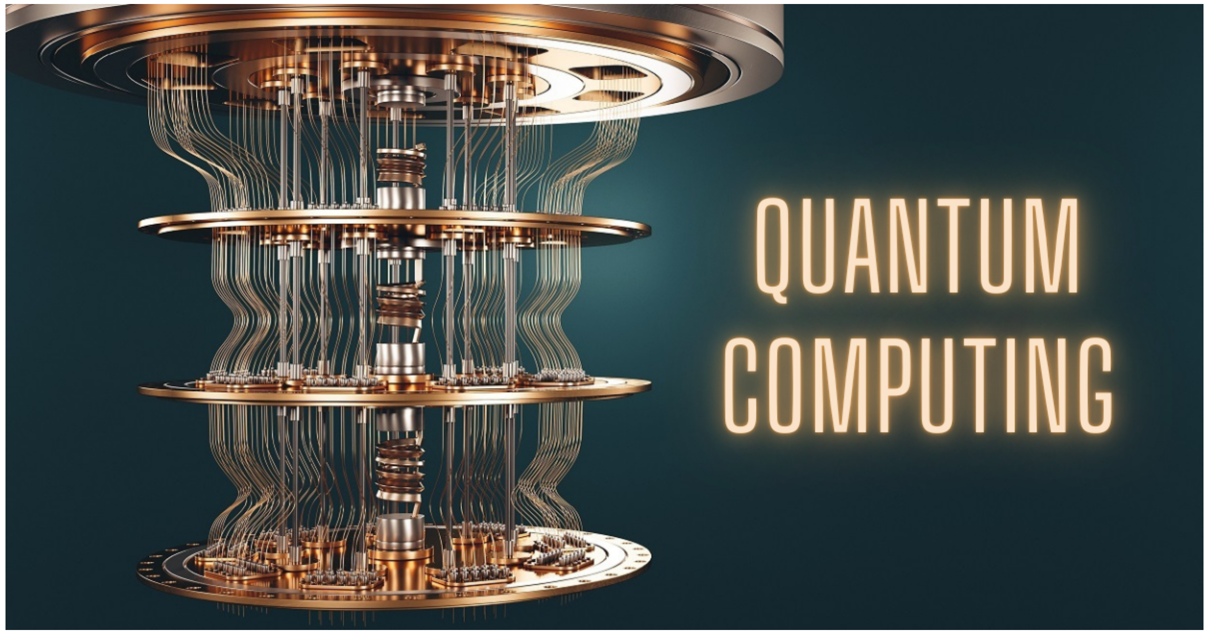
Quantum Computing Lecture 12



PHYS 246 class 11
Fall 2025
J Noronha-Hostler

https://jnoronhahostler.github.io/IntroductionToComputationalPhysics/intro.html

Announcements

- No class next week!
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What is quantum information?

Quantum computing

Entanglement allows exponential speed-up for certain problems

Quantum Networking

Shared entanglement allows secure communication

Quantum sensing

Entangled inputs reduce “shot” noise

Quantum Mechanics intro

Classical vs quantum

Classical mechanics:

State $x, v, \mathscr{R}^{6N}$ vectors

Dynamical equation:

$$F = ma = m\frac{d^2x}{dx^2}$$

Measurement:

x, v are definite!

Quantum mechanics:

State $\Psi(x)$, function $\mathscr{R}^{3N} \to \mathscr{C}$

Dynamical equation:

