

# Joint characterization of phase synchronization in networks with multivariate singular spectrum analysis and vector field phase



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# SUMMARY

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## THE GOAL

## ASSESSMENT TOOLS

VECTOR FIELD PHASE (VFP) | SYNC QUALITY

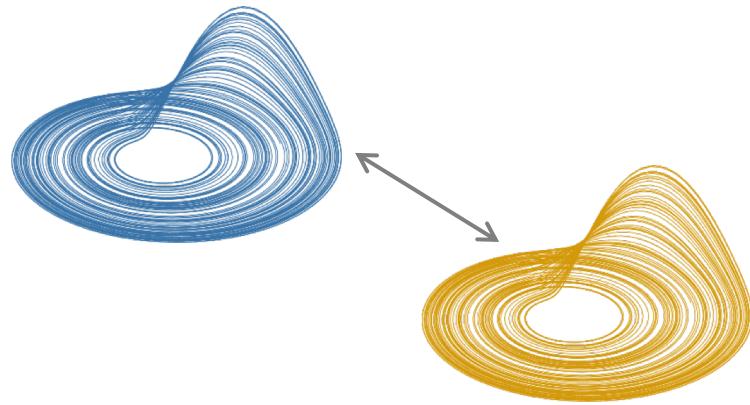
VARIMAX + MULTIVARIATE SINGULAR SPECTRUM ANALYSIS

## EXAMPLE

# THE GOAL

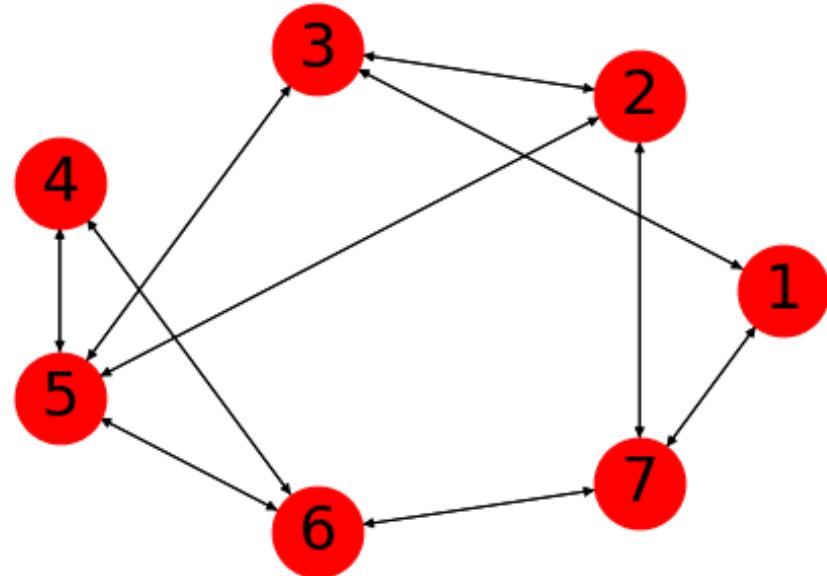
# THE GOAL

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If we decrease the coupling strength...

