PHYS 1444 - Lecture 6
September 16, 2020
Luke Sweeney
UT Arlington

1 Temperature and Resistivity

As temperature increases, so does resistance.

$$\rho = \rho_0 \left[1 + \alpha(T - T_0)\right]$$
$$R = R_0 \left[1 + \alpha(T - T_0)\right]$$

NOTE: α is ONLY valid for a given value of T_0

- $\rho = \rho(T)$ = resistivity at temperature T
- $\rho_0 = \rho(T_0)$ = resistivity at temperature T_0
- α = "Temperature coefficient of resistivity"
- $R = R(T)$ = resistance at temperature T
- $R_0 = R(T_0)$ = resistance at temperature T_0

Example

The resistance of a platinum cylinder is 525Ω at $0\ ^\circ C$. The temperature coefficient of resistivity for platinum is $\alpha = 0.00393\ (^\circ C)^{-1}$ at $20\ ^\circ C$. What is the resistance of the cylinder at $35\ ^\circ C$?

We are given α at the temperature $20^\circ C$, so we need to make sure that $T_0 = 20^\circ C$. We can solve for R_0 in the first equation

$$R_0 = \frac{R}{1 + \alpha(T - T_0)} = \frac{525\Omega}{1 + (0.00393\ ^\circ C^{-1})(35\ ^\circ C - 20\ ^\circ C)} = 569.8\Omega$$

$$R = R_0 \left[1 + \alpha(T - T_0)\right] = (569.8\Omega)\Big[1 + (0.00393\ ^\circ C^{-1})(35\ ^\circ C - 20\ ^\circ C)\Big] = 603\Omega$$

We had to find R_0 at $20\ ^\circ C$ first in order to be able to use α to find R.

Superconductors

Superconductors will have no resistance at a specific very low temperature. This is called the critical temperature, T_c.

2 Electric Power

Power is work divided by time

$$P = \frac{dW}{dt} = (\text{current})(\text{voltage}) = IV$$

For resistors:

$$P = IV = I^2 R = \frac{V^2}{R}$$

Example

When connected to an emf of 120 V, a light bulb consumes 60 W of power. Determine the resistance of the bulb and the current.

$$P = \frac{V^2}{R} \qquad R = \frac{V^2}{P} = \frac{(120\ V)^2}{(60W)} = 240\ \Omega$$

$$P = IV \qquad I = \frac{P}{V} = \frac{(60\ W)}{(120\ V)} = 0.5\ A$$

3 Batteries

A battery is modeled by a series combination of an emf (voltage source) and a resistor (internal resistance).

Batteries are represented in diagrams with a symbol or sometimes a circle. They usually have a positive and negative side.

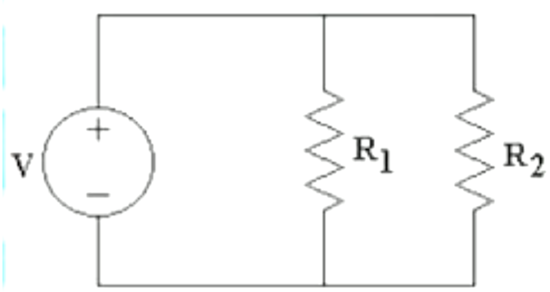
Terminal voltage:

$$V_t = \varepsilon - Ir$$

4 Resistors

Resistors in Parallel

Two resistors in parallel will have the same voltage. The current of the resistors has to equal the current on the source.



Resistors in series are like capacitors in parallel.

$$V_1 = V_2 = V_s$$
$$I_1 + I_2 = I_s$$

$$I_s = I_1 + I_2 = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) V = \left(\frac{1}{R_{EQ}}\right) V$$
$$R_{EQ} = \frac{R_1 R_2}{R_1 + R_2}$$

Resistors in Series

$$V_s = V_1 + V_2$$
$$I_s = I_1 = I_2$$
$$R_{EQ} = R_1 + R_2$$
$$V_s = R_{EQ} I$$

5 Kirchoff Junction Rule (Current)

Kirchoff's Junction Rule (Node Rule): The sum total of currents entering a junction is equal to the sum of the currents leaving that junction.

> Junction: A point where two or more things are joined
>
> Node: A point at which lines or pathways intersect or branch; a central or connecting point.

Charge doesn't build up in a circuit (except locally in capacitors), so this is simply conservation of charge.

Current directions are often unknown, so you can just pick one. If you're wrong, you'll get a negative value for current, which just means it's going the opposite way you chose. The magnitude will be correct.

