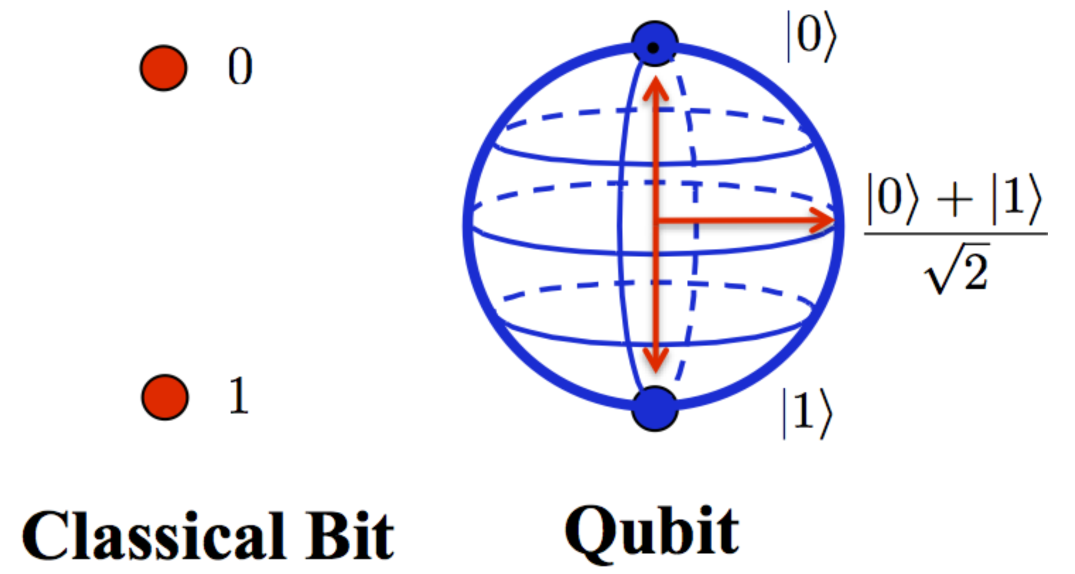
Building a Quantum Qubit
Lecture 14



PHYS 246 class 14
Fall 2025
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https://jnoronhahostler.github.io/IntroductionToComputationalPhysics/intro.html

Announcements

- Last class today
- FINAL!

