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# Orbital motion

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PHYS 246 class 3

<https://lkwagner.github.io/IntroductionToComputationalPhysics/intro.html>

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# Announcements/notes

- 'Dynamics' is due tonight on canvas
- PDF uploads are required to be graded.
- Sorry about the grades being prematurely published (with scary low numbers).  
Canvas decided to publish grades after part I was graded...

```
from google.colab import drive  
drive.mount('/content/drive')  
!cp /content/drive/MyDrive/Colab\ Notebooks/Dynamics.ipynb ./  
!jupyter nbconvert --to HTML "Dynamics.ipynb"
```

# Radial forces

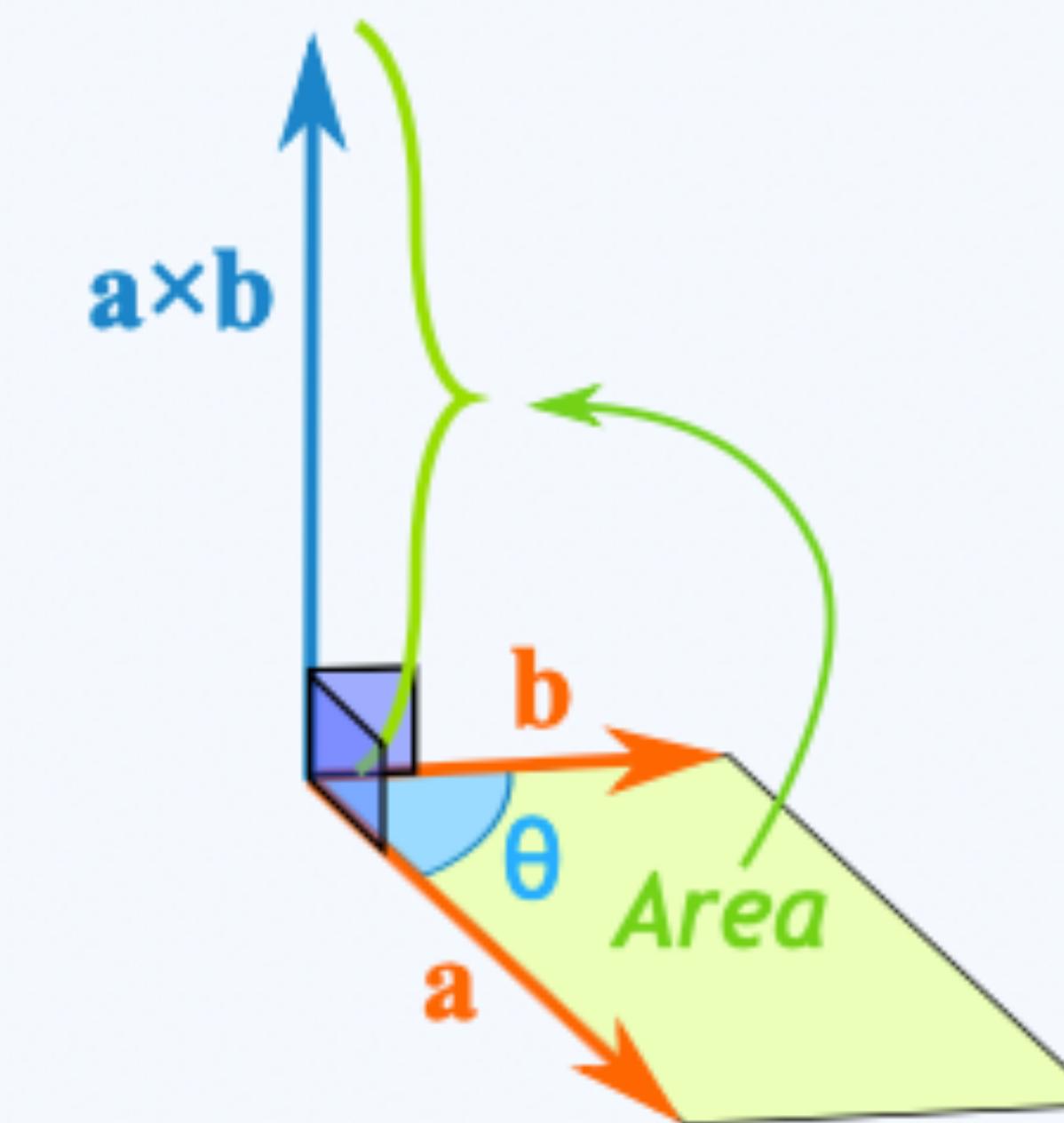
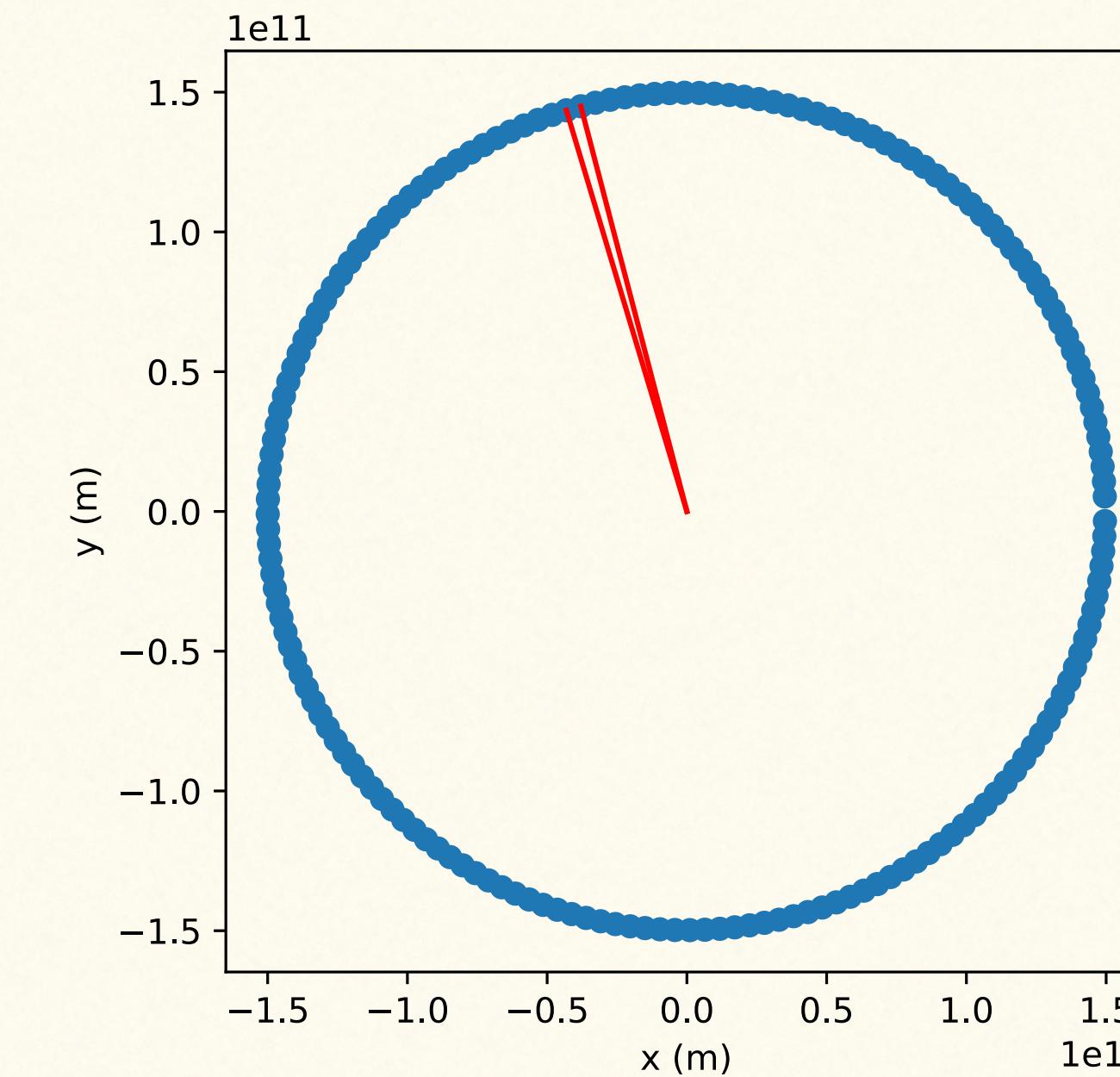
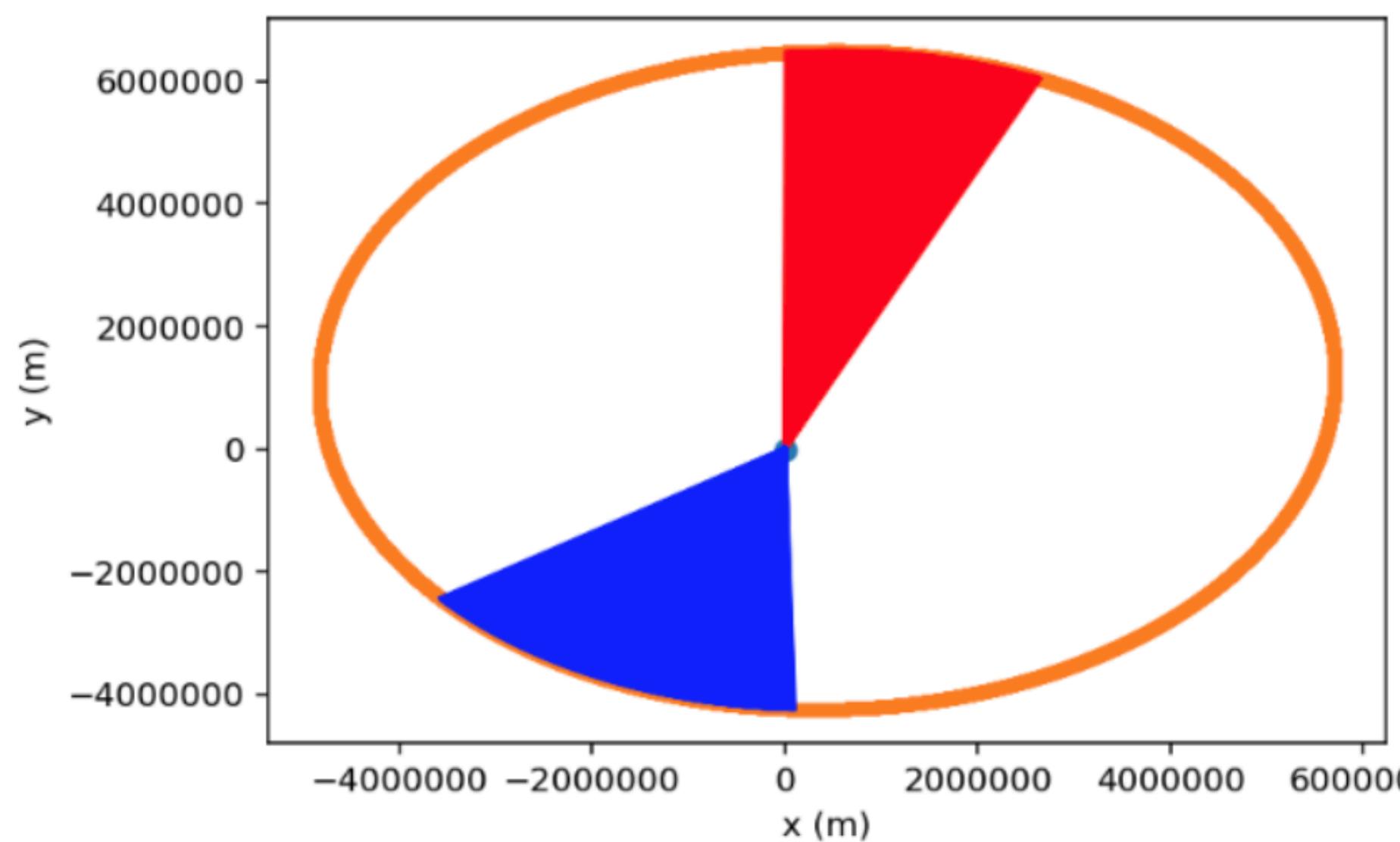
$$V(\vec{r}) = \frac{GMm}{r}$$

