Implementing a quantum gate

PHYS 246 class 14 (last one)

https://lkwagner.github.io/IntroductionToComputationalPhysics/intro.html

```
from google.colab import drive
drive.mount('/content/drive')
!cp /content/drive/MyDrive/Colab\ Notebooks/Dynamics.ipynb ./
!jupyter nbconvert --to HTML "Dynamics.ipynb"
```

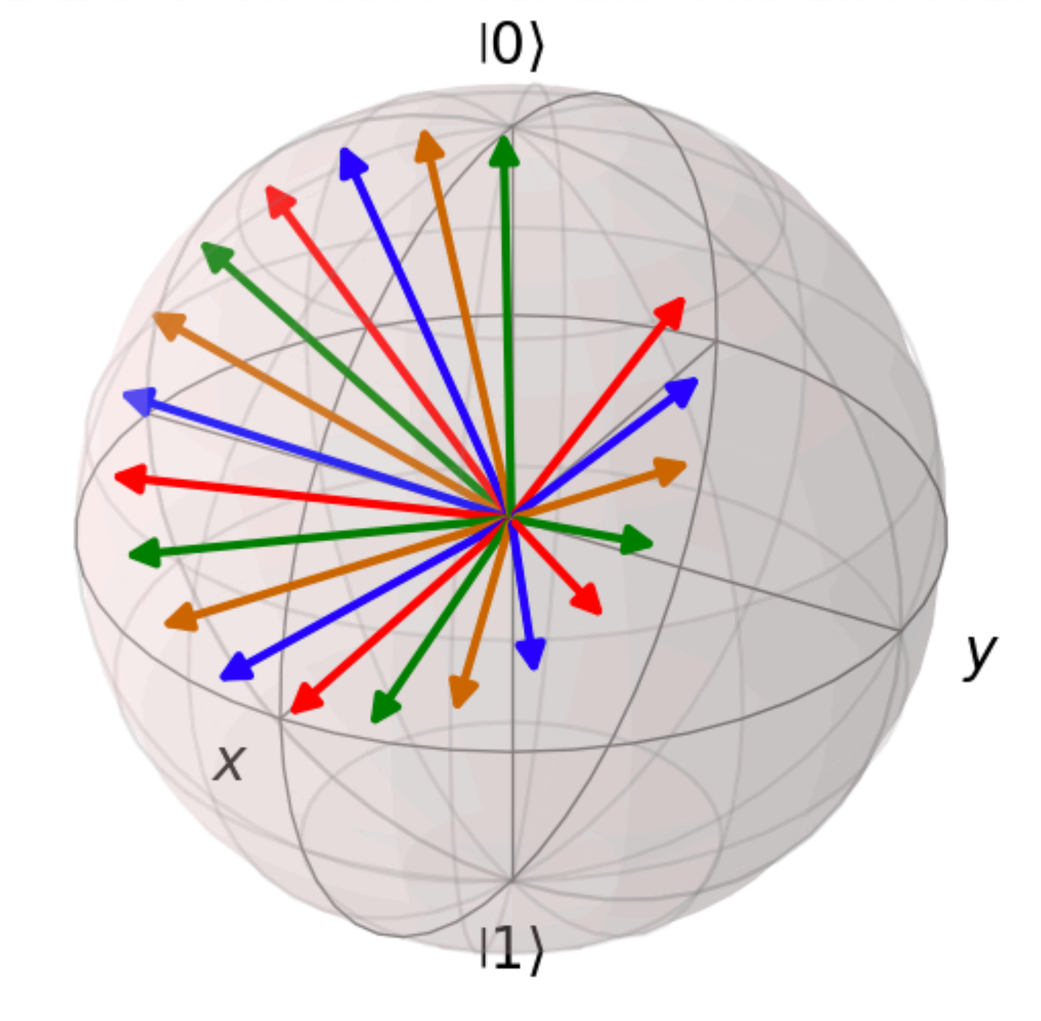
Announcements/notes

- 'Fluids' is due tonight
- For QC2 there are a couple small clarifying updates but no big changes from previous version.
- May 13th NOON -- slides and ipynb file due.Note that the final exam is REQUIRED to obtain a grade in this course.
- Final presentation: Make it 6 minutes for groups of 1-2, 9 minutes for groups of 3. Each person speaks for ~3 minutes.