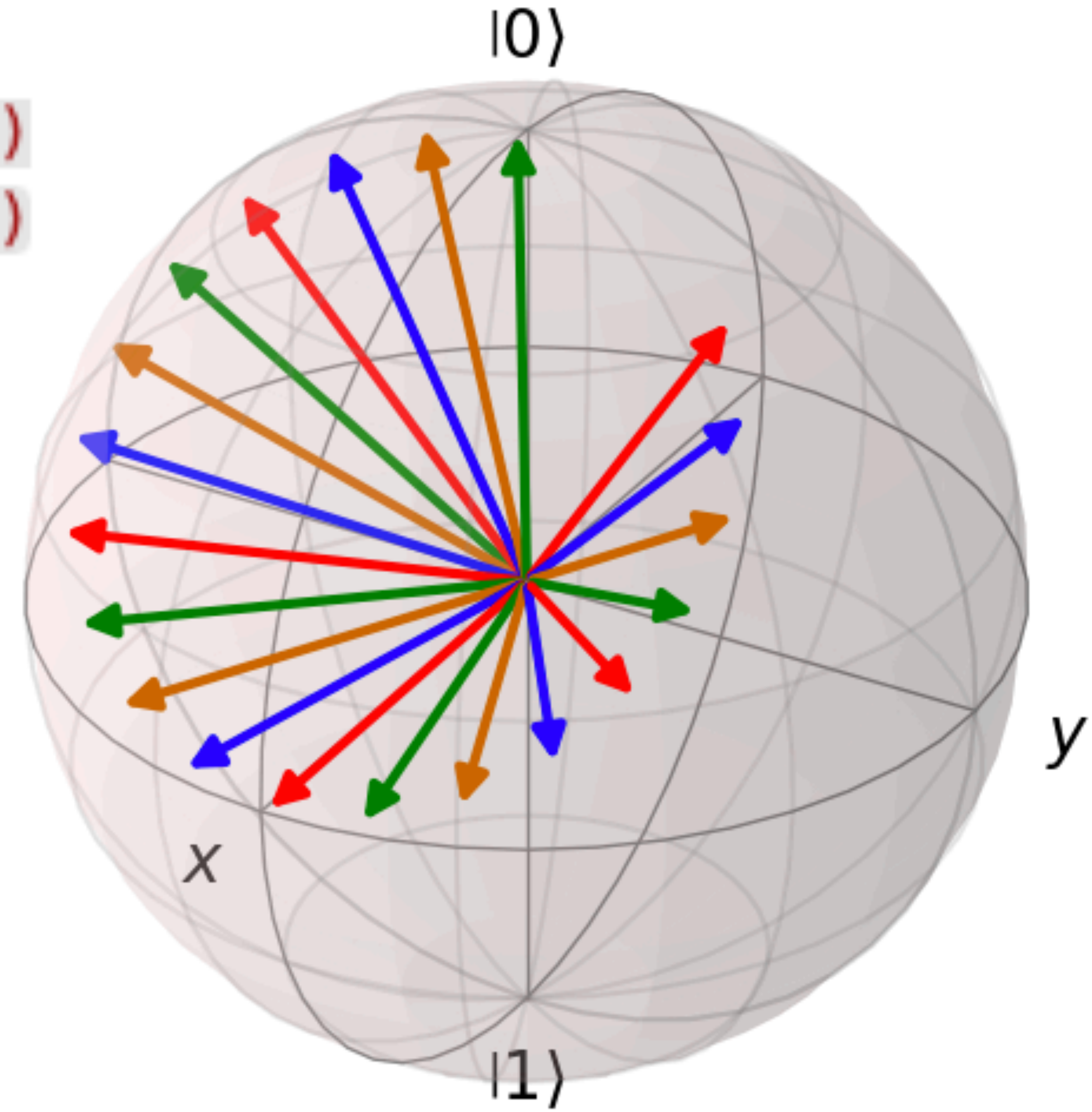
From 1 to n qubits

- Single qubit len(vec_single_qubit)=2
- For n qubits len(vec_n_qubit)=2^n
- If can control how a system evolves from one state to another, can implement abstract gates
- Apply gate to single qubit
    - Use different physical realizations of qubits

Gates

How can we actually get this to happen?

```
circuit.rx(1./50.,0)
circuit.rz(1./50.,0)
```



Implementing a gate on fluxonium

For fixed magnetic flux, ground state is $|0\rangle$, first excited state is $|1\rangle$

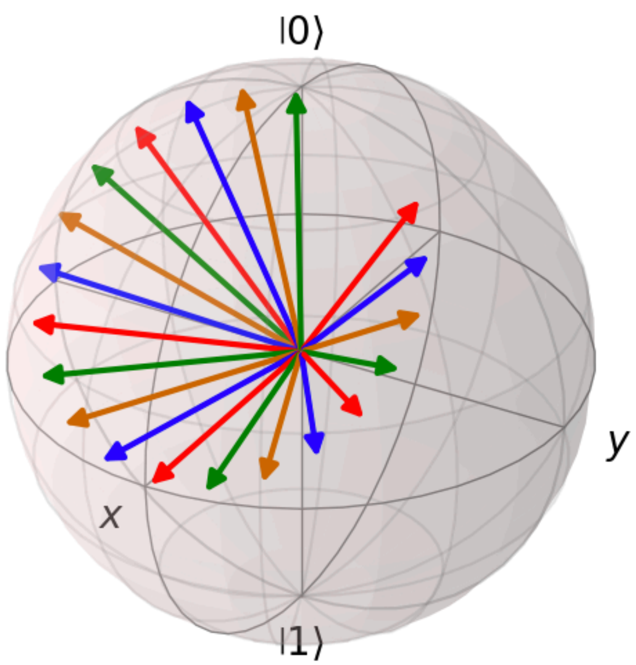
Cool down to $|0\rangle$

$e^{-E_i/kT}$

Change the magnetic flux. Now the state is no longer the ground state and it changes in time.

Measure the state after the right amount of time. Depending on the time you will get some superposition

$\alpha|0\rangle + \beta|1\rangle$



Quantum Mechanics intro

Classical mechanics:

State $x, v, \mathscr{R}^{6N}$ vectors

Dynamical equation:

$$F = ma = m\frac{d^2x}{dx^2}$$

Measurement:

x, v are definite!

Quantum mechanics:

State $\Psi(x)$, function $\mathscr{R}^{3N} \rightarrow \mathscr{C}$

Dynamical equation:

$$i\hbar\frac{\partial\Psi}{\partial t} = \hat{H}\Psi$$

Hamiltonian (similar to energy)

Measurement:

