Dillhoff started the lecture with problem 6 from homework 7 about the camera and projecting object onto the plane. I'm not going to go over it here because I'm running out of time (cry) but it's in this lecture if you want to go over it again.

1 Coordinate Mapping

What defines a coordinate system are the basis vectors. Coordinate systems with wildly different basis vectors will be very different.

Theorem 1 *Let $B = \left\{\vec{b_1}, \cdots, \vec{b_n}\right\}$ be a basis for a vector space V. Then the coordinate mapping $\vec{x} \mapsto [\vec{x}]_B$ is a one-to-one linear transformation from V to $\mathbb{R}^n$.*

There is one point in V that maps to one other point in $\mathbb{R}^n$. If we have a one-to-one linear transformation from some vector space V to another space W, the mapping is called an **isomorphism**.

Example

Consider the space $\mathbb{P}_2$ of polynomial functions and let $B = \left\{1, t, t^2\right\}$ be the standard basis of $\mathbb{P}_2$.

A vector in $\mathbb{P}_2$ has the form

$$\vec{v}(t) = v_0 + v_1 t + v_2 t^2$$

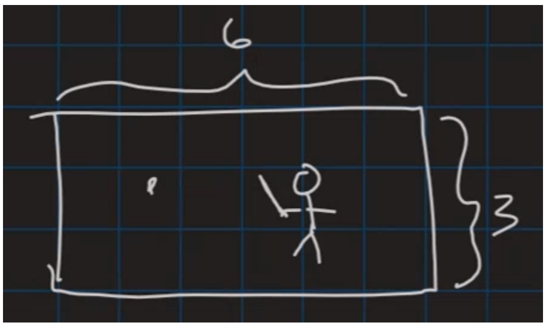
This is a linear combination of coordinates v_0, v_1, v_2 and the basis vectors.

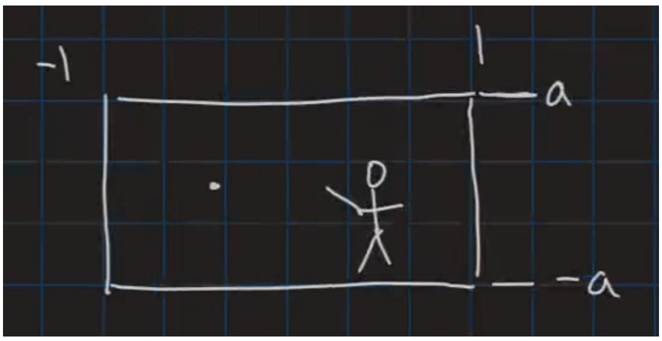
$$[\vec{v}]_B = \begin{bmatrix} v_0 \\ v_1 \\ v_2 \end{bmatrix}$$

This implies all vector operations in $\mathbb{P}_2$ correspond to operations in $\mathbb{R}^3$. These coordinates from $\mathbb{P}_2$ to $\mathbb{R}^3$ are isomorphic.

Example: Rendering and Normalized Device Coordinates (NDC)

Given a scene with objects on a plane, we want to transfer those points into a plane with normalized coordinates. We can take a 6×3 plane with the corner at (0, 0) and translate it to be normalized.





The normalized image on the right shows the same plane with each edge at -1 and 1, with a height of $2a$. From here, we can easily scale the image to whatever resolution we need, in the context of computer graphics.

Example

Coordinate vectors can be used to show linear independence.

Verify that the polynomials $1 + 2t^2$, $4 + t + 5t^2$, and $3 + 2t$ are linearly dependent in $\mathbb{P}_2$.

The coordinate mapping between $\mathbb{P}_2$ and $\mathbb{R}^3$ is (1, 0, 2), (4, 1, 5), and (3, 2, 0):

$$\begin{bmatrix} 1 & 4 & 3 \\ 0 & 1 & 2 \\ 2 & 5 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

The last row shows that there is a free variable in this system, which means these are linearly dependent.