Convex optimization exercises pt. 2 - convex functions

Exercise 1. *Compute the gradient and Hessian of $f(X) = tr(AX)$ (where A and X are symmetric). Show that this function is convex.*

Exercise 2. *Show that*

$$\max(x_1, \ldots, x_n) \leq \textit{log-sum-exp}(x_1, \ldots, x_n) \leq \max(x_1, \ldots, x_n) + \log n.$$

Exercise 3. *Show that the geometric mean is concave. Hint: Compute the Hessian and use Cauchy-Schwarz.*

Exercise 4. *A function is* midpoint-convex *if $f\left(\frac{x+y}{2}\right) \leq \frac{f(x)+f(y)}{2}$. Show that for continuous functions this is equivalent to convexity.*

Exercise 5. *Use the previous exercise to give an alternative proof of convexity of log-sum-exp.*

Exercise 6. *Show that a convex function on an open set is continuous, by induction. First, prove the 1d case, and use it as a building block.*

Exercise 7. *Show that $f(x, y)$ is convex and C is convex, then $g(x) = \inf_{y \in C} f(x, y)$ is convex. Hint: argue that $epi(g)$ is a projection of $epi(f)$.*