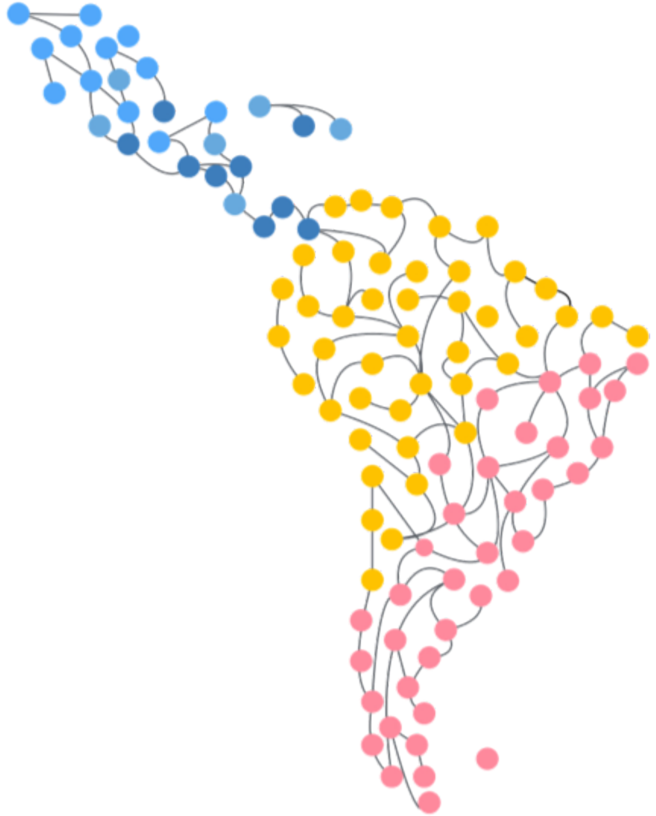

Joint characterization of **phase synchronization** in networks with **multivariate singular spectrum analysis** and **vector field phase**

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SUMARY

THE GOAL

ASSESMENT TOOLS

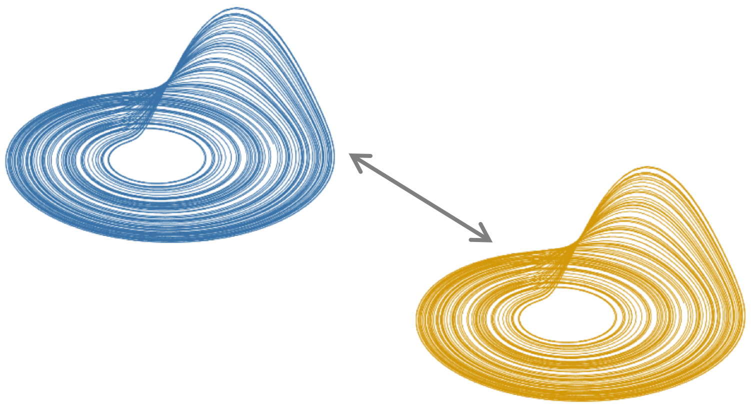
VECTOR FIELD PHASE (VFP) | SYNC QUALITY

VARIMAX + MULTIVARIATE SINGULAR SPECTRUM ANALYSIS

EXAMPLE

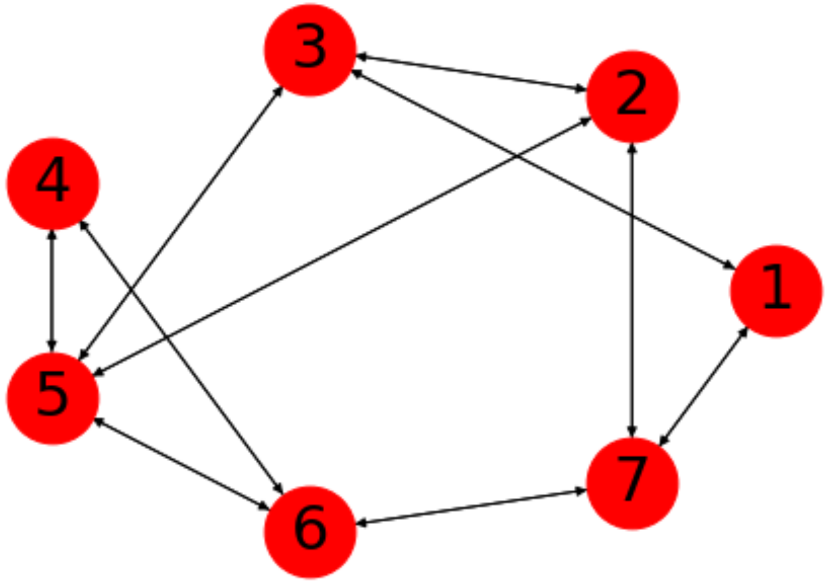
THE GOAL

THE GOAL



If we decrease the coupling strength…

Who will **desynchronize** first?



ASSESMENT TOOLS

- ✓ VECTOR FIELD PHASE (VFP) | SYNC QUALITY
- ✓ VARIMAX + MULTIVARIATE SINGULAR SPECTRUM ANALYSIS

VECTOR FIELD PHASE (VFP) | SYNC QUALITY

Freitas, L., Torres, L. A. B., & Aguirre, L. A. (2018). **Phase definition to assess synchronization quality of nonlinear oscillators**. *Physical Review E*, *97*(5), 052202.

