A Vegetation Model for the Amazonian Rainforest Tapajos

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Abstract

Background: By looking at Articles by Malhi, Galbraith, Delbart, et al., we are able to take a look at the production of trees in the Amazonian forests, and how to create models similar to the ones provided of our own using the values they have provided.

Methods: Using Wolfram's Mathematica, we were able to create a code that allowed us to create our models, as well as input the information necessary to produce such models.

Results: We find that the models project upwards growth in regards to the production of GPP, NPP, and Autotrophic respiration of the site we have decided to put our focus on (Tapajos in Brazil.)

Conclusions: Assuming these ecosystems are left untouched, we can see that there is a projected upwards growth in regards to the production of these systems, and we see that it may not be immediate, but over the course of the next 200 years, the production is going to exist.

1 Introduction

In this report, we focus on the Gross Primary Productivity (GPP) , Net Primary Productivity (NPP) production, Carbon Use Efficiency (CUE), and the carbon cycle in general of the Tapajos site, located near the river in Brazil. Before we approach this topic, we must define some terms. To start, we see that Gross Primary Productivity, or GPP, is defined as "The rate of fixation of CO2 by photosynthesis in the forest canopy" and "the primary measure of carbon supply and metabolic activity in the canopy." (Malhi 2011) We also have Net Primary Productivity defined as the rate of production of biomass by an ecosystem for things such as wood structures or organic compound emissions, amongst other things. The Carbon Use Efficiency is something we cannot ignore as well, and we see that it is defined as the ratio of NPP to GPP, according to Malhi 2011. Using these three things, we are able to take a better look at what Malhi calls the Carbon cycle (which he gives a visual for below:)

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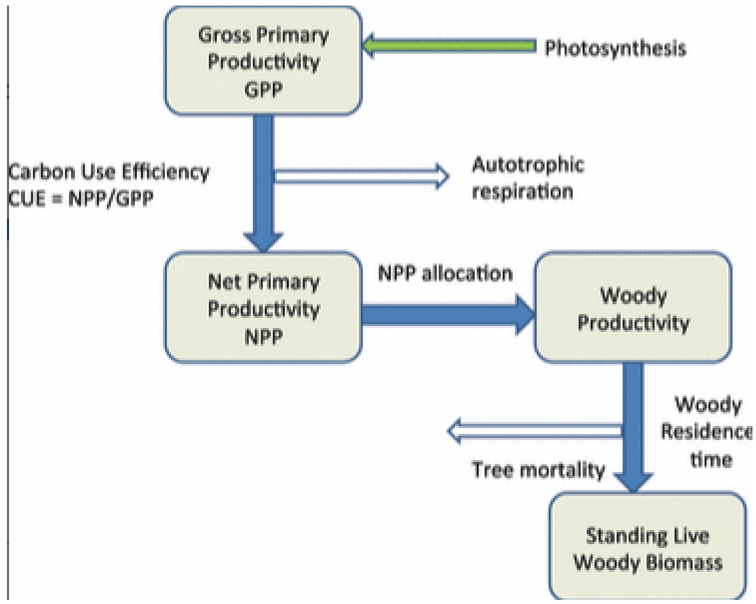

Figure 1: Figure 2 from Malhi 2011 which gives the framework for understanding the carbon cycle and the link between GPP and biomass.

We find that our values for CUE, NPP, GPP, etc. are defined later in the scientific computing section.

