Convex optimization exercises pt. 2 - convex functions, ctd.

Exercise 1. *If f and g are convex, is their product fg convex?*

Exercise 2. *Let $P : \mathbb{R}^k \times \mathbb{R}_{++} \to \mathbb{R}^k$ be the projection function, defined $P(x,t) = x/t$.*

1. *Why is this function called projection?*
2. *Show that the image and the inverse image of a convex set in P is convex (give an intuitive justification as well as a proof using the definition of convexity).*
3. *For $f : \mathbb{R}^k \to \mathbb{R}$, let $g(x,t) = tf(x/t)$, where $t > 0$ and $x/t \in dom(f)$. Show that if f convex, then g also convex, by looking at their epigraphs and using the previous bullet.*
4. *Show that $f(x,t) = ||x||^2/t$ and $f(u,v) = \sum_i u_i \log(u_i/v_i)$ are convex using the previous bullet.*