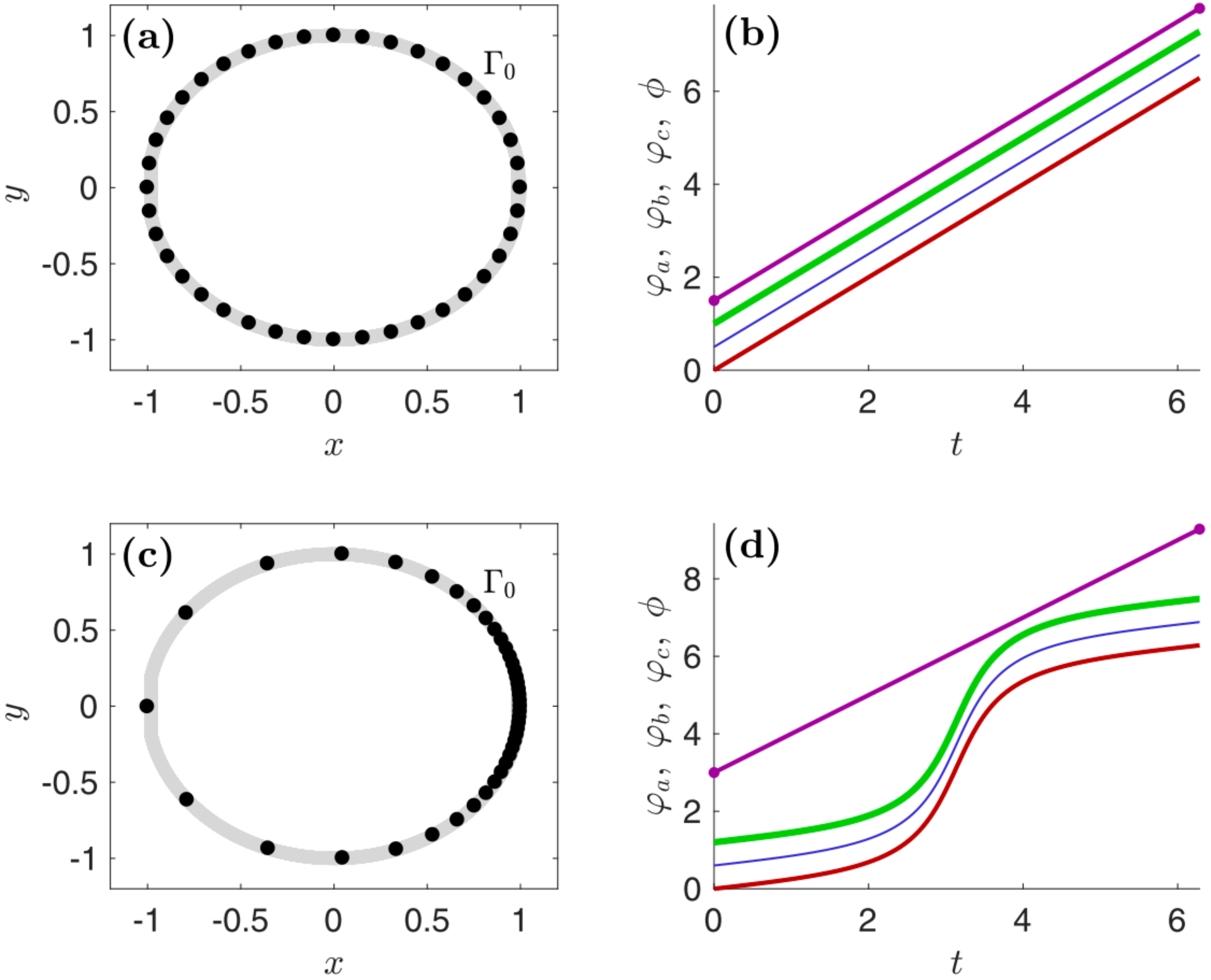
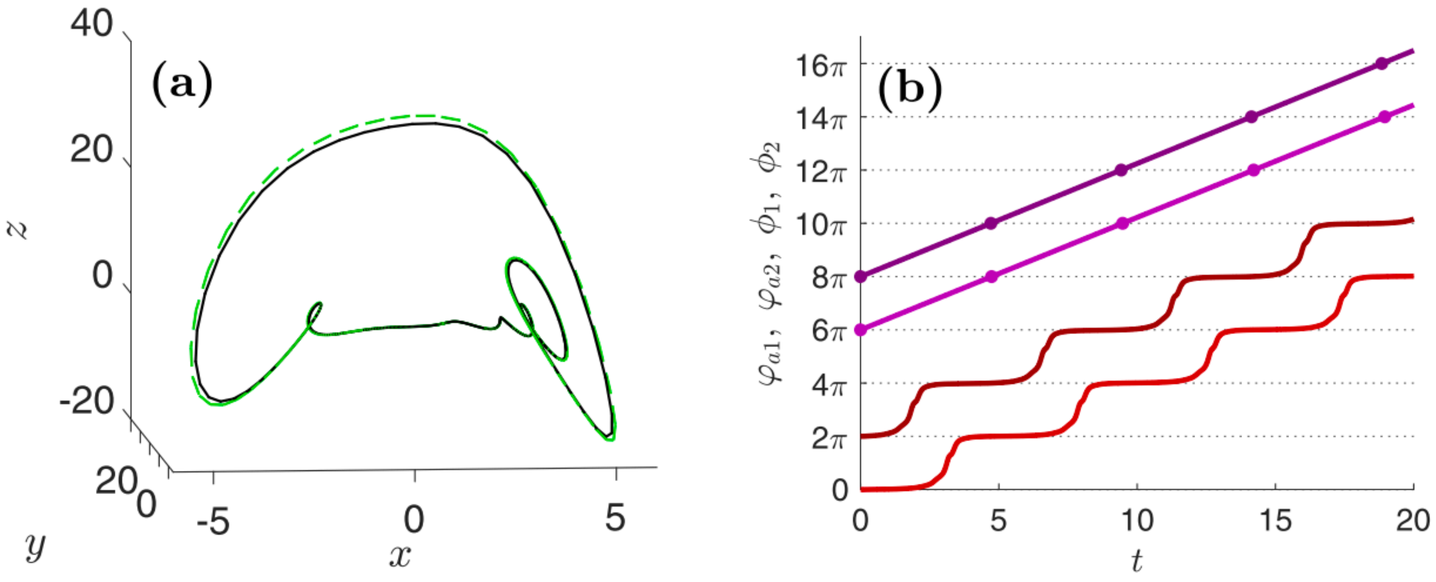


FIG. 1. (a) and (b) Original and (c) and (d) modified Poincaré oscillators: (a) and (c) state space showing evolution on Γ_0 (gray line) and equally time-spaced dots and (b) and (d) phaselike variables vertically displaced for the sake of clarity. Shown, from top to bottom, are φ_a (Poincaré section), φ_b (angle), φ_c (curvature), and ϕ (VFP). The parameters are $(\lambda, p) = (0.5, 1)$, with $\omega = 1$ for the original system, and $\omega = \alpha(1 - 0.9x)$, with $\alpha = 22.646$ for the modified one.

VECTOR FIELD PHASE (VFP) | SYNC QUALITY



Freitas, L., Torres, L. A. B., & Aguirre, L. A. (2018). **Phase definition to assess synchronization quality of nonlinear oscillators**. *Physical Review E*, *97*(5), 052202.

FIG. 3. Results for the cord oscillators (28): (a) steady-state evolution of uncoupled ($\varepsilon = 0$) oscillators 1 (solid black line) and 2 (dashed green line) and (b) phase variables, from top to bottom, φ_{a1}, φ_{a2}, ϕ_1, and ϕ_2.

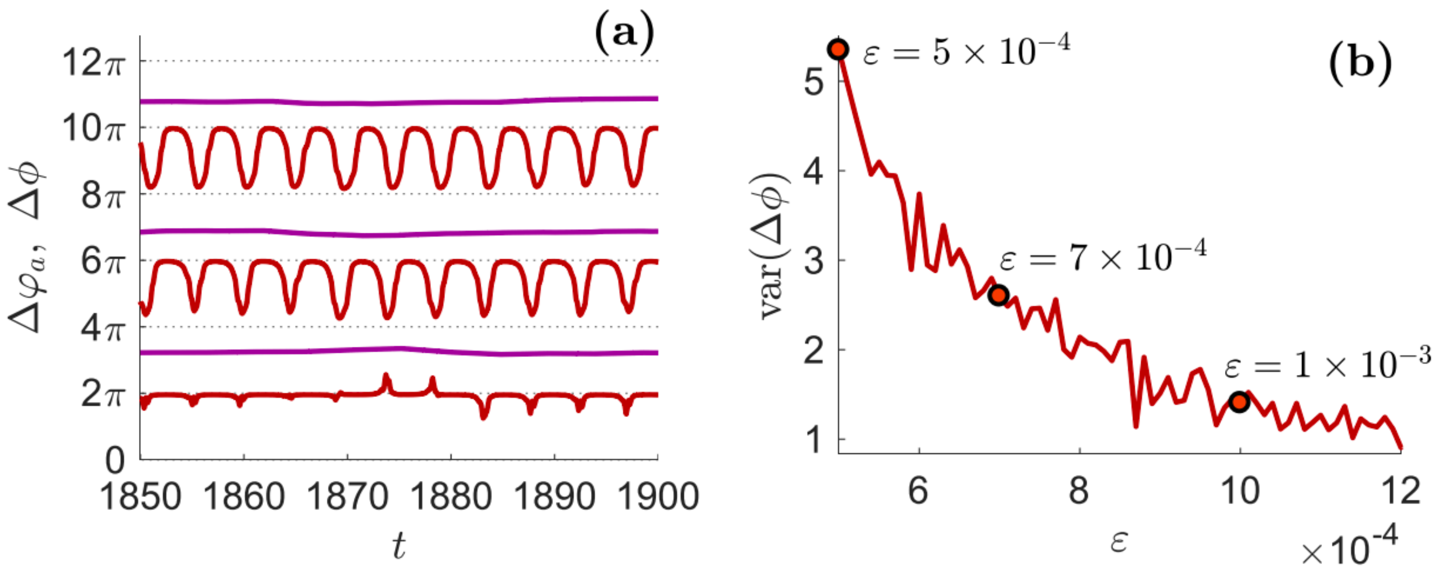
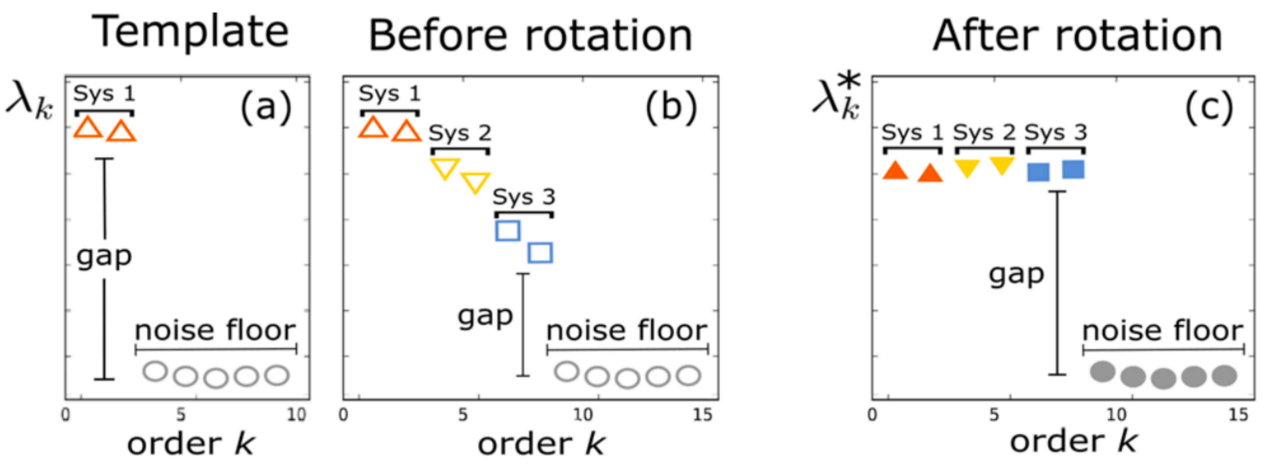