$$i\hbar\frac{\partial\Psi}{\partial t} = \hat{H}\Psi$$

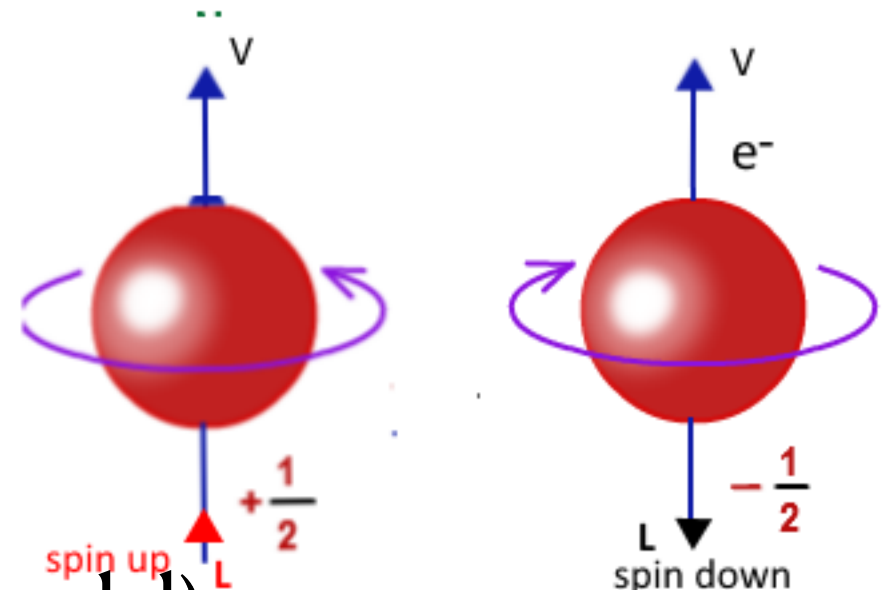
Measurement:

$$\rho(x) = |\Psi(x)|^2$$

$$\rho(p) = |\Psi(p)|^2 = \left| \int e^{ipx/\hbar}\Psi(x)dx \right|^2$$

Two states systems

Spin up and down



Electron spin, other q-bit systems

Can be only up or down.

Classically, $x = [1,0] = \uparrow$ or $[0,1] = \downarrow$ (Ising model)

Then $\Psi(x)$ is specified by two complex numbers $[a, b] = a \uparrow + b \downarrow$.

