Reply of Self Financing

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1. Write wealth W(t), risk stock price S(t) with units a(t), bond price B(t) with units d(t). Then what is W(t) in terms of these quantities?

$$W_t = a_t * S_t + d_t * B_t$$

2. What is the self financing condition? Write it down using your result from 1.

$$\partial W_t = a_t * \partial S_t + d_t * \partial B_t$$

3. What is the new wealth at time W(t+1) in terms of the quantities a,S,B,d before you rebalance the portfolio and after you rebalance assuming zero transaction fees? Why does rebalancing require you to use 2?

   a. Before rebalancing

$$W_{t+1} = a_t * S_{t+1} + d_t * B_{t+1}$$

   b. After rebalancing

$$W_{t+1} = a_{t+1} * S_{t+1} + d_{t+1} * B_{t+1}$$

4. What is alpha(t) in terms of a, S, b, B, W?

   To compute α_t, I used Eq.1 and obtained:

$$\alpha_t = \frac{a_t * S_t}{a_t * S_t + d_t * B_t} = \frac{a_t * S_t}{W_t}$$

   From above equation, we have following relationships

$$a_t = \frac{\alpha_t W_t}{S_t}$$
$$d_t = \frac{(1-\alpha_t) W_t}{B_t}$$

5. Solve for the recurrence equation in return space by using your results from 1, 2, 3, 4 and recreate the wealth recurrence equation.

$$W_{t+1} - W_t = a_t * (S_{t+1} - S_t) + d_t * (B_{t+1} - B_t)$$

   and we substitute the a_t and d_t into the equation

$$W_{t+1} = W_t + W_t \left[\alpha_t * (\frac{S_{t+1}}{S_t} - 1) + (1-\alpha_t) * (\frac{B_{t+1}}{B_t} - 1) \right]$$

   If we further assume $r_t = \frac{B_{t+1}}{B_t} - 1$, we then have

$$W_{t+1} = W_t + W_t \left[\alpha_t * (\frac{S_{t+1}}{S_t} - 1) + (1-\alpha_t) * r_t) \right]$$

So after some steps of calculation, I proved the original equation is correct... and my problem still holds the weird return jump caused by the discrete and imbalanced data inputs.