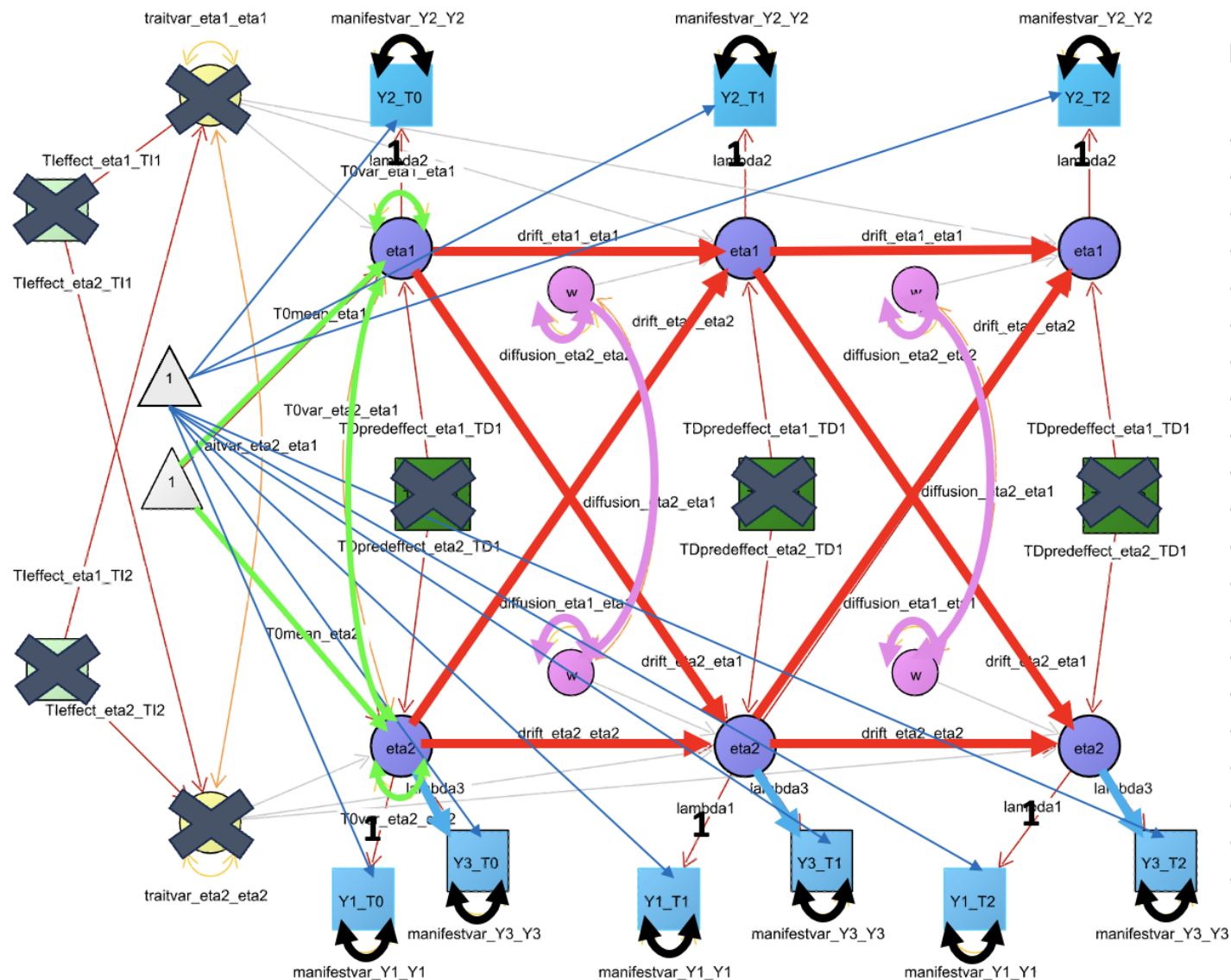
Function to convert to wideformat:

```
1 df_wide <- ctLongToWide(datalong = df_long, id = "id",  
2   time = "time", manifestNames = c("Quality", ...))
```

Function to create time intervals:

```
1 df_wide <- ctIntervalise(datawide = wideexample, Tpoints = 3, n  
2   manifest = 2, manifestNames = c("Quality", ...))
```

**Next: Model set-up
and interpretation
walkthrough**



model from exercise 6

##	\$ctparameters	Value	Matrix
##	stressLoading	0.002977787	LAMBDA
##	T0m_eta1	-3.695248816	T0MEANS
##	T0m_eta2	-19.950426226	T0MEANS
##	drift_eta1	-0.304066770	DRIFT
##	drift_eta2_eta1	-0.227847804	DRIFT
##	drift_eta1_eta2	0.186420890	DRIFT
##	drift_eta2	0.085819017	DRIFT
##	mvarStress	0.229760390	MANIFESTVAR
##	mvarStress2	0.269595124	MANIFESTVAR
##	mvarQuality	0.301769320	MANIFESTVAR
##	diff_eta1	0.792245329	DIFFUSION
##	diff_eta2_eta1	0.908630806	DIFFUSION
##	diff_eta2	1.194224737	DIFFUSION
##	T0var_eta1	1.900422851	T0VAR
##	T0var_eta2_eta1	2.455585775	T0VAR
##	T0var_eta2	3.488468973	T0VAR
##	mm_Stress	11.182382669	MANIFESTMEANS
##	mm_Stress2	0.034551688	MANIFESTMEANS
##	mm_Quality	13.212576945	MANIFESTMEANS

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