FIG. 6. Results for chaotic cord oscillators in a steady state (28): (a) details of $\Delta\varphi_a$ and $\Delta\phi$ of mutually coupled systems in the phase synchronization regime for three values of coupling strengths, from top to bottom, $\varepsilon = 5 \times 10^{-4}$, 7×10^{-4}, and 1×10^{-3} and (b) variance of $\Delta\phi$ according to the coupling strength.

ASSESMENT TOOLS

- ✓ VECTOR FIELD PHASE (VFP) | SYNC QUALITY
- ✓ VARIMAX + MULTIVARIATE SINGULAR SPECTRUM ANALYSIS

VARIMAX + MULTIVARIATE SINGULAR SPECTRUM ANALYSIS



Portes, L. L., & Aguirre, L. A. (2016). **Enhancing multivariate singular spectrum analysis for phase synchronization: The role of observability**. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, *26*(9), 093112

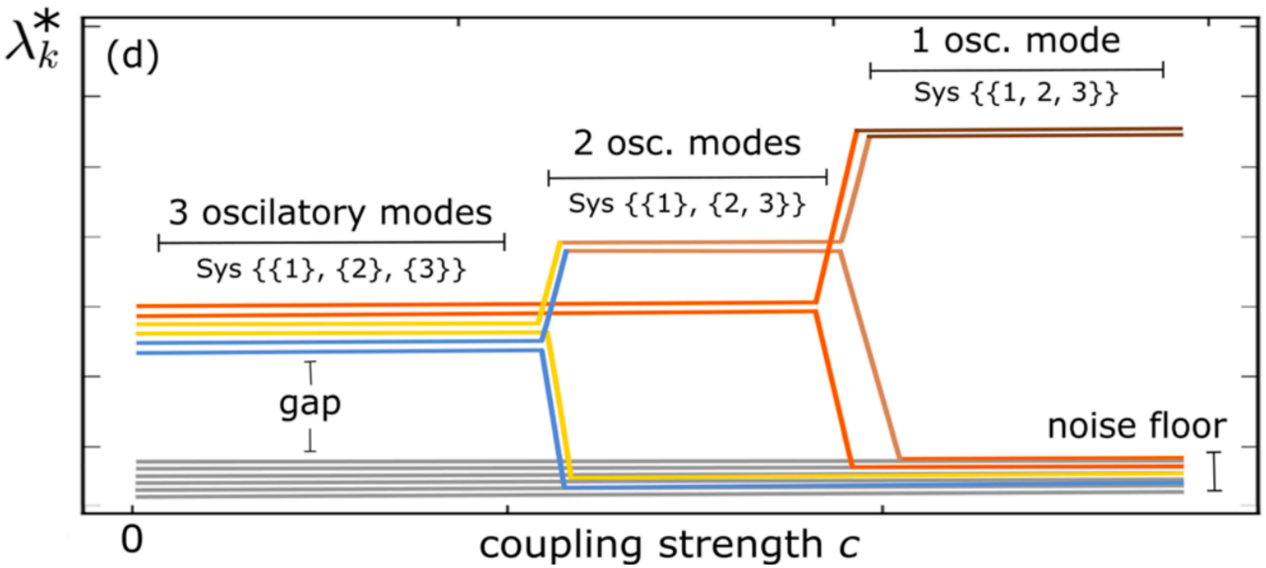
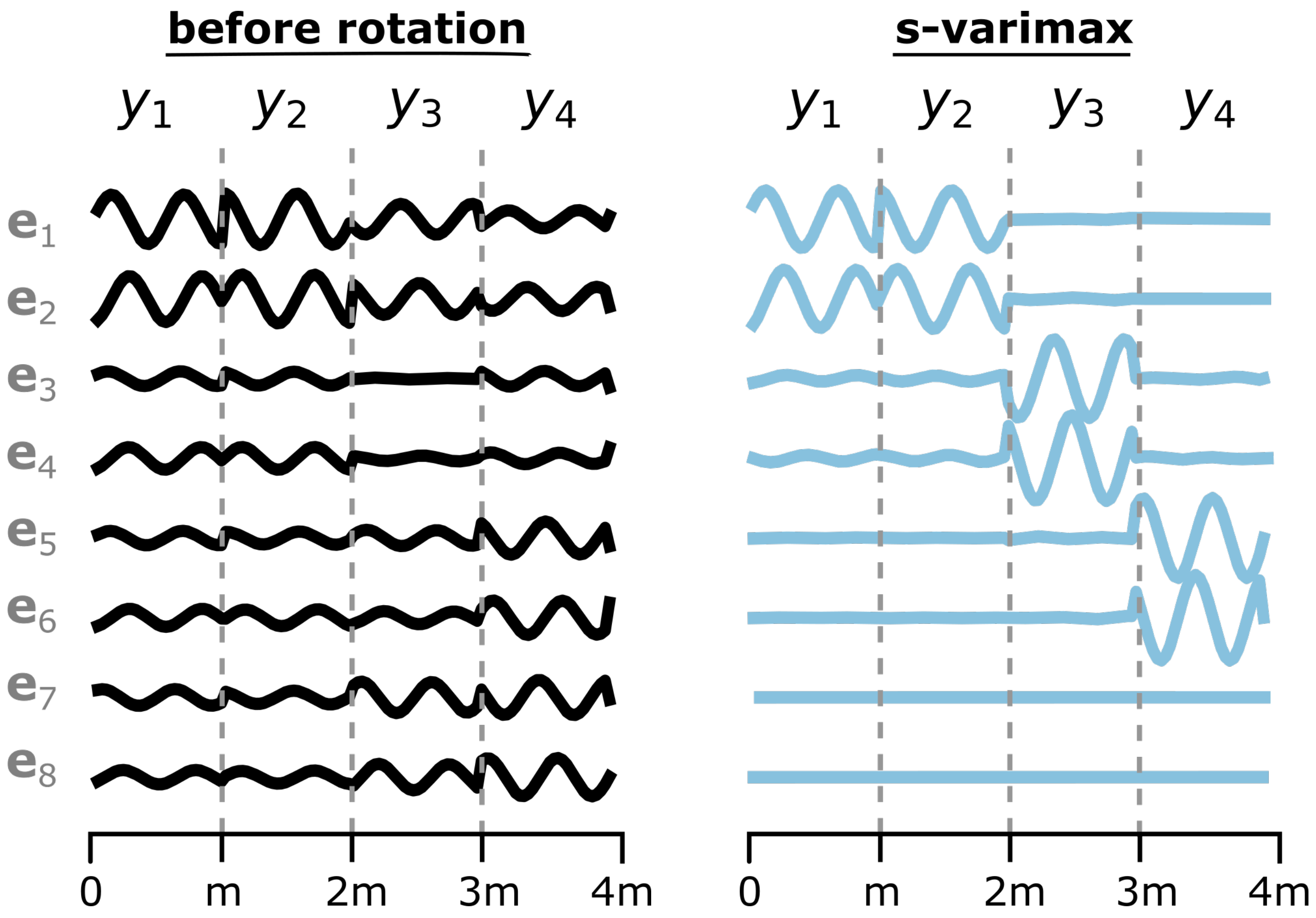
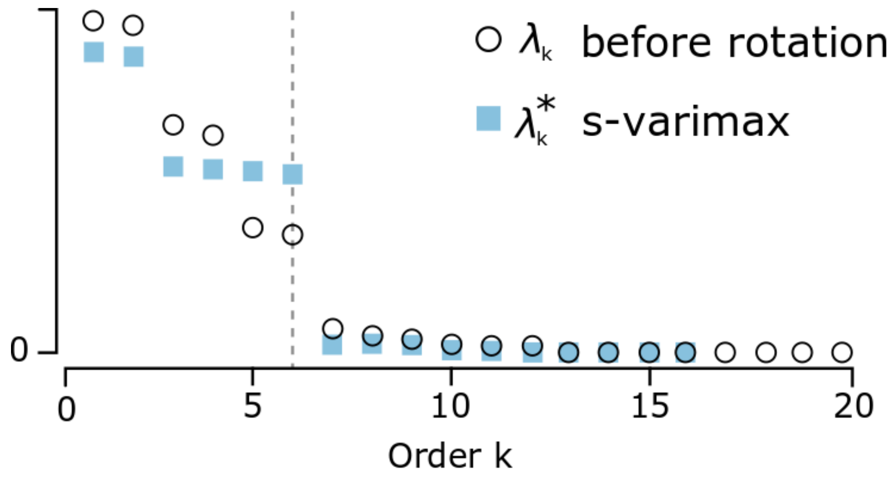