Gates

```
circuit.rx(1./50.,0)
circuit.rz(1./50.,0)
```

How can we get this to actually happen?



Implementing a gate on fluxonium

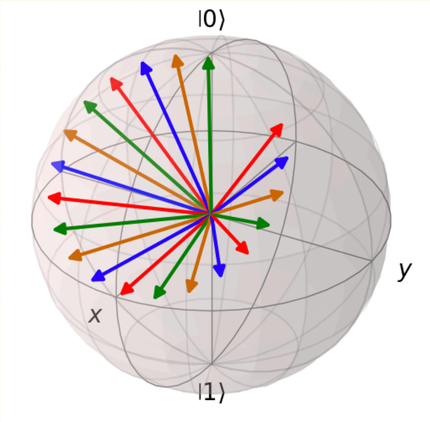
For fixed magnetic flux, ground state is $|0\rangle$, first excited state is $|1\rangle$
Cool down to $|0\rangle$
$e^{-E_i/kT}$

→

Change the magnetic flux. Now the state is no longer the ground state and it changes in time.

→

Measure the state after the right amount of time. Depending on the time you will get some superposition $\alpha|0\rangle + \beta|1\rangle$



Quick quantum intro

Classical mechanics:

State $x, v, \mathscr{R}^{6N}$ vectors

Dynamical equation: $F = ma = m\frac{d^2x}{dx^2}$

Measurement:
x, v are definite!

Quantum mechanics:

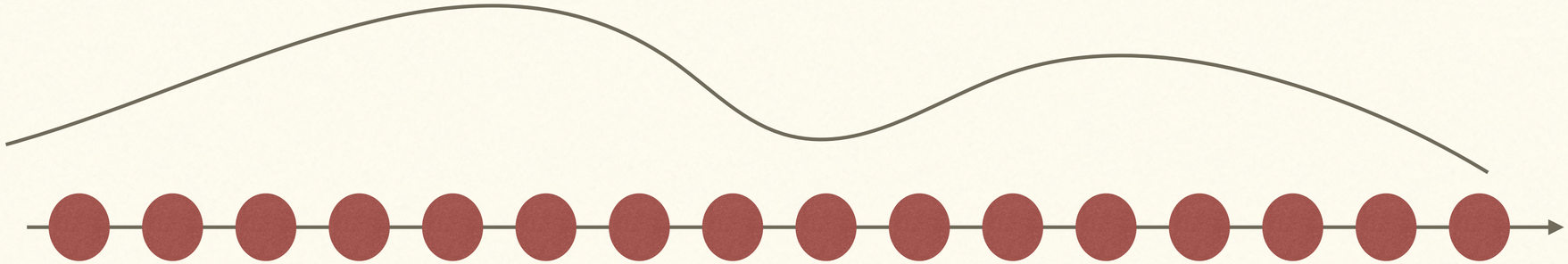
State $\Psi(x)$, function $\mathscr{R}^{3N} \rightarrow \mathscr{C}$