$$\mathbf{F} = -\nabla V$$

# Things to check: discussion

Does Earth orbit in about a year?

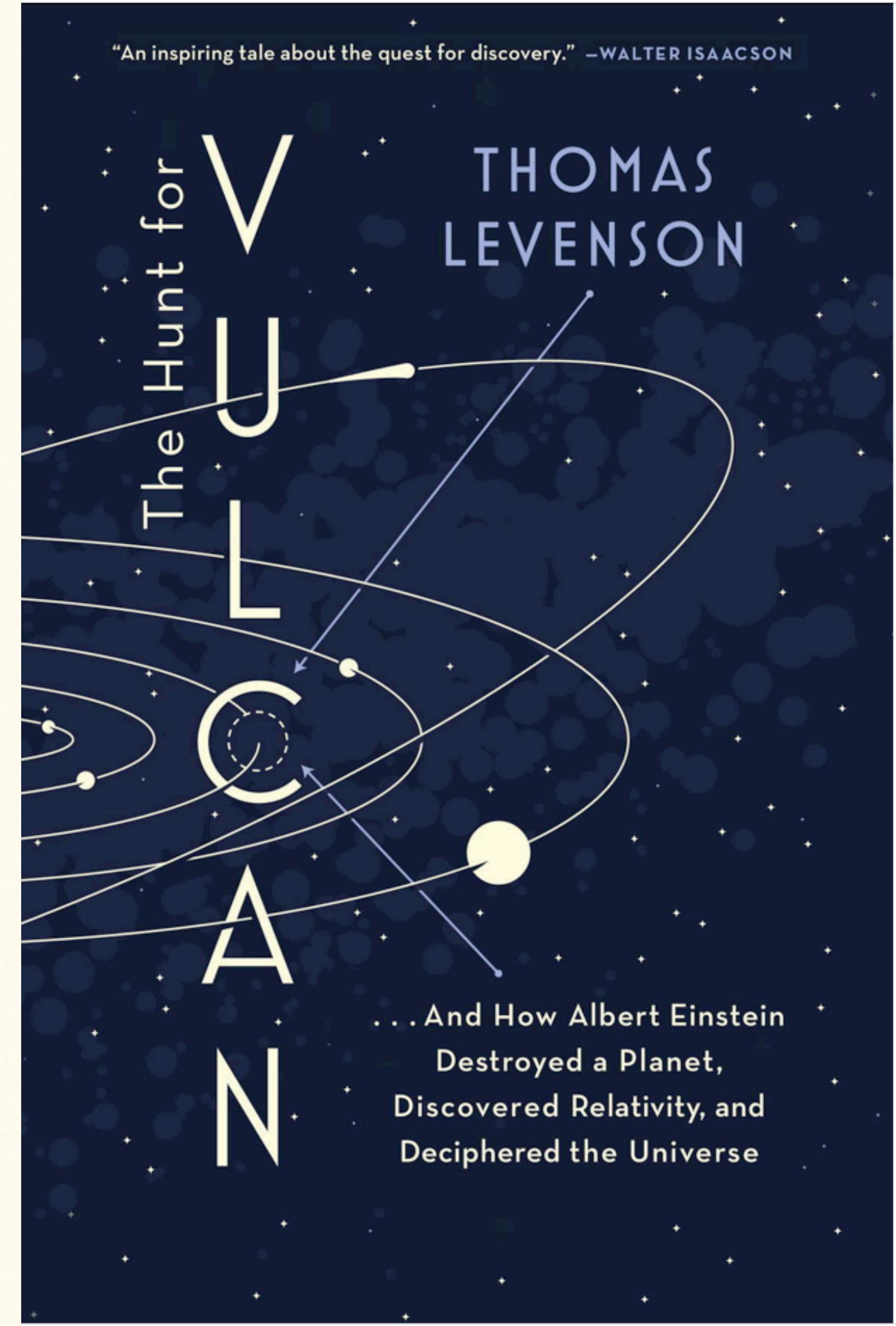
# Kepler integration



"An inspiring tale about the quest for discovery." —WALTER ISAACSON

