Operation Research - Project E

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1. Notations.

Let $m = 0, \cdots, 6$ denote the months ($m = 0$ for February, $m = 1$ for March, $m = 2$ for April, etc.). Introduce the notations:

x_m	the number of arrivals at the beginning of month m
y_m	the number of departures at the end of month m
w_m	the number of workers employed in month m
N_m	the number of workers required in month m

without loss of generosity, we may set $x_0 = 0$. N_m is given for $m = 1, \cdots, 6$. It is also known that $w_0 = 3, y_0 = 0$. It is clear that:

$$w_m = w_{m-1} - y_{m-1} + x_m.$$

2. Considering the restrictions.

(1) Overtime must be limited to 25% of the total hours worked:

$$\frac{1}{w_m} \leq \frac{1.25}{N_m}, \quad m = 1, \cdots, 6;$$

i.e.,

$$N_m \leq 1.25\ w_m, \quad m = 1, \cdots, 6.$$

(2) Every month, at most three workers can arrive at the construction site:

$$0 \leq x_m \leq 3, \quad m = 0, \cdots, 6.$$

(3) The departure of workers to other construction sites is limited to one third of the total staff employed in the month:

$$y_m \leq \frac{1}{3} w_m, \quad m = 1, \cdots, 5.$$

(4) 3 workers remain on site at the end of August：

$$w_6 - y_6 = 3.$$

3. Objective function.

The total cost (the objective function, denoted by f) comes from three aspects:

1) Arrivals of workers： $f_1 = 100(x_0 + \cdots + x_6)$;

2) Departures of workers： $f_2 = 160(y_0 + \cdots + x_6)$;

3) Over/understaffing： $f_3 = 200(|w_0 - N_0| + \cdots + |w_6 - N_6|)$;

then we need to find $x_m,\ y_m$ that minimize $f = f_1 + f_2 + f_3$.

Noticing that absolute value occurred in the objective function, thus we need to replace it, introducing new variable z_m and restriction:

$$-z_m \leq w_m - N_m \leq z_m,$$

we get the new objective function $\bar{f} = f_1 + f_2 + \bar{f}_3$, where $\bar{f}_3 = 200(z_0 + \cdots + z_6)$.

4. Result.

Using AMPL to solve the problem, the result is given as follow:

```
CPLEX 22.1.1.0: iisfind 2
CPLEX 22.1.1.0: optimal integer solution; objective 1780
6 MIP simplex iterations
0 branch-and-bound nodes
tot_cost = 1780

:       w    x    y    :=
Feb     3    0    0
Mar     4    1    0
Apr     6    2    0
May     6    0    0
June    6    0    0
July    6    0    2
Aug     4    0    1
;
```

meaning that: to minimize the total cost, the arrivals from March to August is 1, 2, 0, 0, 0, 0 respectively, and the departures is 0, 0, 0, 0, 2, 1 respectively, and the minimum cost is 1780.