$$\rho(x) = |\Psi(x)|^2$$

$$\rho(p) = |\Psi(p)|^2 = \left| \int e^{ipx/\hbar}\Psi(x)dx \right|^2$$

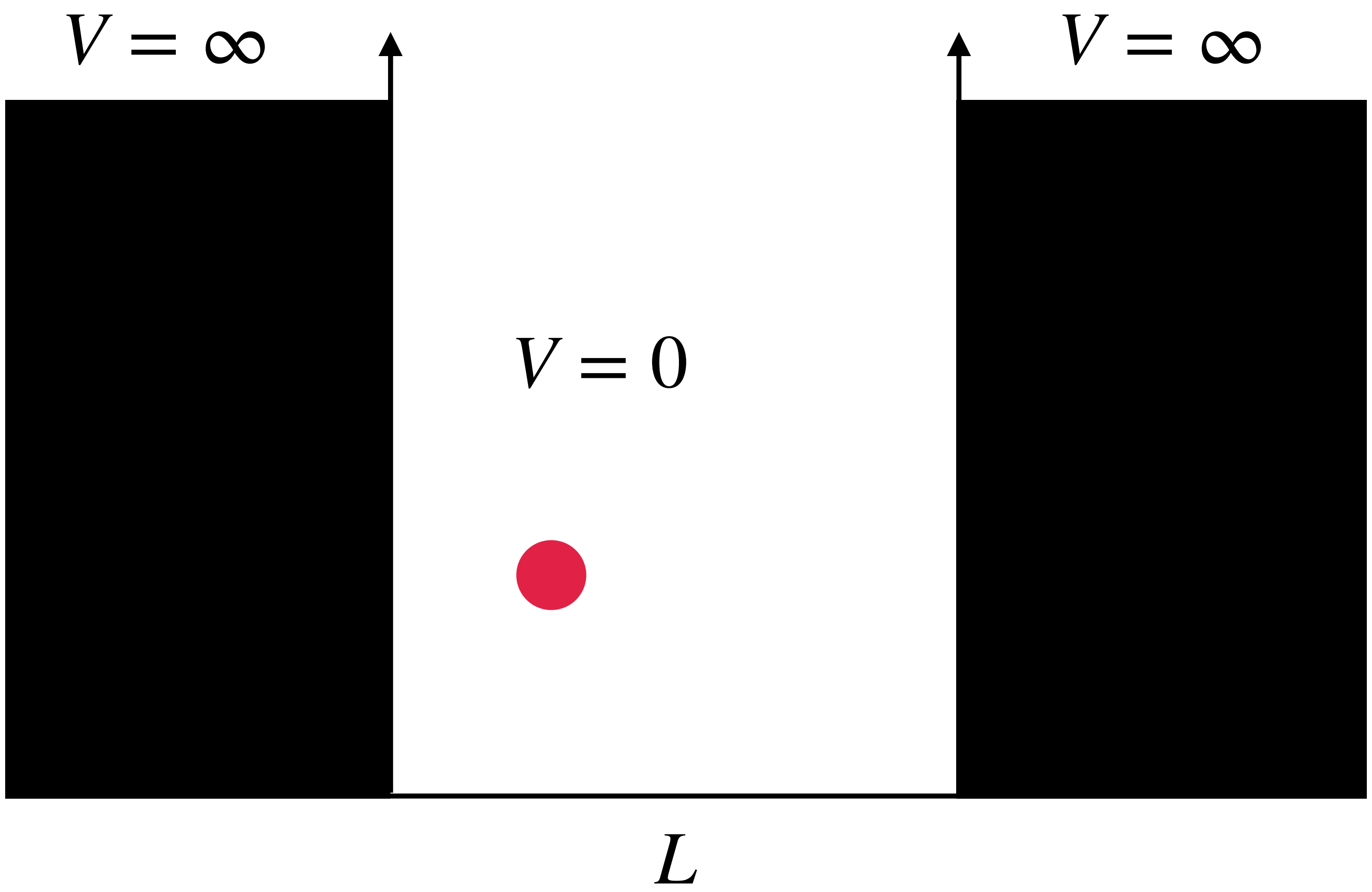
QM particle in an infinite square well

Particle in a box

Energy eigenvalue equation:

$$H\Psi_n(x) = E_n\Psi_n(x)$$

$$H = -\frac{\partial}{\partial x^2}$$



You can only measure energies with a corresponding energy eigenstate.

Question for you: what are the solutions?

Time propagation in QM

Dynamical equation:

$$i\hbar\frac{\partial\Psi}{\partial t} = \hat{H}\Psi$$

Formal solution:

$$\Psi(x,t) = e^{\frac{-i\hat{H}t}{\hbar}}\Psi(x,0)$$

What is the exponential of a matrix??

$$e^A = 1 + A + \frac{1}{2}A^2 + \dots$$

OR

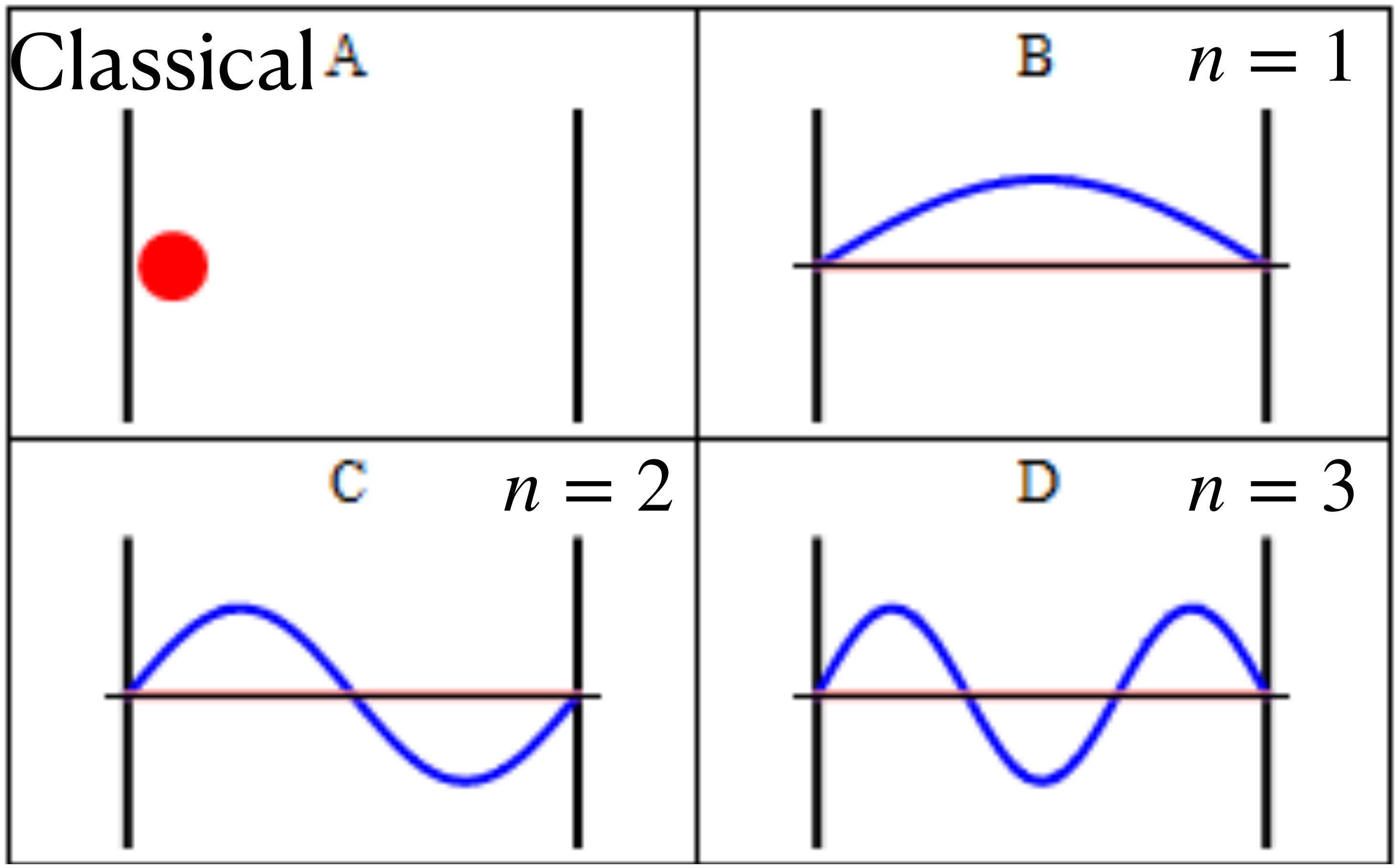
If U is the set of eigenvectors (columns), then