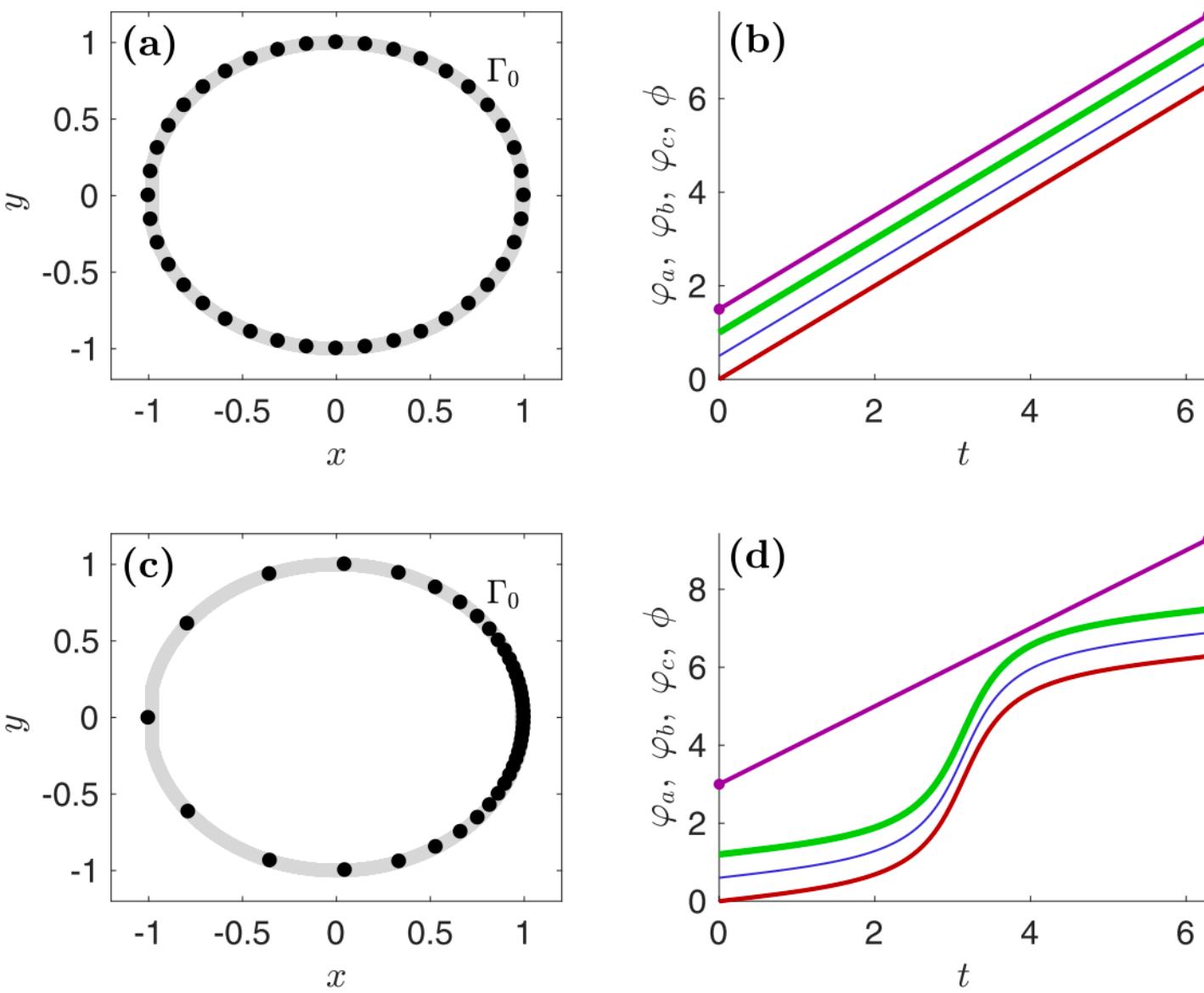
**Who will desynchronize first?**



# ASSESSMENT TOOLS

- ✓ VECTOR FIELD PHASE (VFP) | SYNC QUALITY
- ✓ VARIMAX + MULTIVARIATE SINGULAR SPECTRUM ANALYSIS

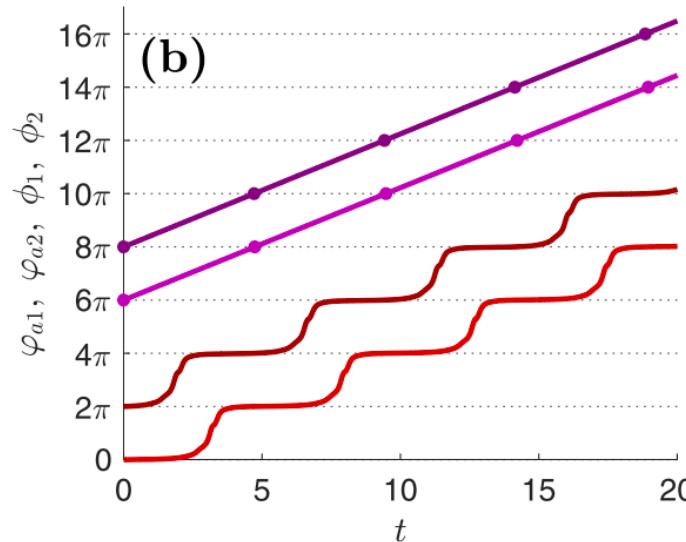
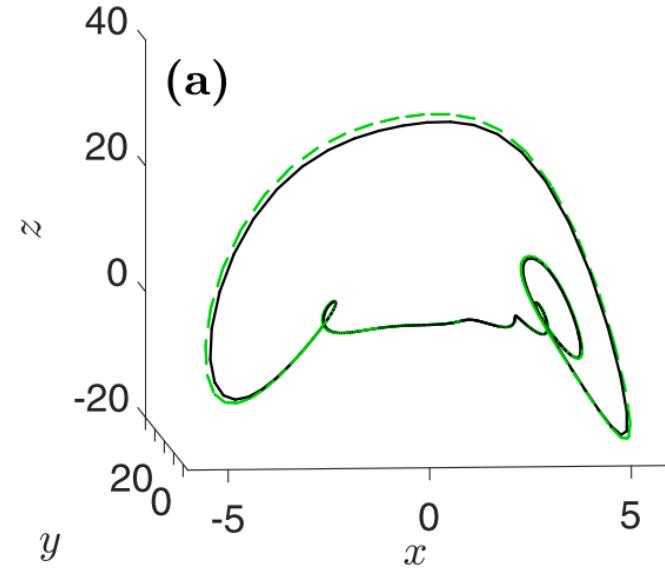
# VECTOR FIELD PHASE (VFP) | SYNC QUALITY



Freitas, L., Torres, L. A. B., & Aguirre, L. A. (2018). **Phase definition to assess synchronization quality of nonlinear oscillators.** *Physical Review E*, 97(5), 052202.

FIG. 1. (a) and (b) Original and (c) and (d) modified Poincaré oscillators: (a) and (c) state space showing evolution on  $\Gamma_0$  (gray line) and equally time-spaced dots and (b) and (d) phaselike variables vertically displaced for the sake of clarity. Shown, from top to bottom, are  $\varphi_a$  (Poincaré section),  $\varphi_b$  (angle),  $\varphi_c$  (curvature), and  $\phi$  (VFP). The parameters are  $(\lambda, p) = (0.5, 1)$ , with  $\omega = 1$  for the original system, and  $\omega = \alpha(1 - 0.9x)$ , with  $\alpha = 22.646$  for the modified one.

# VECTOR FIELD PHASE (VFP) | SYNC QUALITY



Freitas, L., Torres, L. A. B., & Aguirre, L. A. (2018). **Phase definition to assess synchronization quality of nonlinear oscillators.** *Physical Review E*, 97(5), 052202.

FIG. 3. Results for the cord oscillators (28): (a) steady-state evolution of uncoupled ( $\varepsilon = 0$ ) oscillators 1 (solid black line) and 2 (dashed green line) and (b) phase variables, from top to bottom,  $\varphi_{a1}$ ,  $\varphi_{a2}$ ,  $\phi_1$ , and  $\phi_2$ .

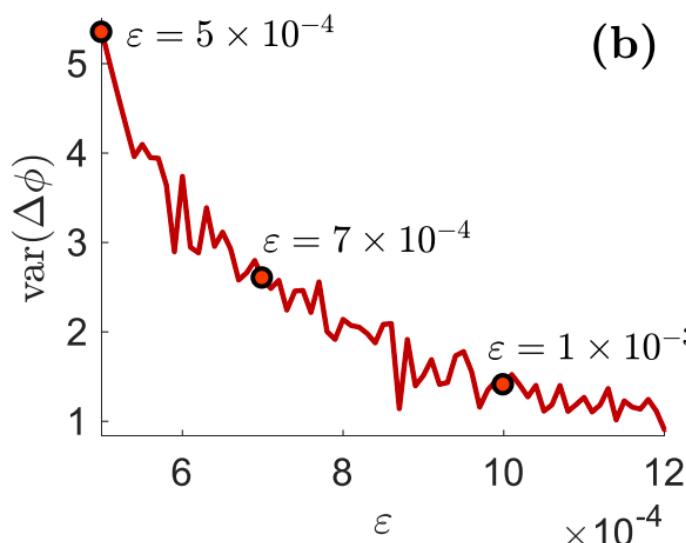
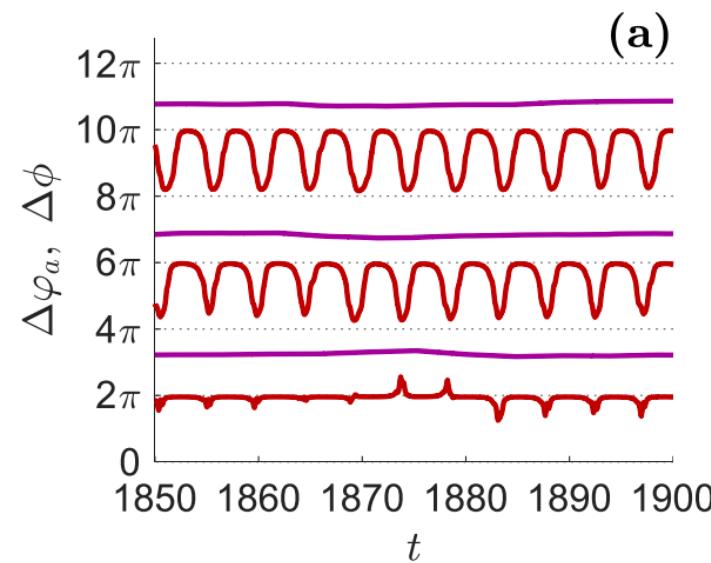
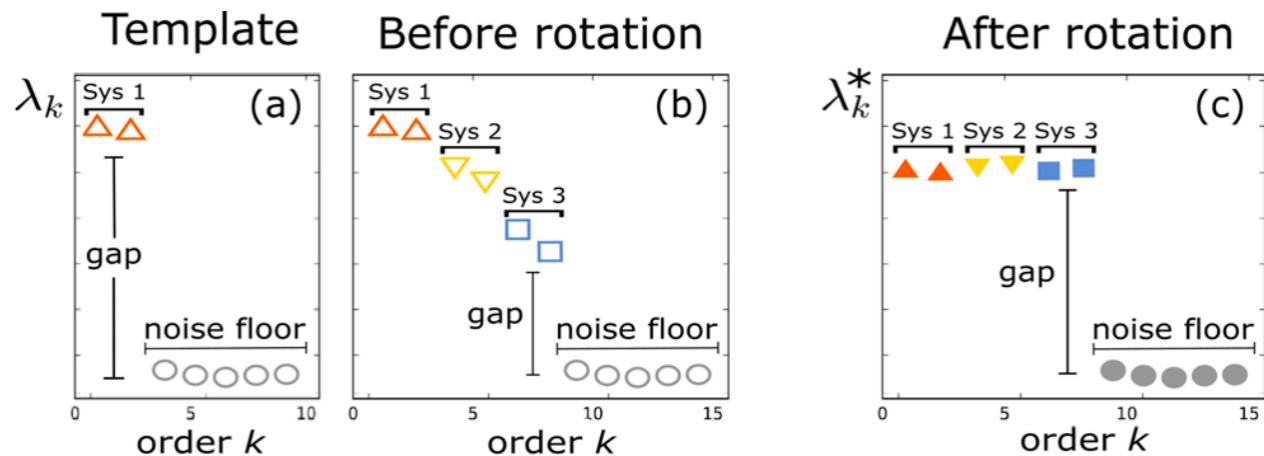


FIG. 6. Results for chaotic cord oscillators in a steady state (28): (a) details of  $\Delta\varphi_a$  and  $\Delta\phi$  of mutually coupled systems in the phase synchronization regime for three values of coupling strengths, from top to bottom,  $\varepsilon = 5 \times 10^{-4}$ ,  $7 \times 10^{-4}$ , and  $1 \times 10^{-3}$  and (b) variance of  $\Delta\phi$  according to the coupling strength.

