Exam information:	Midterm exam
Version:	I
Date:	Friday Nov. 4, 2022
Time:	2:30 pm to 3:30 pm
Duration:	1 hours

Part 1: Multi choice questions:

1. Quicksort is guaranteed to run in time $O(n\ log\ n)$ so long as the pivot is (5 points)
   a. randomly selected.
   b. set to the median of the first, middle, and last array element.
   c. set to the median of the array.
   d. none of the above
   Ans: c

2. In a double hashing solution to cope with a collision, what is not correct? (5 points)
   a. There are two hash functions; first hash function: $h(k) = k\ mod\ N$ and second hash function: $d(k) = q - (k\ mod\ q)$
   b. Second hash function cannot have a zero value
   c. There is no condition for table size to allow probing to all cells
   d. For these hash functions: $q < N$ and q and N are prime
   Ans: c

3. The worst-case running time $T(n)$ for inserting n elements into an initially empty binary search tree (BST) is: (5 points)
   a. $T(n) = O(n^2)$
   b. $T(n) = O(\sqrt{n}\ log\ n)$
   c. $T(n) = O(n)$
   d. $T(n) = O(n\ log\ n)$
   Ans: a

4. When following Kruskal's algorithm, the greedy choice is to: (5 points)
   a. remove the edge of greatest cost from the graph so long as its removal does not disconnect the graph.
   b. add the edge of least cost to the forest so long as its addition does not create a cycle.
   c. add the vertex having least connection cost to the current tree.
   d. remove the vertex having greatest connection cost from the tree.
   Ans: b

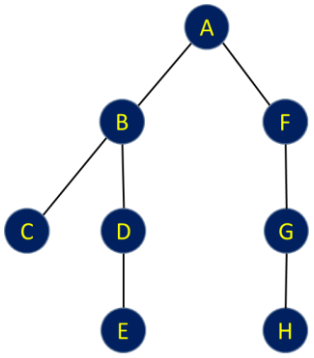
5. Which of the following expressions surely supports the statement $f(n) = \Omega(g(n))$? (5 points)
   a. $g(n) \leq 4f(n)$ for all $n \geq 1$
   b. $g(n) \leq 4f(n)$ for all $n \geq 136$
   c. $f(n) \leq 4g(n)$ for all $n \geq 1$

d. $f(n) \geq 4g(n)$ for all $n \geq 136$
Ans: d

6. Which of the following algorithms does not require a heap for its efficient implementation? (5 points)
   a. Human's algorithm
   b. Dijkstra's algorithm
   c. Kruskal's algorithm
   d. Prim's algorithm
   Ans: c

7. Which function has the least growth rate? (5 points)
   a. $2n^3 + 80$
   b. $n \log n$
   c. $98 \log n$
   d. $n + \log n$
   Ans: c

8. What is the output sequence of a Breath-first search (BFS) traversing over the following graph? (5 points)



   a. A B F C D G E H
   b. A B C D E F G H
   c. A B C F D G E H
   d. A F G H B C D E
   Ans: a

Part 2: Essay questions:

9. Consider an open addressing hash table with 18 slots. Find proper hash function to insert the keys {33, 15, 69, 74, 68, 109, 50, 96} (in the order given) into the table.
   a. Use linear probing for collision resolution. (5 points)
   b. Use quadratic probing for collision resolution. (5 points)

   Ans: proper hash function to save on table is $f(k) = k \ mod \ 18$. Hash values for input values are {15, 15, 15, 2, 14, 1, 14, 6}. There are conflicts on hash table on 15 and 14.

   a: By linier probing at collision place: $f(i) = [(i + j) \ mod \ 18], j = 1,2,3,..$ Therefore, we have following values after addressing the collisions: {15, 16, 17, 2, 14, 1, 18, 6}

   b: By quadratic probing at collision place: $f(i) = [(i + j^2) \ mod \ 18], j = 1,2,3,..$ Therefore, we have following values after addressing the collisions: {15, 16, 1, 2, 14, 5, 18, 6}

   (Students can show hash table and its cells with values)

10. Consider the following algorithm:

```
Algorithm Alg (n)
if (n ≤ 1)
    return false
if (n == 2)
    return true;
if (n%2 == 0)
    return false;
for ( int i = 3; i² ≤ n; i += 2)
    if (n%i == 0)
        return false ;
return true;
```