$$U^\dagger A U = \text{diag}[E_i]$$

and

$$e^A = U^\dagger \text{diag}[\exp(E_i)]U$$

Solution to particle in a box



After boundary conditions, energy is quantized!

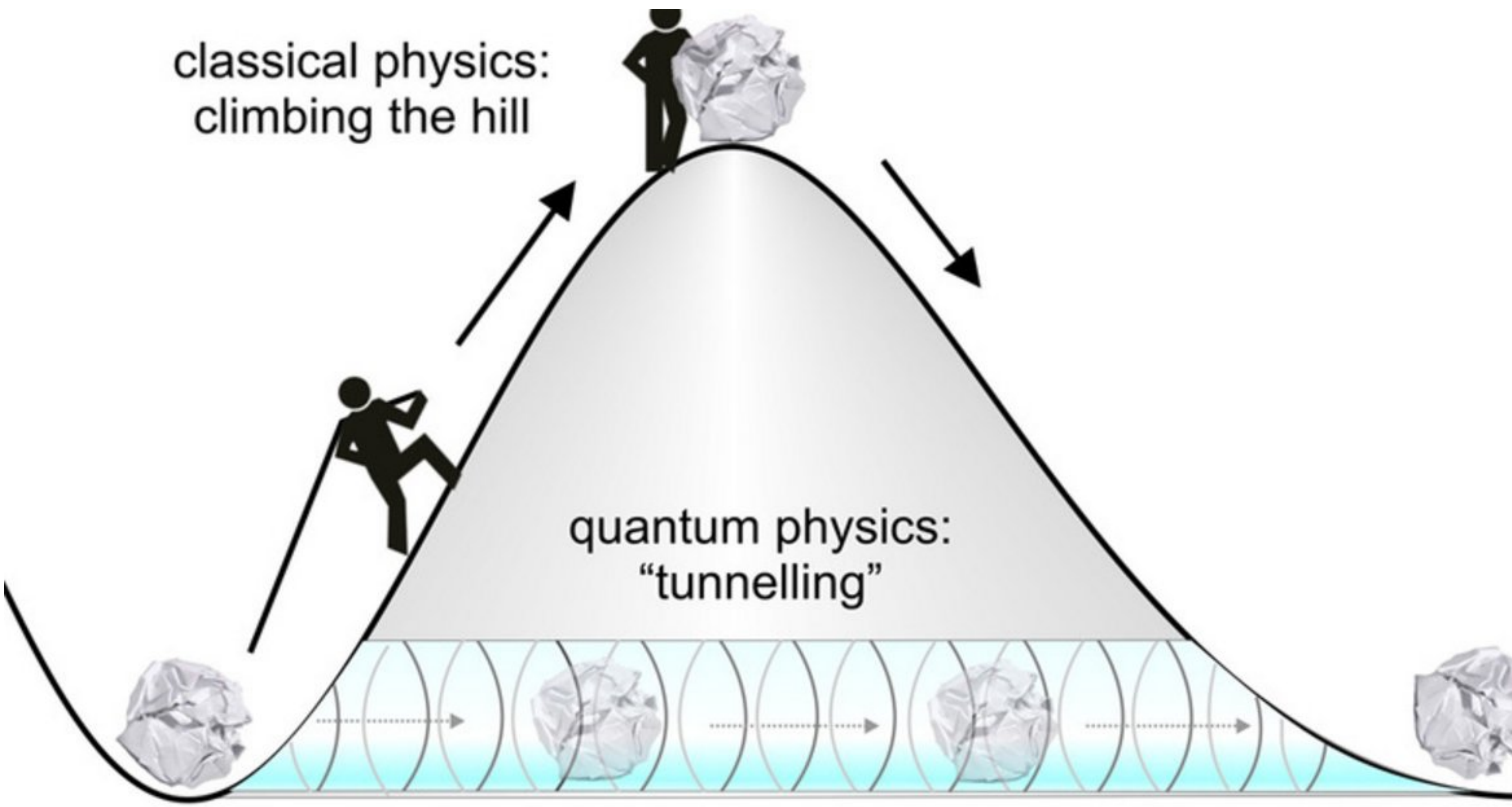
$$\boxed{E_n = \hbar\omega_n = \frac{n^2\pi^2\hbar^2}{2mL^2}}$$