Kirchoff's Loop Rule (Voltage)

Kirchoff's Loop Rule: The sum total of the electric potential (voltages) around any closed loop is zero.

Any electron that is returned to it's original position must have the same potential energy.

Current determines polarity of passive elements (resistors, for example). Current enters at a higher potential and leaves at a lower potential. The sign on voltage must match the choice of current direction.

General Rules:

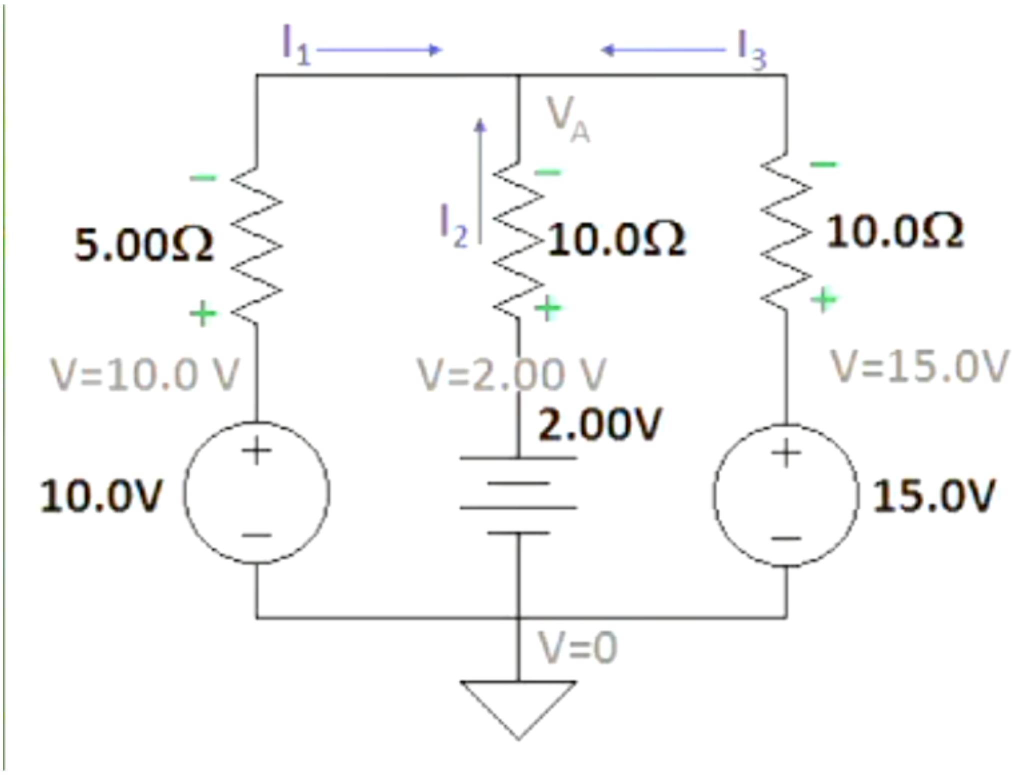
1. Assign a ground (a reference point where $V = 0$), usually at the base of the voltage source.

2. Label node (junction) voltages
3. Label currents
4. Mark resistor polarity

This is easiest to understand through an example.

Example

Take the following circuit



We have assigned the ground point to be at the base of the batteries, where potential (voltage) is 0. We also marked the polarity of the resistors. The positive side of the resistor is nearest the positive side of the battery. We also marked 3 different currents (I_1, I_2, I_3) for each battery.

There are 3 paths we can take, all starting from the ground point.

1. to the left loop and back through the middle
2. to the left and all the way around to the right
3. through the middle and to the right

Remember that going through an element from the positive to the negative side results in a drop in potential (voltage). Going from negative to positive results in an increase in potential.

Starting with the first path (left and through the middle), we get the following equation

$$\underbrace{10.0\ V}_{\text{first battery}} - \underbrace{I_1(5.0\ \Omega)}_{\text{first resistor}} + \underbrace{I_2(10.0\ \Omega)}_{\text{second resistor}} - \underbrace{2.00\ V}_{\text{second battery}} = 0$$

$$10\ V - I_1(5\ \Omega) + I_2(10\ \Omega) - 2\ V = 0$$

We can do the same process for the other paths

2. $10\ V - I_1(5\ \Omega) + I_3(10\ \Omega) - 15\ V = 0$
3. $2\ V - I_2(10\ \Omega) + I_3(10\ \Omega) - 15\ V = 0$

The second path is redundant because all of its segments are covered by other loops. All we really need is 1 and 3 because together they cover the whole circuit. Each equation needs to have at least one unique segment for it to be useful.

