# Risk and Rates of Return

What is Risk? The big picture

- **Risk** is an uncertain outcome or a chance of an adverse outcome

- Related to the probability of earning a low or negative actual return.

- The greater the chance of lower than expected or negative returns is, the riskier the investment is.

What is Risk? The big picture-cont'd…

- Probability distribution: A listing of all possible outcomes, and the probability of each occurrence

- Can be shown graphically.

Firm X

Firm Y

Rate of Return (%)

-70   0   15   100

Expected Rate of Return
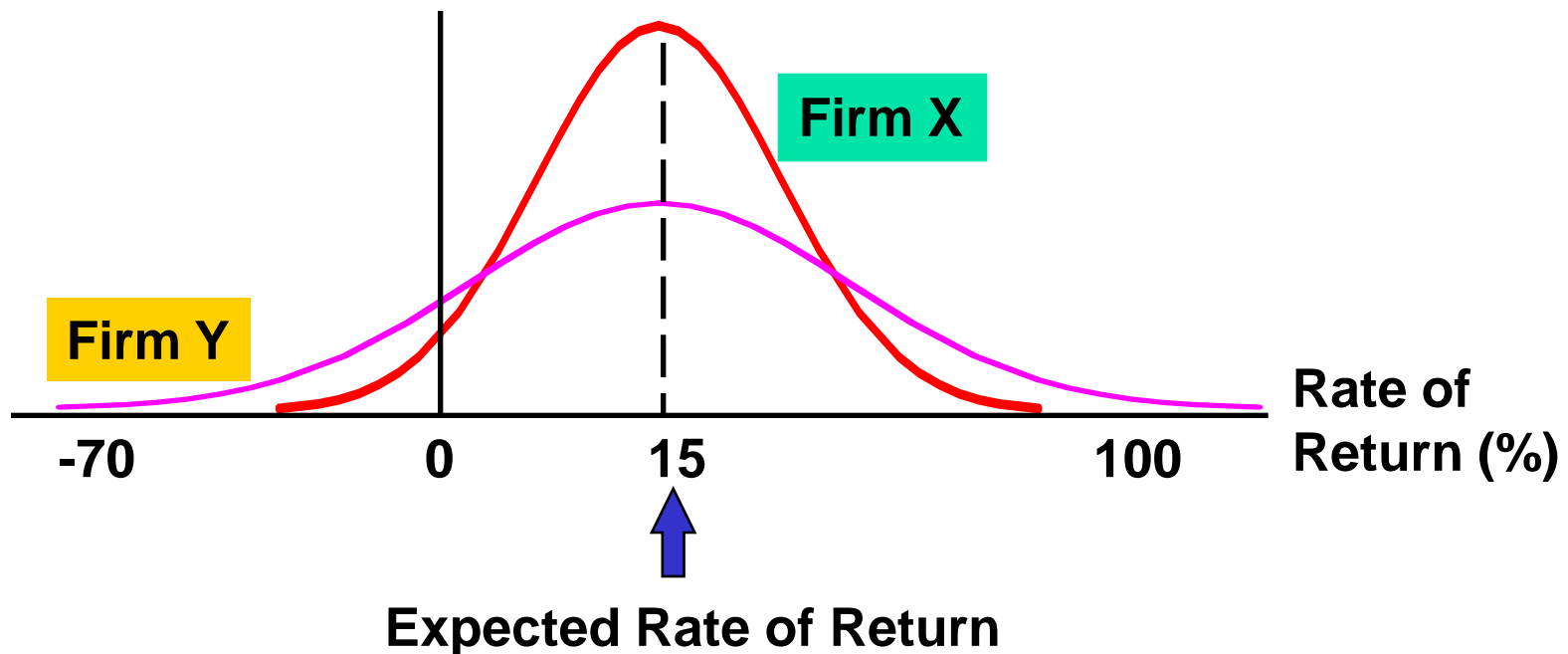
Selected Realized Returns, 1926 – 2004

	Average Return	Standard Deviation
Small-company stocks	17.5%	33.1%
Large-company stocks	12.4	20.3
L-T corporate bonds	6.2	8.6
U.S. Treasury bills	3.8	3.1

What is Risk? The big picture – cont'd…

- Concerned with the riskiness of cash flows from financial assets.

— Stand Alone Risk: Single Asset

relevant risk measure is the *standard deviation* of expected cash flows.

What is Risk? The big picture - cont'd…

— Portfolio Context: A group of assets.
Diversifiable Risk
Market Risk

- Small group of assets with Diversifiable Risk remaining:

 interested in portfolio standard deviation.
  correlation (ρ or r) between asset returns which affects portfolio standard deviation

General comments about risk

- $\sigma \approx 35\%$ for an average stock.
- Most stocks are positively (though not perfectly) correlated with the market (i.e., ρ between 0 and 1).
- Combining stocks in a portfolio generally lowers risk.
- σ_p decreases as stocks added, because they would not be perfectly positively correlated with the existing portfolio.
- Eventually the diversification benefits of adding more stocks dissipates (after about 10 stocks), and for large stock portfolios, σ_p tends to converge to $\approx 20\%$.