Now that we have defined some key terms, it is important to ask: Why is something like the Amazonian Rainforest important when talking about ecology? It is quite simple. Rainforests help stabilize the planet's climate (climate regulation), they act as a home to not just animals, but humans as well, they produce 40% of our Net Primary Productivity (which we will go over later), and act as absorbers of carbon dioxide. In order to take a proper look at this, we use Yadvinder Malhi's 2011 article *The Productivity, Metabolism, and Carbon Cycle of Tropical Forest Vegetation*, In which he discussed things ranging from how ecological studies used to be conducted and how they have evolved, a study on GPP as well as NPP levels reported in the Americas and Asia, as well as their respective CUE (Carbon Use Efficiency) levels that have been estimated in their respective sites, the connection between photosynthesis and the biomass of a forest, and the allocation of NPP in the respective areas on the forest. Galbraith, Malhi, Affum-Baffoe, Castanho, Doughty, Fisher, Lewis, Peh, Phillips, Quesada, Sonke, and Lloyd's article, *Residence times of woody bioimass in tropical forests* is another one that we turn to, where they discuss residence time (denoted τ ,) the large variance of residence time between many tropical forests, the connection between the residence time and impending global environmental change, as well as the ability for slight changes in residence time to affect vegetation models. It is in this Article by Galbraith, Malhi, et al. that we find one of the most important formulas for our report, that is one of the formulas for calculating residence time. Delbart, Malhi, Viovy, Chave, Ciais, and Le Toan's article, *Mortality as a key driver of the spatial distribution of aboveground biomass in Amazonian forest: Results from a dynamic vegetation model* is the last article I used when looking at what was necessary to complete this project. In this article, Delbart, Malhi, et al. discuss the usage of different vegetation models, namely the ORCHIDEE model. Additionally, we take a loot at NPP and how to calculate a new mortality model using the respective NPP of specific sub-compartments. Additionally, we take notice that there is a strong relationship between forests with high productivity (productivity being the growth rate of trees in any given forest) and having higher mortality rates. Using these three articles, we are able to create differential equations that will assist us in projecting the growth, or decrease of GPP, carbon, and NPP of the aforementioned Tapajos site. By taking a look at the carbon cycle, the given equations in the articles, as well as the accompanied figures in these articles, we are able to complete everything we set out to do.

2 Mathematical Model

As mentioned before, using the three articles we have chosen to use, we are able to create the following equations to assist us in graphing/projecting the production in our site. Please note, all functions are with respect to time (t), and the values for these equations will vary on a site-by-site basis.

$$ps(t) = (.85t + 31.4)e^{-0.01t} \tag{1}$$

$$twnpp(t) = cue * photosynthesis(t) \tag{2}$$

$$arnpp(t) = (1 - cue)Photosynthesis(t) \tag{3}$$

$$\tau_w(t) = \frac{W_p}{1.10231 * {NPP_w}^{1.32}} \tag{4}$$

$$\tau_c(t) = \frac{W_c}{1.10231 * {NPP_c}^{1.32}} \tag{5}$$

$$\tau_r(t) = \frac{W_r}{1.10231 * {NPP_r}^{1.32}} \tag{6}$$

$$\frac{d}{dt}Wood = \alpha_w NPP - \frac{W_p}{\tau_w} \tag{7}$$

$$\frac{d}{dt}Canopy = \alpha_c NPP - \frac{W_p}{\tau_w} \tag{8}$$

$$\frac{d}{dt}Roots = \alpha_r NPP - \frac{W_p}{\tau_w} \tag{9}$$

$$\frac{d}{dt}GPP = ps(t) - \alpha_{NPP}NPP - (1 - \alpha_{NPP}R_a) \tag{10}$$

It is important to note what some of these parameters and equations mean. For example, we see that equation (1): ps(t) is our function for photosynthesis, in regards to time, denoted t. We can ask ourselveswhy we went with the choice of our photosynthesis function as it is. We notice that it is related to the total GPP of our respective site (Specifically 31.4 for our respective site, which will be introduced later.) As is the case, we see that this function will be directly related to the site we are putting our focus towards (Tapajos.) Similarly, we see that equation (2) twnpp(t) is defined as the total wood NPP at a time t, which cannot be confused with our function (3) of arnpp(t), which is defined as the total autotrophic respiration NPP at time t. When we look at equations (4) through (6), we see that they are defined as the time of residence functions for our wood, canopy, and root sub-compartments. Similarly, equations (7) through (9) give us NPP values at a time t, where equation (7) is the equation for our wood sub-compartment, equation (8) is the equation for our canopy sub-compartment, and equation (9) denotes the equation for our root sub-compartment. Lastly, equation (10) gives us our GPP production function, in which we will plot the GPP levels over a time t. When we look at how residence time is defined to us, we see that we are told that residence time can be simply thought of as the mortality rate of a tree, i.e. how long the tree survives (or resides) in the ecosystem. In order to start the process of modeling something like residence time, we can take a look at this figure that was provided in Malhi 2011:

