CS310: Paradigms of Programming
Homework 1

1. Calculate and simplify $wp(S4, a > 0 \wedge b > 0)$ for the command

   $S4$: **if** $a > b \rightarrow a \leftarrow a - b$ [] $b > a \rightarrow b \leftarrow b - a$ **end-if**

2. Arrays $f[0:n]$ and $g[0:m]$ are alphabetically ordered lists of names of people. It is known that at least one name is on both lists. Let X represent the first (in alphabetic order) such name. Calculate and simplify the weakest precondition of the following alternative command with respect to predicate R given after it. Assume i and j are within the array bounds.

   $S6:$ **if** $f[i] < g[i] \rightarrow i \leftarrow i + 1$
   $[] f[i] = g[i] \rightarrow skip$
   $[] f[i] > g[i] \rightarrow j \leftarrow j + 1$
   end-if
   $\{R: ordered(f[0:n]) \wedge ordered(g[0:m]) \wedge f[i] \leq X \wedge g[j] \leq X\}$

3. Show that the following loops are correct by showing *all* of the 5 checkpoints given in class (P is the loop invariant, R is the result, Q is the precondition and t is the bound function).

   (a) Q: $0 \leq n$
   P: $0 \leq i \leq n \wedge x \notin b[0:i-1]$
   R: $(0 \leq i < n \wedge x = b[i]) \vee (i = n \wedge x \notin b[0:n-1])$
   t: $n - i$

   program:
   $i \leftarrow 0;$
   do
   $i < n$ **cand** $x \neq b[i] \rightarrow i \leftarrow i + 1$
   end-do

   (b) Q: $0 < n$
   P: $0 < i \leq n \wedge (\mathbf{E}\ p : i = 2^p)$
   R: $0 < i \leq n < 2 * i \wedge (\mathbf{E}\ p : i = 2^p)$
   t: $n - i$

   program:
   $i \leftarrow 1;$
   do
   $2 * i \leq n \rightarrow i \leftarrow 2 * i$
   end-do

   (c) Q: $x \geq 0 \wedge 0 < y$
   P: $0 \leq r \wedge 0 < y \wedge q * y + r = x$
   R: $0 \leq r < y \wedge q * y + r = x$
   t: r

program:
$q, r \leftarrow 0, x;$
do

$\qquad r \geq y \rightarrow r, q \leftarrow r - y, q + 1$

end-do

(d) Q: $X > 0 \wedge Y > 0$
P: $0 < x \wedge 0 < y \wedge gcd(x, y) = gcd(X, Y)$
R: $x = y = gcd(X, Y)$
t: $x + y$

program:
$x, y \leftarrow X, Y;$
do

$\qquad x > y \rightarrow \mathrm{x} \leftarrow x - y;$

$\qquad y > x \rightarrow \mathrm{y} \leftarrow y - x;$

end-do

4. Write a program that, given two fixed integers x and y satisfying $x \geq 0$ and $y > 0$, finds the quotient q and remainder r when dividing x by y. That is, it establishes $0 \leq r \wedge r < y \wedge q * y + r = x$. The program may not use multiplication or division. Develop the invariant of the loop by deleting a conjunct.

5. (Binary Search). Write a program that, given fixed x and fixed, ordered (by $\leq$) array $b[1:n]$ satisfying $b[1] \leq x < b[n]$, finds where x belongs in the array. That is, for a fresh variable i the program establishes

$$R : 1 \leq i < n \wedge b[i] \leq x < b[i+1]$$

The execution time of the program should be proportional to $log\ n$.

After writing the program, incorporate it in a program for a more general search problem: with no restriction on the value x, determine i to satisfy

$$(i = 0 \wedge x < b[1]) \vee$$
$$(1 \leq i < n \wedge b[i] \leq x < b[i+1]) \vee$$
$$(i = n \wedge b[n] \leq x)$$