Illustrating diversification effects of a stock portfolio
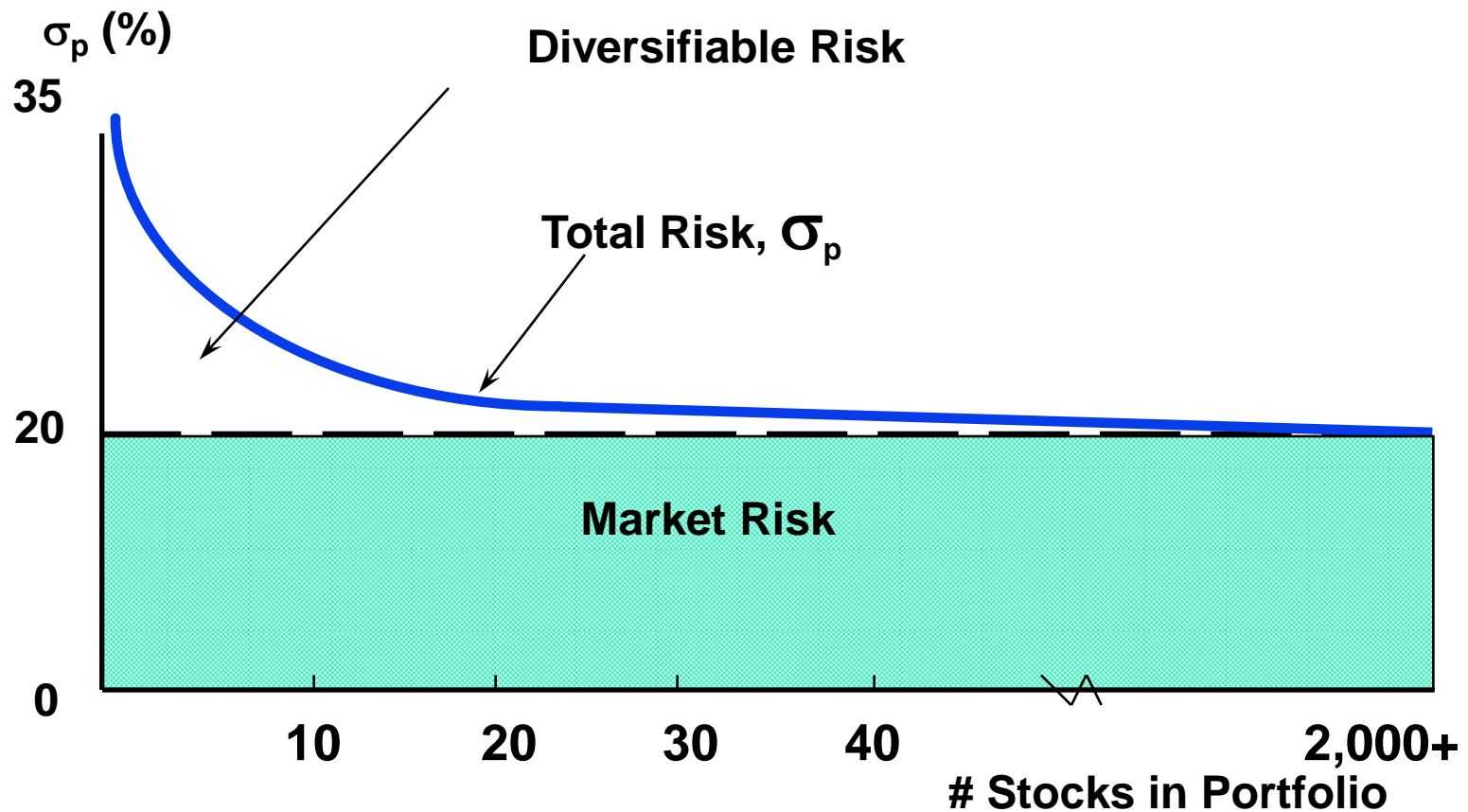
Well-diversified Portfolio

Large Portfolio (10-15 assets) eliminates diversifiable risk for the most part.

Interested in **Market Risk** that is the risk that cannot be diversified away.

The relevant risk measure is **Beta** that measures the riskiness of an individual asset in relation to the market portfolio.