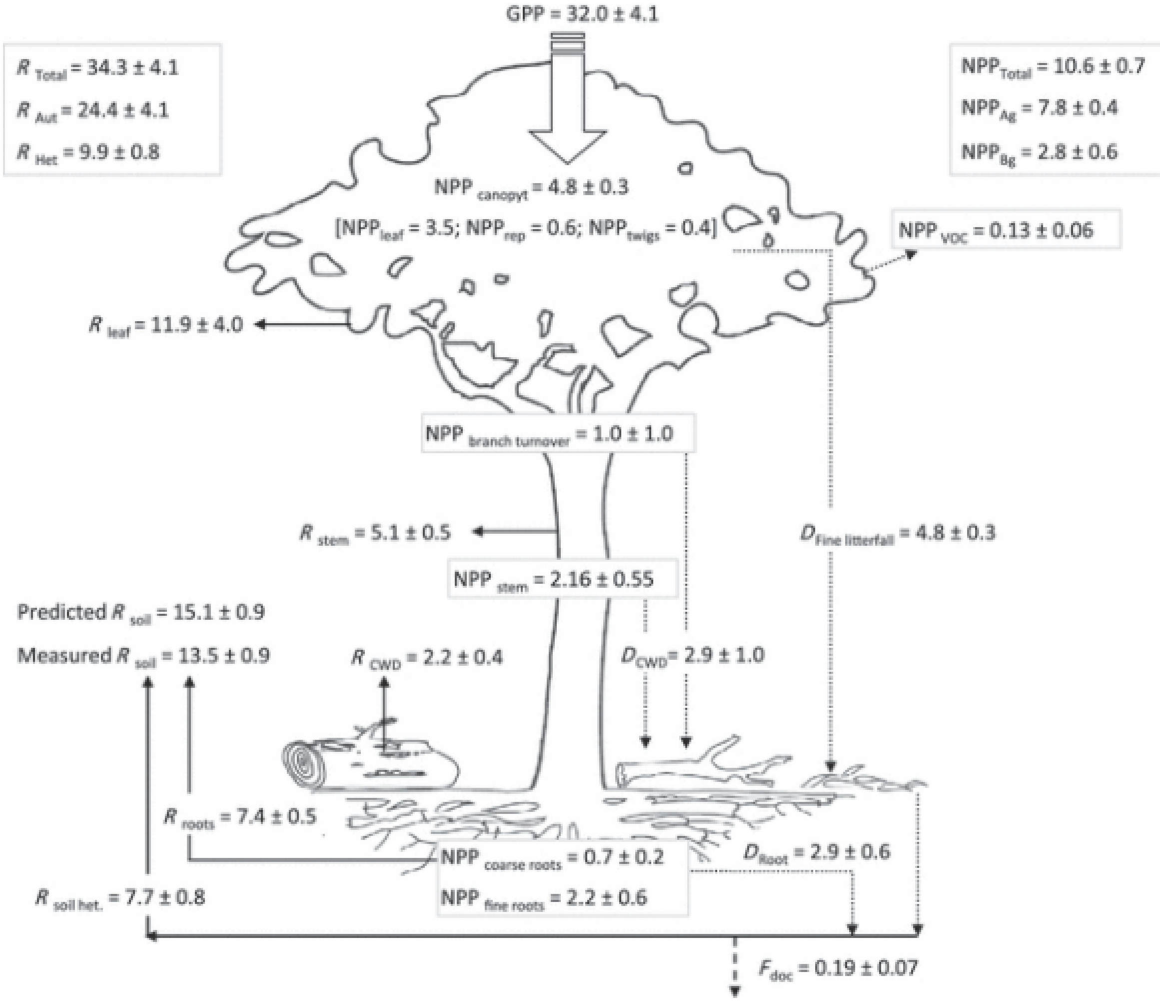

Figure 2: Figure 1 from Malhi 2011, where we are given NPP and Autotrophic/Heterotrophic Respiration values, as well as their respective sub-compartments. We can see that there is autotrophic respiration for the leaves, stems, soil, roots, woody mortality, heterotrophic respiration for the soil and NPP values for the canopy, wood, and roots sub-compartments.

Using these functions and terms that we have defined, we should be able to follow along with the carbon cycle. We can see that the cycle starts with our GPP, which is defined as "The rate of fixation of CO2 by photosynthesis in the forest canopy, and as such is the primary measure of carbon supply and metabolic activity in the canopy," in Malhi 2011. As mentioned earlier, we defined CUE as the ratio of NPP to GPP, and in this scenario, we can also state that CUE is our autotrophic respiration. Of course, this varies from site to site, and in the case of the site we are focusing on (Tapajos, we have a CUE of 0.46.) From this, we can develop our autotrophic respiration values, as well as NPP values. From this point, we can then look at the respective NPP and autotrophic respiration allocation, as explained or displayed in Malhi 2011.

