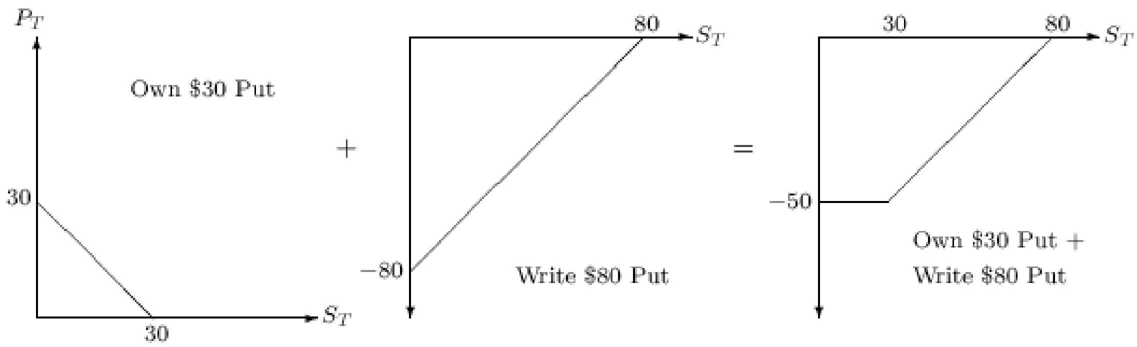
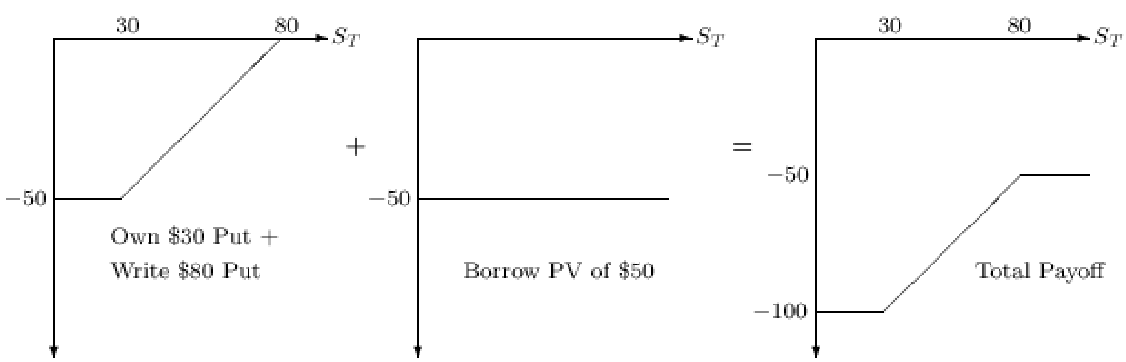
Assignment 2
Solution

1. a. First, let us add the expiration date payoff diagram from buying the put with $30 strike and writing the put with $80 strike:

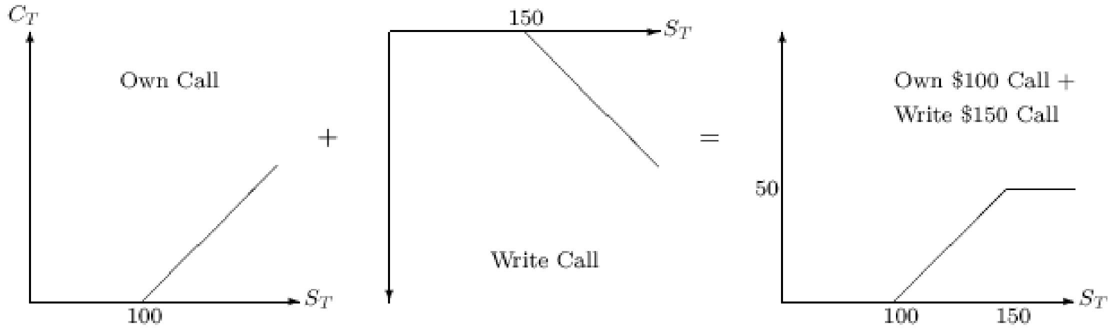


Now, we add this payoff diagram to the expiration date payoff diagram that results from your borrowing: Since you borrowed the present value of $50 risk-free, you will have to repay $50 for certain on the expiration date, i.e., your expiration date payoff from your borrowing is -$50 no matter what the expiration date stock price turns out to be.