Wavefunction (before applying boundary conditions)

$$\psi(x,t) = \left[A\sin(kx) + B\cos(kx)\right] e^{iwt}$$

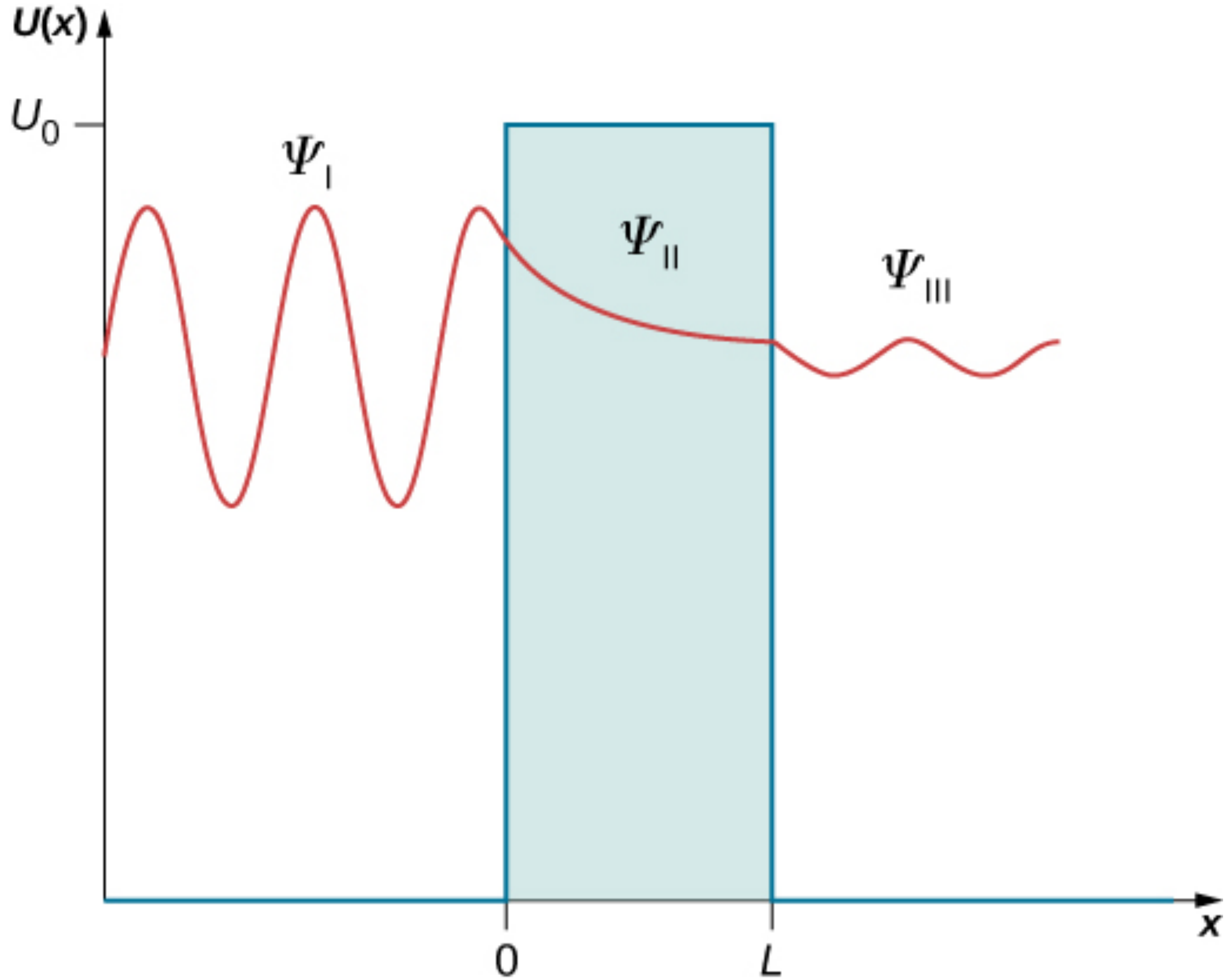
Quantum tunneling

Passing through walls!