Holding Period (Realized) Return

$$HPR = \frac{(Selling\ Price - Buying\ Price + Dividens)}{Buying\ Price}$$

HPR = Capital Gains Yield (CGY)+ Dividend Yield (DY)

Example: Bought at $50, Receive $3 in dividends, current price is $54

HPR =

CGY =

Expected Return: Single Asset

Expected Rate of Return given a probability distribution of possible returns(k_i): E(k)

$$E(k) = \sum_{i=1}^{n} P_i k_i$$

Realized or Average Return on Historical Data:

$$k = \frac{1}{n} \sum_{i=1}^{n} k_i$$

Standard Deviation

- Relevant Risk Measure for single asset

$$\text{Variance} = \sigma^2 = \sum_{i=1}^{n} P_i (k_i - E(k))^2$$

Standard Deviation = Square Root of Variance

$$\text{Sample Variance} = S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (k_i - E(k))^2$$

Example 1

- Exp. Return and s

State of Economy	Probability	MAD Inc.	Contrary Co. (CON)
Boom	0.25	80%	5%
Normal	0.60	25%	10%
Recession	0.15	-25%	15%

MAD: E(k) =

CON: E(k) =

Example 2

- Standard Deviation

MAD s:

$s^2 = 0.25(80\%\text{-}31.25\%)^2 + 0.60(25\%\text{-}31.25\%)^2 + 0.15(\text{-}25\%\text{-}31.25\%)^2$

$= 1092.1875\%$

$s = (1092.1875\%)^{(1/2)} = 33.0\%$

Contrary s:

$s^2 =$

$s =$

Coefficient of Variation

Most investors are ***Risk Averse***, meaning they don't like risk and demand a higher return for bearing more risk.

The ***Coefficient of Variation*** (**CV**) scales risk per unit of expected return.

$$CV = s/E(k)$$

Coefficient of Variation – cont'd…

Back to our Example: CV

MAD Inc.

E(k) = 31.25%

s = 33.0%

CV =

Contrary Co.

E(k) = 9.5%

s = 3.1%

CV =

Portfolio Risk and Return

$E(k_p) = \sum w_i E(k_i)$ = weighted average of the expected return of each asset in the portfolio

In our example, MAD $E(k) = 31.25\%$ and CON $E(k) = 9.5\%$

What is the expected return of a portfolio consisting of 40% MAD and 60% CON?

$E(k_p) = \sum w_i E(k_i) =$

Portfolio Risk

Looking at a 2-asset portfolio for simplicity, the riskiness of a portfolio is determined by the relationship between the returns of each asset over different states of nature or over time.

This relationship is measured by the correlation coefficient(ρ):

$-1 <= \rho <= +1$

All else constant: Lower ρ = less portfolio risk

Portfolio Risk - cont'd…

- Example Portfolios

State of Economy	Probability	MAD Inc.	Contrary Co. (CON)	MAD-CON Portfolio
Boom	0.25	80%	5%	35.0%
Normal	0.60	25%	10%	16.0%
Recession	0.15	-25%	15%	-1.0%

$s^2 =$

$s = (128.76\%)^{(1/2)} = 11.35\%$

Market Risk

As more and more assets are added to a portfolio, risk measured by s decreases.

However, we could put every conceivable asset in the world into our portfolio and still have risk remaining.

This remaining risk is called *Market Risk* and is measured by *Beta*.

The Concept of Beta

Beta(b) measures how the return of an individual asset (or even a portfolio) varies with the market.

b = 1.0 : same risk as the market

b < 1.0 : less risky than the market

b > 1.0 : more risky than the market

Beta is the slope of the regression line (y = a + bx) between a stock's return(y) and the market return(x) over time, b from simple linear regression.

Portfolio Beta

The beta of a portfolio of stocks is equal to the weighted average of their individual betas:

$$\mathbf{b_p} = \sum w_i \mathbf{b_i}$$

Example

The Kish Investment Fund, in which you plan to invest some money, has total capital of $500 million invested in five stocks:

Stock	Investment	Stock's beta
A	$160 M	0.5
B	$120 M	2.0
C	$80 M	4.0
D	$80 M	1.0
E	$60 M	3.0

The current risk-free rate is 6 percent, whereas market returns have the following estimated probability distribution for the next period:

Probability	Market return
0.1	7%
0.2	9%
0.4	11%
0.2	13%
0.1	15%

Q. What is the portfolio beta?

Example-cont'd…

Relating Market Risk and Return: the Capital Asset Pricing Model (CAPM) and Security Market Line (SML) equation

The main assumption is investors hold well diversified portfolios = only concerned with market risk.

A stock's risk premium = measure of market risk * market risk premium.

CAPM/SML Equation

RP_M = market risk premium = $k_M - k_{RF}$

RP_i = stock risk premium = $(RP_M)b_i$

$$k_i = k_{RF} + (k_M - k_{RF})b_i = k_{RF} + (RP_M)b_i$$

From the above example:

$k_{RF} = 6\%$, $k_M = 11\%$, $b(fund) = 1.8$

$k(fund) =$

SML Equation – cont'd…

Suppose Bridger Nelson, the president, receives a proposal for a new stock. The investment needed to take a position in the stock is $50 million, it will have an expected return of 15 percent, and its estimated beta coefficient is 2.0. Should the new stock be purchased? At what expected rate of return should the fund be indifferent to purchasing the stock?