# ASSESSMENT TOOLS

- ✓ VECTOR FIELD PHASE (VFP) | SYNC QUALITY
- ✓ VARIMAX + MULTIVARIATE SINGULAR SPECTRUM ANALYSIS

# VARIMAX + MULTIVARIATE SINGULAR SPECTRUM ANALYSIS



Portes, L. L., & Aguirre, L. A. (2016). **Enhancing multivariate singular spectrum analysis for phase synchronization: The role of observability.** *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 26(9), 093112

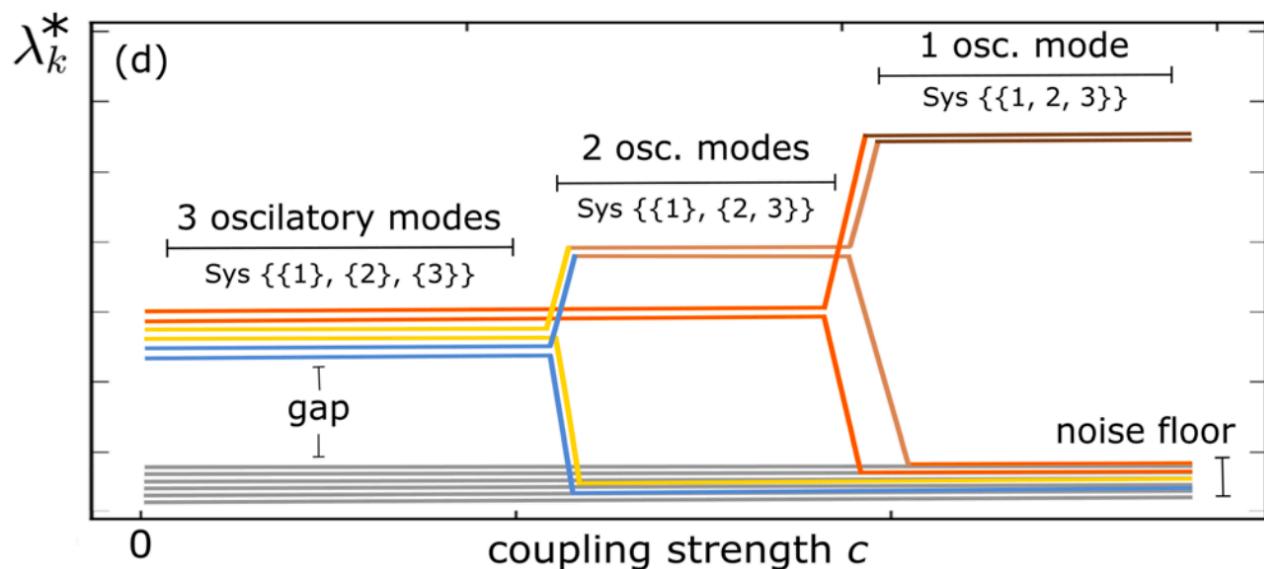
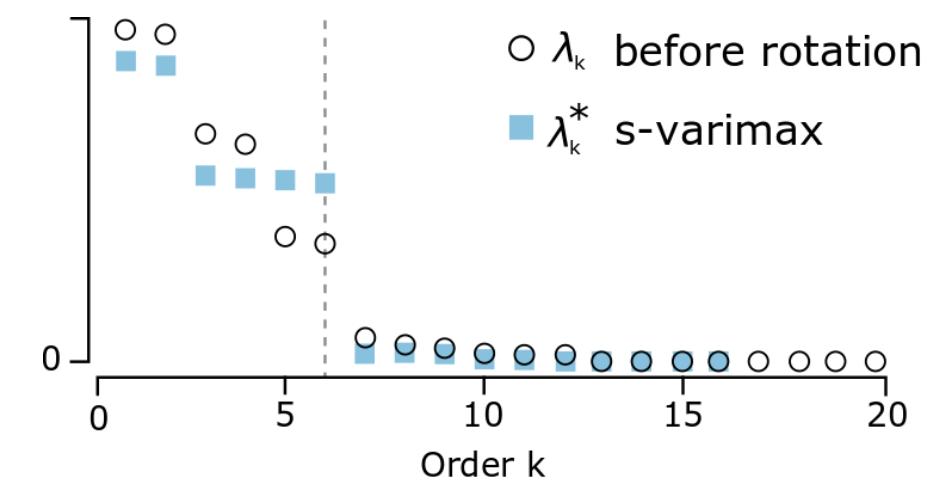
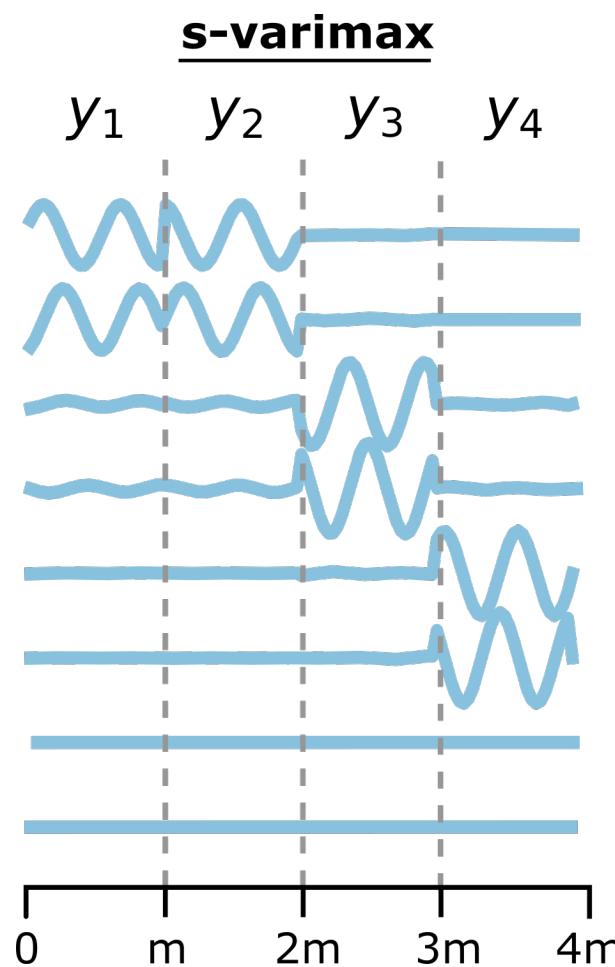
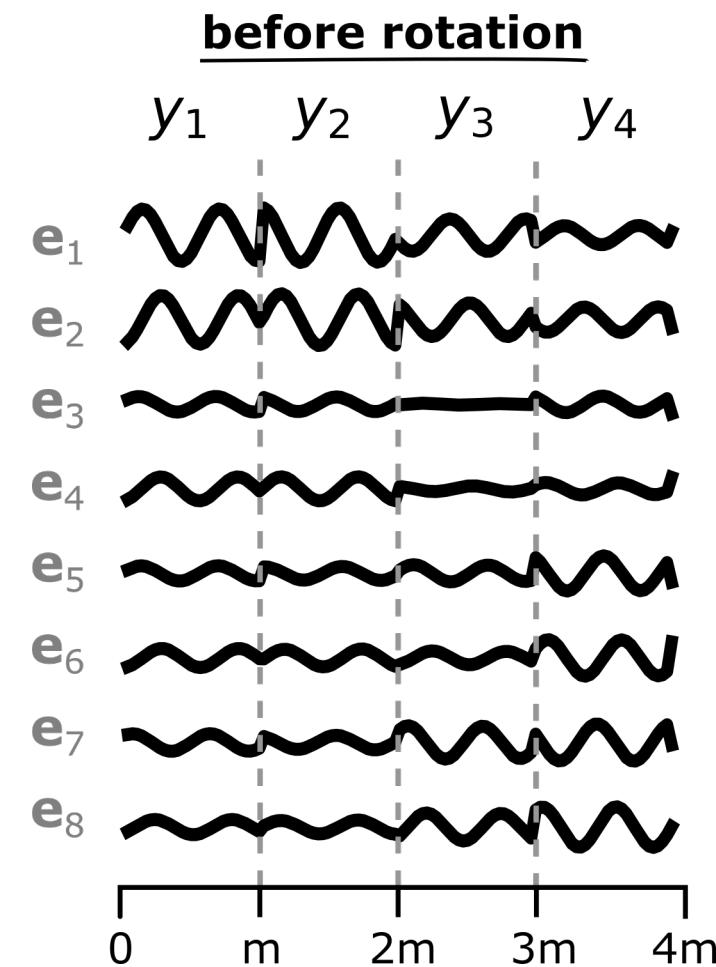