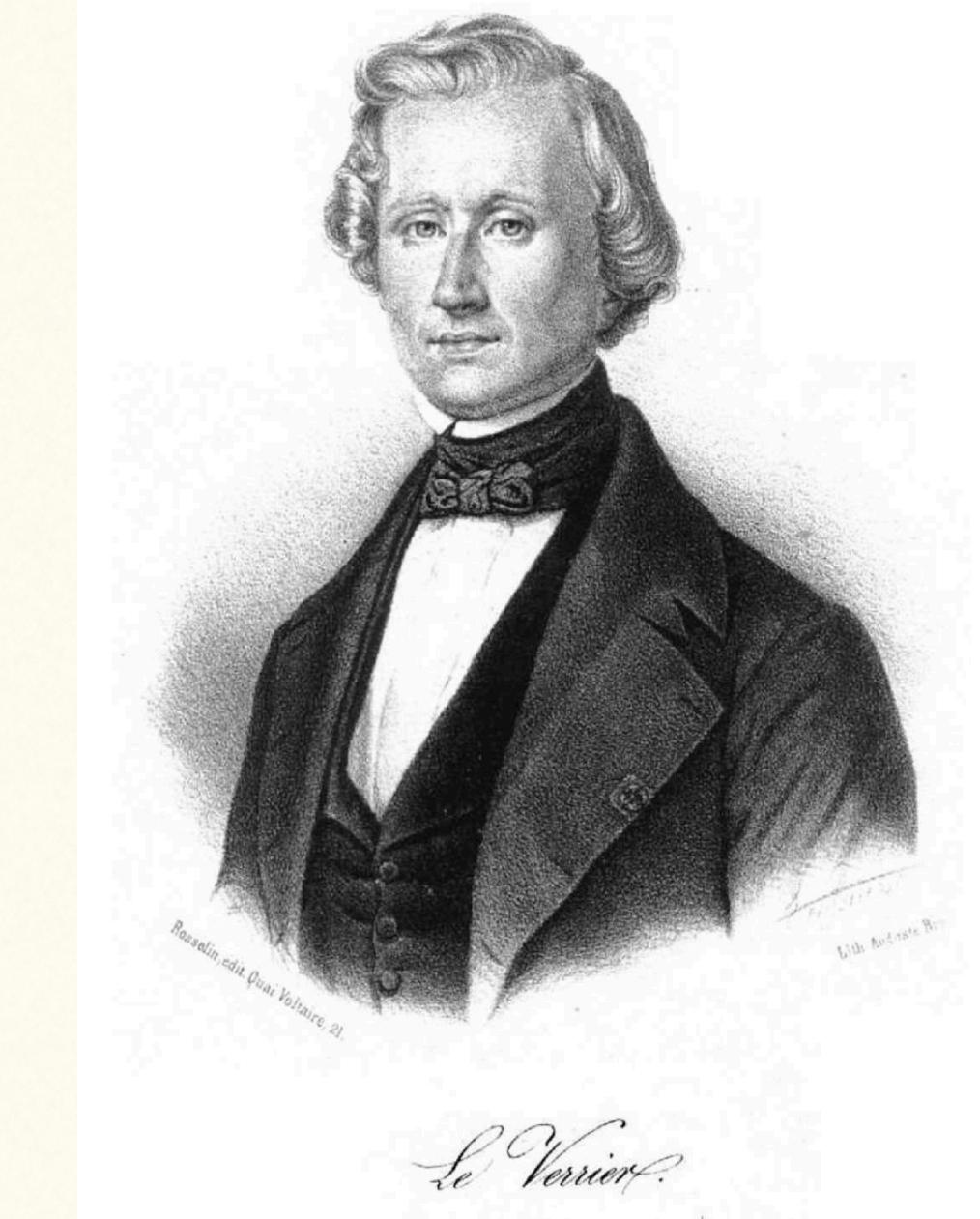
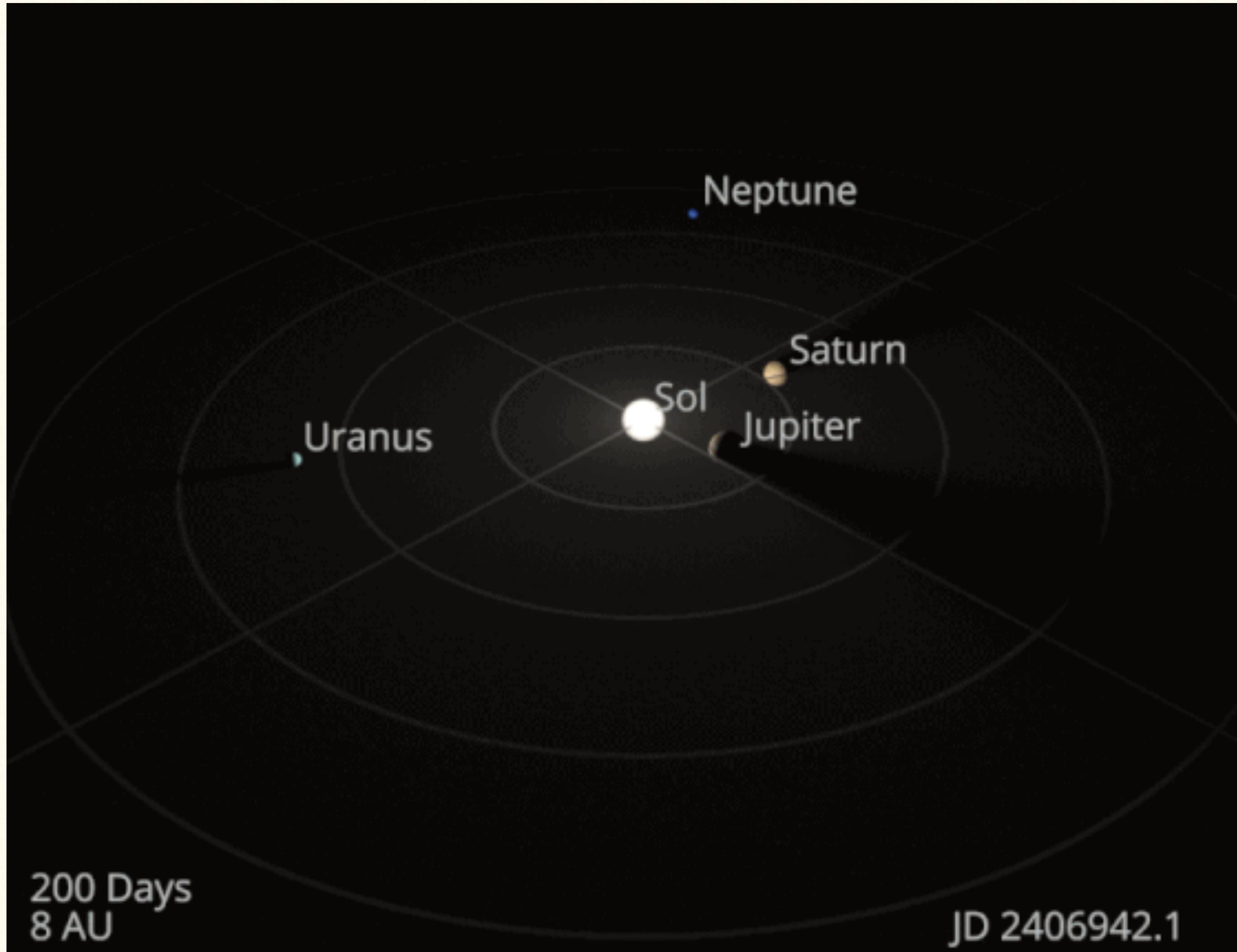
In the 1800's, the universe was thought to be mechanistic according to Newton's laws.

