Exercises: linear programs

1. Show that all the forms of linear programs below are equivalent.

$$\begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & Ax \leq b \end{array} \qquad \begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & Ax \geq b \end{array}$$

$$\begin{array}{lll} \text{minimize} & c^T x & \\ \text{subject to} & Ax & \geq b \\ & x & \geq 0 \end{array} \qquad \begin{array}{lll} \text{minimize} & c^T x & \\ \text{subject to} & Ax & = b \\ & x & \geq 0 \end{array}$$

2. Show that the following optimization problem (which is not even convex!) can be transformed to an equivalent linear program.

$$\begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & Gx \leq h \\ & Ax = b, \end{array}$$

where $f(x) = \frac{c^T x + d}{e^T x + f}$ with $\mathrm{dom} f = \{x \in \mathbb{R}^n \mid e^T x + f > 0\}$. Here, $d, f \in \mathbb{R}$, $c, e \in \mathbb{R}^n$, $h \in \mathbb{R}^m$, $b \in \mathbb{R}^p$, $G \in \mathbb{R}^{m \times n}$, $A \in \mathbb{R}^{p \times n}$ are given numbers/vectors/matrices and $x \in \mathbb{R}^n$ is a variable.