3 Results

For the most part, we are able to see that the production of GPP and NPP are expected to trend upwards over the course of 200 years, which is in line with what many articles say about what these South American sites heading into the future. When using the models mentioned prior, we get plots that look like the following:

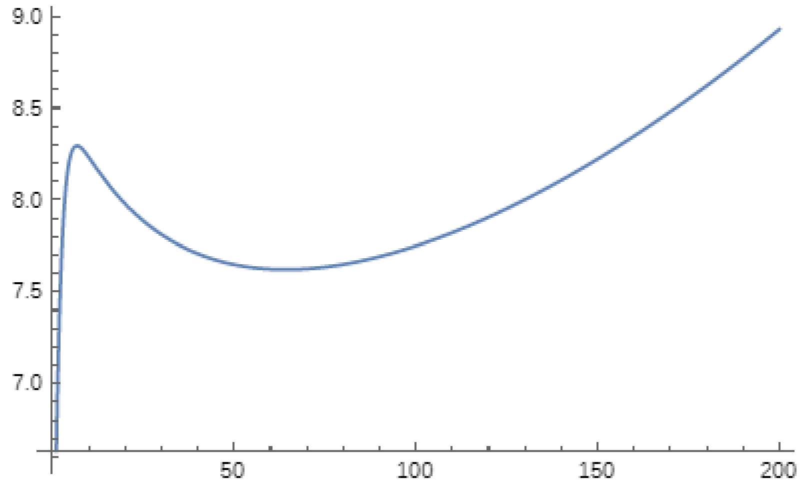

Figure 3: The graph we use to decide that NPP production with regards to the canopy sub-compartment of the tree is trending upwards.

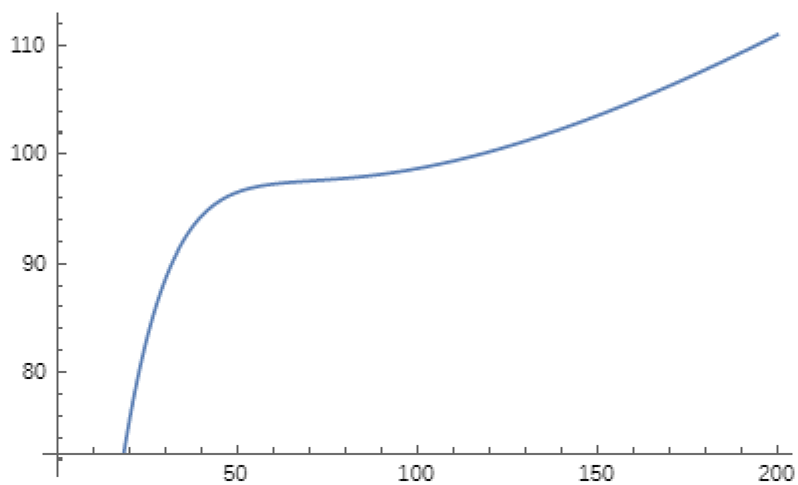

Figure 4: Similar to the canopy sub-compartment, we can see that the wood sub-compartment of the tree has a projected upwards growth in regards to its production.