Probability we measure the state $\uparrow$: $\dfrac{|a|^2}{|a|^2 + |b|^2}$

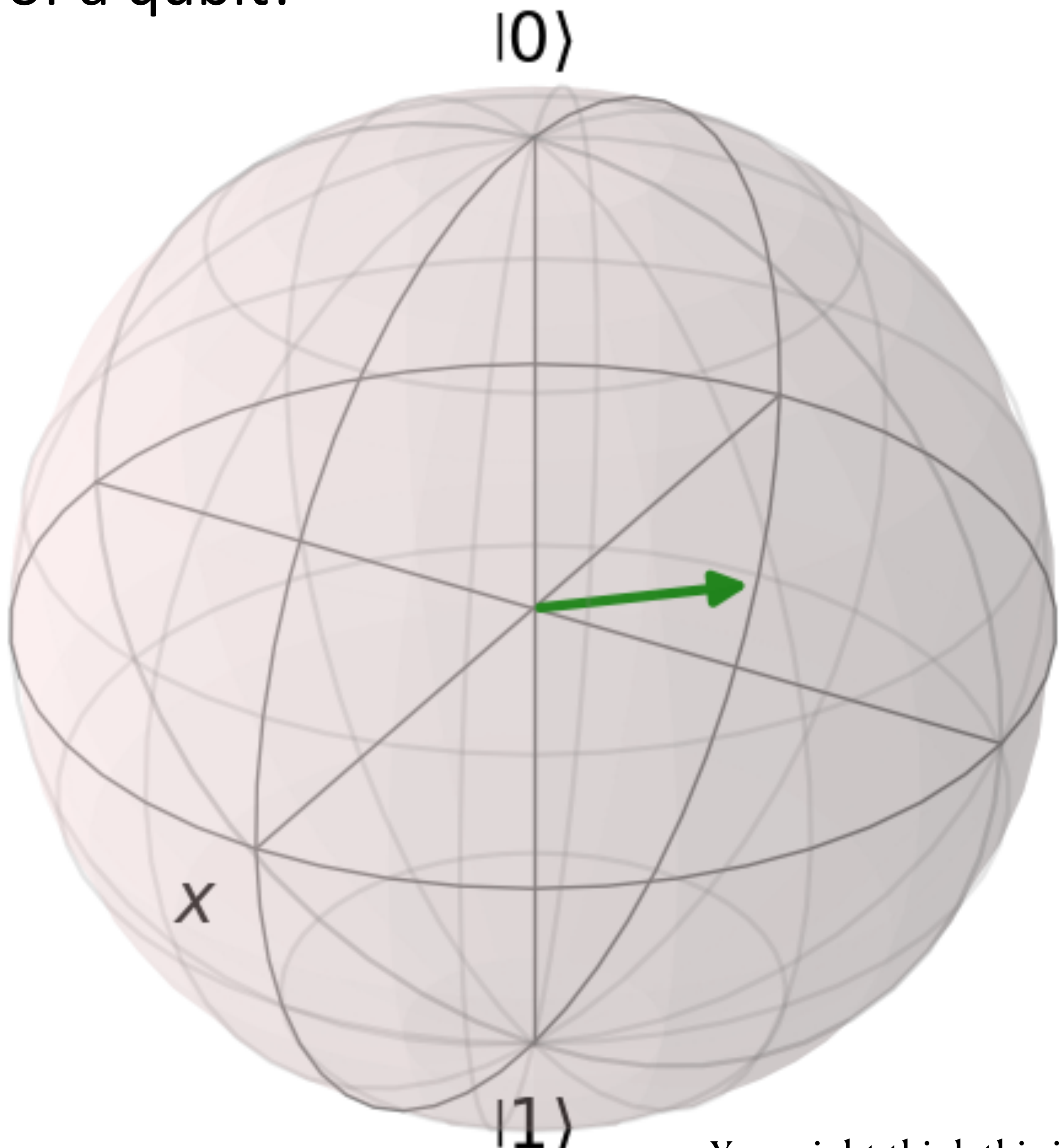
Normalization: set $|a|^2 + |b|^2 = 1$

Entangled states of N qubits described by 2^N real numbers!
Exponential growth: computers can't simulate even moderately sized quantum systems
quantum systems can "compute" problems that are intractable on computers!

Bloch sphere

Cannot measure the full quantum state of a qubit!