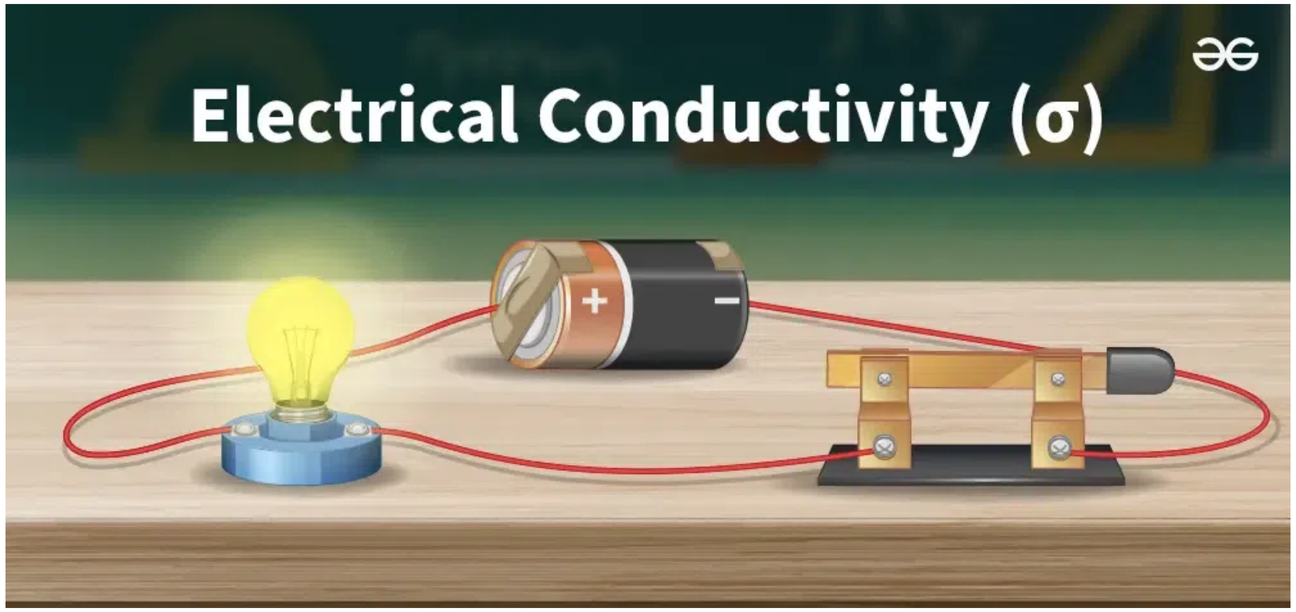
Wave function through a barrier

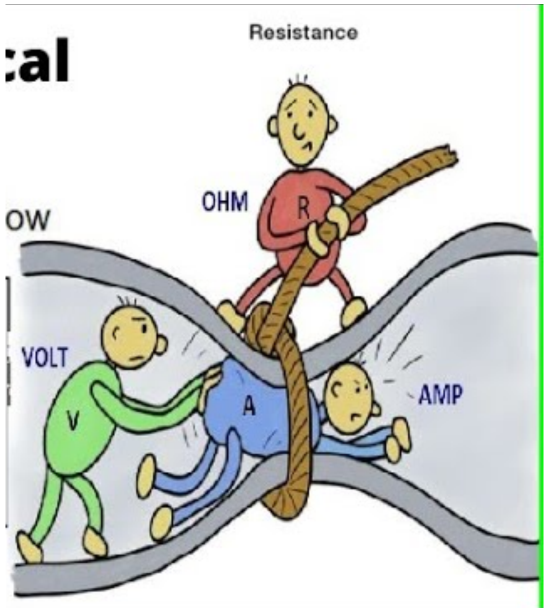
More on quantum tunneling



Superconductors

Vanishing electrical resistance, magnetic fields are expelled





- Ordinary metal conductor: resistance ↓ as temperature ↓
- Superconductor: critical temperature T_c
  - $T < T_c$, resistance = 0
- Macroscopic state
- Why does this happen?

