Time Value of Money

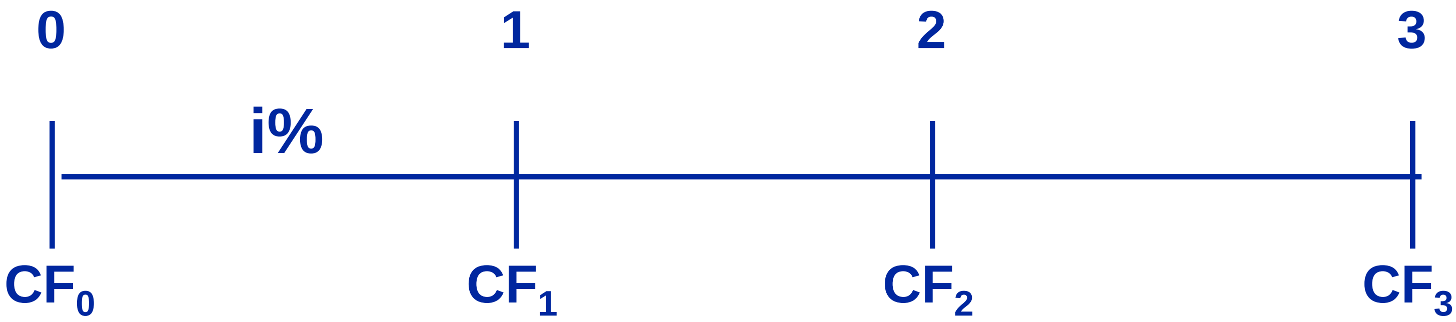
Topics Overview

- Future Value of Single Cash Flow (Lump Sum)
- Present Value of a Single Cash Flow
- Future and Present Value of Annuities
- Future and Present Value of Uneven Cash Flows
- Non-annual Interest Compounding

Terms and Short Hand

- PV = Today's amount of money
- FV_n = Amount of money n years from today
- i = nominal annual interest rate
- n = number of years (or number of periods for your calculator)
- PMT = periodic annuity cash flow
- Time Line = important graphical representation of cash flows

Time lines show timing of cash flows



Tick marks at ends of periods, so Time 0 is today; Time 1 is the end of Period 1; or the beginning of Period 2

Example

Draw a time line for the following cases!

- Time line for a $100 lump sum due at the end of year 2
- Time line for an ordinary annuity of $100 for 3 years
- Time line for uneven CFs -$50 at t = 0 and $100, $75, and $50 at the end of Years 1 through 3.

Future Value of a Single CF

- Want to know what some amount invested today will be worth at some future point.
- Good for finding the future cost of something by inflating today's cost.
- Finding FVs is ***compounding***

Example

- What is the FV of an initial $100 after 3 years if i=10%?

– Time line,

Example-continued…

- The general FV formula is

$$FV_n = PV(1+i)^n \text{ or } PV(FVIF_{i,n})$$

Four Ways to Find FVs

- Solve the equation with a regular calculator, using the general formula
- Use tables
- Use a spreadsheet like EXCEL
- Use a financial calculator

Using Future Value Tables

- From Future value Table, Future Value of $1 at the End of n Periods, we find the corresponding "future value interest factor (FVIF)".

– Period = 3, Interest rate = 10%

  then the corresponding FVIF = $(1+i)^n$ =

- From the general formula for FV,

$$FV_3 = PV(1 + i)^3 = PV(FVIF_{10\%,\ 3\ yrs})$$

$$=$$

Using a Spreadsheet, EXCEL

- FV = (rate, nper, pmt, pv, type)

- Rate: interest rate per period
- Nper: total number of payments
- Pmt: the payment made each period
- PV: the present value, or the lump sum amount that a series of future payments worth now
- Type: 0 if the payment is made at the end of period,1 if the payment is made at the beginning of the period (annuity due)

Using a Spreadsheet, EXCEL-continued…

So,

- Put rate = , nper = , pmt = , pv = , type = ,
- then FV = $

Using Financial Calculator

- Financial calculators solve this equation:

$$FV_n = PV(1+i)^n$$

There are 4 variables. If 3 are known, the calculator will solve for the 4th.

Using Financial Calculator-continued…

- Here's the setup to find FV:

– Inputs:

N =   ,   I/Y =   ,   PV =   , PMT =   , CMP FV=   ?