Linear superposition of states



$\uparrow$ is $[1,0] = |0\rangle$

$\downarrow$ is $[0,1] = |1\rangle$

$\rightarrow$(x) is $[1,1] = |0\rangle + |1\rangle$

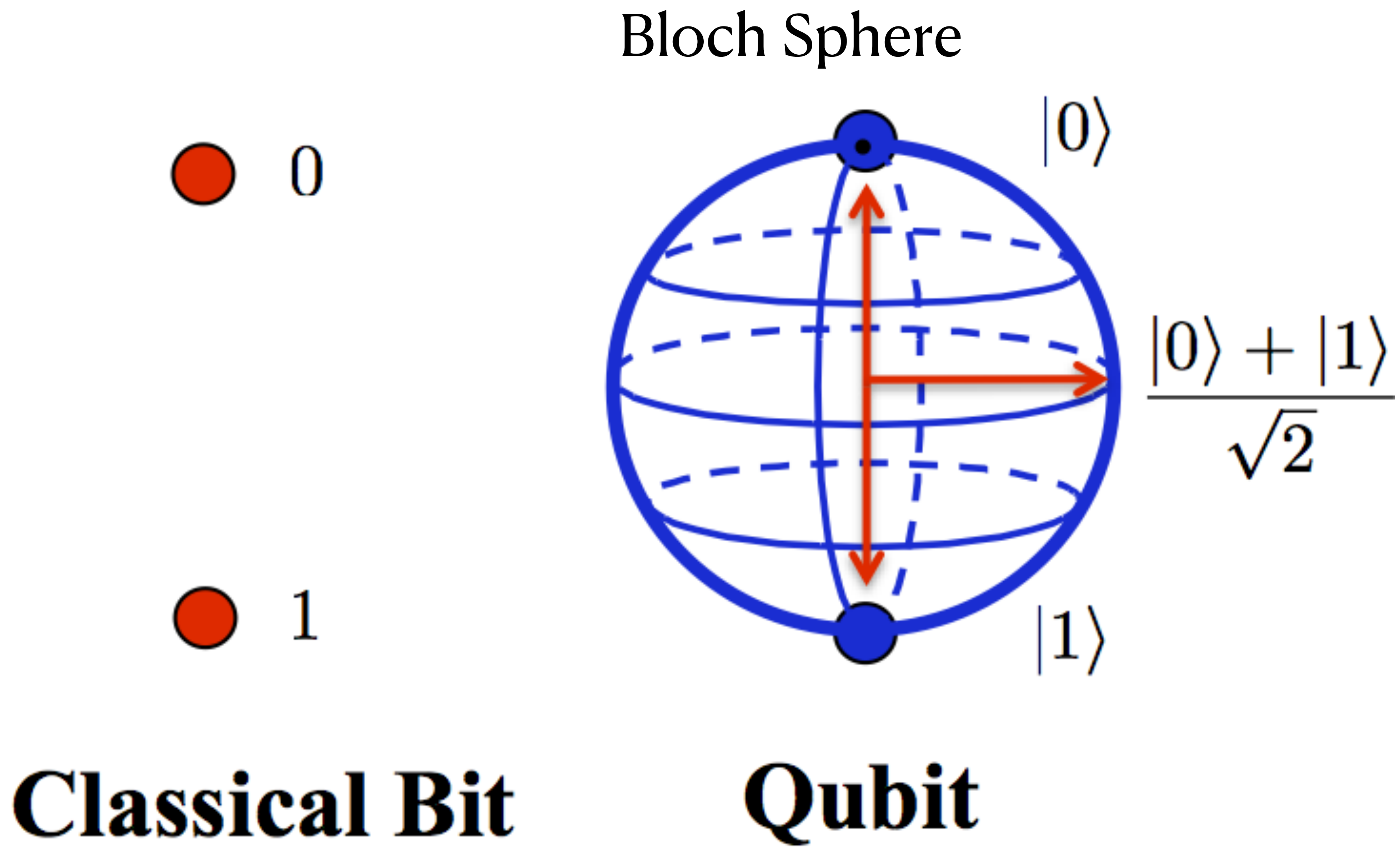
and so on

Spin direction	State
$\hat{z}$	$\uparrow$
$-\hat{z}$	$\downarrow$
$\hat{x}$	$\frac{1}{\sqrt{2}}(\uparrow + \downarrow)$
$-\hat{x}$	$\frac{1}{\sqrt{2}}(\uparrow - \downarrow)$
$\hat{y}$	$\frac{1}{\sqrt{2}}(\uparrow + i \downarrow)$
$-\hat{y}$	$\frac{1}{\sqrt{2}}(\uparrow - i \downarrow)$

You might think this is 4d since there are two complex numbers, but an overall phase does not change anything, so the states can be represented in 3d.

Qubits

Classical vs quantum computing basics



Multiple two-state system

Classically, the phase space is $\uparrow\uparrow, \uparrow\downarrow, \downarrow\uparrow, \downarrow\downarrow$

Each gets multiplied by a different complex number:

$\Psi = a\uparrow\uparrow + b\uparrow\downarrow + c\downarrow\uparrow + d\downarrow\downarrow$ (four complex numbers)

Entanglement

Einstein–Podolsky–Rosen

Consider the state $\frac{1}{\sqrt{2}} \uparrow\uparrow - \frac{1}{\sqrt{2}} \downarrow\downarrow$.

P(both up) = 1/2

P(both down) = 1/2

P(one up, one down) = 0

P(one down, one up) = 0

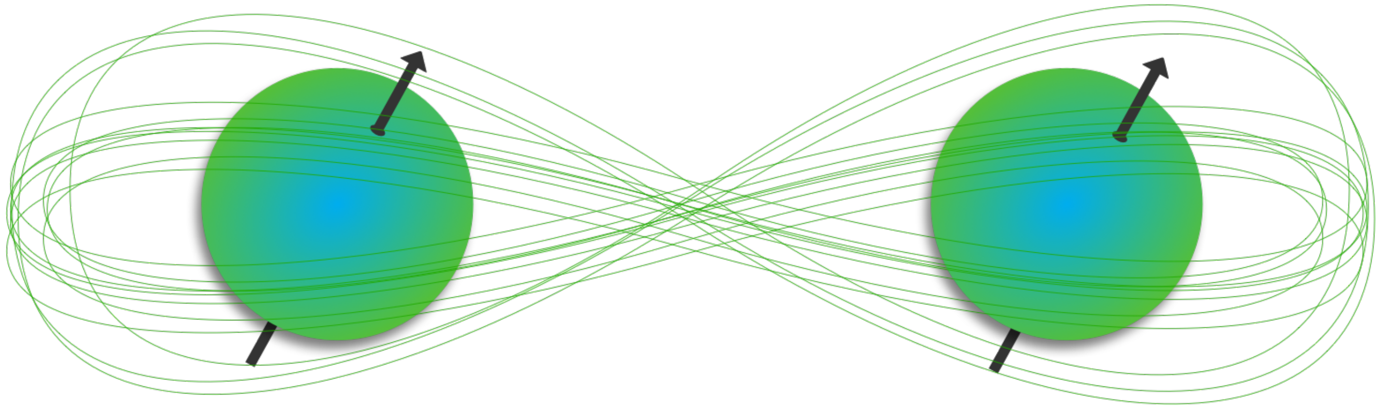
This is called an entangled state

(sometimes a 'cat' state)