A major project was to compute the behavior of the solar system based on Newton's gravitation.



...And How Albert Einstein  
Destroyed a Planet,  
Discovered Relativity, and  
Deciphered the Universe

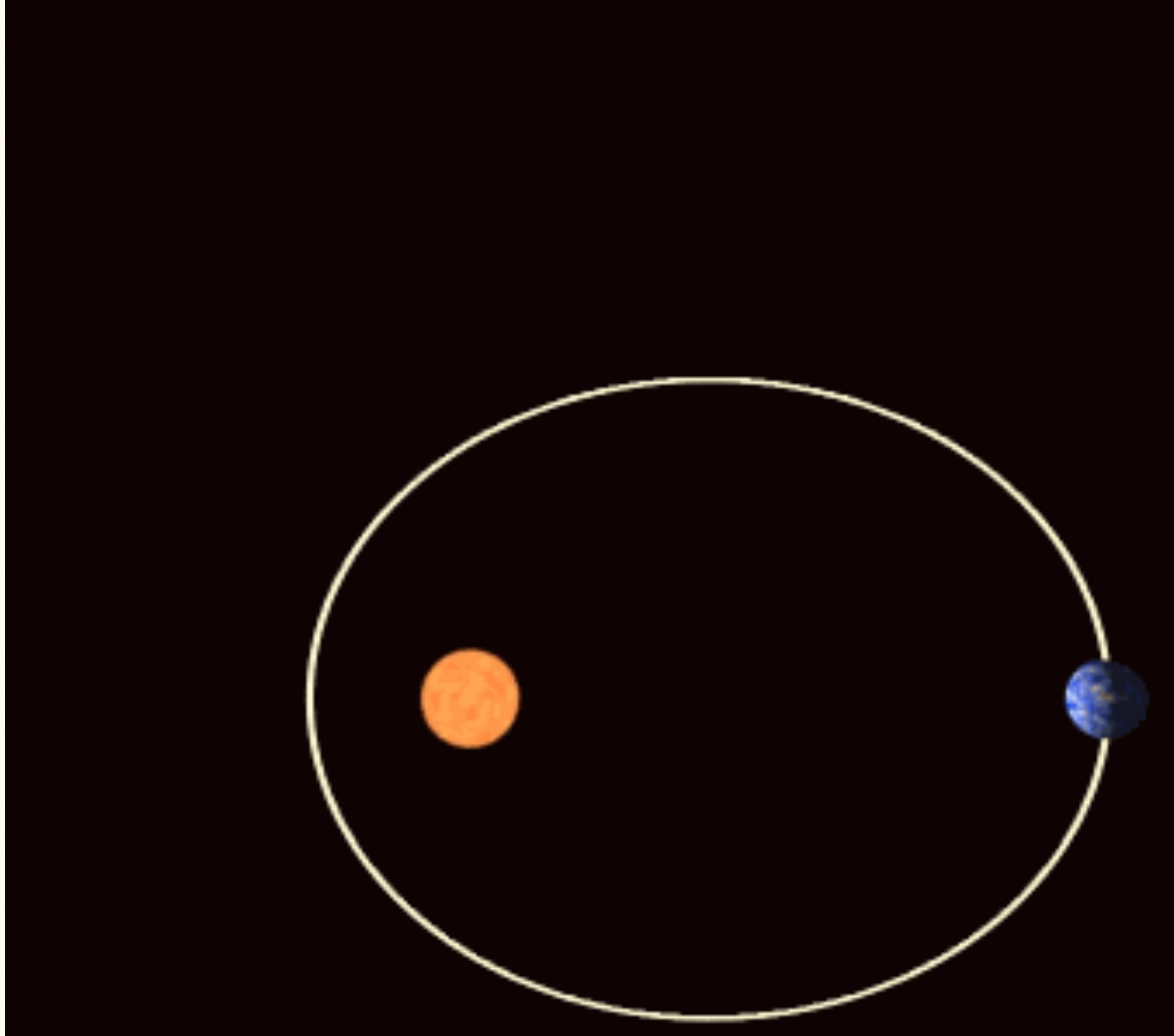
# Finding Neptune



Uranus was too slow!