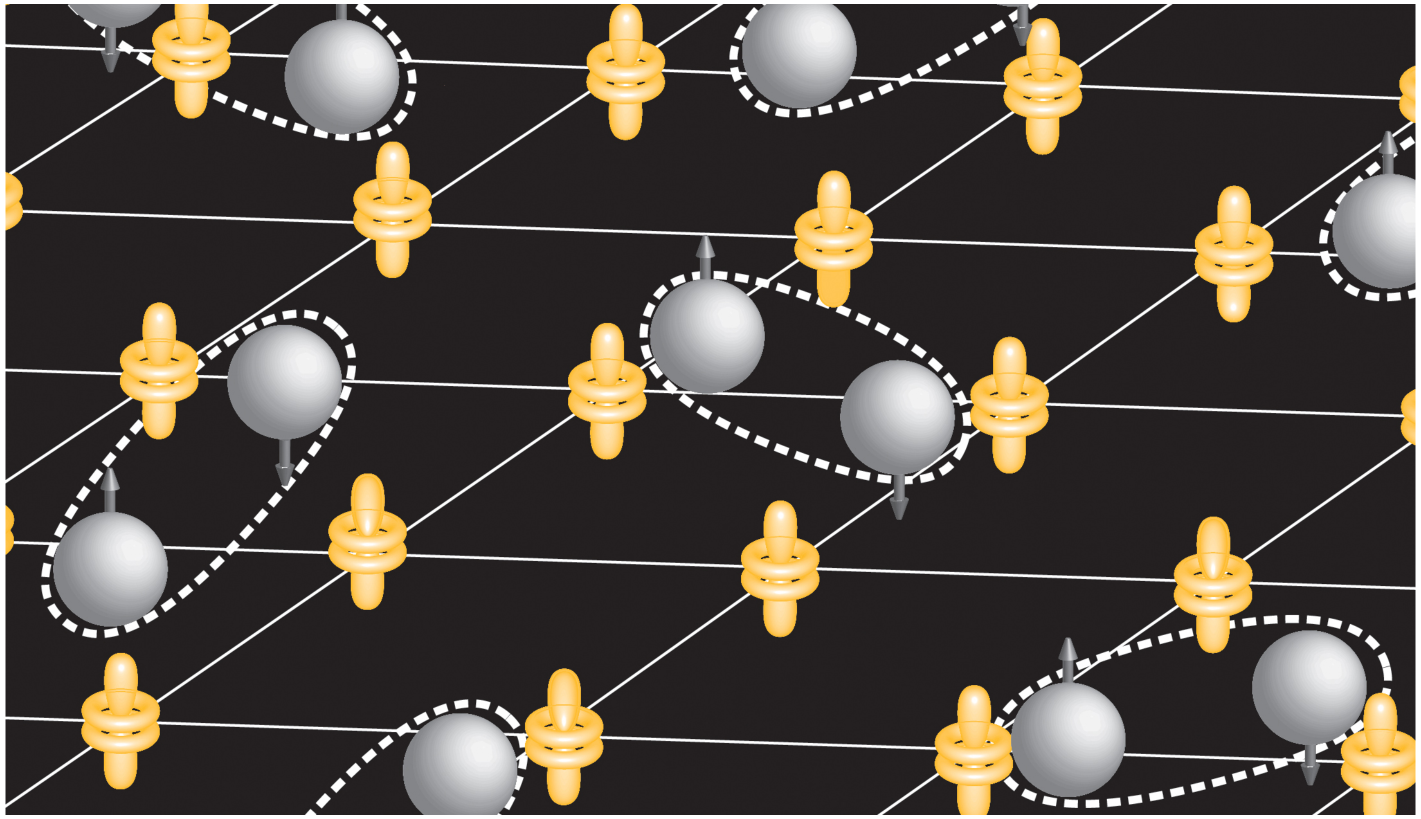
Electron pairing

At very cold temperatures, electrons pair up (↑ ↓). This state is described as an amplitude and phase

$|\Psi| e^{i\theta}$

amplitude is how many electrons are paired up (n)

phase is related to the current



Superconductor circuit

All quantum...

Held at 20 mK or so..

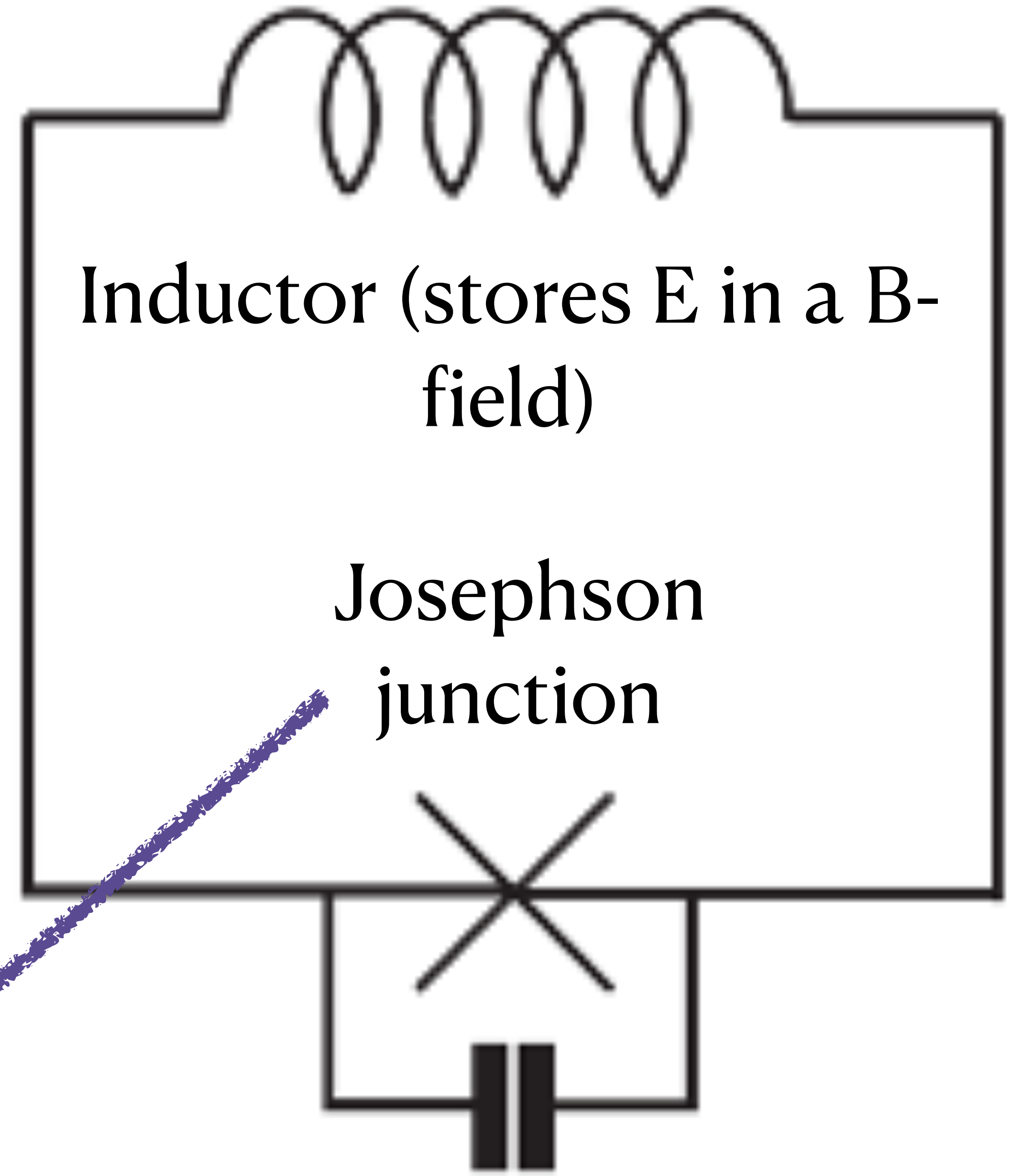
A few hundred μm across

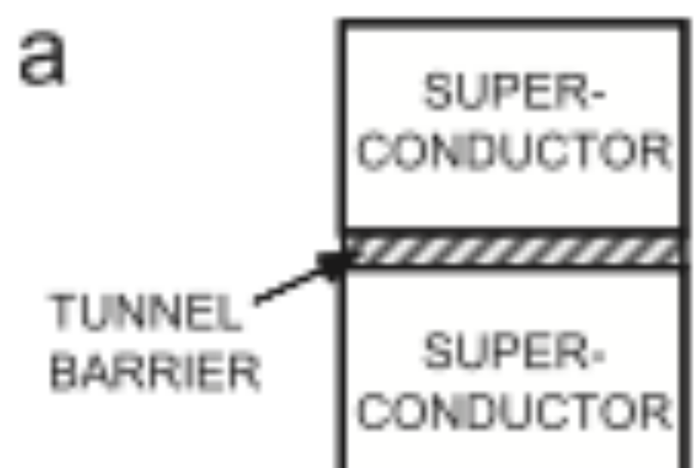
Quantum!