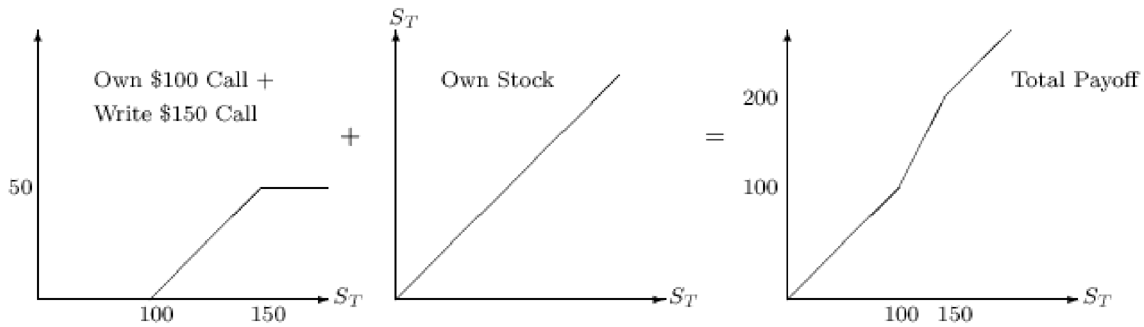


b.

First, let us add the expiration date payoff diagrams for the call option with $100 strike price that you buy and the call option with $150 strike price that you write (or sell):



Now we add this payoff diagram to the expiration date payoff diagram that results from your purchase of a share of stock:



2. There are two ways to answer this question. We can construct a portfolio that replicates the payoffs from the put option directly. An alternative is to figure out the price of a call option with the same exercise price and the same expiration date and then apply the put-call parity to compute the put price. In this question, we try the first way.

We are given that the stock is currently at \$51 and it will go up or down by 10%over the next period to \$56.10 or \$45.90. Furthermore, we are given a put option with K =\$53 ⇒we can draw the binomial processes for the stock and the put as we do below. Remember, it is a trivial exercise to calculate the expiration date (end-of-period) option prices.

Stock price (t=0) = \$51	Stock price (t=1) = \$56.10
	Stock price (t=1) = \$45.90

Corresponding option payoff	At t=1, \$0
	At t=1, \$7.10

Let the portfolio that replicates the put option be to buy δ shares of stock now and to borrow x dollars at the risk-free rate (to be repaid next period). We know that the option delta can be calculated as: δ =spread of option prices/spread of stock prices =0-7.10/56.10-45.90 = -0.696.

Therefore, one part of the replicating portfolio is to buy -0.696 shares or to short sell 0.696 shares. And we will have to payback the stock at the end of period (at the expiration date of option).

Now, let's see the following payoffs.

	Stock price = 56.10	Stock price = 45.90
Put option	$0	$7.10
Short selling 0.696 shares	-$39.05	-$31.95
We need	$39.05 more	$39.05 more

So we need to lend PV(39.05) at risk free rate (7%) so that we can have cash inflow of 39.05.

Therefore, the portfolio that mimics the option payoff should consist of (short sell 0.696 shares of stock and lend about $36.50 at 7%.

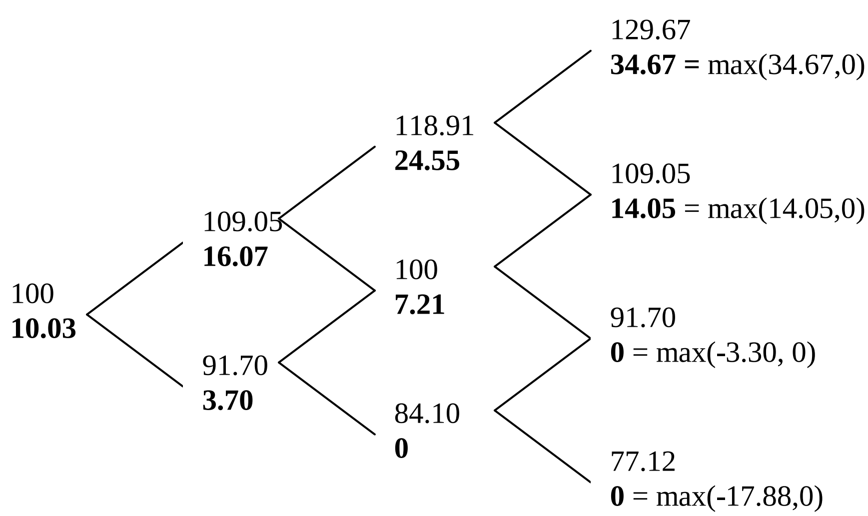
The cost of constructing this portfolio will be (-0.696*51 + 36.50) = $1.00

3. We are given the following information: S=$100, E=$95, r_f=8%, σ=30%, and Δt=1/12

Let's figure out u, d, a, and p first. $u = e^{\sigma\sqrt{\Delta t}} = e^{0.3\sqrt{1/12}} = 1.090463, d = 1/u = 0.917042$

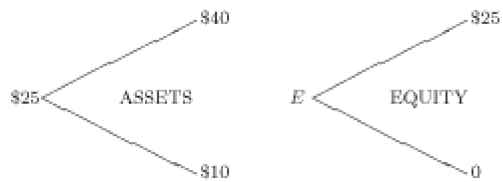
$a = e^{r\Delta t} = e^{0.08(1/12)} = 1.006689\ p = \dfrac{a-d}{u-d} = 0.516933,\ \ 1-p = 0.483067$