– Output:

FV = $

- NOTE: Don't forget to set P/Y = 1.00

FV example

- Under the new NBA collective bargaining agreement, the most a free agent with 10 years of experience can be offered for 2015 is a pro-rated \$14 million. If the free agent resigns with his current team, he can receive an annual increase of 12.5% a year. However, if he signs with another team, he can only receive a 10% annual increase in salary.

  SuperStar of the Chicago Bulls is a free agent looking for a 5-year contract (the 5th year of the contract starts 4 years from today, n = 4). Let's assume both the Bulls and another team are willing to offer him the maximum \$14 million and maximum allowable annual salary increases over the life of the 5-year contract.

  What is the difference in SuperStar's 5th (last) year salary between the Bulls and the other team?

FV example -Answer

- SuperStar's Contract Breakdown

Season	N	Bulls	Other team	
I/Y		12.5%	10%	
2015	0	$14 M	$14 M	PV
16-17	1			
17-18	2			
18-19	3			
19-20	4			
Reported total				

Present Value of a Single Cash Flow

- When you need to know how much some future amount is worth today.
- All of the valuation models we will look at this semester are based on the idea of today's price is equal to the present value of the future expected cash flows.
- Handy for figuring out how much to save today to reach some future goal.

Present Value of a Single Cash Flow-continued…

- Solve $FV_n = PV(1 + i)^n$ for PV:

The general formula is,

$$PV = \frac{FV_n}{(i + i)^n} = FV_n\left(\frac{1}{1+i}\right)^n$$

Or

$$PV = FV_n(PVIF_{i,n})$$

Example

- What is the PV of $100 due in 3 years if i = 10%?

— Finding PVs is discounting, and it's the reverse of compounding.
— Time line,

- Numerical solution:

PV =

Example-continued…

- Using Present Value Tables

— From the table, Present Value of $1 Due at the End of n Periods, we find the corresponding "present value interest factor (PVIF)".

— Period = 3, Interest rate = 10%
then the corresponding PVIF = $1/(1+i)^n$ =

— From the general formula for PV,

$$PV = FV_3 \left(\frac{1}{(1+i)^3}\right) = FV(PVIF_{10\%,\ 3\ yrs})$$

$$=$$

Example-continued…

- Using a Spreadsheet, EXCEL

PV = (rate, nper, pmt, pv, type)

- Rate: interest rate per period
- Nper: total number of payments
- Pmt: the payment made each period
- PV: the present value, or the lump sum amount that a series of future payments worth now
- Type: 0 if the payment is made at the end of period, 1 if the payment is made at the beginning of the period (annuity due)

So,
Put rate = , nper = , pmt = , FV = , type = ,
then FV =

PV Example

- The Seattle Mariners are trying to figure out a way to have enough money to re-sign all-stars Ken Griffey, Jr. and Alex Rodriguez each to new $100 million contracts four years from today. Being a small market team, the Mariners plan to ask local billionaire Bill Gates to deposit a single amount today that will be invested at 9%.

How much money do the Mariners need to get from Bill Gates? What if the money can be invested at 12%?

Finding Number of Periods

If sales grow at 20% per year, how long before sales double?

Solve for n:
Time line,

Finding Number of Periods-continued…

- Using Equation

  $FV_n = 1(1 + i)_n$;

  =

  n =

- Using EXCEL,

  NPER(rate, pmt, pv, fv, type)

Put rate = , pmt = , pv = , fv = , type = ,

then, NPER =

Finding Rate of Return or Interest Rate

A broker offers you an investment (a zero coupon bond) that pays you $10,000 five years from now for the cost of $7,000 today.

What is your annual rate of return?