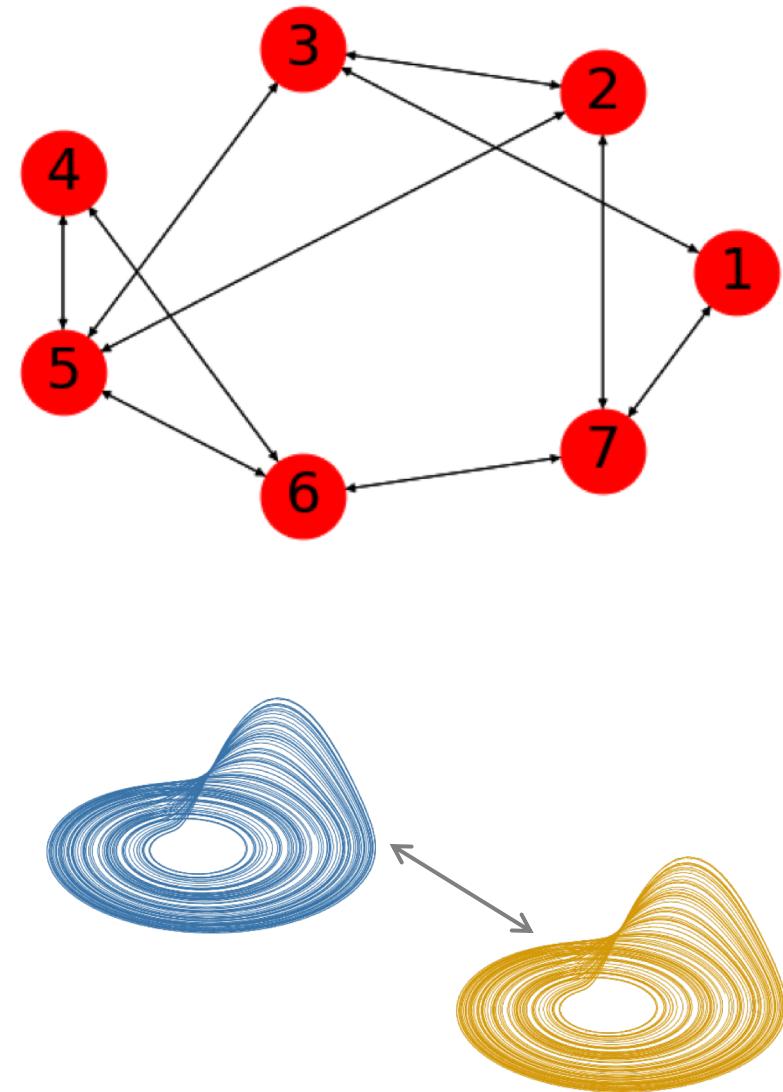
FIG. 1. Schematic representation of M-SSA for three idealized coupled and detuned phase oscillators. (a) The template of a *single oscillator* reveals that the particular fingerprint is the presence of a single pair of nearly equal eigenvalues  $\lambda$ , which are associated with a single oscillatory mode. (b) Each *uncoupled oscillator* is represented by pairs of almost equal singular values  $\lambda_k$ , due to the fact that 3 distinct oscillatory modes are presented in the global system. (c) After SVR, the leading modified variances  $\lambda_k^*$  are more aligned, increasing the gap. (d) Finally, the spectra for an increasing coupling strength show the sequential coupling of the oscillators, as they start gradually to share oscillatory modes.

# VARIMAX + MULTIVARIATE SINGULAR SPECTRUM ANALYSIS



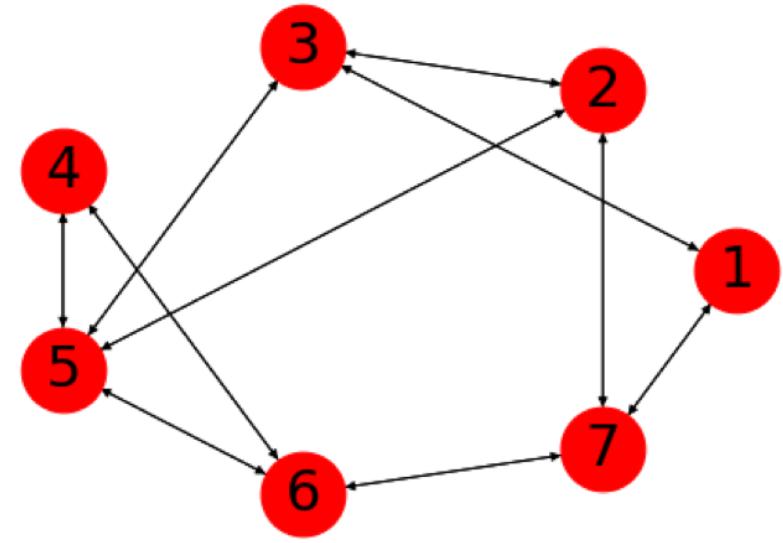
# EXAMPLE

✓ COUPLED ROSSLER OSCILLATORS

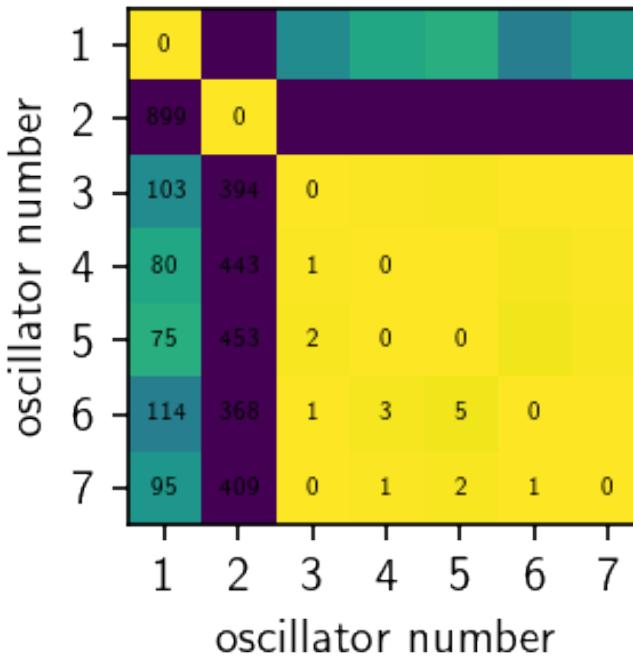


If we decrease the coupling strength...