Measuring one spin will also collapse the state of the other spin

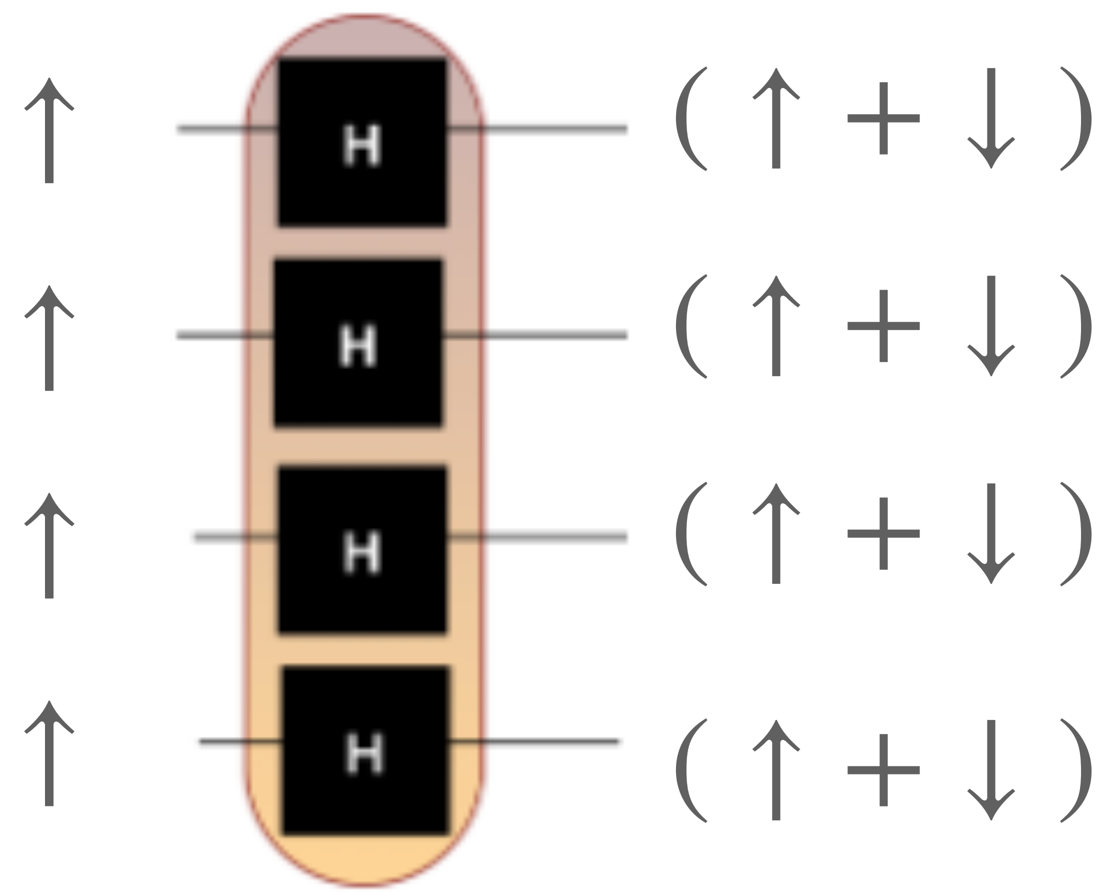
Discussion:

Is $\frac{1}{2}(\uparrow\uparrow + \uparrow\downarrow + \downarrow\uparrow + \downarrow\downarrow)$ entangled?



Gates

In quantum computing, we idealize the time dependence of the wave function to 'gates'



