CSE 2315 - Lecture 1.2
September 1, 2020
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1 Propositional Logic

Propositional logic is deriving a logical conclusion by combining many propositions and using formal logic to determine the truth of arguments.

An **argument** is a sequence of statements in which the conjunction of the initial statements (called the premises/hypotheses) is said to imply the final statement (the conclusion). An argument can be represented symbolically as

$$(P_1 \wedge P_2 \wedge P_3 \wedge \cdots \wedge P_n) \to Q$$

A valid argument is when the truth of the hypotheses leads to the truth of the conclusion.

A **argument is valid** *if* whenever the hypotheses are all true, the conclusion must also be true. In other words, $(P_1 \wedge P_2 \wedge \cdots \wedge P_n) \to Q$ is a tautology.

How to Arrive at a Valid Argument

a **proof sequence** is a sequence of wffs in which each wff is either a hypothesis or the result of applying one of the formal system's derivation rules to earlier wffs in the sequence.

2 Rules for Propositional Logic

- Equivalence Rules
  - Allow individual wffs to be rewritten
  - Truth preserving rules
- Inference Rules
  - Allow new wff to be derived
  - Work only in one direction

Equivalence Rules

Expression	Equivalent To	Abbreviation for Rule
$R \vee S$ $R \wedge S$	$S \vee R$ $S \wedge R$	Commutative - comm
$(R \vee S) \vee Q$ $(R \wedge S) \wedge Q$	$R \vee (S \vee Q)$ $R \wedge (S \wedge Q)$	Associative - ass
$(R \vee S)'$ $(R \wedge S)'$	$R' \wedge S'$ $R' \vee S'$	De Morgan's Laws - De Morgan
$R \to S$	$R' \vee S$	Implication - imp
R	$(R')'$	Double negation - dn
$P \leftrightarrow Q$	$(P \to Q) \wedge (Q \to P)$	Equivalence - equ

Inference Rules

From	Can Derive	Abbreviation for Rule
R, $R \to S$	S	Modus ponens - mp
$R \to S$, S'	R'	Modus tollens - mt
R, S	$R \wedge S$	Conjunction - con
$R \wedge S$	R, S	Simplification - sim
R	$R \vee S$	Addition - add

From	Can Derive	Name / Abbreviation
$P \to Q$, $Q \to R$	$P \to R$	Hypothetical syllogism- hs
$P \vee Q$, P'	Q	Disjunctive syllogism- ds
$P \to Q$	$Q' \to P'$	Contraposition- cont
$Q' \to P'$	$P \to Q$	Contraposition- cont
P	$P \wedge P$	Self-reference - self
$P \vee P$	P	Self-reference - self
$(P \wedge Q) \to R$	$P \to (Q \to R)$	Exportation - exp
P, P'	Q	Inconsistency - inc
$P \wedge (Q \vee R)$	$(P \wedge Q) \vee (P \wedge R)$	Distributive - dist
$P \vee (Q \wedge R)$	$(P \vee Q) \wedge (P \vee R)$	Distributive - dist

Examples

- Simplify $(A' \vee B') \vee C$

1.	$(A' \vee B') \vee C$	hyp
2.	$(A \wedge B)' \vee C$	1, De Morgan
3.	$(A \wedge B) \rightarrow C$	2, imp

To prove an argument of the form

$$P_1 \wedge P_2 \wedge \cdots \wedge P_n \rightarrow R \rightarrow Q$$

The deduction method allows for the use of R as an additional hypothesis and thus prove

$$P_1 \wedge P_2 \wedge \cdots \wedge P_n \wedge R \rightarrow Q$$

Example

Prove $(A \rightarrow B) \wedge (B \rightarrow C) \rightarrow (A \rightarrow C)$. We can use the deduction method to rewrite in the form $(A \rightarrow B) \wedge (B \rightarrow C) \wedge A \rightarrow C$

1.	$A \rightarrow B$	hyp
2.	$B \rightarrow C$	hyp
3.	A	hyp
4.	B	1, 3, mp
5.	C	2, 4, mp