Time line,

Finding Rate of Return or Interest Rate-continued…

- Using Equation

$$PV = \frac{FV}{(1+i)^n}$$

- Using EXCEL

RATE(nper, pmt, pv, fv, type, guess)

nper = , pmt = , pv = , fv = , type = , guess =

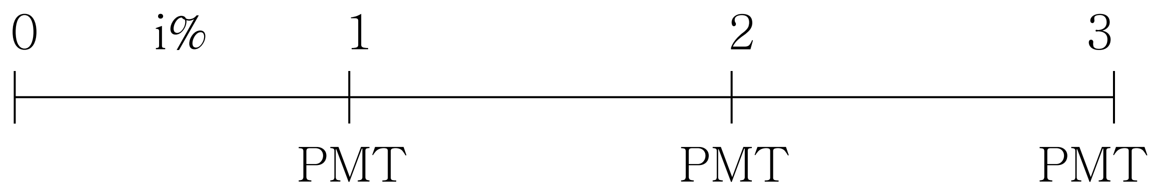
then, RATE =

Annuities

A series of equal periodic cash flows

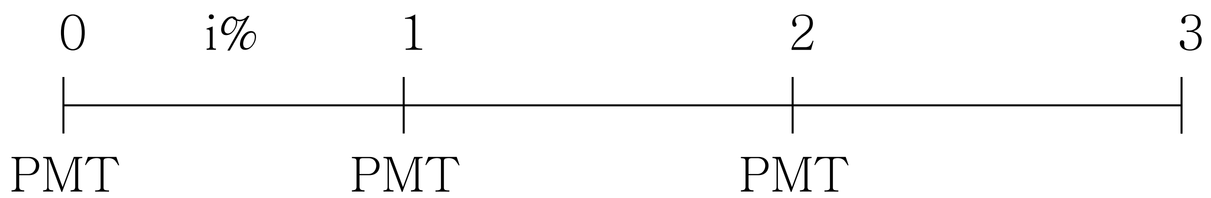
Two types:

- Ordinary (or Deferred Annuity)



Annuities-continued…

- Annuity-due



FV of Annuities-Ordinary Annuity

What is the FV of a 3-year ordinary annuity of $100 at 10%?

Time line,

- Using Equation,

$$FV_1 = PV(1+i) =$$

$$FV_2 = PV(1+i)^2 =$$

$$FVA_3 = \$100 + \$110 + \$121 = \$331$$

FV of Annuities- Ordinary Annuity-cont'd…

- Using Future Value Interest Factor Annuity Table

$$FVA_n = PMT(FVIFA_{i,n}) = PMT\left(\frac{(1+i)^n - 1}{i}\right) = \$$$

- Using EXCEL

FV(rate, nper, pmt, pv, type)

Put rate = , nper = , pmt = , pv = , type = , then

$FVA_3 = \$$

FV of Annuities- Annuity Due

What is the FV of a 3-year annuity due of $100 at 10%?

Time line,

- Using Equation,

$FV_1 = PV(1+i) =$

$FV_2 = PV(1+i)^2 =$

$FV_3 = PV(1+i)^3 =$

FVA_3(Annuity-due) = \$110 + \$121 + \$133.10 = \$364.1

PV of Annuities- Ordinary Annuity

What is the PV of a 3-year ordinary annuity of $100 at 10%?
Time line,

- Using Equation

$$PV_1 = FV_1/(1+i) =$$

$$PV_2 = FV_2/(1+i)^2 =$$

$$PV_3 = FV_3/(1+i)^3 =$$

PVA_3 = $90.91 + $82.64 + $75.13 = $248.68

PV of Annuities- Ordinary Annuity-cont'd…

- Using Present Value Interest Factor Annuity Table,

$$PVA_n = PMT(PVIFA_{i,n}) = PMT\left(\frac{1-\frac{1}{(1+i)^n}}{i}\right) = PMT\left(\frac{1}{i} - \frac{1}{i(1+i)^n}\right)$$

$$= \$$$

- Using EXCEL,

PV(rate, nper, pmt, pv, type)

Put rate = , nper = , pmt = , fv = , type = , then
PVA_3 = $248.69

PV of Annuities- Annuity Due

What is the PV of a 3-year annuity due of $100 at 10%?

Time line,

- Using Equation,

$$PV_1 = FV_1/(1+i) =$$

$$PV_2 = FV_2/(1+i)^2 =$$

PVA_3 = \$100 + \$90.91 + \$82.64 = \$273.55

PV of Annuities- Annuity Due-cont'd…

- Using Present Value Interest Factor Annuity Table,

$$PVA_n(\text{Annuity due}) = PMT(PVIFA_{i,n})(1+i) =$$

$$Or$$

$$= PMT\left(\frac{1 - \frac{1}{(1+i)^n}}{i}\right)(1+i) = 273.55$$

- Using EXCEL,

PV(rate, nper, pmt, fv, type)

Put rate = , nper = , pmt = , fv = , type = , then
FV = $273.55

Example 1

How much would you have at age 65 if you deposit $2,000 at the end of each year in an account paying 10% annually starting at:

(A) Age 36?