Le Verrier spent years computing what new planet could cause it — prediction of Neptune!

# Precession of Mercury



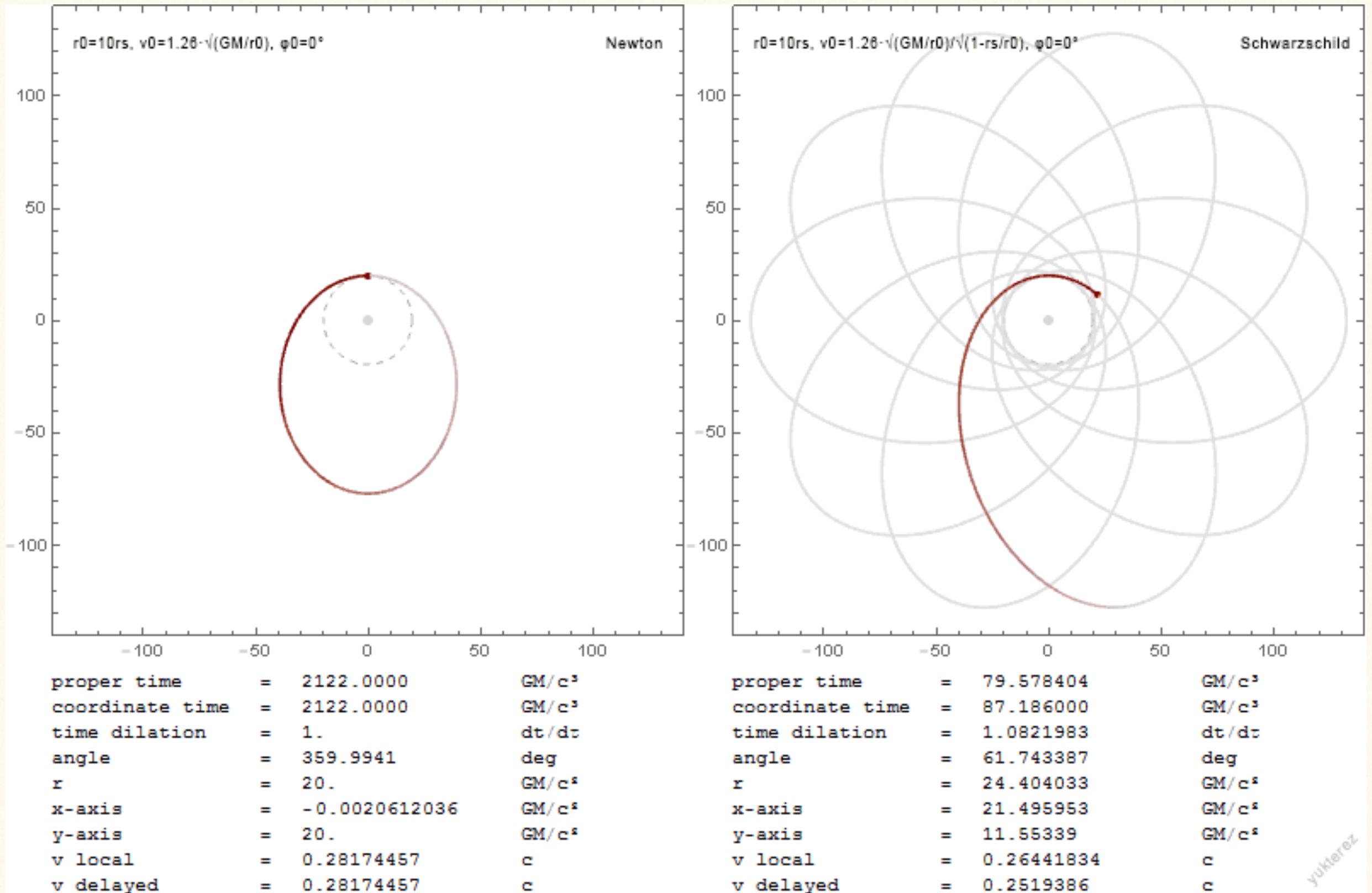
Mercury precesses 565 arcseconds/100 years.

Total due to the planets: 526.7 arcseconds/100 years

Le Verrier took years to do all the calculations!

Ultimately resolved by general relativity in 1915.

Accurate calculation made the detailed “laws” valuable (and falsified them).



We are just going to ignore the planetary contributions to precession.

Our objective: find the correction to the precession due to general relativity.

# Derivation of the GR correction term

$$\frac{(1 - \frac{r_s}{4R})^2}{(1 + \frac{r_s}{4R})^2} dt^2 - (1 + \frac{r_s}{4R})^2(dx^2 + dy^2 + dz^2)$$