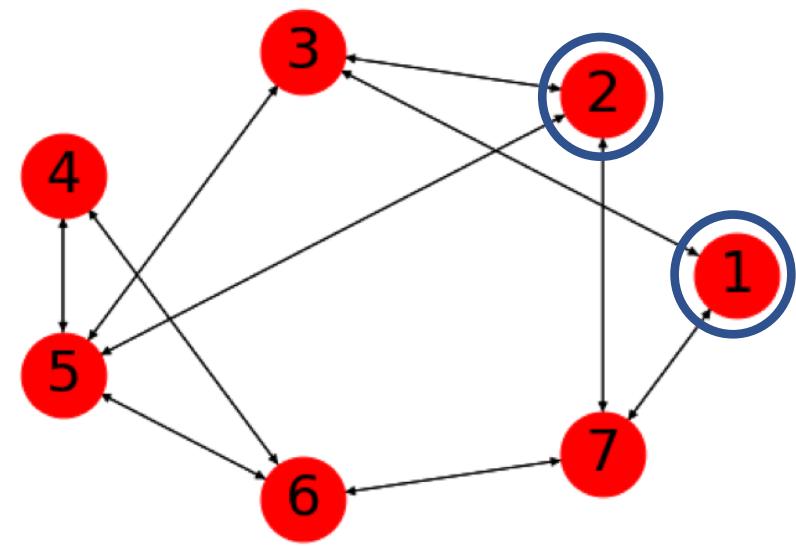
**Who will desynchronize first?**



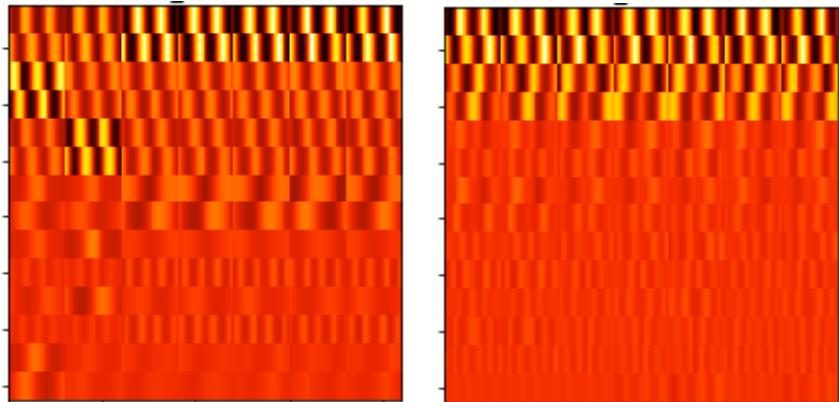
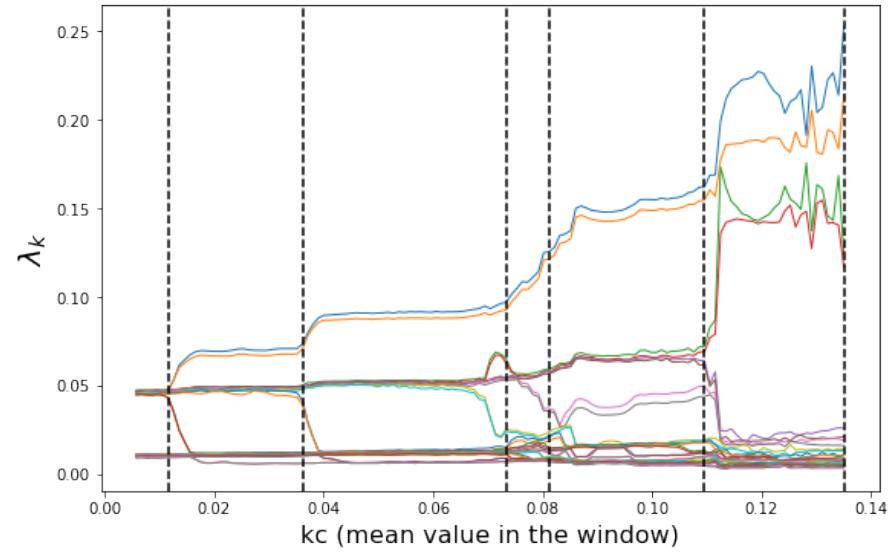
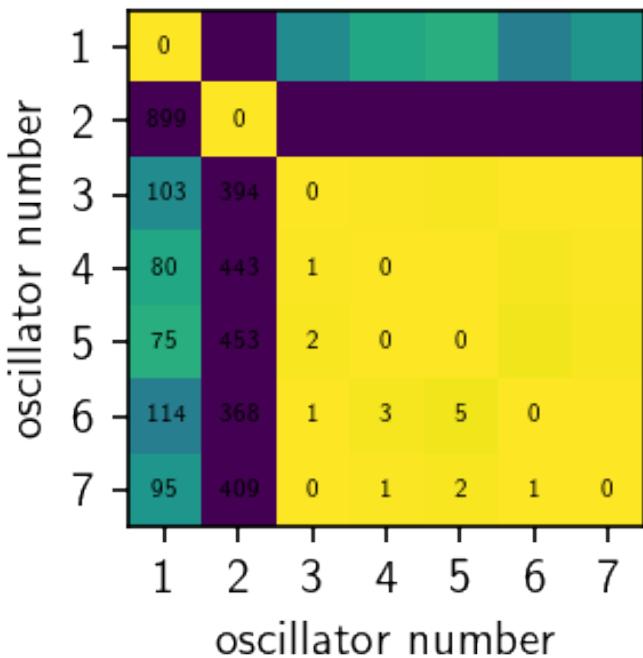
Energy of VFP-phase difference  
(time index 29000:38999)

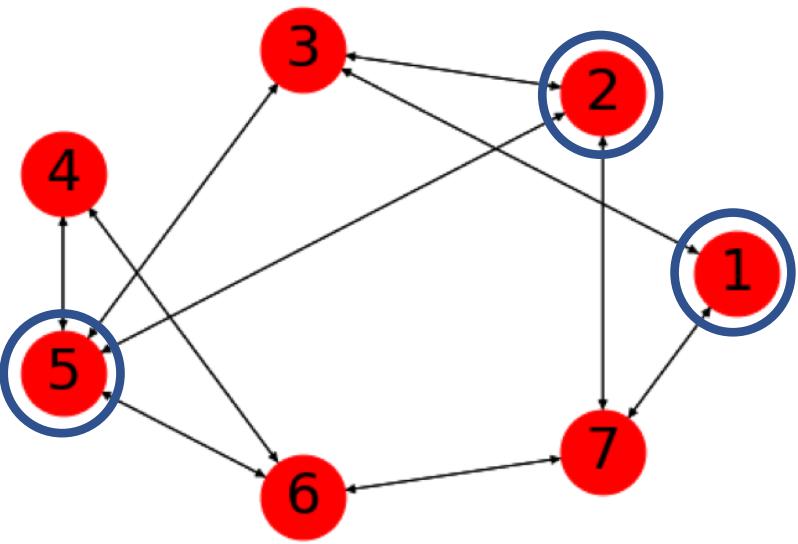


1 and 2 will!

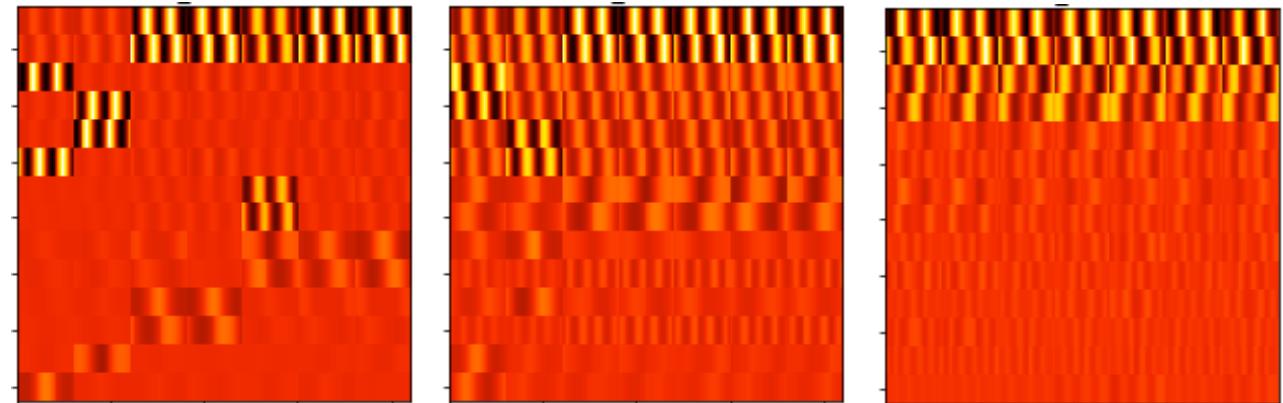
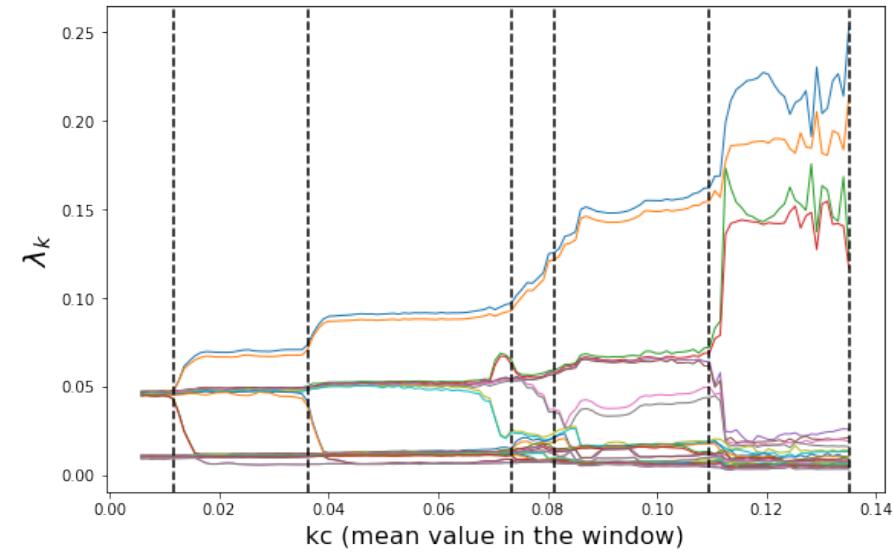
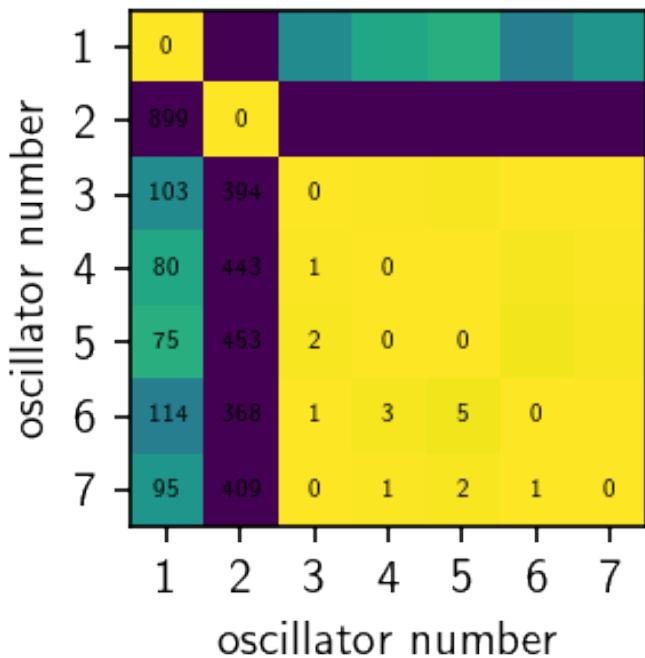