(B) Age 25?

Example 1-Solution

(A) $n =$

$FVA_{29} =$

(B) $n =$

$FVA_{40} =$

Example 2

The Illinois State Lottery switched from a 20-year annuity-due payout of the Lotto grand prize to one lump sum payment at the beginning of 1998. As a result a “fresh grand prize starts at $1 million rather than $2 million.

Is the state lottery losing money if they were previously buying annuities with an 8% discount rate?

Uneven Cash Flows

- Example:

We want to find the present and future value of the following series of cash flows at 10%

\$100 in year 1, \$300 in year 2, \$300 in year 3, and -\$50 in year 4.

Time line,

Uneven Cash Flows-cont'd…

- Using Equation,

$PV_1 =$
$PV_2 =$
$PV_3 =$
$PV_4 =$

PV = \$90.91 + \$247.93 + \$225.39 - \$34.15 = \$530.08

Uneven Cash Flows-cont'd…

- Using EXCEL,

NPV(rate, value1, value2,…………)

Put rate = 0.1, value1 = 100, value2 = 300, value3 = 300, value4 = -50, then

NPV = $530.09

Uneven Cash Flows-cont'd…

- NOTE:

Will the FV of a lump sum be larger or smaller if we compound more often, holding the stated i% constant?

Why?

If compounding is more frequent than once a year--for example, semiannually, quarterly, or daily--interest is earned on interest more often.

Non-Annual Interest Compounding

- Annual Percentage Rate (APR):
  - Nominal, or stated, or quoted, rate per year
  - i_{nom}
- Effective Annual Rate (EAR):
  - the actual annual interest rate earned or paid
  - $EAR = (1 + \frac{i_{nom}}{m})^m - 1$, where m = # of compounding periods per year

Non-Annual Interest Compounding-cont'd

- Periodic Rate:
  - i_{per}

APR should be stated in contracts. Periods per year (m) should also be given.

Ex) 8%, quarterly

8%, daily interest (365 days)

Periodic Rate (i_{per})

- $i_{per} = i_{nom}/m$, where m = # of compounding periods per year

- m = 1 for annually, 2 for semi annually, 4 for quarterly, 12 for monthly, 360 or 365 for daily compounding

- Examples:

8% quarterly: i_{Per} =

8% daily (365): i_{Per} =

Effective Annual Rate (EAR = EFF%)

- The annual rate that causes PV to grow to the same FV as under multi-period compounding.
- Example:

  EAR for 10%, semiannual:

  FV = $(1 + i_{Nom}/m)^m = (1.05)^2 = 1.1025$.

  EAR = 10.25% because $(1.1025)^1 = 1.1025$.

  Any PV would grow to same FV at 10.25% annually or 10% semiannually.

- An investment with monthly payments is different from one with quarterly payments. Must put on EFF% basis to compare rates of return. Use EFF% only for comparisons.
- Banks say "interest paid daily." Same as compounded daily.

Effective Annual Rate (EAR = EFF%)-cont'd…

- How do we find EAR for a nominal rate of 10%, compounded semiannually?

$$EAR = (1 + \frac{i_{nom}}{m})^m - 1$$

EAR_{Annual} = 10%.

EAR_Q =

EAR_M =

$EARD_{(360)}$ =

Effective Annual Rate (EAR = EFF%)-cont'd…

- Can the effective rate ever be equal to the nominal rate?

Yes, but only if annual compounding is used, i.e., if $m = 1$. If $m > 1$, EAR will always be greater than the nominal rate.

When is each rate used?

- i_{Nom} : Written into contracts, quoted by banks and brokers. Not used in calculations or shown on time lines.

- i_{Per} : Used in calculations, shown on time lines.

  If i_{Nom} has annual compounding, then $i_{Per} = i_{Nom}/1 = i_{Nom}$.

- EAR : Used to compare returns on investments with different payments per year. (Used for calculations if and only if dealing with annuities where payments don't match interest compounding periods.)

