PHYS 1444 - Lecture 3
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These notes are probably incomplete, please don't rely on them.

1 Electric Flux

Electric flux is the dot product of an electric field and an area. To find an area vector, the magnitude is the area and the direction is perpendicular to the surface.

Flux is similar to flow. The analogy he provides is rain falling into a barrel. If the surface area of the opening of a barrel is 100 in^2 and 2 in of rain falls, then $(2\ in)(100\ in^2) = 200\ in^3$ of rain falls into the barrel. However, if the rain is coming at an angle of 30°, then $(2\ in)(100\ in^2)\cos(30°) = 173\ in^3$ falls into the barrel.

Remember that the dot product of two vectors is

$$\overrightarrow{a} \cdot \overrightarrow{b} = |\overrightarrow{a}| \cdot |\overrightarrow{b}| \cos(\theta)$$

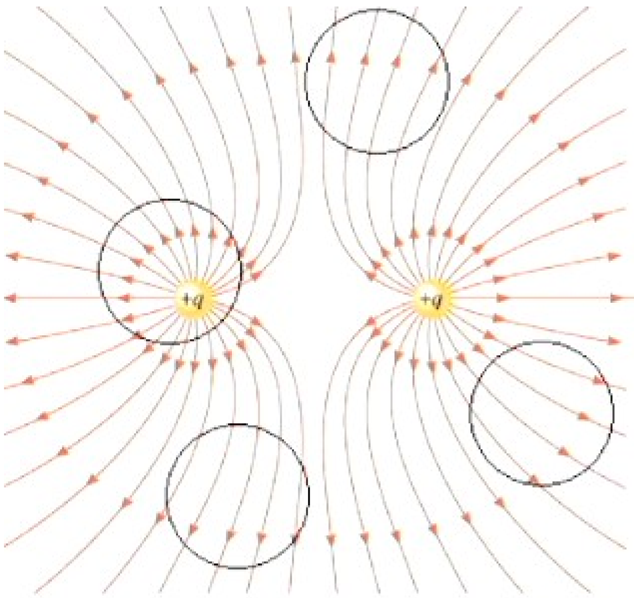
So the electric flux Φ_E

$$\Phi_E = \overrightarrow{E} \cdot \overrightarrow{A}$$

where $\overrightarrow{A}$ is the area vector.

2 Gauss's Law

$$\Phi_E = \oint \overrightarrow{E} \cdot d\overrightarrow{A} = \frac{Q_{ENC}}{\varepsilon_0}$$



The flux in any enclosed area is proportional to the charge in that area. In some of these circles, there is an equal amount of charge entering and leaving the area (lines going in and out) leaving the area with no net charge, but others only have lines going out.

ε_0 is the permittivity of free space

$$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/(Nm^2)$$
$$k = \frac{1}{4\pi\varepsilon_0} \approx 9.00 \times 10^9$$

Example

- The geometric center of a cube is located at the origin. If a 30 nC point charge is also at the origin, what is the electric flux that passes through one of the cube's faces?

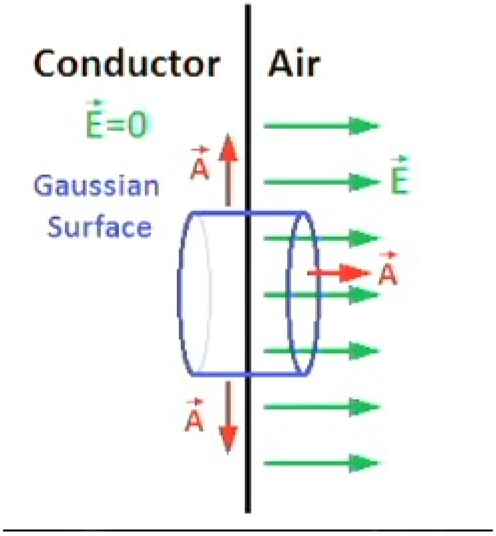
  The electric flux is equal to the enclosed charge. We can find the enclosed charge of the cube, then divide by 6 to get the flux on one face. Remember that ε_0 is a constant.

$$\Phi_E = \frac{Q_{ENC}}{\varepsilon_0}$$
$$Q_{ENC} = 30 \times 10^{-9}\ C$$
$$\Phi_E = \frac{30 \times 10^{-9}\ C}{(6)(\varepsilon_0)} = 5.65\frac{Nm^2}{C}$$

- A conductor has a surface charge density of $\sigma = 3.25\ \mu C/m^2$. Determine the electric field at the surface.