Curvature of space

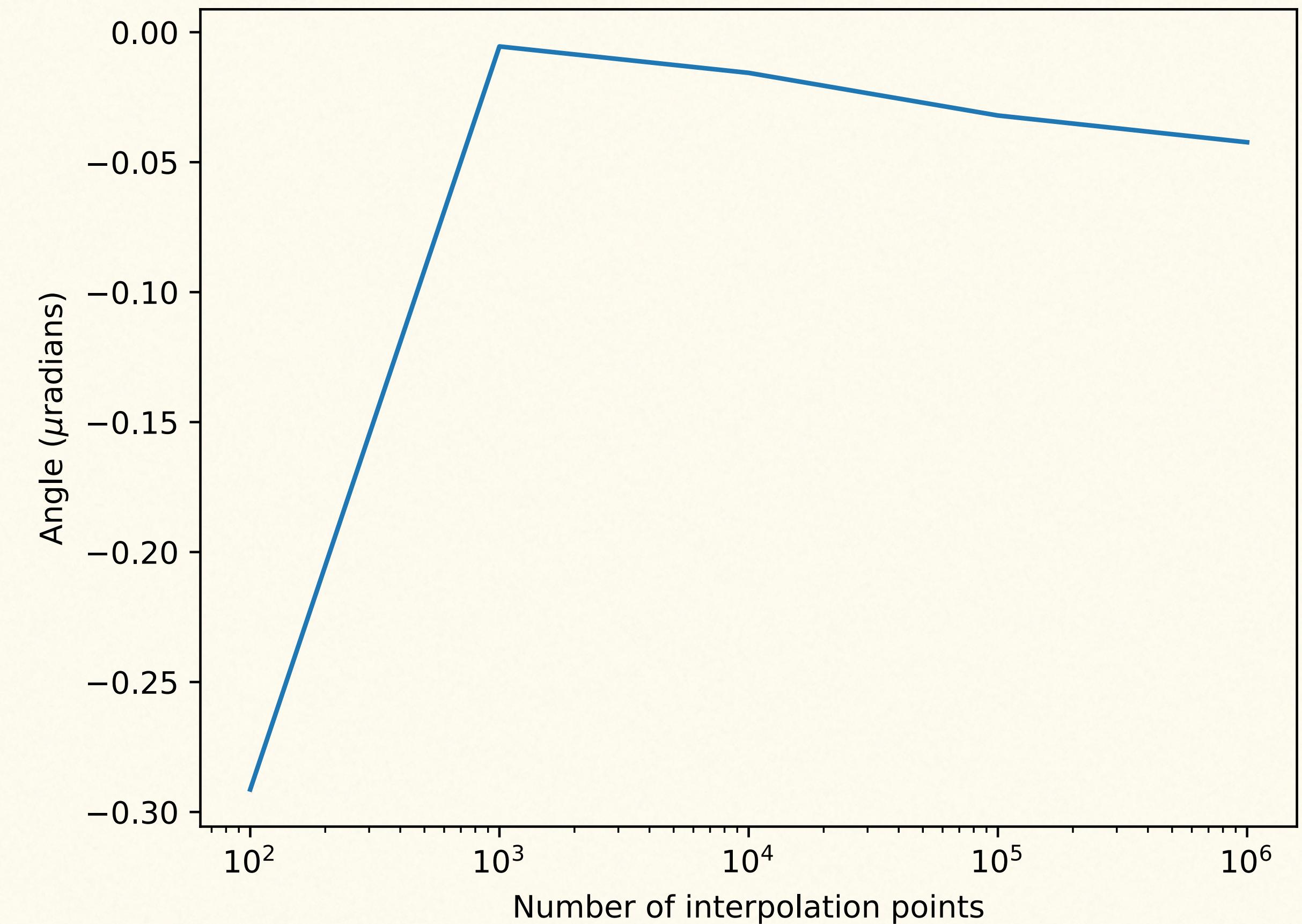
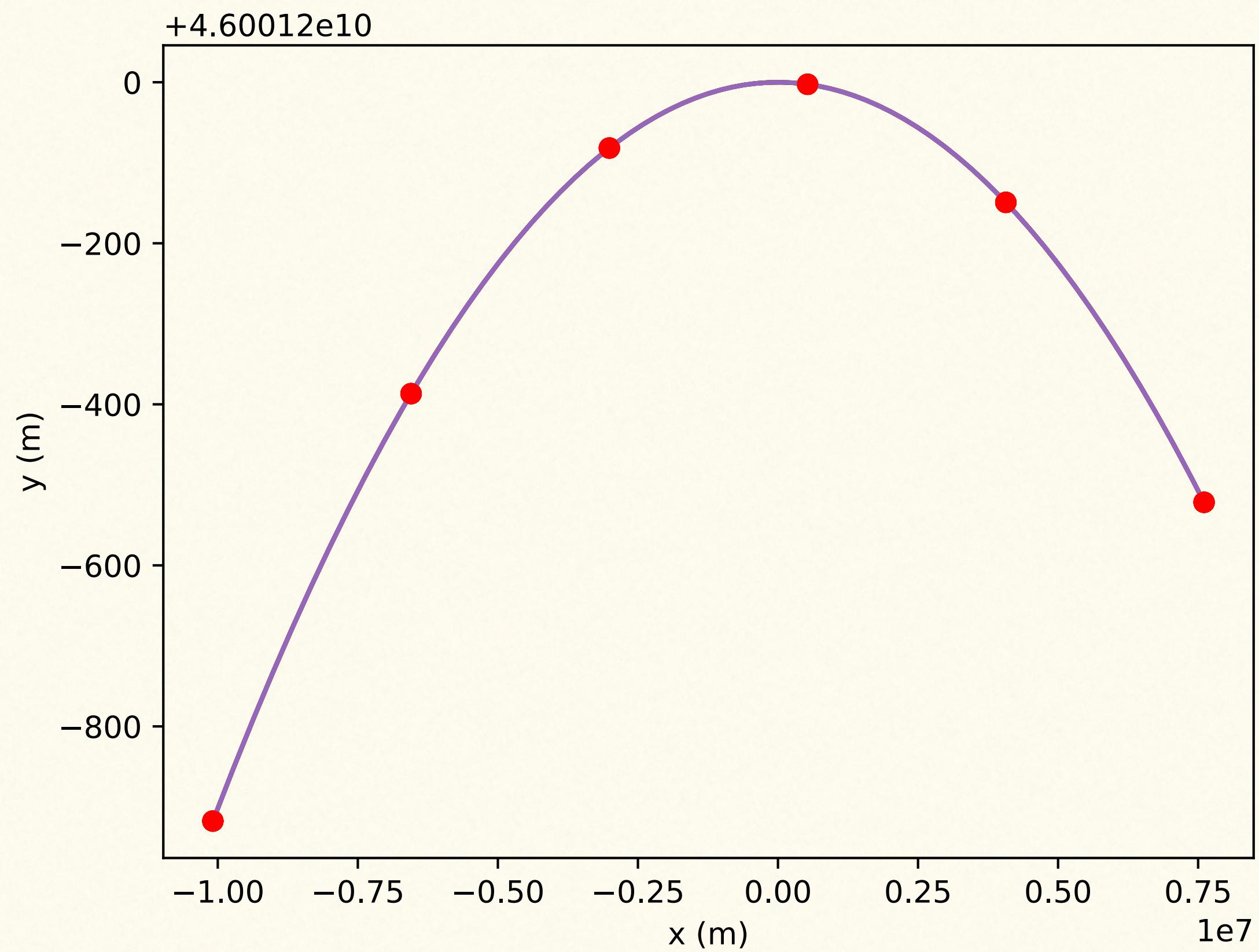
$$F/m = \left( -\frac{GM_\odot}{r^2} - \frac{3r_s \left( v_{\text{perihelion}} r_{\text{perihelion}} \right)^2}{2r^4} \right) \frac{\vec{r}}{r}$$