Recall, the SML equation is:

$$k_i = k_{RF} + (k_M - k_{RF})b_i = 6\% + (11\% - 6\%)b$$

Req Ret(new stk)=

SML Equation – cont'd…

E(k) = 15%, Add stock?

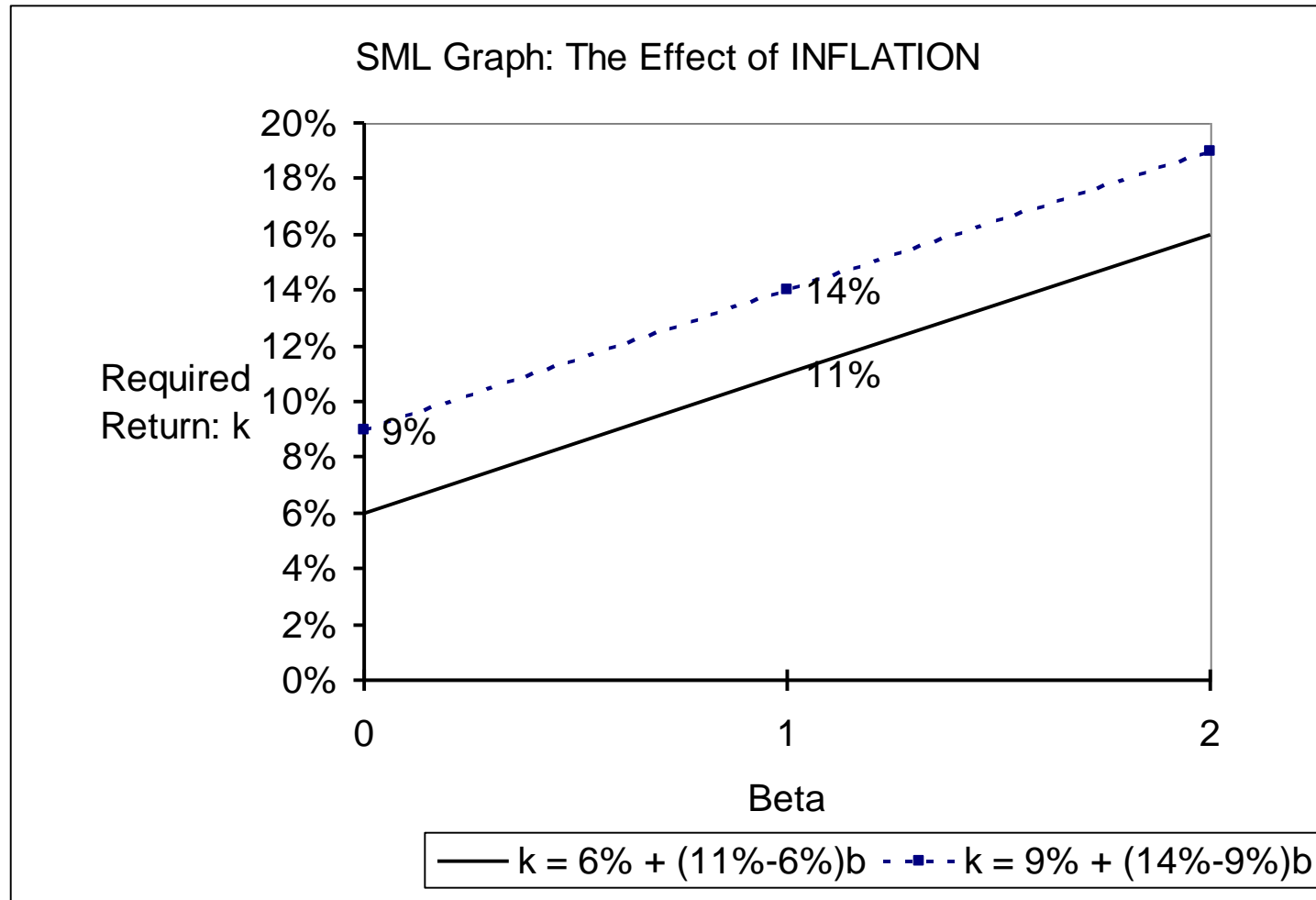, expected return < req. return

SML Graph

Changes to SML Equation

What happens if inflation increases?

What happens if investors become more risk adverse about the stock market?

Check out the following graphs as base SML = 6% + (11%-6%)

Changes to SML Equation –cont'd…

SML Graph: The Effect of INFLATION

Changes to SML Equation –cont'd…

SML: Change in Risk Aversion