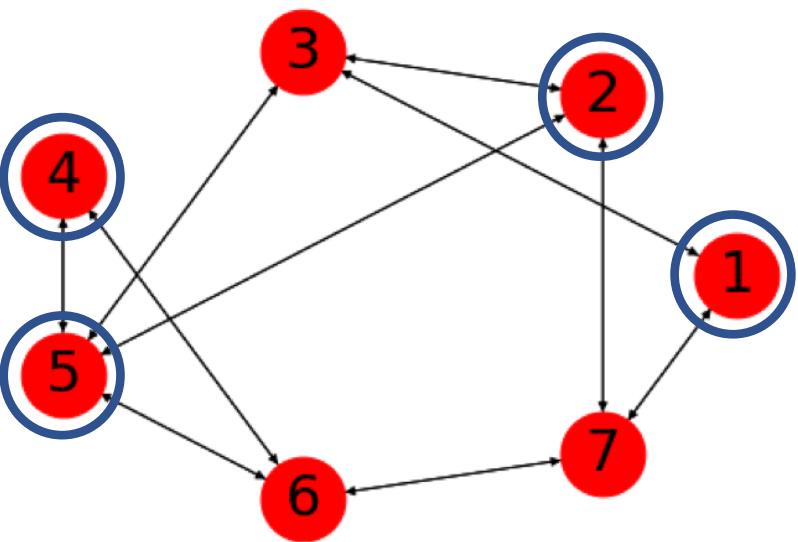
Energy of VFP-phase difference  
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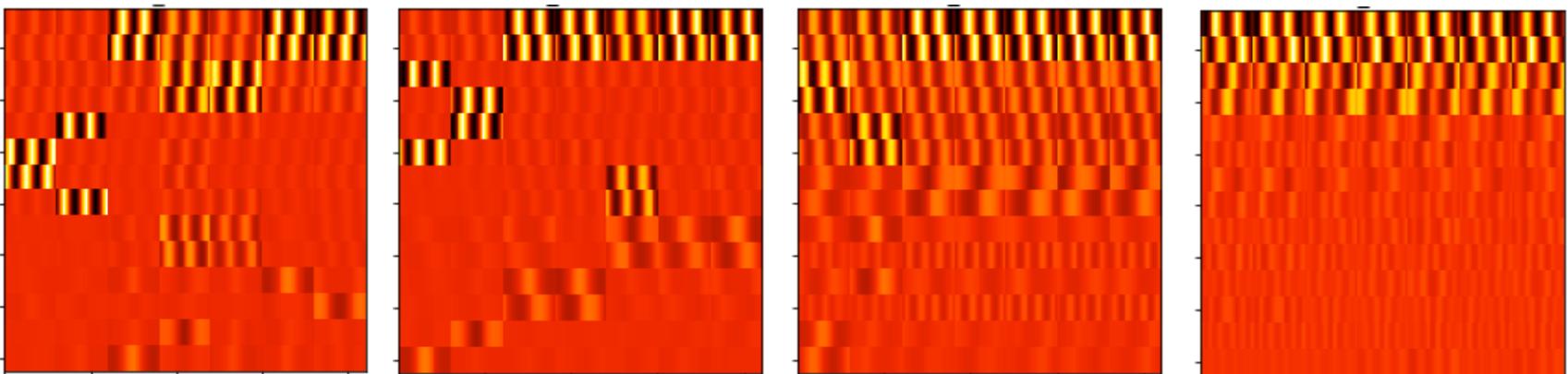
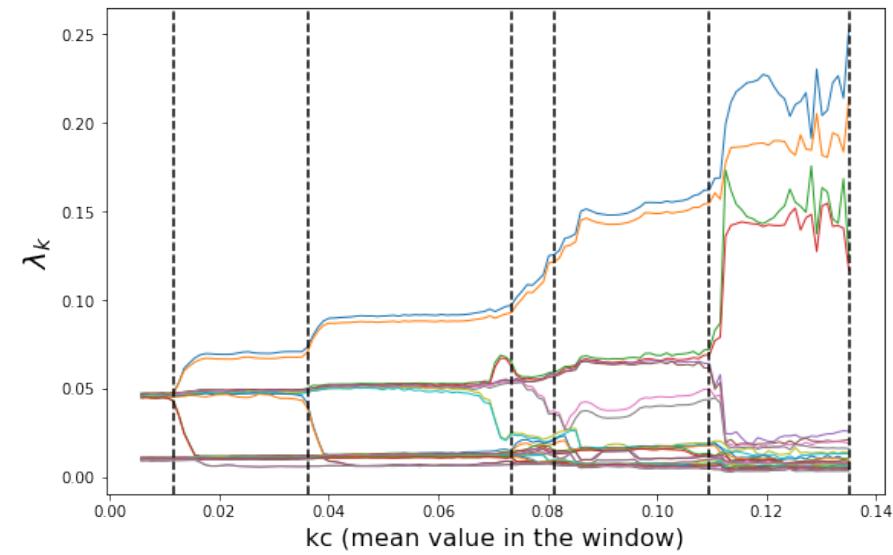
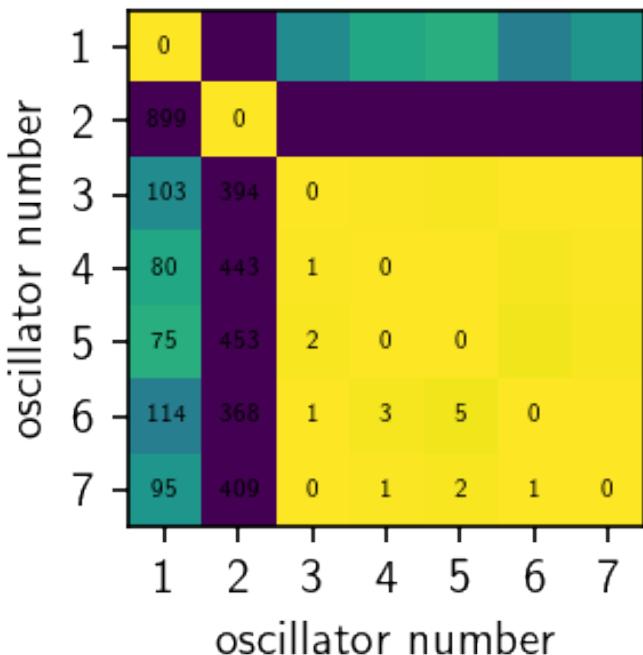