FIG. 1. Schematic representation of M-SSA for three idealized coupled and detuned phase oscillators. (a) The template of a *single oscillator* reveals that the particular fingerprint is the presence of a single pair of nearly equal eigenvalues λ, which are associated with a single oscillatory mode. (b) Each *uncoupled* oscillator is represented by pairs of almost equal singular values λ_k, due to the fact that 3 distinct oscillatory modes are presented in the global system. (c) After SVR, the leading modified variances λ_k^* are more aligned, increasing the gap. (d) Finally, the spectra for an increasing coupling strength show the sequential coupling of the oscillators, as they start gradually to share oscillatory modes.

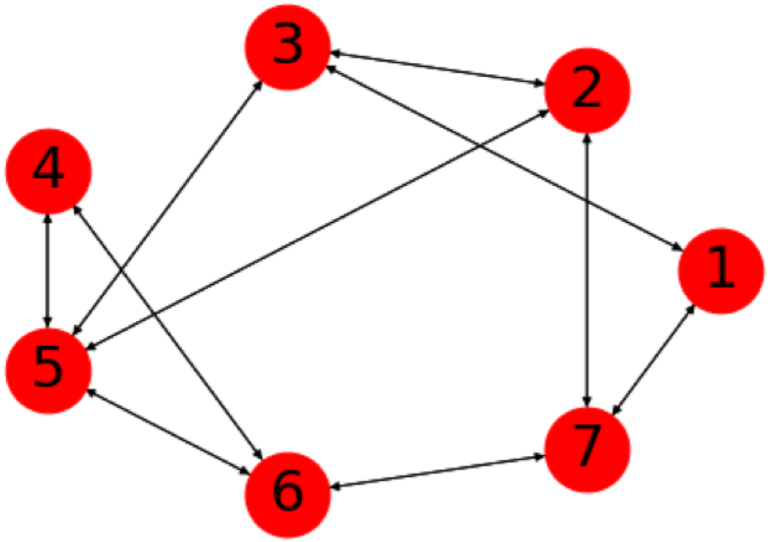
VARIMAX + MULTIVARIATE SINGULAR SPECTRUM ANALYSIS

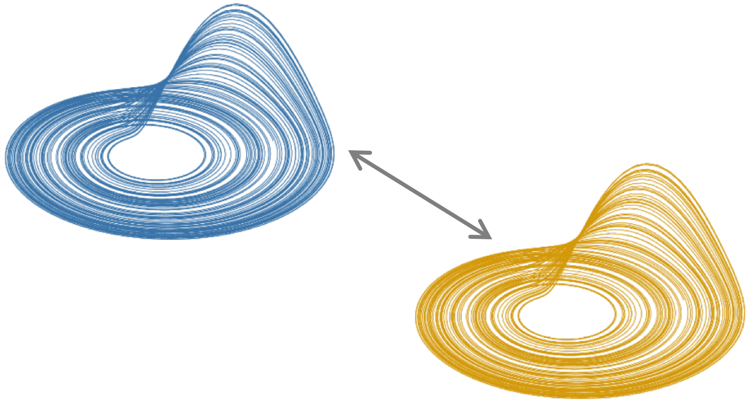




EXAMPLE

- ✓ COUPLED ROSSLER OSCILLATORS



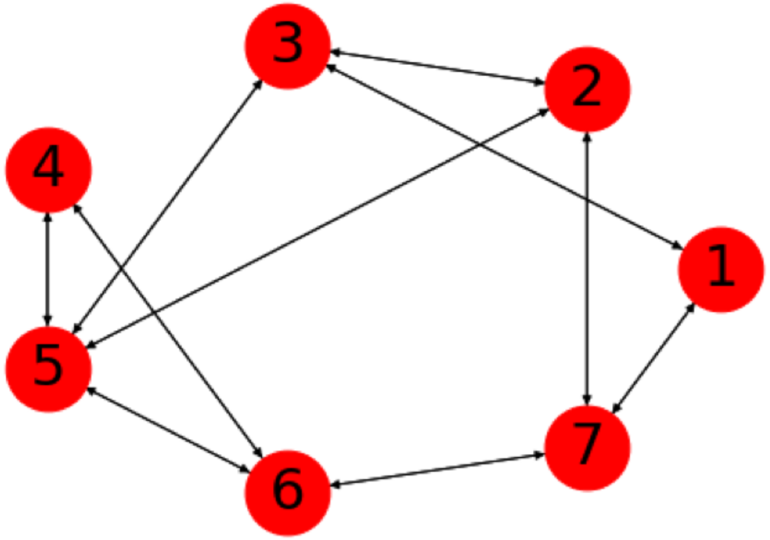


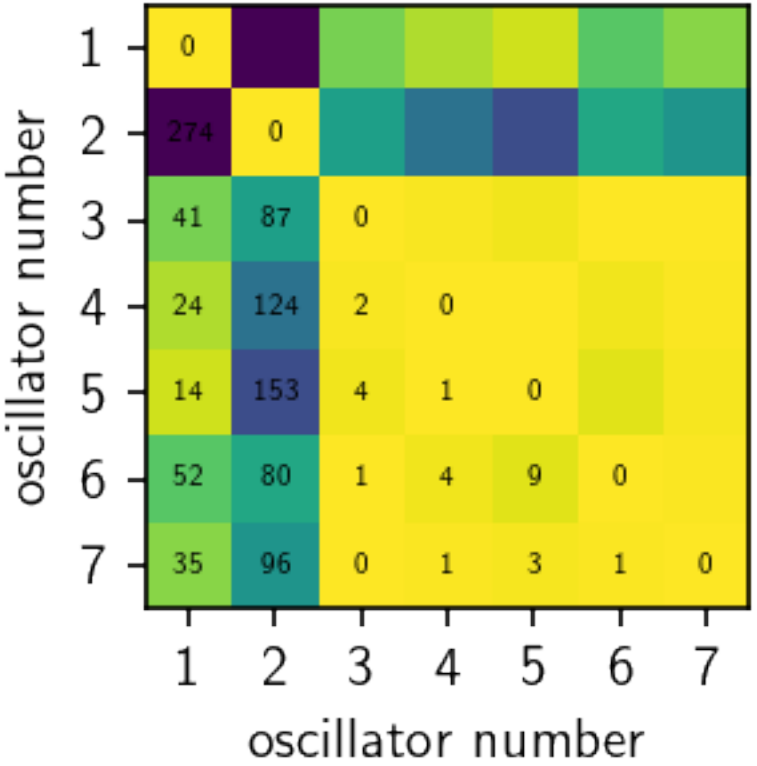
If we decrease the coupling strength…

Who will **desynchronize** first?