Phase across the junction -> current/velocity

Number that cross the junction-> position

Stores electricity





Capacitor (stores electrical field)

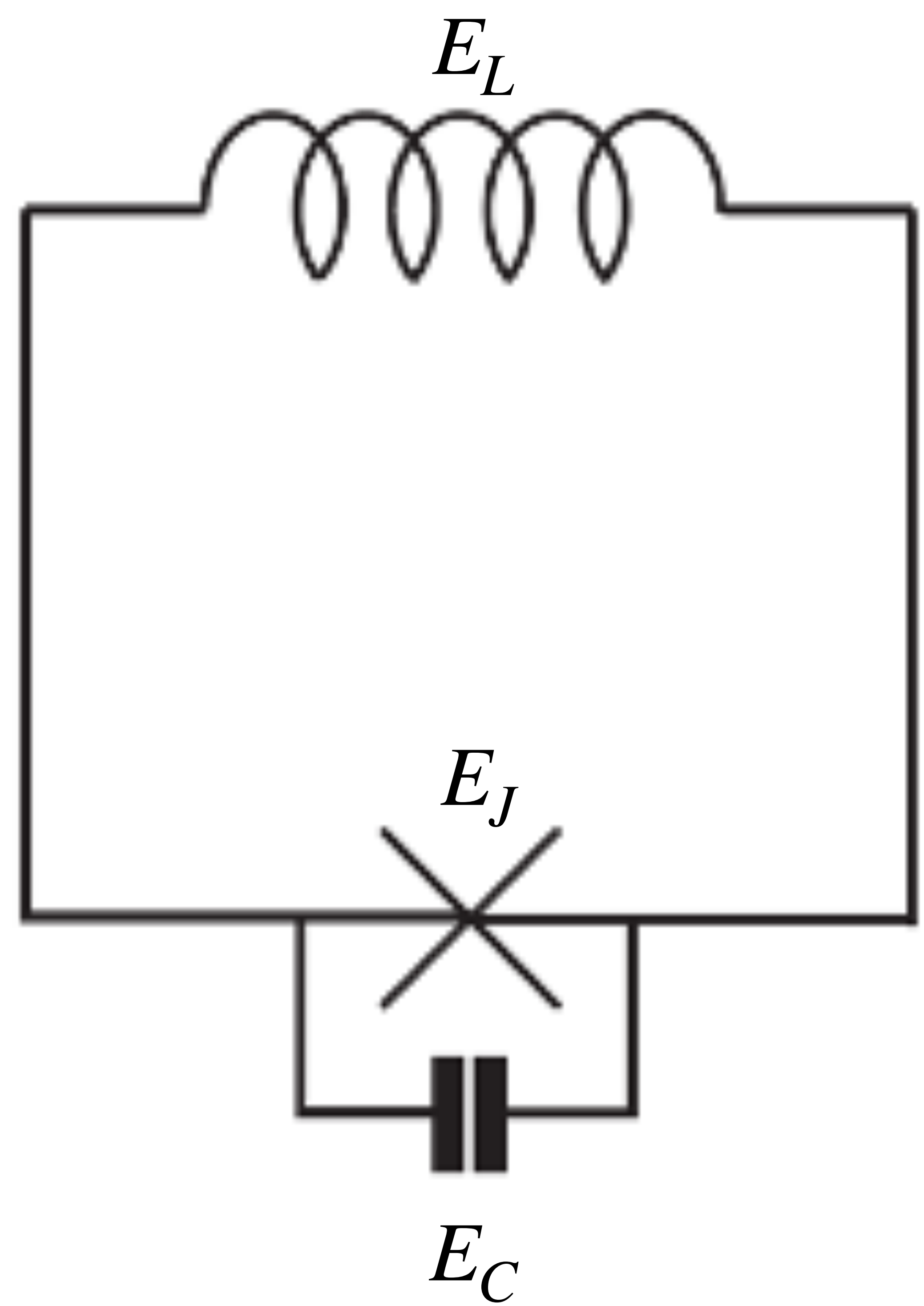
Fluxonium system

Inductor energy: $\frac{1}{2}LI^2 \equiv \frac{1}{2}E_L\phi^2$

Capacitor energy:
$\frac{1}{2}\frac{Q^2}{C} \equiv 4E_C\hat{n}^2 = -4E_C\frac{\partial^2}{\partial\phi^2}$

Magnetic flux: ϕ_{ext} (written in appropriate units of $\frac{hc}{e}$)

Josephson junction: $-E_J\cos(\phi - \phi_{ext})$



NOTE: we do a change of variables for a fixed external flux, of $\phi \to \phi - \phi_{ext}$

More info

If you do

eigs, vecs = np.linalg.eigh(H)

The eigenvector i is given by vecs[:,i]

Input: square matrix H

Output: 2 arrays (w,v)
w 1-D array of the eigenvalues in ascending order

v 2-D array where each column v[:,I] represents the normalized eigenvector corresponding to eigenvalue w[I]