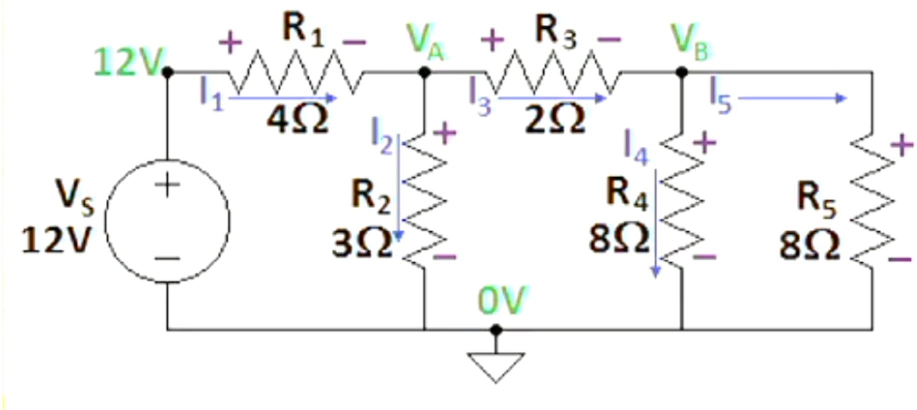
We can use 1 and 3 together with $I_1 + I_2 + I_3 = 0$ to find the currents in the circuit. It's basically just some algebra. You should try it, the answers are

$$I_1 = 0.15\ A$$
$$I_2 = -0.725\ A$$
$$I_3 = 0.575\ A$$

Another Example

This example is pretty long, just bear with me.

What is the current running through R_5?



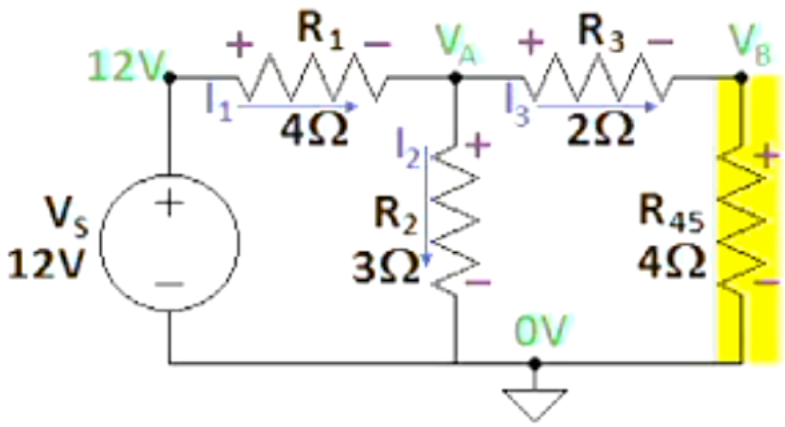
Using Kirchoff's rules through each branch would be a lot of work. We can combine resistors to make this easier.

Before we do that, we label everything we need to. Ground on the bottom, the polarity of the resistors, and a few points of potential (voltage). We want to find V_b so we can use it in Ohm's law to find the current through R_5.

First we can start combining resistors to collapse the circuit down into a simpler form, then expand it again.

Let's combine R_4 and R_5 into R_{45}:

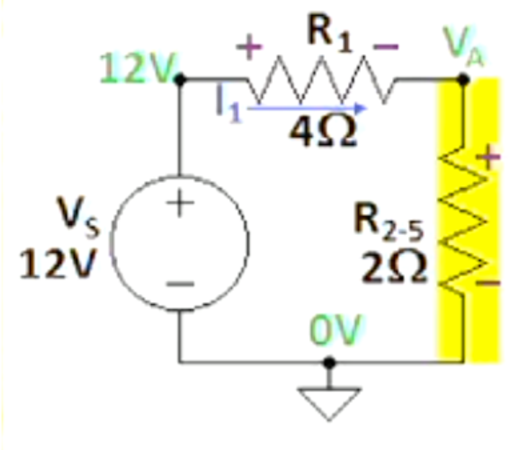
$$R_{45} = \frac{R_4 \cdot R_5}{R + 4 + R_5} = 4\ \Omega$$