Energy of VFP-phase difference

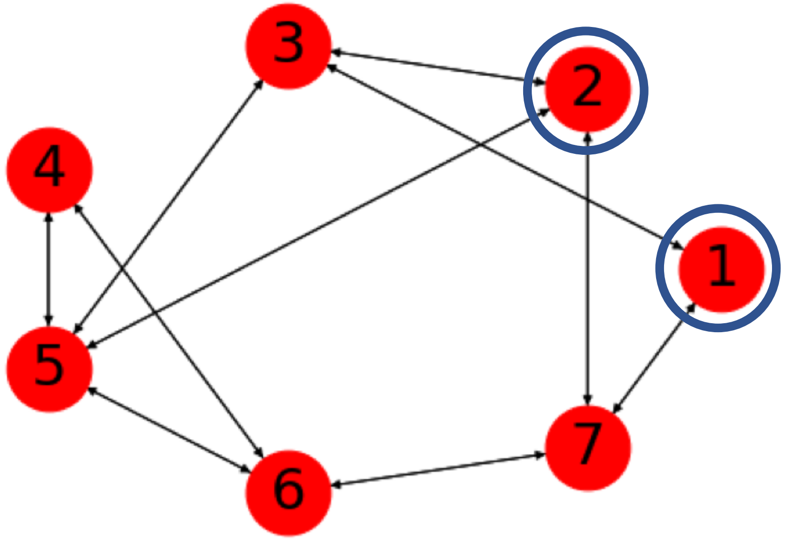
1 and **2** will!

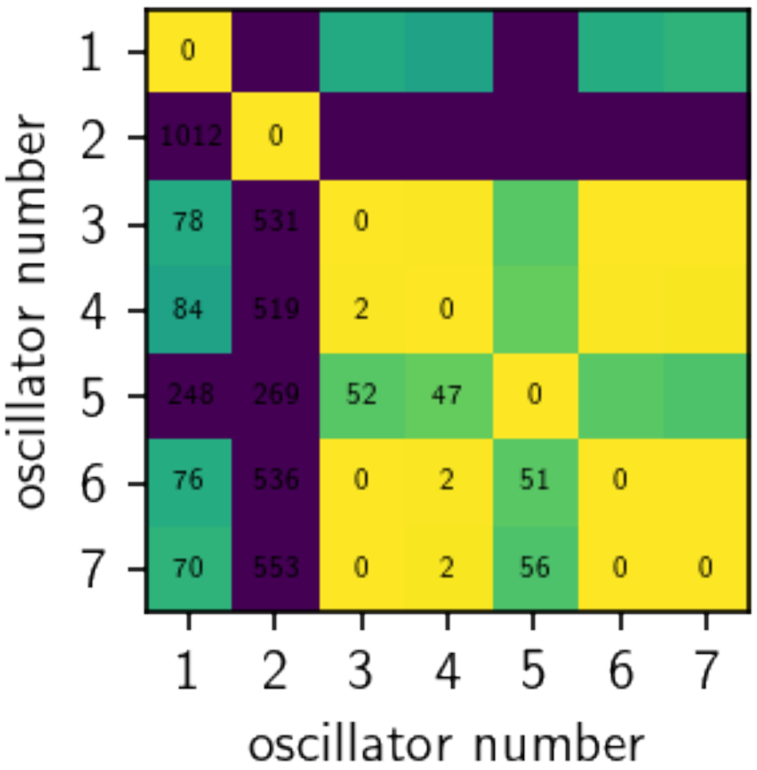


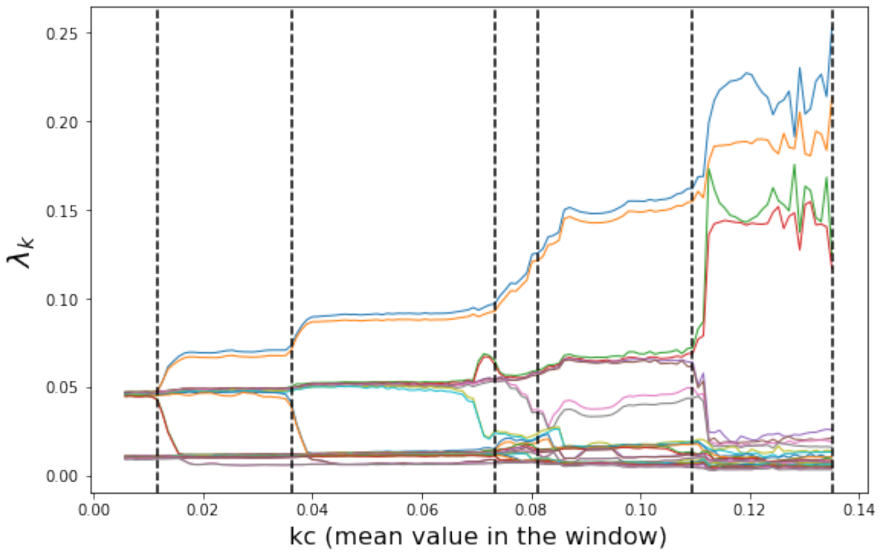


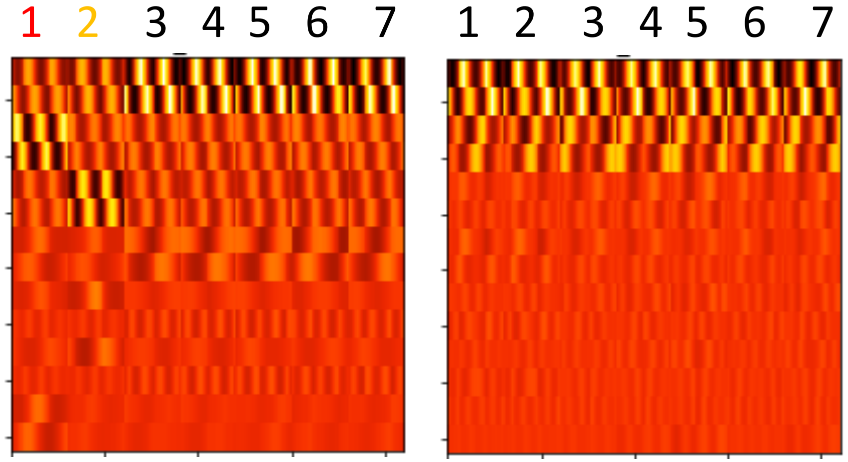
Energy of VFP-phase difference

5 will!



