CSE 2315 - Proof Techniques
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# 1 Informal Proofs

Usually we'll prove conjectures about a specific domain of objects, but not universally true. We can introduce hypotheses that are only true for that domain.

Proofs can be informal and conversational, as long as formal proof can back it up.

**Inductive reasoning** is drawing a conclusion from a hypothesis based on experience. Hence the more cases you find where $Q$ follows from $P$, the more confident you are about the conjecture $P \to Q$.

**Deductive reasoning** is finding a counterexample to disprove the conjecture or prove the conjecture true.

### Proof Techniques

If you can't disprove a conjecture by finding a counterexample, then try to prove that it is true.

Possible methods of proof:

1. Proof by exhaustion - given a finite domain, prove the conjecture for all objects in the domain.
   - This is good for computer or small domains, but impractical on large or infinite domains.
2. Direct proof - equivalence and inference rules
   - Using this can get long and is sometimes unnecessary
3. Proof by contraposition
   - Using the fact that $(Q' \to P') \to (P \to Q)$ is a tautology, we can use contraposition to prove an argument. Flip both arguments and flip the implication and prove that.
4. Proof by contradiction
   - 0 is any contradiction (false)
   - $(P \wedge Q' \to 0) \to (P \to Q)$ is a tautology
   - To prove by contradiction, take the hypothesis and conclusion $P \to Q$. Assert that the conjunction of $P$ and $Q'$ implies a contradiction. $(P \wedge Q' \to 0)$
   - As an example, take "if a number added to itself gives itself, then that number is 0". This is true, and we can prove it by contradiction. Take the hypotheses

$P = x + x = x$

$Q' = x \neq 0$

Try to prove $(P \wedge Q' \rightarrow 0)$

5. Serendipity - fortuitous happening or something happening by chance or good luck.

**Proof Techniques Summary**

| **Proof Technique** | **Approach to prove** $P \rightarrow Q$ | **Remarks** |
|---|---|---|
| Exhaustive Proof | Demonstrate $P \rightarrow A$ for all cases | May be used only for finite domains |
| Direct Proof | Assume P, deduce Q | The standard approach, usually the thing to try |
| Proof by contraposition | Assume $Q'$, derive $P'$ | Use this if $Q'$ as a hypothesis seems to give more ammunition than $P$ would. |
| Proof by contradiction | Assume $P \wedge Q'$, deduce a contradiction | Use this when Q says something is not true |
| Serendipity | Not really a proof technique | Fun to know |