Then, we construct a binomial tree for the evolution of underlying asset value, stock price. And then, we compute the payoffs of the call option at the expiration date. Now we start working backward to value the call today. Then the tree should look as follows. (Top numbers are stock prices and bottom numbers are option value)



Therefore, the (theoretical) call price = $10.03

4. We know that the firm 's assets are currently worth \$25 million and they will be worth either \$40 million or \$10 million next period. This is shown in the figure below. We know that the firm 's equity is a call option on the underlying assets of the firm with a strike price equal to the face value of debt (\$15 million). Since we know the strike price and the expiration date asset values, we can easily compute the payoffs to equity on the expiration date. This is also shown in the figure below. (To understand the payoffs to equity, note that when the projects succeed and the assets are worth \$40 million, shareholders will exercise their call option by paying out the \$15 million owed to bondholders and taking ownership of the assets ⇒their payoff=\$40 -\$15 =\$25 million. But if the projects fail and the assets are worth only \$10 million (less than the \$15 million debt owed), the shareholders will declare bankruptcy and walk away from the firm and so they will get nothing.)



In order to calculate the current equity (call) value, we use the risk-neutral approach. Recognize that u =40 /25 =1 .6and d =10 /25 =0.4. Given r_f =10%, we can calculate the risk-neutral probability as:

$$p = \frac{a-d}{u-d} = \frac{1.10-0.40}{1.60-0.40} = 0.58$$

We can use now calculate the equity value as the present value of its future expected payoff discounted at the risk-free rate:

$$E = \frac{0.58 \times 25 + 0.42 \times 0}{1.10} = \$13.18 \text{ million}$$

5. a. Stock price (ex-right) = (13+2) / (1+0.5) = \$10
Subscription price = 2 / 0.5 = \$4
Right's price = 13-10 = \$3
= (10-4) / 2 = \$3

b. Stock price (ex-right) = (13+2) / (1+0.25) = \$12
Subscription price = 2 / 0.25 = \$8
Right's price = 13-12 = \$1
= (12-8) / 4 = \$1

c. The stockholders' wealth is the same between the two arrangements.

6. The NPV of the refunding is the difference between the gain from refunding and the refunding costs.

Gain = B $(r_1 - r_2) / r_2$
Cost = (CB) / F + K
Where C = the call premium
F = the face value
B = the par value of the old bonds
K = the issuing costs
r_1 = the coupon rate of the old bonds and
r_2 = the coupon rate of the new bonds.

Gain = \$500 million (0.09 - 0.07) / 0.07 = \$142,857,143
Cost = 90 (\$500 million) / \$1,000 + \$80 million = \$125 million
NPV = \$142,857,143 - \$125,000,000 = \$17,857,143

7.

Payoff at expiration	S = \$25	S = \$35
1. Call	\$0	\$3 × 100 = \$300
2. Stock (N shares)	25 N	\$35N
Borrow (\$25N/1.05)	-25 N	-\$25N
Net payoff	\$0	\$10N

Duplicating amount = \$25 N / 1.05
where \$10 N = \$300
N = 30 shares
Borrow \$25 × 30 shares / 1.05 = \$714.29.
Thus, buying one call contract

= (1) buy 30 shares of stock	\$900
(2) borrow \$714.29	-\$714.29
	\$185.71

Call option value = \$185.71
Call price per share = \$1.857
From put-call parity,
P = C + PV(E) - S
= \$1.857 + \$32 / 1.05 - \$30 = \$2.333

8. The equity of the firm is regarded as a call option.

Payoff at expiration (million)	\$250	\$650
1. Call	\$0	\$350
2. Buy N shares	\$250N	\$650N
Borrow	-\$250N	-\$250N
Net payoff	\$0	\$400N

To equate, \$400N = \$350
N = 0.875 shares of assets.

Thus, borrowing amount = $250 × 0.875 / 1.07 = $204.44 million
Call value = Value of 0.875 shares of the asset + Borrowing $204.44 million
= $350 million - $204.44 million
= $145.56 million
The value of the equity
= $145.56 million
The value of the debt
= $400 - $145.56
= $254.44 million

9. First, if we ignore the potential dilution effects on stock price,

$d_1 = [\ln(\$22 / \$20) + (0.05 + (0.005)^2 / 2)] / 0.005$
$= 29.07$
$d_2 = 29.07 - 0.005$
$= 29.06$
$N(d_1) = 1$
$N(d_2) = 1$
$C = \$22 \times 1 - \$20\, e^{-0.05} \times 1$
$= \$2.98$

Consider the dilution effect on stock price.

$S^* = 22 + (5/40)*2.98 = 22.37$
$d_1 = 32.42$
$d_2 = 32.41$

$C^* = \$22.37 \times 1 - \$20\, e^{-0.05} \times 1$
$= \$3.35$

$W = C^*(N_W/N) = \$3.35*(1/(1+5/40)) = \$2.96$