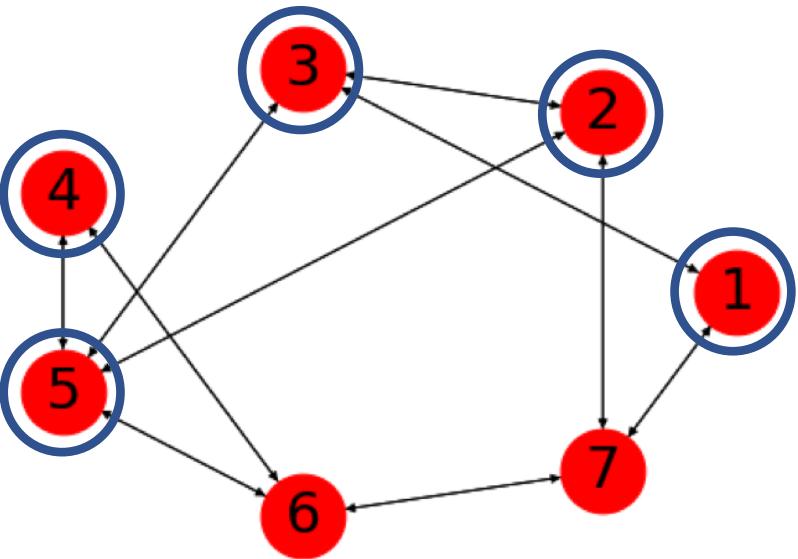
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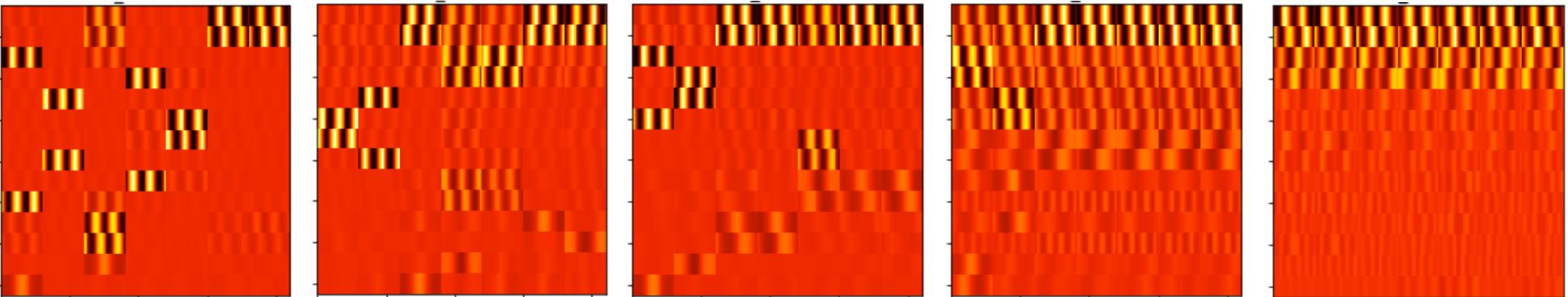
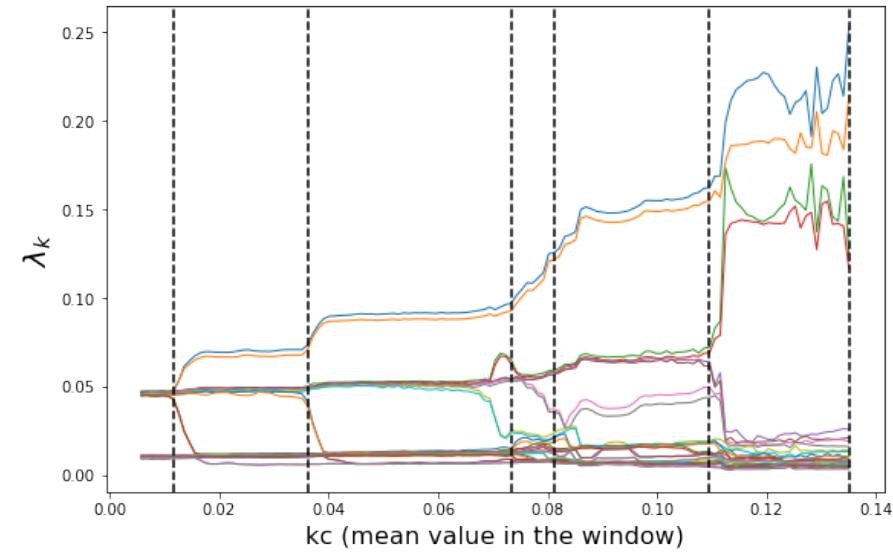
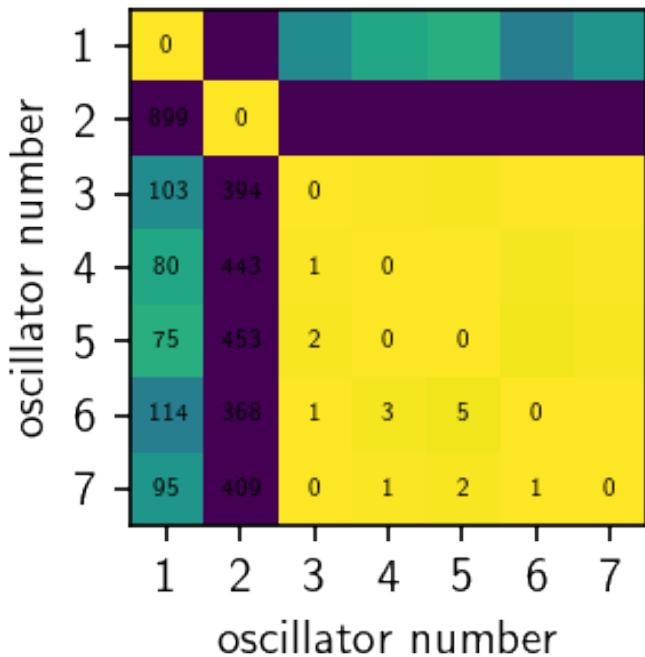


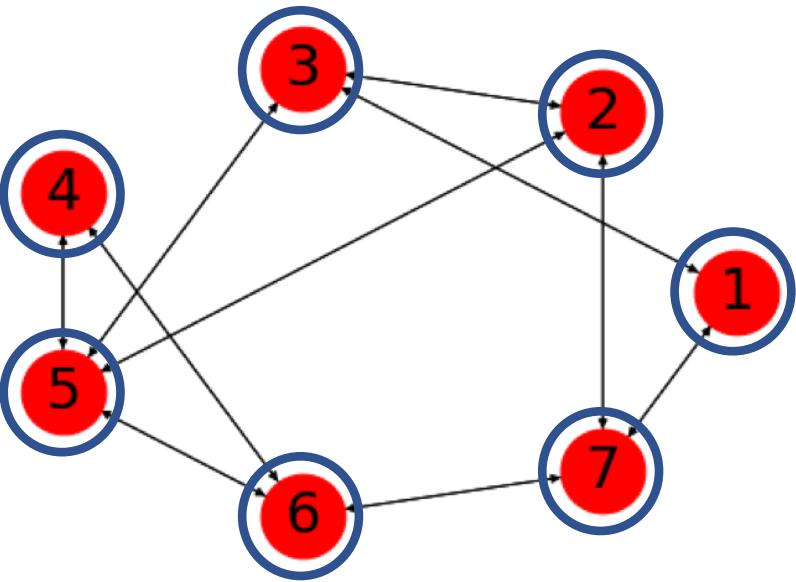
Energy of VFP-phase difference  
(time index 29000:38999)