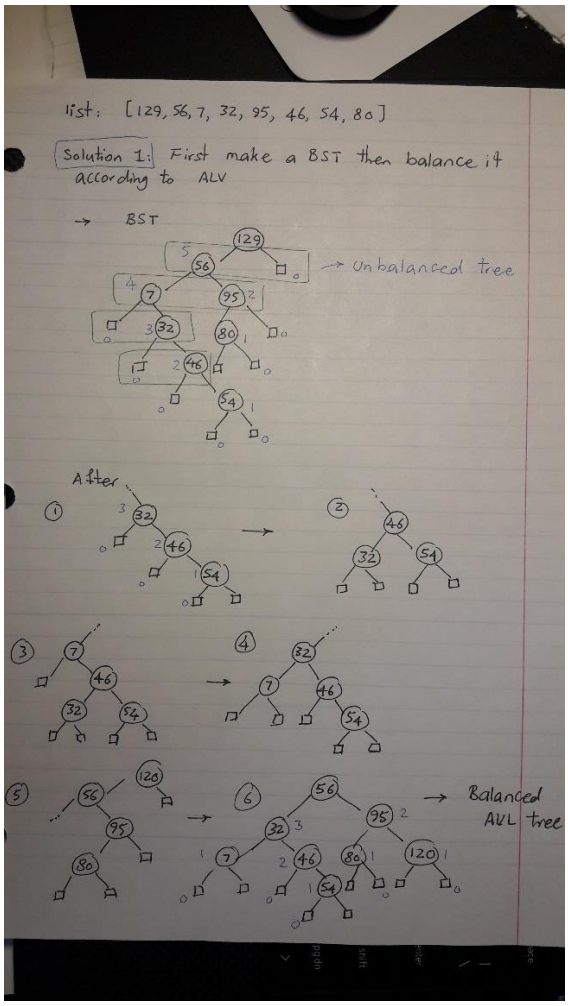
   a. What does Alg compute? (5 points)
   b. What is O() complexity? (5 points)

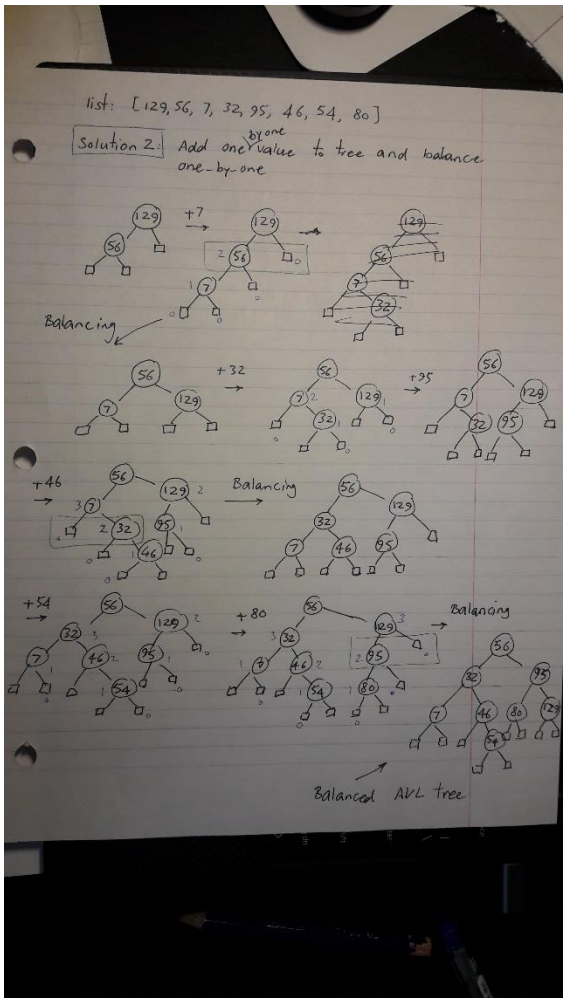
   Ans a: The function returns whether n is a prime number.

   Ans b: The complexity/cost is determined by the loop, which makes $O(\sqrt{n})$ iterations, each of which has constant cost. Thus, the complexity is O($\sqrt{n}$)

11. a: Make an AVL tree of a list of [129,56,7,32,95,46,54,80] (10 points)
    b: What is the Insertion complexity of this AVL tree? (5 points)

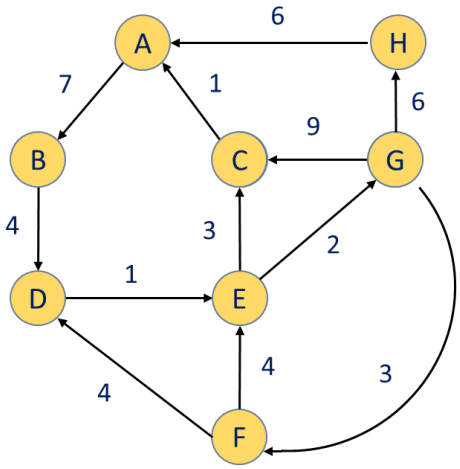
Ans a:





Ans b: Insertion complexity of this AVL tree is $O(\log n)$.

12. Use Dijkstra algorithm and find the shortest path to all nodes from the starting point node E. (10 points)



Ans:

Vertex	Shortest distance from E	Previous vertex
E	0	-
A	4	C
B	11	A
C	3	E
D	9	F
F	5	G
G	2	E
H	8	G

13. For a given integer array $a[]$ with n integers,

    a. Write an algorithm to find if there are three indices i, j, and k (not necessarily distinct) such that $a[i] + a[j] + a[k] == 0$. (10 points)

    b. What is time complexity of your algorithm? (5 points)

    Ans: there is no unique answer for this question. I want to challenge students and find their understanding and novelty to solve the problem. To evaluate an algorithm, three items are considered, namely Correctness, Performance, and Ending in a finite time. Consider these three properties and grade the suggested algorithms.

Good luck,
Mahdi Firoozjaei
November 2022