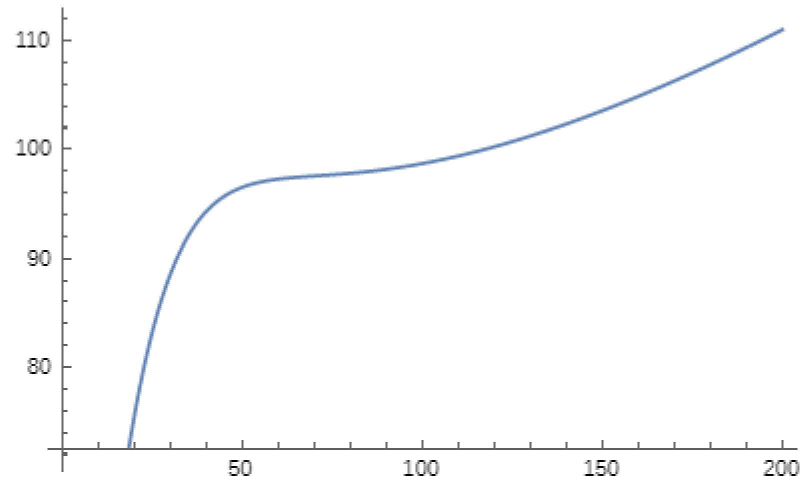

Figure 5: Expected production of the root sub-compartment of our tree

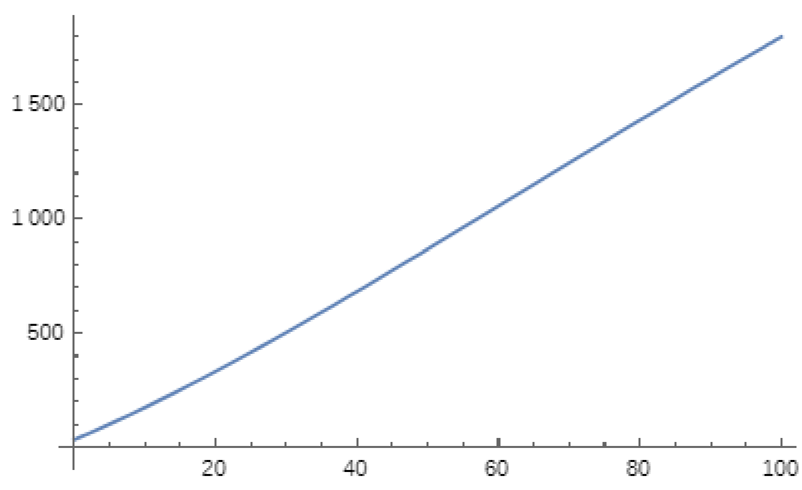

Figure 6: A plot of our upwards projection of GPP

4 Scientific Computing

Using Wolfram's Mathematica, we were able to create a code that allowed for us to be able to compute what we were looking for. By establishing equations, values, initial conditions, etc., we were able to produce the graphs that were made in the previous section. With code that resembles the following, we are able to enable Mathematica to give us graphs similar to the ones we saw in the Results section.

In order to use these equations established in the *Mathematical Model* section to their fullest potential, we must turn towards Malhi's 2011 article. In this article, he provides us with many of the values needed in order to put these equations to use. Initially, to find the total amount of NPP in our given site, we use this table:

Site	Lat.	Long.	Elevation	GPP_{td}	GPP_{bu}	NPP	CUE	Sources
Mae Klong	14.58	98.85	160	32.3				Hirata *et al.* (2008)
Sakaerat	14.48	101.92	535	38.1				Gamo *et al.* (2005)
*Pasoh**	2.97	102.30	75–150	31.2		12.8	0.41	Takanashi *et al.* (2005b), Kosugi *et al.* (2008), Kira (1978)
Palangkaraya	−2.35	114.03	30	33.0				Hirano *et al.* (2007)
Xishuangbanna	21.93	101.27		25.9	26.0	8.8	0.34	Tan *et al.* (2010)
Caxiuanã			15	31.2	31.4	10.0	0.32	Malhi *et al.* 2009b
Tapajós			200	31.4	29.3	14.4	0.46	Malhi *et al.* 2009b
Manaus			90	30.4	29.9	10.1	0.33	Malhi *et al.* 2009b
Caxiuanã drought control	−1.72	−51.46	15	31.2	33.0	9.5	0.30	Metcalfe *et al.* 2010
Caxiuanã drought	−1.72	−51.46	15	27.0		7.4	0.27	Metcalfe *et al.* 2010
Tono			1000		24.9	7.3	0.29	Malhi *et al.*, unpublished
San Pedro			1500		26.9	10.8	0.40	Malhi *et al.*,

Figure 7: Table 1 from Malhi's 2011 article *The Productivity, Metabolism, and Carbon Cycle of Tropical Forest Vegetation.*

From here, when we focus on the Tapajos site, we see that we have initial NPP values of 14.4, CUE values of 0.46, and GPP values of 31.4 (since we are focusing on the GPP acquired from the top-down method.) Similarly, when we look in Malhi 2011, we are presented with this figure 1, which was shown earlier:

Looking back at figure 1 from Malhi 2011, we can see that the Autotrophic respiration NPP is divided into different sections, where the leaves take up 0.4877 (48.77%), the stem takes .209 (20.9%) , and the roots take .303 (30.3%) of the total Autotrophic respiration NPP. Similarly, in the same article, Malhi 2011 states that "... allocation of NPP between its three main components is relatively invariant and fairly close to equal partitioning (mean 34 +- 6% , 39 +- 10 % for wood, 27 +- 11% for fine roots..." These values yield 5.61602 for the wood sub-compartment, 4.89601 for the canopy sub-compartment, and 3.88801 for the root sub-compartment when we multiply the total NPP by 34, 39, and 27 % for the wood, canopy, and root sub-compartments of our site (Tapajos) respectively. Using these, we can calculate the residence time, which is strongly related to tree mortality, but not the sole factor.