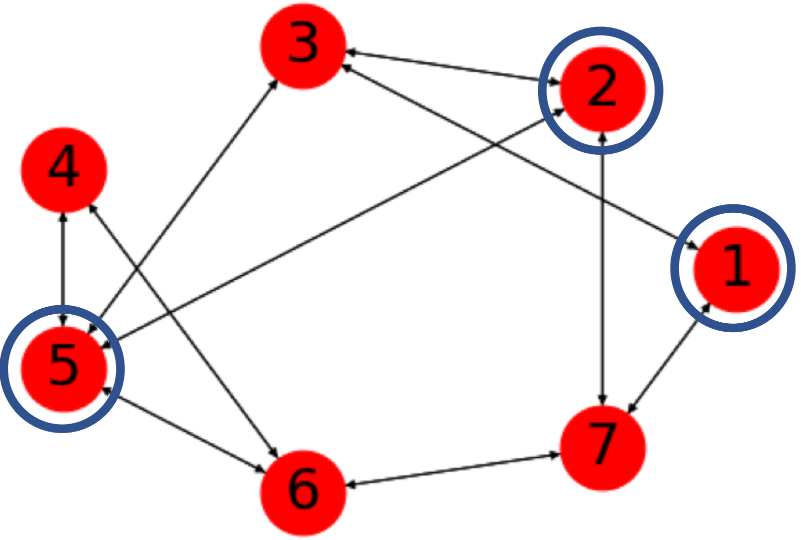


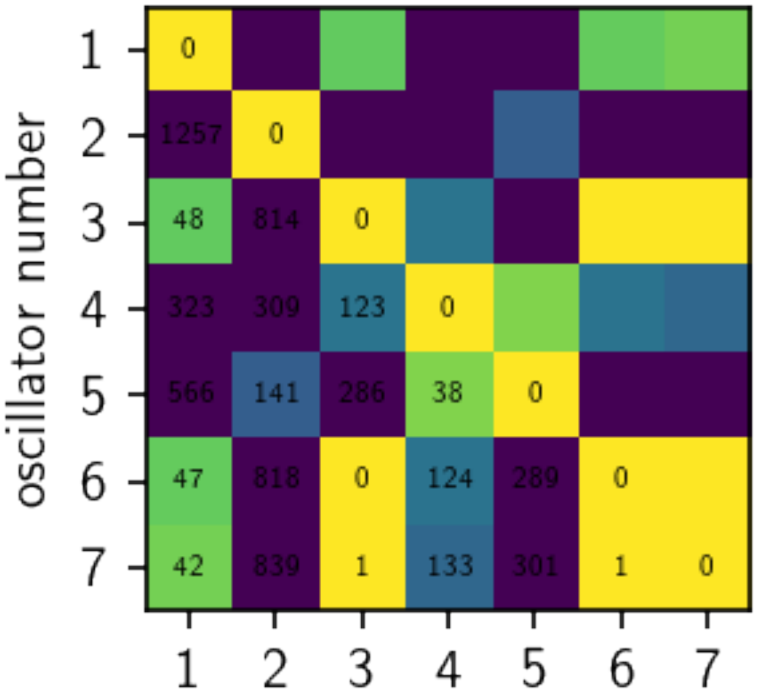


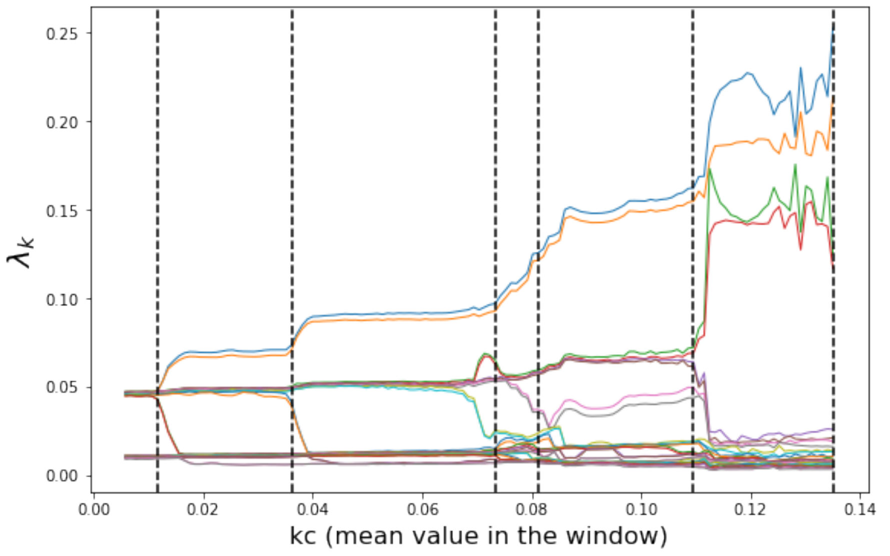


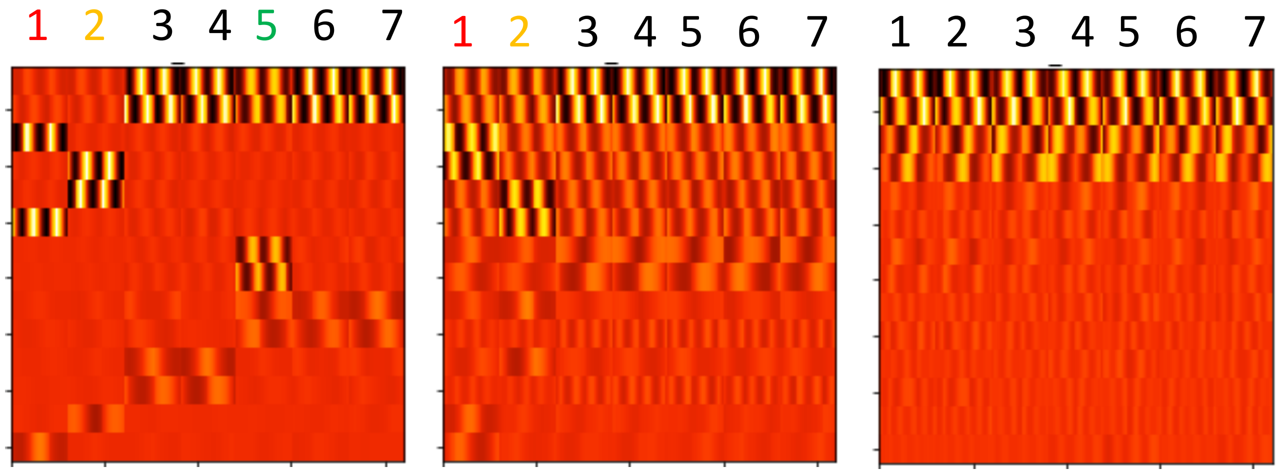
Energy of VFP-phase difference

4 will!