FV and PV with non-annual interest compounding

n = number of years

m = number of times interest is paid per year

i = nominal annual rate (APR)

- Single CF

$$FV_{nm} = PV(1+\frac{i}{m})^{nm} \text{ or } PV(FVIF_{i/m,\,nm})$$

$$PV = FV_{nm}(PVIF_{i/m,\,nm})$$

FV and PV with non-annual interest compounding-cont'd…

- Example:

FV of $100 after 3 years under 10% semiannual compounding? Quarterly?

$$FV_{nm} = PV\left(1+\frac{i_{Nom}}{M}\right)^{nm}$$

Semiannual compounding,

$FV_6 =$

Quarterly compounding,

$FV_{12} =$

FV and PV with non-annual interest compounding-cont'd…

- Non-annual annuities
  - Ordinary:

$FVA_{nm} = PMT\ (FVIFA_{i/m,nm})$

$PV = PMT\ (PVIFA_{i/m,nm})$

  - Annuity-due:

$FVA_{nm}(\text{Annuity-due}) = PMT\ (FVIFA_{i/m,nm})(1+i/m)$

$PV(\text{Annuity-due}) = PMT\ (PVIFA_{i/m,nm})(1+i/m)$

FV and PV with non-annual interest compounding-cont'd…

- Example:

You want to buy a $200,000 house. You have $40,000 to put down on a 7.0% APR 30-year mortgage. What is your monthly payment?

- Solution

Loan Amount =

n = , m = , nm = = N, I/m or i_{PER} =

PVA = Loan Amount =

$160,000 = PMT(PVIFA$_{7\%/12,\ 360}$)

PMT = $1,064.48

Example

- What is the value at the end of Year 3 of the following CF stream if the quoted interest rate is 10%, compounded semiannually?

Time line:

Payments occur annually, but compounding occurs each 6 months.

So we can't use normal annuity valuation techniques.

Example-cont'd…

Compound Each CF

$$FVA_3 = 100(1.05)^4 + 100(1.05)^2 + 100 = \$331.80$$

Example-cont'd…

- What's the PV of this stream?

PV_3 = $247.59

Case1

A father is planning a savings program to put his daughter through college. His daughter is now 13 years old. She plans to enroll at the university in 5 years, and it should take her 4 years to complete her education. Currently, the cost per year (for everything-food, clothing, tuition, books, transportation, and so forth) is $12,500, but a 5% annual inflation rate in these costs is forecasted. The daughter recently received $7,500 from her grandfather's estate; this money, which is invested in a bank account paying 8% interest, compound-ed annually, will be used to help meet the costs of the daughter's education. The rest of the coasts will be met by money the father will deposit in the savings account. He will make 6 equal deposits to the account, one deposit in each year from now until his daughter starts college. These deposits will begin today and will also earn 8% interest, compounded annually.

Case1-cont'd…

a. What will be the present value of the cost of 4 years of education at the time the daughter becomes 18?

[Hint: Calculate the future value of the cost (at 5%) for each year of her education, then discount 3 of these costs back 9 (at 8%) to the year in which she turns 18, then sum the 4 costs]

Case1-cont'd…

b. What will be the value of the $7,500 which the daughter received from her grandfather's estate when she starts college at age 18?

[hint: Compound for 5 years at an 8% annual rate]

Case1-cont'd…

c. If the father is planning to make the first of 6 deposits today, how large must each deposit be for him to be able to put his daughter through college?

[Hint; An annuity due assumes interest is earned on all deposits; however, the 6th deposit earns no interest---therefore, the deposits are an ordinary annuity]

Case2

You are thinking about buying a car, and a local bank is willing to lend you $20,000 to buy the car. Under the terms of the loan, it will be fully amortized over 5 years (60 months), and the nominal rate of interest will be 12 %, with interest paid monthly. What would be the monthly payment on the loan? What would be the effective rate of interest on the loan?

Case3

It is now January 1, 2009. You plan to make 5 deposits of $100 each, one every 6 months, with the first payment being made today. If the bank pays a nominal interest rate of 12%, but uses semiannual compounding, how much will be in your account after 10 years?

Case4

Ten years from today you must make a payment of $1,432.02. To prepare for this payment, you will make 5 equal deposits, beginning today and for the next 4 quarters, in a bank that pays a nominal interest rate of 12 %, quarterly compounding. How large must each of the 5 payments be?