NPP of Total Wood Sub-compartments

The following are the proportional factors of the total wood NPP for the sub-compartments of wood: wood (including stem and branches), canopy, and root.

```
In[-]:= αnppw = .39;
In[-]:= αnppc = .34;
In[-]:= αnppr = .27;
```

A) Now, we define the NPP function for the wood sub-compartment of the total wood compartment.

```
In[-]:= wnpp[t_] := αnppw * twnpp[t];
In[-]:= wnpp[0]
Out[-]= 5.61602
```

This is the ic for wood NPP. We will assign a name to it for use in the DE's.

```
In[-]:= wnppic = 5.61602;
```

B) Next, we define the NPP function for the canopy sub-compartment of the total wood compartment.

```
In[-]:= cnpp[t_] := αnppc * twnpp[t];
In[-]:= cnpp[0]
Out[-]= 4.89601
In[-]:= cnppic = 4.89601;
```

C) Next, we define the NPP function for the root sub-compartment of the total wood compartment.

```
In[-]:= rnpp[t_] := αnppr * twnpp[t];
In[-]:= rnpp[0]
Out[-]= 3.88801
In[-]:= rnppic = 3.88801;
```

Figure 8: An example of the mathematica code used to define our NPP values

In order to find our residence time functions, we must look towards Delbart 2010, where we find this figure:

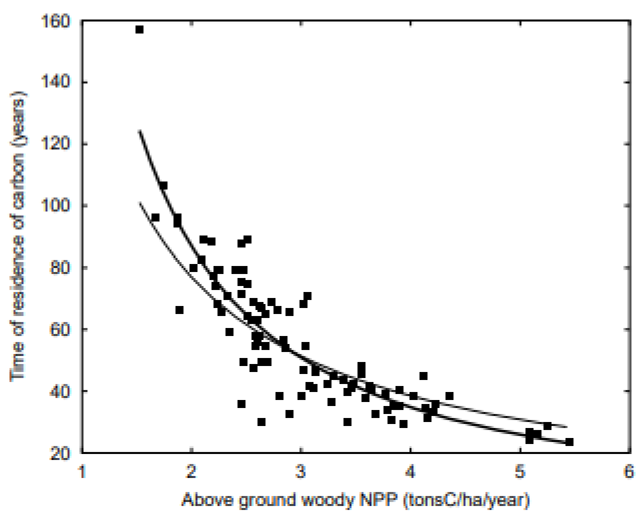

Figure 9: Figure 5 from Delbart 2010

In Delbart 2010 article, we see that this is the best overall fit for residence time, and we see that it is defined as

$$\frac{217}{NPP_{AGW}^{1.32}} \tag{11}$$

Where we see that our equations are multiplied by constants. In the case of above ground woody biomass, we have 217. Using this constant, we are able to find the constant multipliers for our canopy and root sub-compartments by having inverse functions and using them to compare to the Woody productivity as shown below:

```
In[•]:= wp = 217;

In[•]:= canopyinverse = 1 / 0.67

Out[•]= 1.49254

In[•]:= woodinverse = 1 / 0.05

Out[•]= 20.

In[•]:= rootinverse = 1 / 0.47

Out[•]= 2.12766

In[•]:= Solve[{canopyinverse / woodinverse == c / wp}, c]

Out[•]= {{c → 16.194}}

In[•]:= Solve[{rootinverse / woodinverse == r / wp}, r]

Out[•]= {{r → 23.0851}}

In[•]:= wc = 16.194;
        wr = 23.0851;
```

Figure 10: Image of how we found our constant multipliers for canopy and root functions

Using these values, we are now able to define our time of residence functions as such:

With Mathematica, we are able to graph these functions and show the productivity over the course of the next 120 years.

Time of Residence Functions

Time of residence for wood sub-compartment.

In[∘]:= $\tau w[t\_] := \frac{w_p}{(1.10231 \times wnpp[t])^{1.32}};$

Time of residence for canopy sub-compartment.

In[∘]:= $\tau c[t\_] := \frac{w_c}{(1.10231 * cnpp[t])^{1.32}};$

Time of residence for root sub-compartment.

