PHYS 1444 - Lecture 4
September 10, 2020
Luke Sweeney
UT Arlington

Important Shit

- Electric potential $V = \overrightarrow{E} \cdot d$
- Electric potential for a point charge $\Delta V = kQ\left[\frac{1}{r_b} - \frac{1}{r_a}\right]$
- To get the electric field from potential V, take the partial derivative for each dimension

  $E_x = -\frac{\partial V}{\partial x}$
  $E_y = -\frac{\partial V}{\partial y}$
  $E_z = -\frac{\partial V}{\partial z}$
- Capacitance (Farads)

  $Q = CV$
  $E = \frac{Q}{\varepsilon_0 A}$
  $C = \frac{\varepsilon_0 A}{d}$
  Dielectrics add a "dielectric constant" k, which is another factor $C = \frac{k\varepsilon_0 A}{d}$

1 Electric Potential

Electric potential in a uniform electric field, electric potential V is given by

$$V = \underbrace{\overrightarrow{E}}_{\text{electric field}} \cdot \underbrace{d}_{\text{distance}}$$

If the electric field is changing, you can use an integral. The electric potential difference between two points is

$$\Delta V = -\int_{P_{init}}^{P_{final}} \overrightarrow{E} \cdot \overrightarrow{dl}$$

(Be careful about the sign)

For a Point Charge

For a point charge, the electric field is given by Coulomb's law $E = kQ/r^2$

$$\Delta V = -\int_{r_a}^{r_b} \overrightarrow{E} \cdot dr = -\int_{r_a}^{r_b} k\frac{Q}{r^2} dr$$

And if Q is constant, then

$$-\int_{r_a}^{r_b} k\frac{Q}{r^2} = -kQ\int_{r_a}^{r_b}\frac{1}{r^2}dr = -kQ\left[-\frac{1}{r}\right]_{r_a}^{r_b}$$

$$\Delta V = kQ\left[\frac{1}{r_b} - \frac{1}{r_a}\right]$$

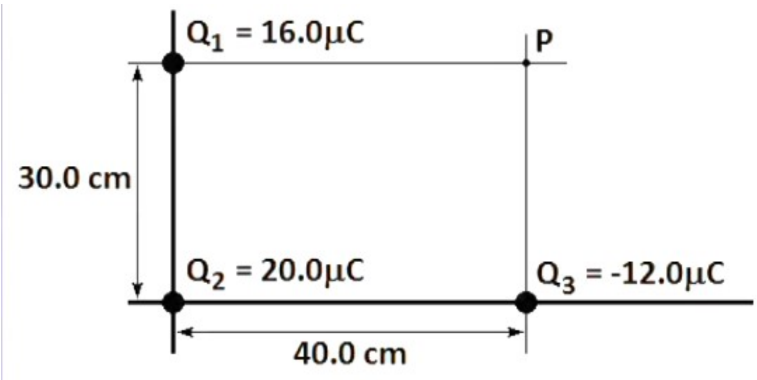
Examples

- What is the potential at the point (2.25 m, 1.50 m) measured with respect to the origin in a region with a uniform electric field $E = (4.00\ N/C)\hat{\imath} + (2.00\ N/C)\hat{\jmath}$?
  This is not a point charge, just a uniform electric field

$$\Delta V = -\int_{P_{init}}^{P_{final}} \overrightarrow{E}\cdot dl = -\overrightarrow{E}\cdot\left[\int_{P_{init}}^{P_{final}} dl\right] = -\overrightarrow{E}\cdot\overrightarrow{l}$$

$$-\left[(4.00\ N/C)\hat{\imath} + (2.00\ N/C)\hat{\jmath}\right]\cdot\left[(2.25\ m)\hat{\imath} + (1.50\ m)\hat{\jmath}\right] = -12.0\ V$$

- Determine the electric potential at the point P due to the 3 point charges



  Because electric potential is a scalar, we can just use the point charge potential equation from above and add them together.

$$V_{\text{point charge}} = k\frac{Q}{r}$$

$$V_{total} = V_1 + V_2 + V_3 = k\left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2} + \frac{Q_3}{r_3}\right)$$

$$(9\times 10^9)\left(\frac{16\times 10^{-6}}{0.4} + \frac{20\times 10^{-6}}{\sqrt{0.4^2+0.3^2}} + \frac{-12\times 10^{-6}}{0.3}\right) = 360\ kV$$

Electric Potential Energy

Above is electric potential. Electric potential *energy* is the same thing, multiplied by charge. Electric potential can be found at any point in space within an electric field, while electric potential energy requires a charge.

2 Getting E from V

You can get the electric field from electric potential using a partial derivative. This is pretty easy to understand with an example. To take a partial derivative, you take the derivative with respect to one variable and assume all other variables are constant.

$$dV = -\overrightarrow{E} \cdot dl$$

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

Example

In a certain region of space the electric potential is given by $V(x, y, z) = y^2 + 2.5xy - 3.5xyz$. Determine the electric field.

We just need to take the partial derivative for each variable

$$E_x = -(2.5y - 3.5yz) = 3.5yz - 2.5y$$
$$E_y = -(2y + 2.5x - 3.5xz) = 3.5xz - 2.5x - 2y$$
$$E_z = -(-3.5xy) = 3.5xy$$

$$E_x\hat{\imath} + E_y\hat{\jmath} + E_z\hat{k} = (3.5yz - 2.5y)\hat{\imath} + (3.5xz - 2.5x - 2y)\hat{\jmath} + (3.5xy)\hat{\jmath}$$

3 Equipotential Surfaces

Meaning a surface where the potential is the same all over.

1. No work is done by electrical forces when moving along an equipotential surface.
2. The electric field is always perpendicular ($\perp$) to the surface.