Then we can add R_3 and R_{45} because they're in parallel, then combine R_{345} with R_2

$$R_{345} = 2\ \Omega + 4\ \Omega = 6\ \Omega$$

$$R_{2-5} = \frac{R_{345} \cdot R_2}{R_{345} + R_2} = 2\ \Omega$$



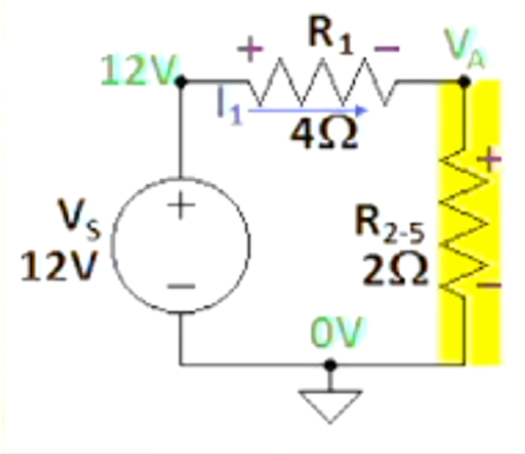
Finally, we can combine R_1 and R_{2-5} to get R_{EQ} (not pictured)

$$R_{EQ} = R_1 + R_{2-5} = 6\ \Omega \quad \text{(total resistance of the circuit)}$$

Now we're going to start expanding to get back to our original circuit. Using Ohm's law, we can find I_1

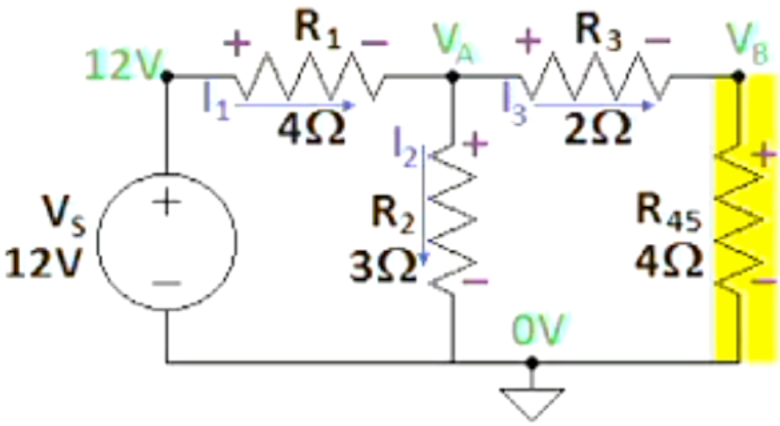
$$I_1 = \frac{V_s}{R_{EQ}} = \frac{12\ V}{6\ \Omega} = 2\ A$$

Now that we have I_1, we can go back to this picture to find V_A



$$V_A = 12\ V - (R_1 I_1) = 12\ V - (4\ \Omega \cdot 2\ A) = 4\ A$$

We can back out one more picture to



Now we need to find I_3. We know that $I_2 + I_3 = I_1$, so let's find I_2 with Kirchoff's law

$$I_2 = \frac{V_A}{R_2} = \frac{4\ V}{3\ \Omega} = \frac{4}{3}\ A$$

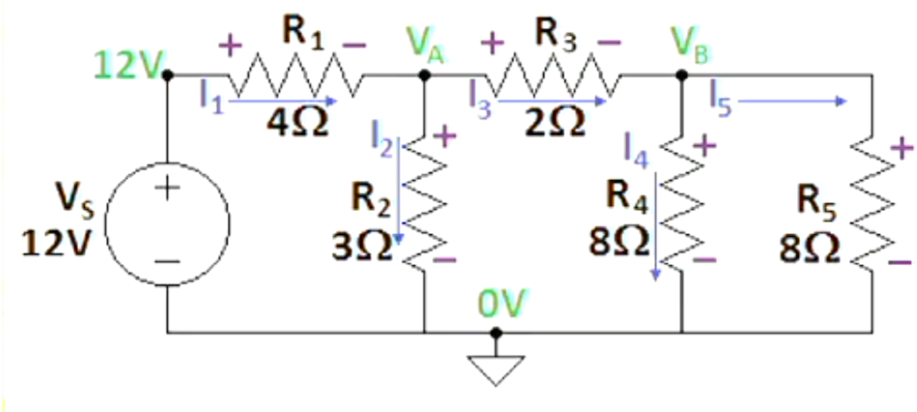
$$I_3 = I_1 - I_2 = 2 - \frac{4}{3} = \frac{2}{3}\ A$$

V_B can be found with Ohm's law again

$$V_B = V_A - (I_3 \cdot R_3) = 4\ A - \left(\frac{2}{3}\ A \cdot 2\ \Omega\right) = \frac{8}{3}\ V$$

Remember above to start with the initial voltage before the resistor (V_A) and subtract the drop in voltage due to the resistor.