Dynamical equation:
$i\hbar\frac{\partial\Psi}{\partial t} = \hat{H}\Psi$

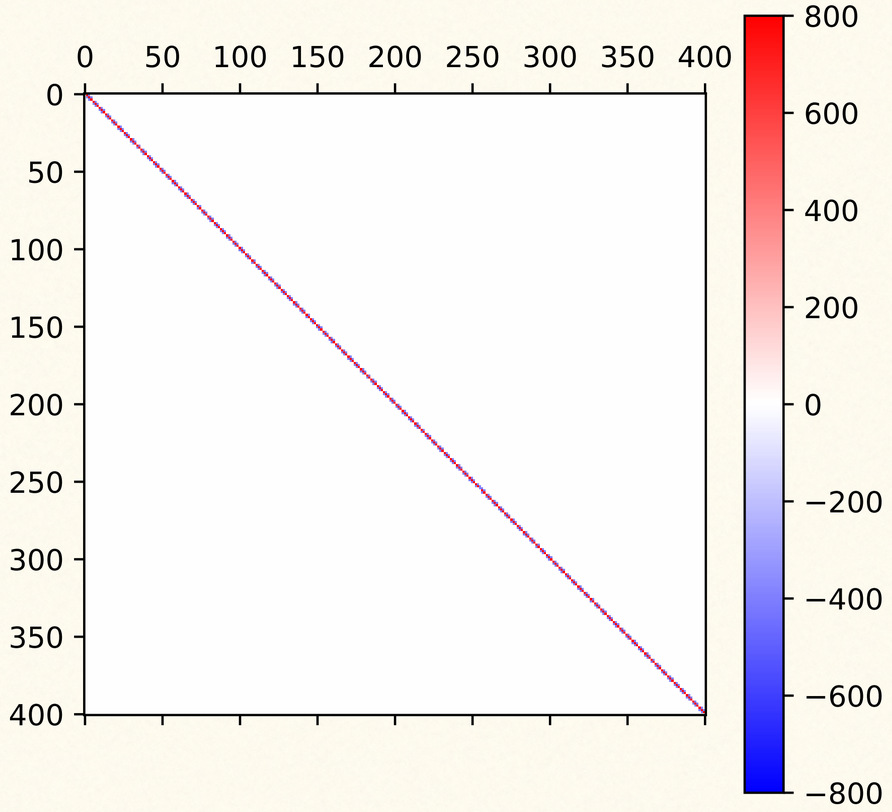
Measurement:
$\rho(x) = |\Psi(x)|^2$

$$\rho(p) = |\Psi(p)|^2 = \left| \int e^{ipx/\hbar}\Psi(x)dx \right|^2$$

Hamiltonian for a discrete representation



$$-\frac{\partial^2}{\partial x^2} = \lim_{\delta \to 0} \frac{f(x+\delta) + f(x-\delta) - 2f(x)}{\delta^2}$$



Solutions for infinite square well (particle in a box)

Energy eigenvalue equation:

$$H\Psi_n(x) = E_n\Psi_n(x)$$

$$H = -\frac{\partial}{\partial x^2}$$

You can only measure energies with a corresponding energy eigenstate.

Question for you: what are the solutions?

Time propagation in QM

Dynamical equation:

$$i\hbar\frac{\partial\Psi}{\partial t} = \hat{H}\Psi$$

Formal solution:

$$\Psi(x,t) = e^{\frac{-i\hat{H}t}{\hbar}}\Psi(x,0)$$

What is the exponential of a matrix??

$$e^A = 1 + A + \frac{1}{2}A^2 + \ldots$$

OR

If U is the set of eigenvectors (columns), then

$$U^\dagger A U = \text{diag}[E_i]$$

and

$$e^A = U^\dagger \text{diag}[\exp(E_i)]U$$

Superconductors

Macroscopic state.

At very cold temperatures, electrons pair up. This state is described as an amplitude and phase

$$|\Psi| e^{i\theta}$$

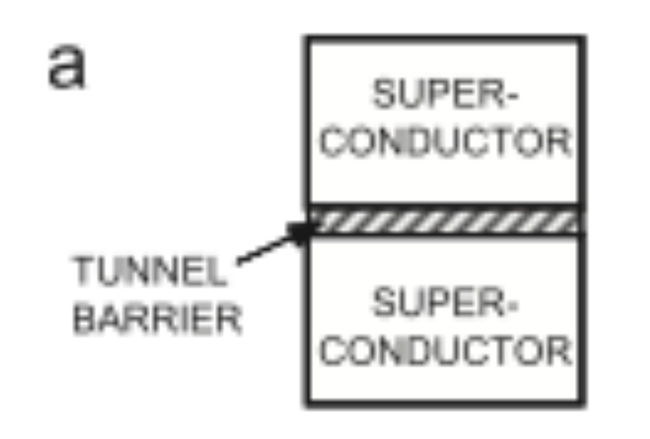
amplitude is how many electrons are paired up (n)
phase is related to the current

