ESCUELA SUPERIOR DE CÓMPUTO
PRIMER EXAMEN PARCIAL DE ECUACIONES DIFERENCIALES
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INSTRUCCIONES: Resolver todos los problemas. No se permite el uso de formularios. Apagar teléfonos celulares.

1. Resolver

$$\frac{dy}{dx} = Sen(x - y)$$

(3.0 puntos)

2. La pendiente de una familia de curvas en cualquier punto (x, y) está dada por

$$\frac{dy}{dx} = \frac{x + y}{x}$$

Halle la ecuación del miembro que pasa por el punto $(3,0)$. *(Use el método de la ecuación Homogénea).* **(3.0 puntos)**

3. La corriente I, en amperes, de un circuito eléctrico satisface la ecuación diferencial

$$\frac{dI}{dt} + 3I = e^{-2t}$$

Donde t es el tiempo. Si $I(0) = 5$, encuentre $I = I(t)$.
(*Use el método de exactitud*) **(3.0 puntos)**

4. Resuelva el problema anterior usando otro método. **(1.0 punto)**

$$\frac{dy}{dx} = \operatorname{sen}(x-y)$$

$$v = x - y$$

$$v + y = x$$

$$y = x - v$$

$$\frac{dy}{dx} = 1 - \frac{dv}{dx}$$

$$1 - \frac{dv}{dx} = \operatorname{sen}(v)$$

$$1 = \operatorname{sen}(v) + \frac{dv}{dx}$$

$$1 - \operatorname{sen}(v) = \frac{dv}{dx}$$

$$\frac{dv}{dx} = 1 - \operatorname{sen}(v)$$

$$dv = (1 - \operatorname{sen} v)\, dx$$

$$\frac{dv}{1 - \operatorname{sen} V} = dx$$

$$\int \frac{dv}{1 - \operatorname{sen} V} - \int dx = 0$$

$$\tan(v) + \sec(v) - x = C$$

$$\tan(x-y) + \sec(x-y) - x = C$$

3.0

$-2500 = 0$

$m = \pm\sqrt{2500}$

$m_{1,2} =$

$y_c = s$

$\sin^2(v) + \cos^2(v) = 1$

$\cos^2(v) = 1 - \sin^2(v)$

$$\int \frac{dv}{1-\text{sen}(v)} = \int \frac{dv}{1-\text{sen}(v)} \cdot \frac{1+\text{sen}(v)}{1+\text{sen}(v)}$$

$$= \int \frac{1+\text{sen}(v)}{1-\text{sen}^2(v)}\,dv = \int \frac{1+\text{sen}(v)}{\cos^2(v)}$$

$$= \int \frac{1}{\cos^2(v)}\,dv + \int \frac{\text{sen}(v)}{\cos^2(v)}\,dv$$

$$= \int \sec^2(v)\,dv + \int \text{sen}(v)\cdot\cos^{-2}(v)\,dv$$

$$= \tan(v) + \int \text{sen}(v)\cdot[\cos(v)]^{-2}\,dv$$

$u = \cos(v)$

$du = -\text{sen}(v)\,dv$

$$= \tan(v) + (-1)\int (-1)\,\text{sen}(v)\cdot[\cos(v)]^{-2}\,dv$$

$$= \tan(v) - \frac{[\cos(v)]^{-2+1}}{-2+1} + C = \tan(v) - \frac{[\cos(v)]^{-1}}{-1} + C = \tan(v) + [\cos(v)]^{-1}$$

$$= \tan(v) + \sec(v)$$

$\frac{dy}{dx} =$

$ec(v)\ \frac{dy}{dx} = \frac{x+y}{x}$

$\frac{dy}{dx} = \frac{x}{x} + \frac{y}{x}$

$\frac{dy}{dx} = 1 + \frac{y}{x}$

$\frac{y}{x} - \ln|x| = c$

$p(3,0)$

$x = 3$
$y = 0$

$\cancel{\frac{0}{3}} - \ln|3| = c$

$-\ln|3| = c$

$\ln|3^{-1}| = c$

$\ln|\frac{1}{3}| = c$

$\frac{y}{x} - \ln|x| = \ln|\frac{1}{3}|$

$\frac{rx+ry}{rx} = \frac{\cancel{r}(x+y)}{\cancel{r}x}$

∴ Es homogenea de gradocero

$\frac{dy}{dx} = \frac{x+y}{x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$

$\frac{dy}{dx} = \frac{1 + y/x}{x \cdot \frac{1}{x}}$

$\frac{dy}{dx} = \frac{1 + y/x}{1}$

$\frac{dy}{dx} = 1 + \frac{y}{x}$

$v = \frac{y}{x}$

$vx = y$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = 1 + v$

$x\frac{dv}{dx} = 1 + \cancel{v} - \cancel{v}$

$\frac{dv}{dx} = \frac{1}{x}$

$dv = \frac{1}{x}\,dx$

$dv - \frac{1}{x}\,dx = 0$

$\int dv - \int \frac{1}{x}\,dx = 0$

$v - \ln|x| = c$

$\frac{y}{x} - \ln|x| = C$

$\frac{dI}{dt} + 3I = e^{-2t}$ $\quad I(0) = 5$

$\frac{dI}{dt} + 3I = e^{-2t}$ $\quad y(0) = 5$

$(0,5)$

¿(t, I) ó (t, y)?