Now we can back out a picture again

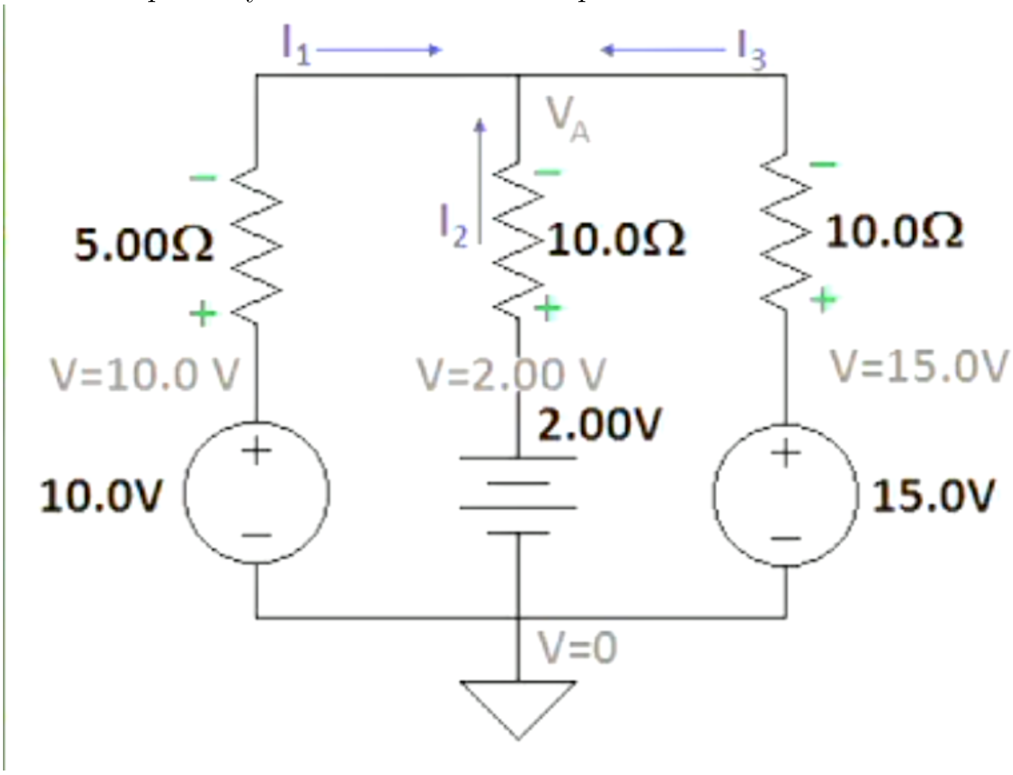


Because R_4 and R_5 have the same resistance, and parallel resistors have the same voltage, the current will be split between them. So $I_3 = I_4 + I_5$ and $I_4 = I_5$

$$I_5 = \frac{1}{2} I_3 = \frac{1}{3}\ A$$

6 Nodal Analysis

There's a simpler way to solve the first example from above



There's only one unknown voltage in the circuit (V_A), we know all the others. We can use the following equation to find V_A and then solve for the currents.

$$I_R = \frac{V_{HI} - V_{LO}}{R}$$

This means the voltage on the higher side of a resistor minus the voltage on the low side, all over the resistor is the current through that resistor.

We can use that equation and $I_1 + I_2 + I_3 = 0$ to find V_A then the currents

$$\left(\underbrace{\frac{10\ V - V_A}{5\ \Omega}}_{I_1}\right) + \left(\underbrace{\frac{2\ V - V_A}{10\ \Omega}}_{I_2}\right) + \left(\underbrace{\frac{15\ V - V_A}{10\ \Omega}}_{I_3}\right) = 0$$

We can get a common denominator and solve for V_A. (I'll drop units here as well)

$$(20 - 2V_A) + (2 - V_A) + (15 - V_A) = 0$$
$$V_A = 9.25\ V$$

Now we can plug that back into each current equation

$$I_1 = \frac{10 - 9.25}{5} = 0.15\ A$$
$$I_2 = \frac{2 - 9.25}{10} = -0.725\ A$$
$$I_3 = \frac{15 - 9.25}{10} = 0.575\ A$$

Last important note

Voltage sources (batteries) can also be combined. The following circuit is the same as the circuit we were just working with