Effective force  
(approximate)

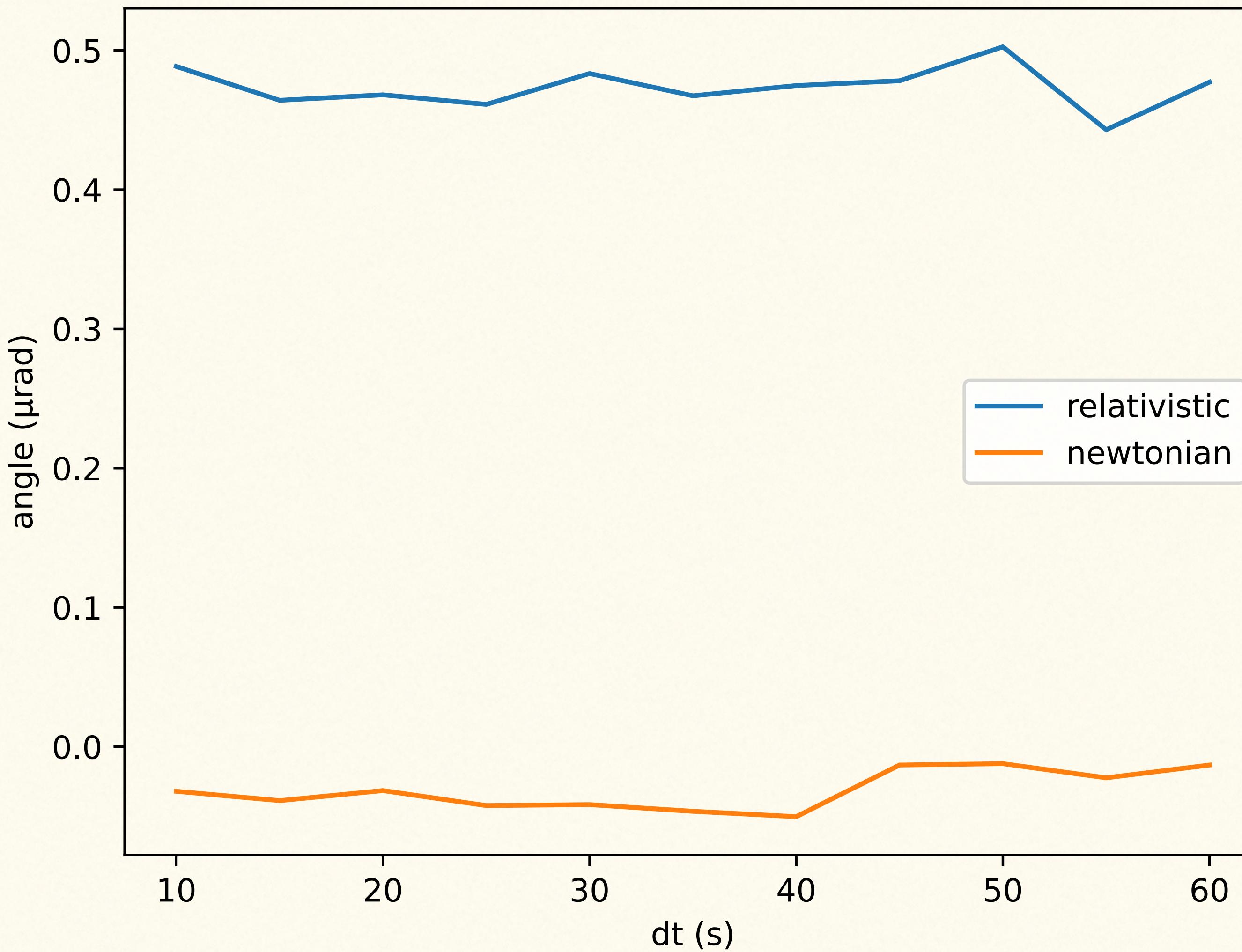


Karl Schwarzschild

# Interpolation: use $10^4$ or $10^5$ points.



# dt convergence



We choose dt of 60 seconds because the results don't really depend on it.