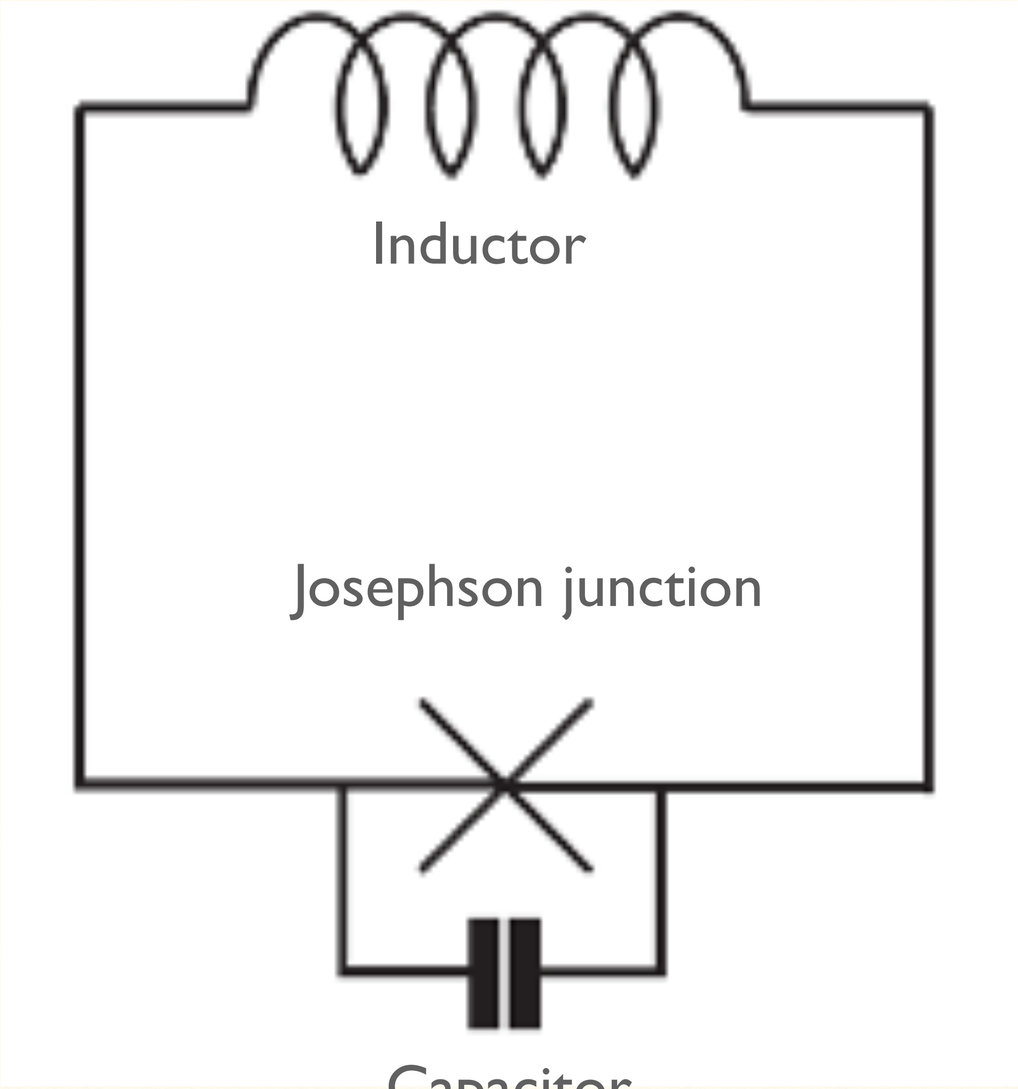
Quantum description of a superconductor circuit

Held at 20 mK or so..
A few hundred μm across

Quantum!