Hadamard gate (change of basis):

$$H\uparrow = \frac{1}{\sqrt{2}}(\uparrow + \downarrow)$$

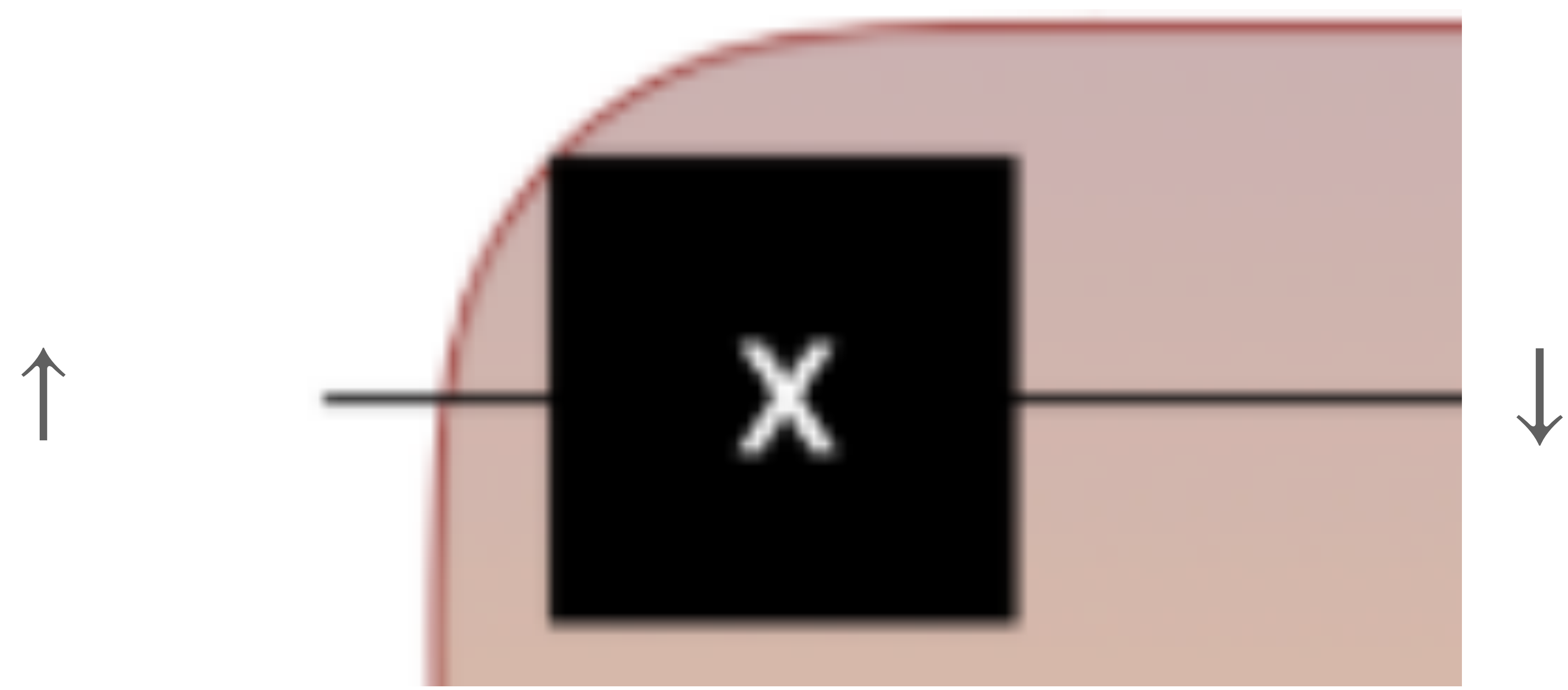
$$\uparrow\uparrow\uparrow\uparrow \qquad \frac{1}{4}(\uparrow + \downarrow)(\uparrow + \downarrow)(\uparrow + \downarrow)(\uparrow + \downarrow)$$