In[∘]:= $\tau r[t\_] := \frac{w_r}{(1.1023 \times rnpp[t])^{1.32}};$

Figure 11: An example of the mathematica code used to solve our equations for time of residence functions

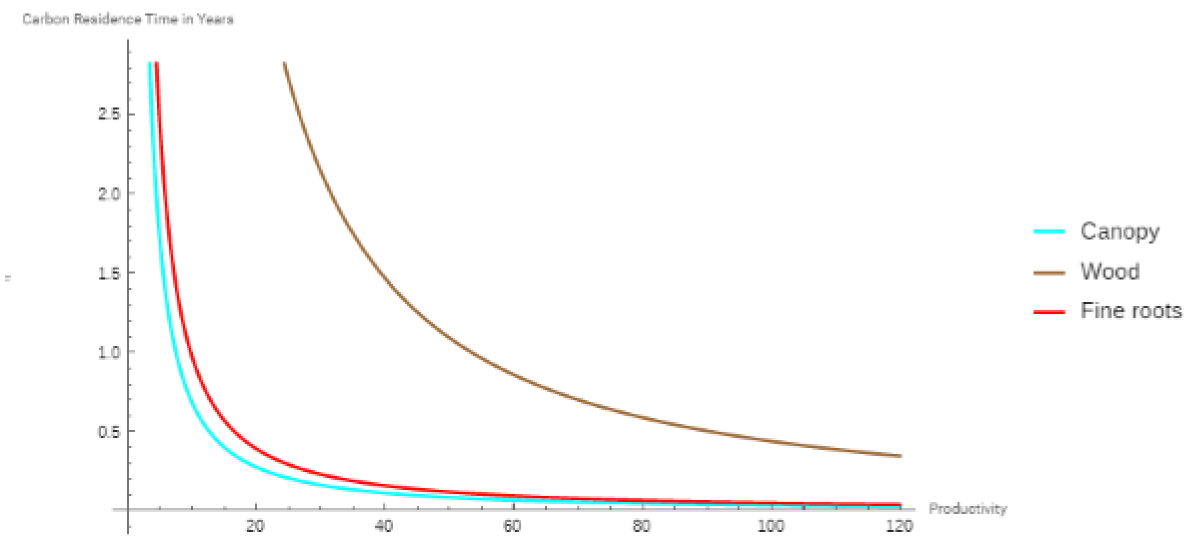

Figure 12: An image of the graphing of the time of residence functions for wood, root, and canopy sub-compartments.

Similarly, we can graph our times of residence using the formulas shown in figure 9 to receive a plot that looks like the following:

From this point on, we can then use Mathematica to define our differential equations and plot them, to receive outputs that look like what we got in the Results section of this report. An example of this code would look like this:

As we can see, we have three different differential equations for the wood, canopy, and root sub-compartments, where solncanopy is our function for the NPP production in our canopy, solnwood is the function for NPP production of the wood sub-compartment, and solnroots is the function of NPP production for the root sub-compartment. Based on

