Convex optimization exercises pt. 1 - convex sets

Exercise 1. *Show that affine image and inverse image of a convex set is convex.*

Exercise 2. *Show that a set $S \subseteq \mathbb{R}^n$ is closed w.r.t. affine combinations iff it is of the form $S = \{x : Ax = b\}$.*

Exercise 3. *Prove the separating hyperplane theorem: If A, B convex and closed and A bounded, then there exists a hyperplane strictly separating A and B. Hint: Show that $\min_{a \in A, b \in B} ||a - b||$ exists. Show that the hyperplane perpendicular to ab and cutting it in half is strictly separating.*

Exercise 4. *Design a separating oracle for S_+^n. I.e. for a given matrix M that is not PSD, give a hyperplane separating M from S_+^n.*