$p(x) = 3$

$\mu = e^{\int 3\,dx} = e^{3x}$

$e^{3x}\frac{dy}{dx} + 3y \cdot e^{3x} = e^{-2x} \cdot e^{3x}$

$e^{3x}\frac{dy}{dx} + 3y \cdot e^{3x} = e^{3x-2x}$

$e^{3x}\frac{dy}{dx} + 3y\, e^{3x} = e^{x}$

0.5

$\frac{d}{dx}\left[e^{3x}\, y\right] = e^{x}$

$e^{3x}y = e^{x} + C$

$e^{3\cdot 0}\cdot 5 = e^{0} + C$

$e^{0}\cdot 5 = e^{0} + C$

$5 = 1 + C$

$5 - 1 = C$

$C = 4$

$e^{3x}y = e^{x} + 4$

$y = \frac{e^{x} + 4}{e^{3x}}$

$y = \frac{e^{x}}{e^{3x}} + \frac{4}{e^{3x}}$

$y = e^{x}\cdot e^{-3x} + \frac{4}{e^{3x}}$

$y = e^{-2x} + \frac{4}{e^{3x}}$

Remplazando:

$I = e^{-2t} + \frac{4}{e^{3t}}$

$+3I = e^{-2t}$

¿(t, I) ó (x, y)?

$$\frac{dy}{dx} + 3y = e^{-2x}$$

$$\frac{dy}{dx} = e^{-2x} - 3y$$

$$dy = (e^{-2x} - 3y)\,dx$$

$$(e^{-2x} - 3y)\,dx - dy = 0$$

$$(e^{-2x} - 3y)\,dx + (-1)\,dy = 0$$

$$M(x,y) = e^{-2x} - 3y$$

$$N(x,y) = -1$$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}\left[e^{-2x} - 3y\right] = \frac{\partial}{\partial y}\left[e^{-2x}\right] - \frac{\partial}{\partial y}\left[3y\right]$$

$$\frac{\partial M}{\partial y} = 0 - 3 = -3$$

$$-3 \neq 0$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}\left[-1\right] = 0$$

$$\frac{1}{(-1)}\left[-3-0\right] = \frac{-3}{-1} = 3 = g(x)$$

$$\frac{1}{N}\left[\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right] = g(x)$$

$$\mu = e^{\int 3\,dx} = e^{3x}$$

$$(e^{-2x} - 3y)\,dx + (-1)\,dy = 0$$

$$e^{3x}(e^{-2x} - 3y)\,dx + (e^{3x})(-1)\,dy = 0$$

$$\left[e^{3x}\cdot e^{-2x} - 3y\,e^{3x}\right]dx + \left[-e^{3x}\right]dy = 0$$

$$\left[e^{x} - 3y\,e^{3x}\right]dx + \left[-e^{3x}\right]dy = 0$$

$$M_1(x,y) = e^{x} - 3y\,e^{3x}$$

$$N_1(x,y) = -e^{3x}$$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}\left[e^{x} - 3y\,e^{3x}\right] = \cancelto{0}{\frac{\partial}{\partial y}\left[e^{x}\right]} - \frac{\partial}{\partial y}\left[3y\,e^{3x}\right]$$

$$\frac{\partial M}{\partial y} = -\frac{\partial}{\partial y}\left[3y\,e^{3x}\right] = -e^{3x}\frac{\partial}{\partial y}3y = -3e^{3x}y$$

$$\frac{\partial M_1}{\partial y} = -3e^{3x}y$$

$$N_1(x,y) = -e^{3x}$$

$$\frac{\partial N_1}{\partial x} = \frac{\partial}{\partial x}\left[-e^{+3x}\right] = -e^{3x}\cdot 3 = -3e^{3x}$$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$$

$$u(x,y) = \int e^{x} - 3y e^{3x}\, \partial x$$

$$u(x,y) = \int e^{x}\, \partial x - \int 3y e^{3x}\, \partial x$$

$$u(x,y) = e^{x} - 3y \int e^{3x}\, \partial x$$

$$u(x,y) = e^{x} - 3y \cdot \frac{1}{3} \int e^{3x} \cdot 3\, \partial x$$

$$(x,y) = e^{x} - \frac{3y}{3} e^{3x} + f(y)$$

$$(x,y) = e^{x} - y e^{3x} + f(y)$$

$e^x - y e^{3x} = c$

$y(0) = 5$

$(0, 5)$

$e^0 - 5 e^{3 \cdot 0} = c$

$1 - 5 = c$

$c = -4$

$e^x - y e^{3x} = -4$

$e^x = y e^{3x} - 4$

$e^x + 4 = y e^{3x}$

$y = \frac{e^x + 4}{e^{3x}}$

1.5

$y = \frac{e^x}{e^{3x}} + \frac{4}{e^{3x}}$

$y = e^x \cdot e^{-3x} + \frac{4}{e^{3x}}$

$y = e^{-2x} + \frac{4}{e^{3x}}$

• Remplazando

$I = e^{-2t} + \frac{4}{e^{3t}}$