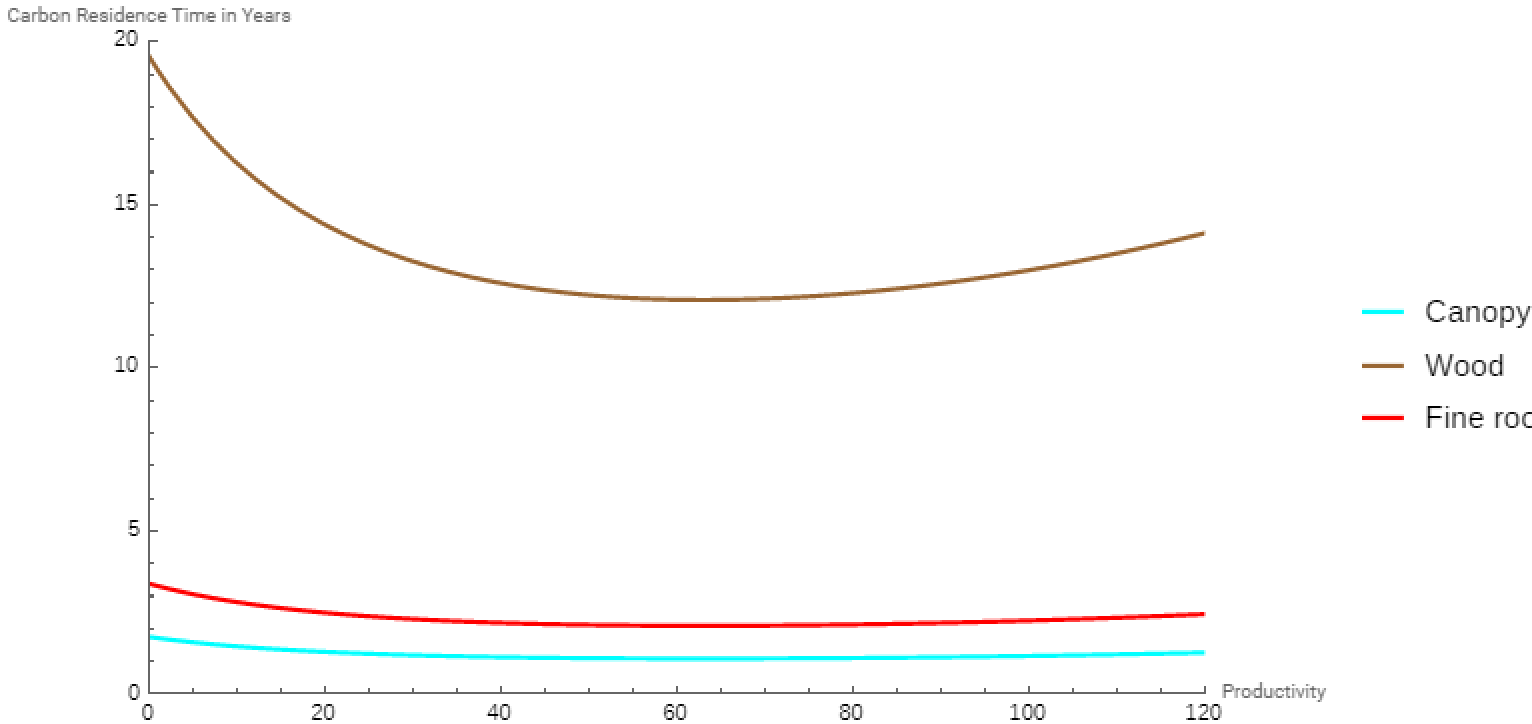

Figure 13: An image of the graphing of the carbon residence time for wood, root, and canopy sub-compartments

```
solncanopy[can_, NPP_, residencec_] := αNPPc * NPP[t] - can/residencec[t];

solnwood[wood_, NPP_, residencew_] := αNPPw * NPP[t] - wood/residencew[t];

solnroots[root_, NPP_, residencer_] := αNPPr * NPP[t] - root/residencer[t];
```

Figure 14: A section of the code used to display a vector field containing our differential equations for finding NPP values of our wood, canopy, and root sub-compartments.

this, we can say that our function *solncanopy* is used to plot the production of NPP for the canopy sub-compartment of the tree. We can also say that *solnwood* will be used to plot the NPP production for the wood sub-compartment of our tree. And similarly, we say that *solnroots* will be used to plot the root sub-compartments NPP production, relative to the tree we are using. If we look at the function itself, we can see that we have NPP as a function of time being multiplied by αNPP_c for the canopy function, αNPP_w for the wood function, and αNPP_r for the root function. This is because, as mentioned before, the allocation of the NPP is dependant on the part of the tree, and αNPP_c, αNPP_r, and αNPP_w are just the percentage of NPP that belongs to the canopy, roots, and wood, respectively. Similarly, we have the productivity of wood, canopy, and roots deined as "wood", "can", and "root", divided by the time of residence functions residencew, residencec, and residencer, which we have defined earlier as well. After doing all this, we are able to receive the results we received with the plots.

As a result, we see that the articles mentioned all played a part in assisting in making it possible to display such plots, as they provided initial conditions, parameters, and the information needed to create the equations.

5 Discussion

As we have mentioned before, by taking a look at the articles published by Malhi, Galbraith, Delbart, et al., we are able to get a better understanding of the production of trees in the Amazonian forests, which enables us to graph and produce the conclusions we came up with. Of course, we have stated that all the subcompartments of the trees are projected to grow upwards, looking into the future. This is assuming that these ecosystems are being left untouched. However, there are still questions to be asked, such as what if climate change affects the values as we know it, or perhaps it changes how the production in these ecosystems may look like in the near future. We have already noticed changes in the forests, at least compared to what earlier scientists and explorers have made out to mention. We must also take into consideration the movements in place to protect these forests, and how that may affect these numbers in the other direction.

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6 Acknowledgements

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