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Q1:

$$z = f(x,y) = ax + by + c$$

$$\nabla f(z) = < \frac{d}{dx}, \frac{d}{dy} >$$

$$\boxed{= < a + by, b + ax >}$$

Q2:

$$z = f(x_1, x_2, \cdots x_N)$$

$$= a_1x_1 + a_2x_2 + \cdots a_Nx_N + d$$

$$\nabla f(z) = < \frac{d}{dx_1}, \frac{d}{dx_2}, \cdots \frac{d}{dx_N} >$$

$$\boxed{= < a_1 + a_2x_2 + \cdots + a_Nx_N, a_1x_1 + a_2 + \cdots + a_Nx_N, a_1x_1 + a_2x_2 + a_3 + \cdots + a_Nx_N, a_1x_1 + a_2x_2 + a_3x_3 + \cdots + a_N)}$$

Q3:

$$z = f(x,y) = A(x - x_0)^2 + B(y - y_0)^2 + C$$

$$\boxed{\frac{df(x,y)}{dx} = 2A(x - x_0) \qquad \frac{df(x,y)}{dy} = 2B(y - y_0)}$$

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Q4:

① $x^T = (3, 1; 4)$

② $y^T = \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix}$

③ $B^T = \begin{pmatrix} 3 & 5 & 1 \\ 5 & 2 & 4 \end{pmatrix}$

④ $x \cdot x = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} = 9 + 1 + 16 = 26$

⑤ $x \cdot y^T = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} = 6 + 5 + 4 = 15$

⑥ $X \times y = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} \times (2 \; 5 \; 1) = \begin{pmatrix} 6 & 15 & 3 \\ 2 & 5 & 1 \\ 8 & 20 & 4 \end{pmatrix}$

⑦ $y \times X = (2 \; 5 \; 1) \times \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} = 6 + 5 + 4 = 15$

⑧ $A \times X = \begin{pmatrix} 4 & 5 & 2 \\ 3 & 1 & 5 \\ 6 & 4 & 3 \end{pmatrix} \times \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 12+5+8 \\ 9+1+20 \\ 18+4+12 \end{pmatrix} = \begin{pmatrix} 25 \\ 30 \\ 34 \end{pmatrix}$

⑨ $A \times B = \begin{pmatrix} 4 & 5 & 2 \\ 3 & 1 & 5 \\ 6 & 4 & 3 \end{pmatrix} \times \begin{pmatrix} 3 & 5 \\ 5 & 2 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 12+25+2 & 20+10+8 \\ 9+5+5 & 15+2+20 \\ 18+20+3 & 30+8+12 \end{pmatrix} = \begin{pmatrix} 39 & 38 \\ 19 & 37 \\ 41 & 50 \end{pmatrix}$

⑩ B.reshape(1,6) $= (3 \; 5 \; 5 \; 2 \; 1 \; 4)$

Q5.

$y = mx + b$

$f = \min SSE = \min \sum_{i=1}^{n} \varepsilon_i^2 = \min \sum_{i=1}^{n} (y_i - mx_i - b)^2$

$\boxed{\text{for } b:}$

$$\frac{d}{db} \min SSE = \frac{d}{db} \sum_{i=1}^{n} -2(y_i - mx_i - b) = 0$$

$$-\frac{2}{n} \sum_{i=1}^{n} y_i + \frac{2m}{n} \sum_{i=1}^{n} x_i + \frac{2b}{n} \sum_{i=1}^{n} 1 = 0$$

$$-\frac{1}{n} \sum_{i=1}^{n} y_i + \frac{m}{n} \sum_{i=1}^{n} x_i + \frac{b}{n} \sum_{i=1}^{n} 1 = 0$$

$$-\bar{y} + m\bar{x} + b = 0$$

$$\therefore b = \bar{y} - m\bar{x}$$

Q5 continued

$\boxed{\text{for } m:}$

$$\min SSE = \min \sum_{i=1}^{n} (y_i - mx_i - (\bar{y} - m\bar{x}))^2$$

$$\frac{d}{dm} \min SSE = \frac{d}{dm} \sum_{i=1}^{n} (y_i - \bar{y} - m(x_i - \bar{x}))^2$$

$$= \sum_{i=1}^{n} 2(y_i - m(x_i - \bar{x}) - \bar{y})(\bar{x} - x_i) = 0$$

$$\frac{2}{n}(y_i - \bar{y})(x_i - \bar{x}) - \frac{2m}{n}(x_i - \bar{x})^2 = 0$$

$$\frac{1}{n}(y_i - \bar{y})(x_i - \bar{x}) - \frac{m}{n}(x_i - \bar{x})^2 = 0$$

$$cov(X,Y) - m \cdot Var(X) = 0$$

$$m = \frac{cov(X,Y)}{Var(X)}$$

from $\boxed{\text{for } b}$:

$$b = \bar{y} - m\bar{x}$$

$$= \bar{y} - \frac{cov(X,Y)}{Var(X)}\bar{x}$$