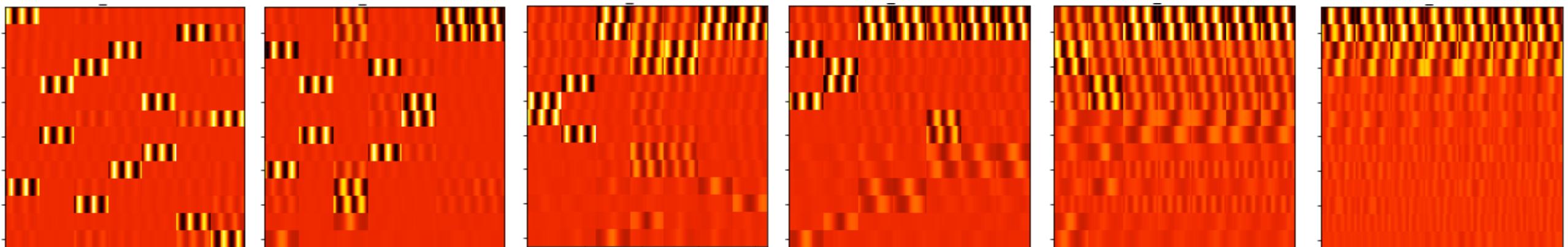
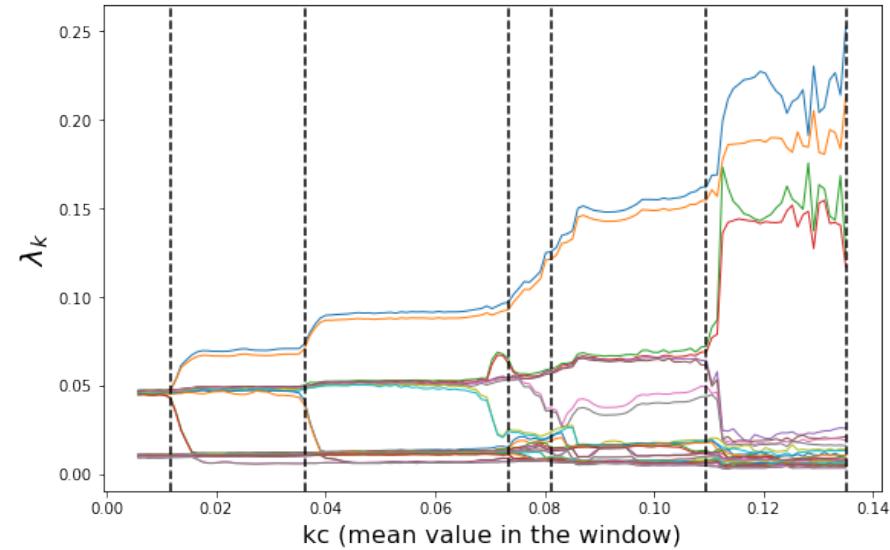
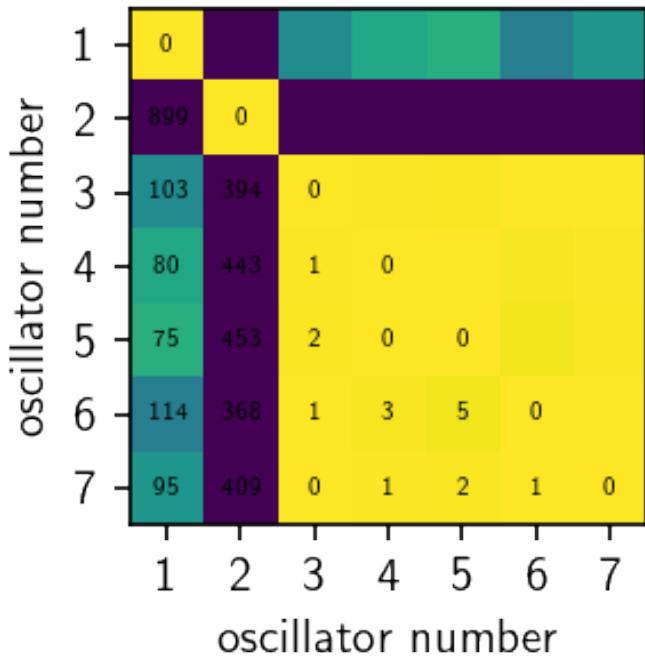


Energy of VFP-phase difference  
(time index 29000:38999)





Energy of VFP-phase difference  
(time index 29000:38999)



# REFERENCES

## SYNC QUALITY ASSESSMENT

- Freitas, L., Torres, L. A. B., & Aguirre, L. A. (2018). Phase definition to assess synchronization quality of nonlinear oscillators. *Physical Review E*, 97(5), 052202. <https://doi.org/10.1103/PhysRevE.97.052202>
- Aguirre, L. A., & Freitas, L. (2017). Control and observability aspects of phase synchronization. *Nonlinear Dynamics*, 91(4), 1–15. <https://doi.org/10.1007/s11071-017-4009-9>

## Varimax + MSSA FOR PHASE SYNC CHARACTERIZATION

- Groth, A., & Ghil, M. (2011). Multivariate singular spectrum analysis and the road to phase synchronization. *Physical Review E*, 84(3), 036206. <https://doi.org/10.1103/PhysRevE.84.036206>
- Portes, L. L., & Aguirre, L. A. (2016). Enhancing multivariate singular spectrum analysis for phase synchronization: The role of observability. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 26(9), 093112. <https://doi.org/10.1063/1.4963013>
- Portes, L. L., Aguirre, L. A., \bf Portes, L. L., & Aguirre, L. A. (2016). Matrix formulation and singular-value decomposition algorithm for structured varimax rotation in multivariate singular spectrum analysis. *Physical Review E*, 93(5), 052216. <https://doi.org/10.1103/PhysRevE.93.052216>

- Portes, L. L., & Aguirre, L. A. (2019). Impact of mixed measurements in detecting phase synchronization in networks using multivariate singular spectrum analysis. *Nonlinear Dynamics*, 96(3), 2197–2209. <https://doi.org/10.1007/s11071-019-04917-7>

- Aguirre, L. A., Portes, L. L., & Letellier, C. (2017). Observability and synchronization of neuron models. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 27(10), 103103. <https://doi.org/10.1063/1.4985291>

For a trajectory  $\gamma(t; \mathbf{x}_0)$  the *Vector Field Phase* (VFP) is given by:

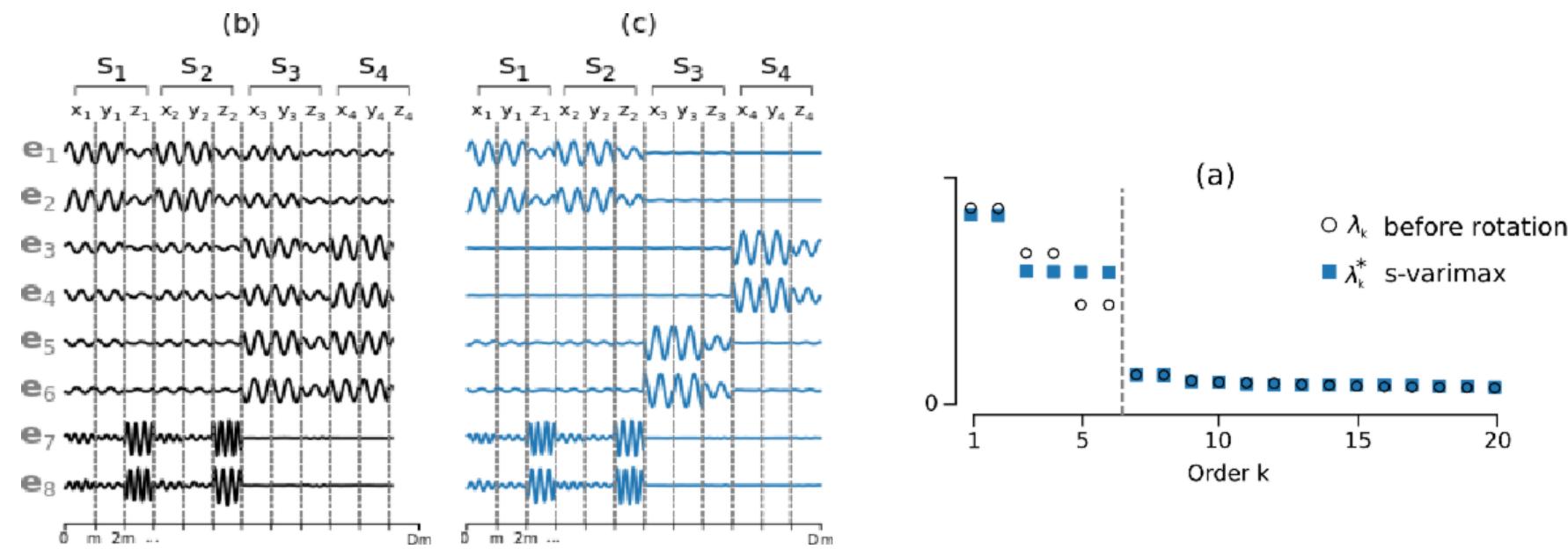
$$\phi(t) = \phi_0 + \int_{\gamma(t; \mathbf{x}_0)} c(\mathbf{x}) \mathbf{f}^\top(\mathbf{x}) d\mathbf{x},$$

where  $c(\mathbf{x})$  is a positive function.

$$c(\mathbf{x}) = \frac{2\pi}{\ell \|\mathbf{f}(\vec{\mathbf{x}})\|}$$

Arc length

# VARIMAX + MULTIVARIATE SINGULAR SPECTRUM ANALYSIS



Groth, A., & Ghil, M. (2011). Multivariate singular spectrum analysis and the road to phase synchronization. *Physical Review E*, 84(3), 036206.

Portes, L. L., & Aguirre, L. A. (2016). Enhancing multivariate singular spectrum analysis for phase synchronization: The role of observability. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 26(9), 093112