Phase across the junction -> current/velocity
Number that cross the junction-> position





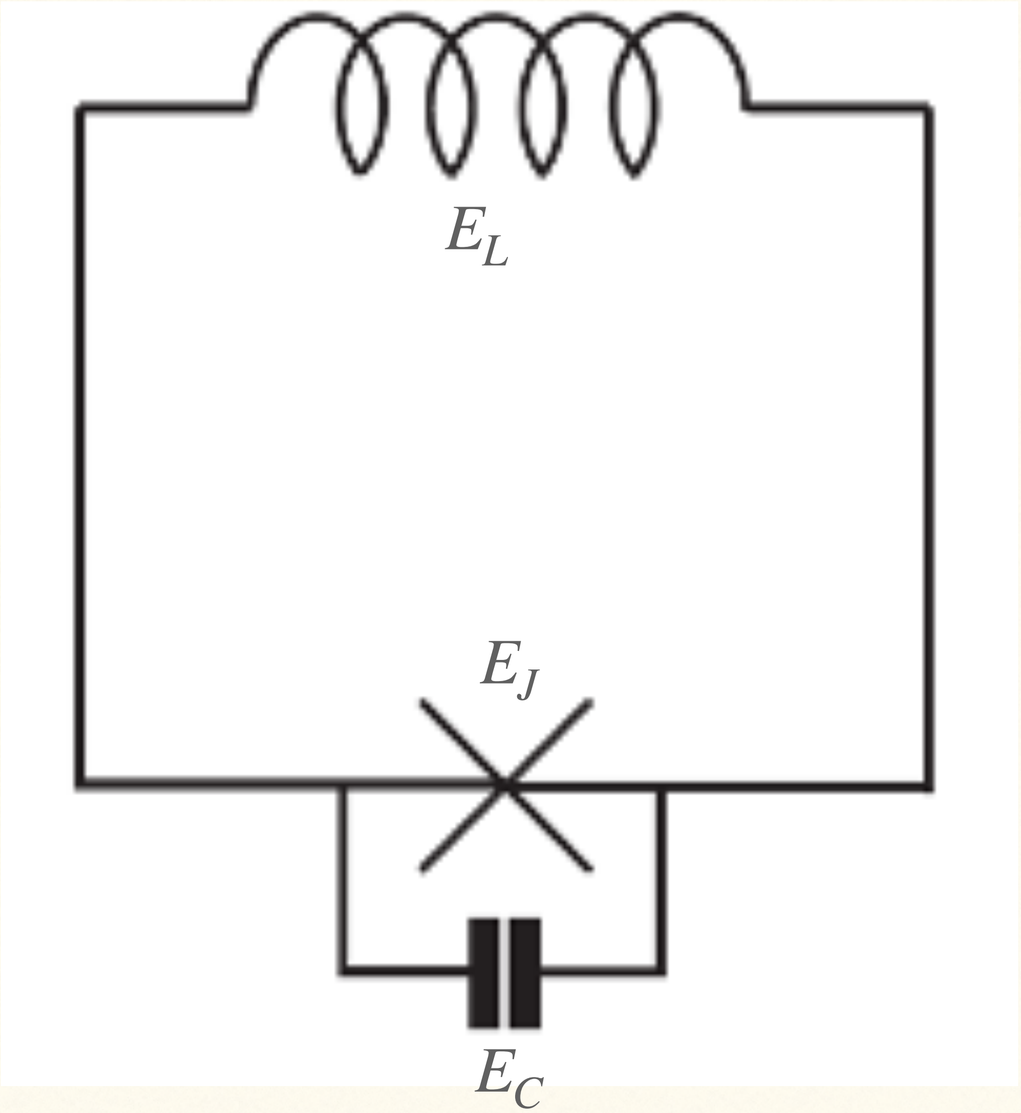
Fluxonium system

Inductor energy: $\frac{1}{2}LI^2 \equiv \frac{1}{2}E_L\phi^2$

Capacitor energy: $\frac{1}{2}\frac{Q^2}{C} \equiv 4E_C\hat{n}^2 = -4E_C\frac{\partial^2}{\partial\phi^2}$

Magnetic flux: ϕ_{ext} (written in appropriate units of $\frac{hc}{e}$)

Josephson junction: $-E_J\cos(\phi - \phi_{ext})$



NOTE: we do a change of variables for a fixed external flux, of $\phi \to \phi - \phi_{ext}$

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More info

If you do
eigs, vecs = np.linalg.eigh(H)

The eigenvector i is given by vecs[:,i]

https://arxiv.org/abs/1610.03438