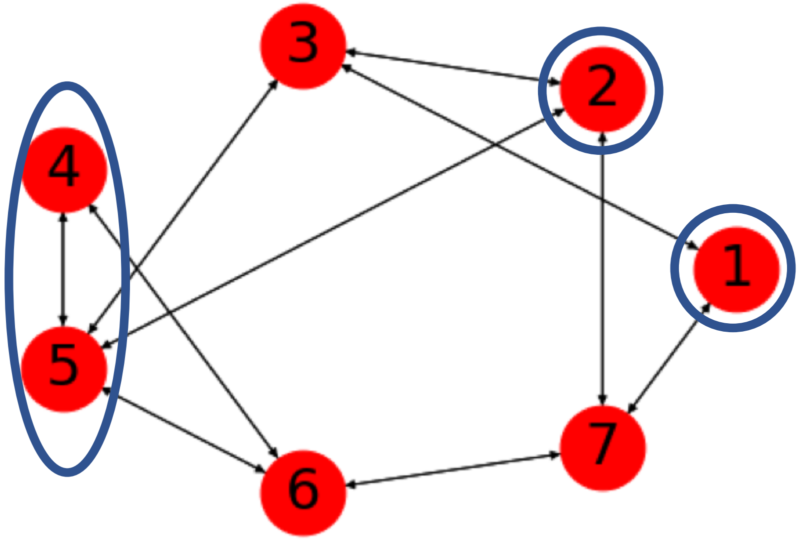


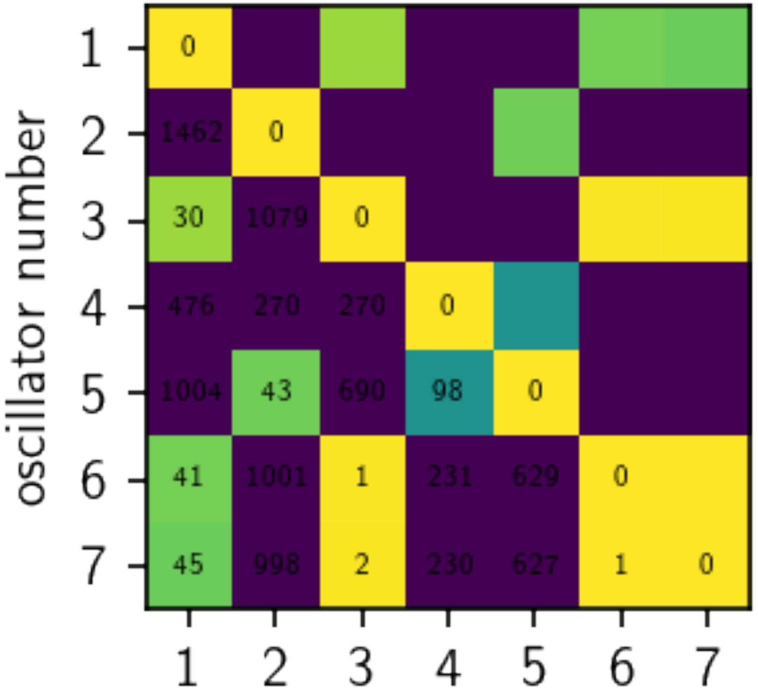


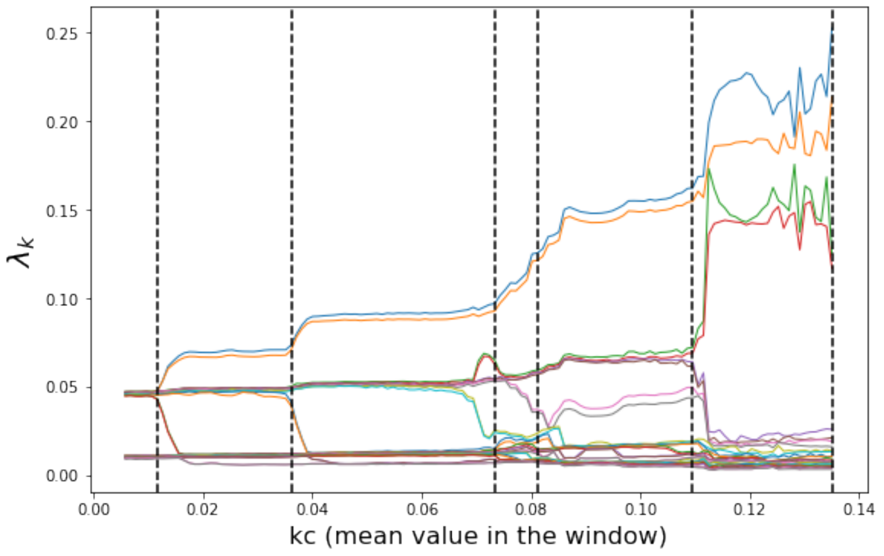


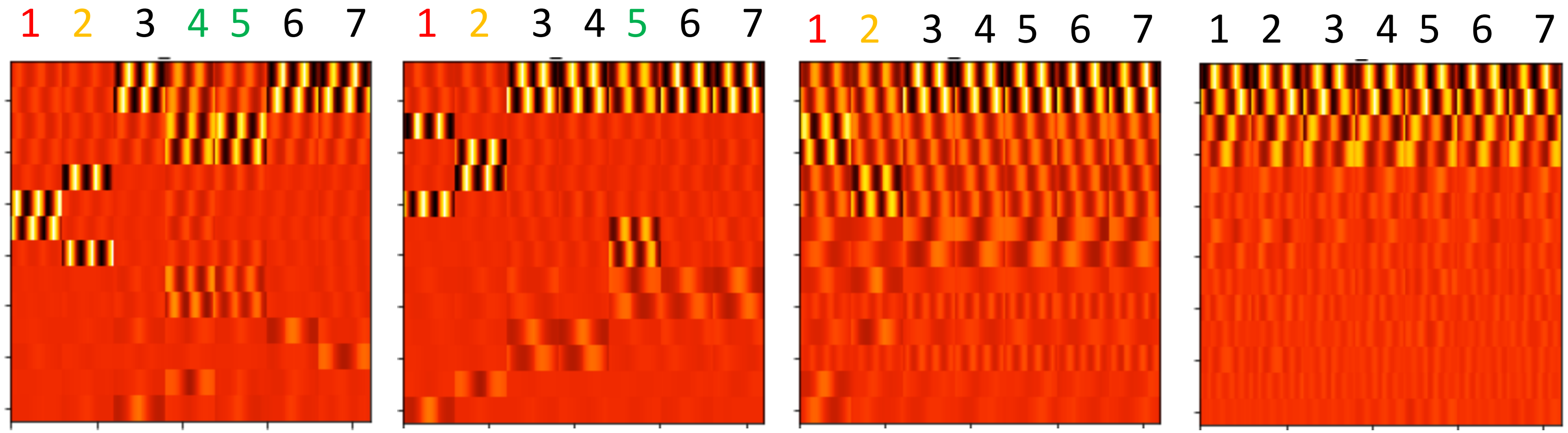
Energy of VFP-phase difference

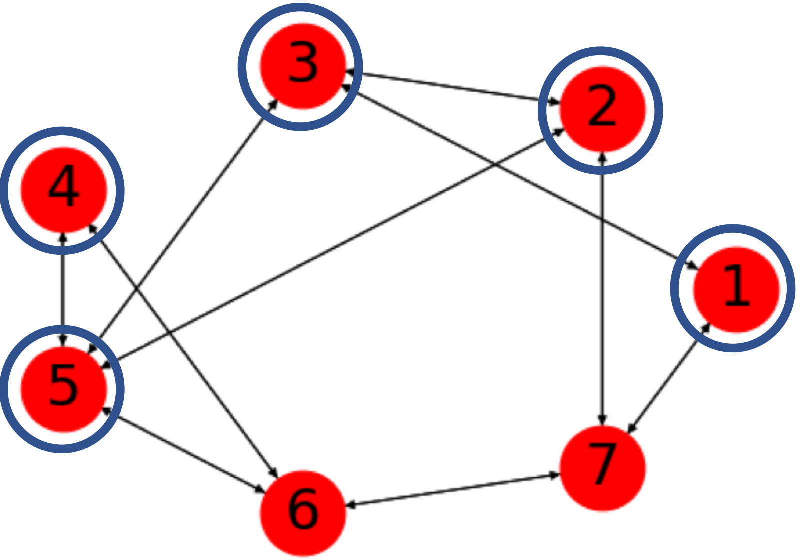
3 (and 4-5) will!