4 Capacitance

The relationship between potential and charge is capacitance. The units for capacitance are Farads. $1\ F = 1\ C/V$

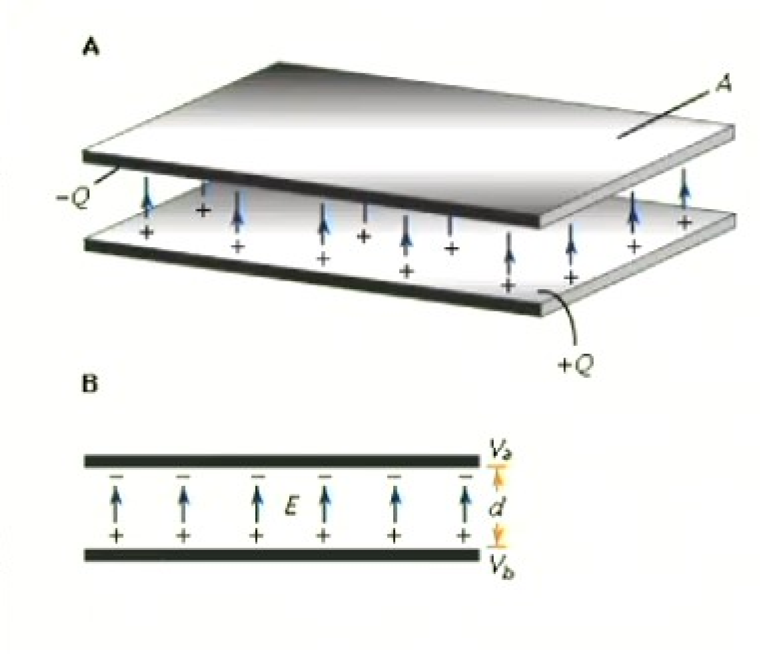
$Q =$ Charge

$C =$ Capacitance

$V =$ electric potential

$$Q = CV$$

The standard capacitor is the parallel plate capacitor. Two plates are separated by a distance d with potential across them, because they have charge on them. There is a constant electric field between them.



The magnitude of the electric field (assuming constant) at the surface of a conductor in this case would be

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

The potential would be $V = \vec{E} \cdot \vec{d}$

$$V = Ed = \frac{Qd}{\varepsilon_0 A} = \left(\frac{d}{\varepsilon_0 A}\right) Q$$

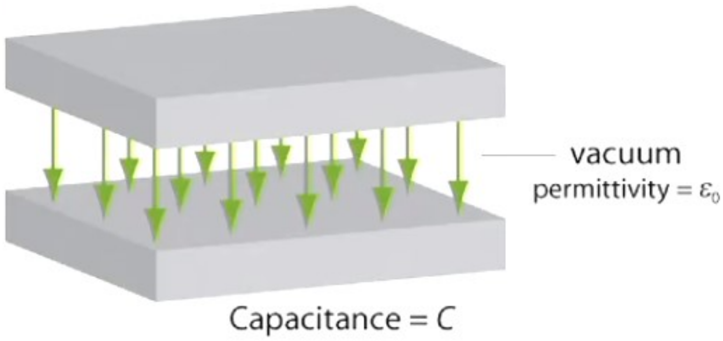
Using the $Q = CV$ equation above, we can find C

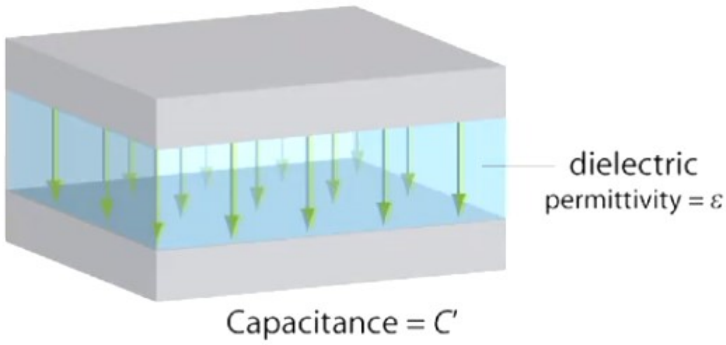
$$C = \frac{Q}{V}$$

$$C = \frac{Q}{\left(\frac{d}{\varepsilon_0 A}\right) Q} = \frac{\varepsilon_0 A}{d}$$

Dielectrics

A dielectric is an insulating material placed between the capacitor plates.





The only difference in these is that we have a new value of ε. Given a dielectric constant k, $\varepsilon = k\varepsilon_0$. Putting that into the original capacitance equation gives

$$C = \frac{k\varepsilon_0 A}{d} = kC_0$$

k will always be greater than 1, so the capacitance of a dielectric will always be greater as well. In an air filled capacitor $k = 1$, which he won't tell us in the formula sheet.

Examples

- What voltage is required to store 7.2×10^{-5} C of charge on the plates of a 6.00 μF capacitor.

  (Changing the charge to 72 μC means the μ cancels out)

$$Q = CV$$
$$V = \frac{Q}{C} = \frac{72 \times 10^{-6}\ C}{6 \times 10^{-6}\ F} = 12.0\ V$$

- A parallel plate capacitor has a capacitance of 7.00 μF when filled with a dielectric. The area of each plate is 1.5 m^2 and the separation between the plates is 1×10^{-5} m. What is the dielectric constant of the dielectric material?

  Rearranging the equation for the capacitance of a dielectric for k gives us

$$k = \frac{C \cdot d}{\varepsilon_0 A} = \frac{(7 \times 10^{-6}\ F)(1 \times 10^{-5}\ m)}{\varepsilon_0 (1.5\ m^2)} = 5.3\ \text{[unitless]}$$