x-gate

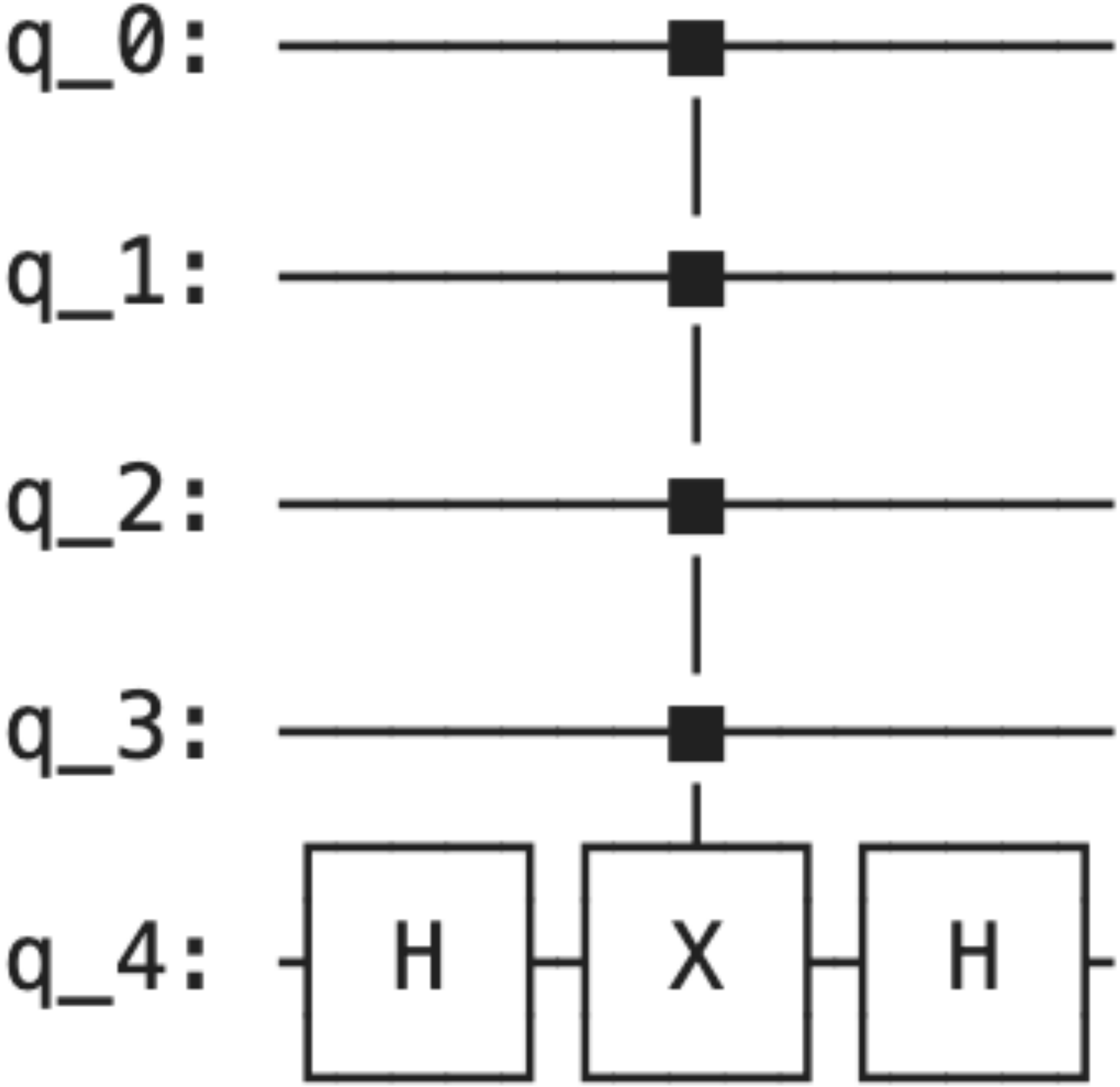
The classical NOT gate (flips the spin)

Named after the Pauli matrix

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$



Control gates (CNOT)



Often the only way we introduce entanglement!

Apply an x on qbit 4 if 0, 1, 2, and 3 are all 1.

Best choice depends on your architecture

quiskit and circuits

Coding help

```
def Mark(r,N):
    circuit=QuantumCircuit(N,N)
    circuit.barrier()


    myString=np.binary_repr(r,width=N)[::-1]
    for i in range(0,len(myString)):
        if myString[i]=='0':
            circuit.x(i)
    circuit.barrier()

    circuit.h(N-1)
    circuit.mcx(list(range(0,N-1)), N-1,mode='noancilla')
    circuit.h(N-1)
    circuit.barrier()

    for i in range(0,len(myString)):
        if myString[i]=='0':
            circuit.x(i)
    circuit.barrier()
    return circuit
```

Create an empty circuit with N bits

Barrier separates pieces of circuit

Adds a gate to the circuit

Qiskit is an open-source Python tool for working with quantum computers

Classical search

N items.

We would like to find one element that matches $f(i) = 1$ ($f(i) = 0$ for all other elements)

Classically you go through items until you find the right one. On average $N/2$.

$\mathcal{O}(N)$

Databases, factorization, find the winning chess move, find the path for a robot to move in...

Grover's algorithm

Quantum search

Scales as $\mathcal{O}(\sqrt{N})$.

One of the few (only?) useful quantum algorithms with a proven speedup over classical algorithms.