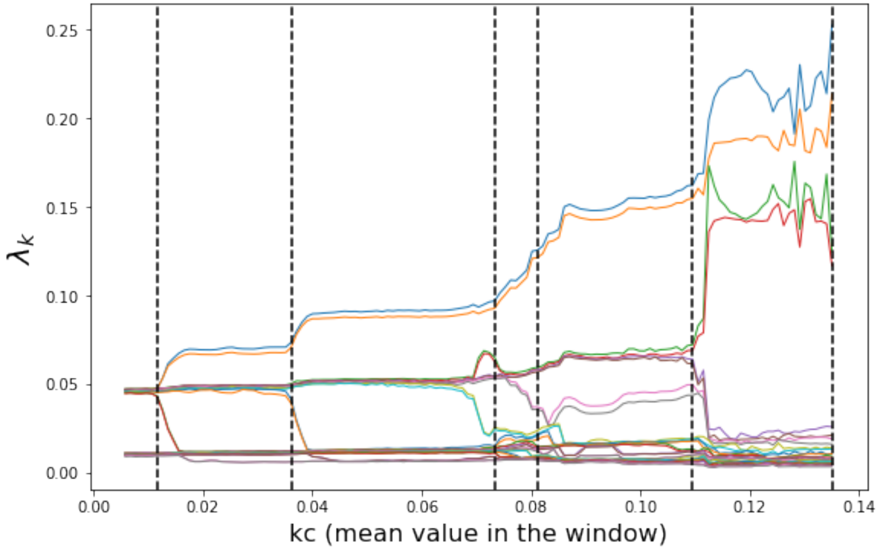


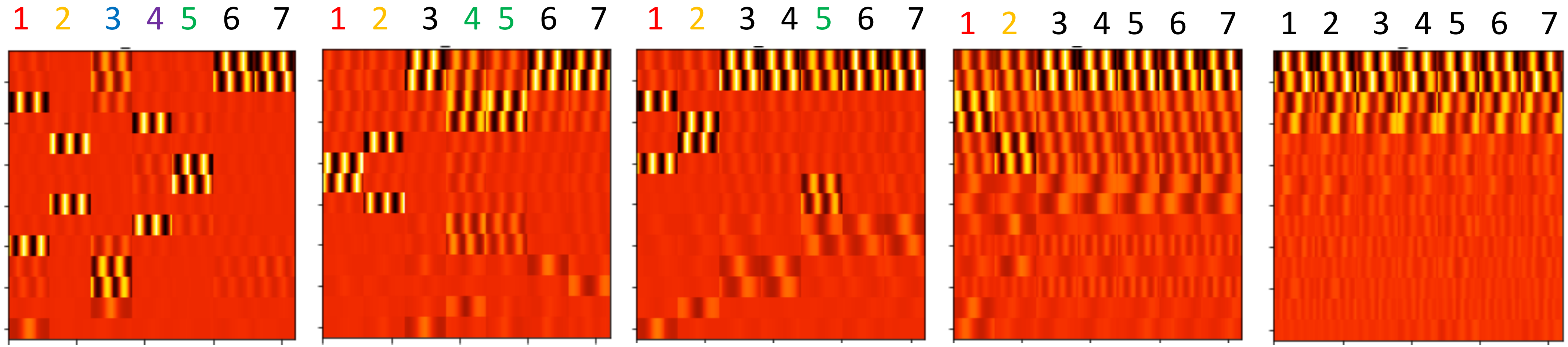


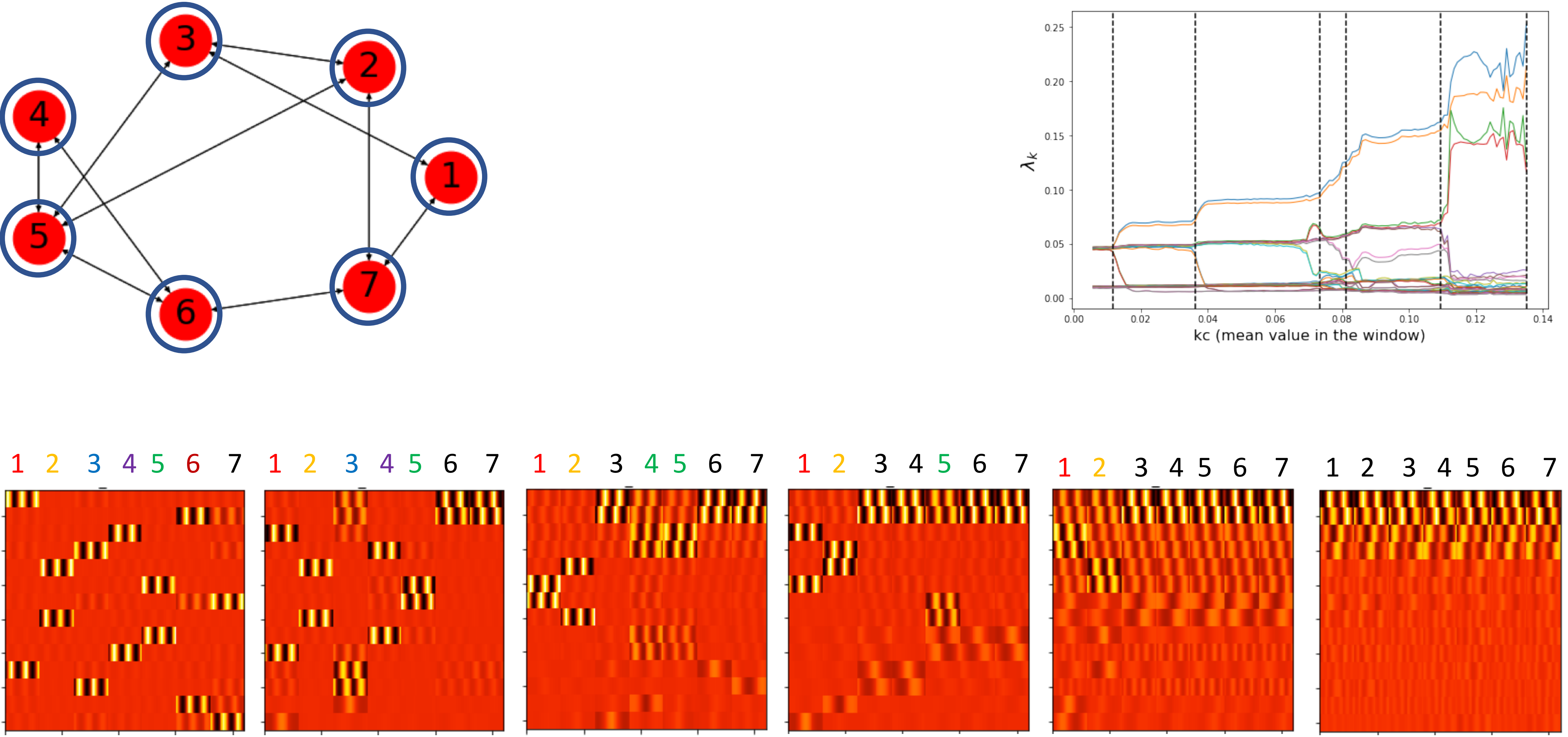





3
2
4
1
5
7
6


0.25
0.20
0.15
0.10
0.05
0.00
λ_k
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
kc (mean value in the window)


1 2 3 4 5 6 7
1 2 3 4 5 6 7
1 2 3 4 5 6 7
1 2 3 4 5 6 7
1 2 3 4 5 6 7



3
2
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5
7
6
0.25
0.20
0.15
0.10
0.05
0.00
λ_k
0.00 0.02 0.04 0.06 0.08 0.10 0.12 0.14
kc (mean value in the window)
1 2 3 4 5 6 7
1 2 3 4 5 6 7
1 2 3 4 5 6 7
1 2 3 4 5 6 7
1 2 3 4 5 6 7
1 2 3 4 5 6 7

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Varimax + MSSA FOR PHASE SYNC CHARACTERIZATION

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VECTOR FIELD PHASE (VFP) | SYNC QUALITY

For a trajectory $\gamma(t;\boldsymbol{x_0})$ the *Vector Field Phase* (VFP) is given by:

$$\phi(t) = \phi_0 + \int_{\gamma(t;\boldsymbol{x_0})} c(\boldsymbol{x})\, \boldsymbol{f}^\top(\boldsymbol{x})\, d\boldsymbol{x}\,,$$

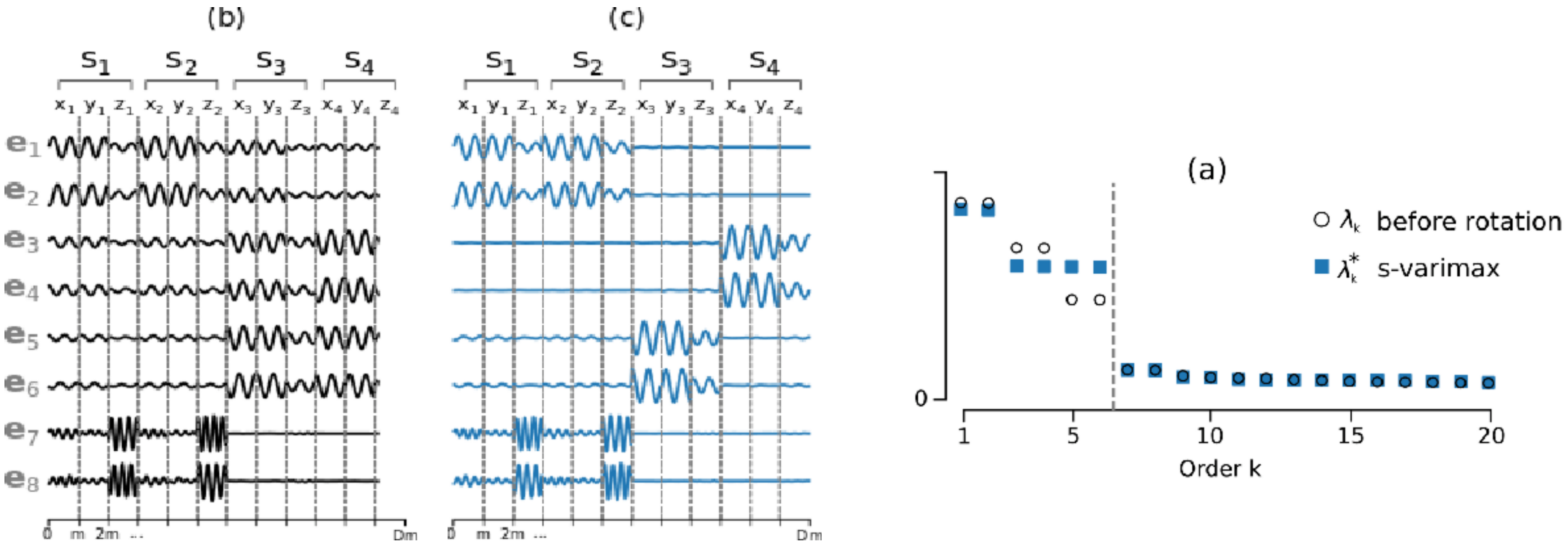
where $c(\boldsymbol{x})$ is a positive function.

$$c(\boldsymbol{x}) = \frac{2\pi}{\ell\ \|\boldsymbol{f}(\vec{x})\|}$$

Arc length

Freitas, L., Torres, L. A. B., & Aguirre, L. A. (2018). **Phase definition to assess synchronization quality of nonlinear oscillators**. *Physical Review E*, *97*(5), 052202

VARIMAX + MULTIVARIATE SINGULAR SPECTRUM ANALYSIS



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