  I don't really understand this but I'll try my best. The surface is an infinite plane, so we can use a cylinder to find the electric field. Remember that

$$\oint \overrightarrow{E} \cdot d\overrightarrow{A} = \frac{Q_{ENC}}{\varepsilon_0}$$



Spurlock says we need to do 3 integrals; one for the back side (circle), one for the sides (tube), and one for the front side (circle). I *think* this is because the inside, surface, and just outside a conductor will have different charges.

For the first integral (back side), the charge E is 0 (because it's a conductor). So the integral is just 0.

$$\oint \overrightarrow{E} \cdot d\overrightarrow{A} = \oint (0) \cdot d\overrightarrow{A} = 0$$

For the second integral, the angle of the field created by the tube is 90°. Taking the dot product of that electric field and the electric field created by the conductor (the $\overrightarrow{E} \cdot d\overrightarrow{A}$ from inside the integral), you get 0 (because $\cos(90°) = 0$.

For the third integral, it's the same as the first but $\overrightarrow{E}$ isn't 0 this time. The area vector points to the right, perpendicular to the surface, and so does the electric field. This means the cos from the dot product is $\cos(0°) = 1$ so we can basically ignore it.

$$\oint \overrightarrow{E} \cdot d\overrightarrow{A} = \oint (\overrightarrow{E} \cdot d\overrightarrow{A} \cdot \cos(0°))$$

Because the electric field is constant over the surface, we can pull $\overrightarrow{E}$ outside of the integral.

$$\oint \overrightarrow{E} \cdot d\overrightarrow{A} = \overrightarrow{E} \oint d\overrightarrow{A} = EA$$

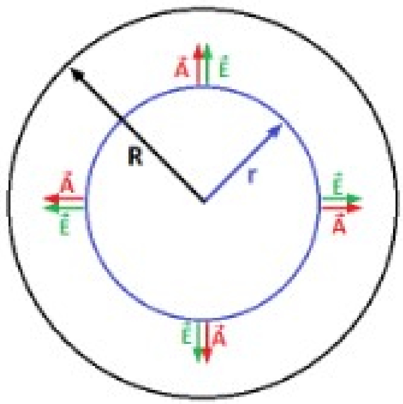
Now we can put the result of the integral back into the first equation and substitute σ for the area and charge enclosed (because σ is charge / area).

$$EA = \frac{Q_{ENC}}{\varepsilon_0}$$

$$E = \frac{Q_{ENC}}{\varepsilon_0 A} = \frac{\sigma}{\varepsilon_0}$$

- A non-conducting sphere of radius R has a non-uniform charge distribution $\rho = \alpha + \beta r$, where r is the distance from the sphere's center. Determine the electric field as a function of r.

$$\oint \overrightarrow{E} \cdot d\overrightarrow{A} = \frac{Q_{ENC}}{\varepsilon_0}$$



Because we're working with a sphere, the area vector and electric field vector are parallel (perpendicular to the surface, we can ignore the dot product and treat it like multiplication

$$\oint \overrightarrow{E} \cdot d\overrightarrow{A} = \oint E \cdot dA$$

He says E is constant (which doesn't make sense to me because it says its a non-uniform charge distribution???), so it can be pulled out from the integral.

$$\oint E \cdot dA = E \oint dA = EA \qquad A_{sphere} = 4\pi r^2$$

$$EA = E(4\pi r^2)$$

I have no idea

3 Electrical Potential

Electrical potential is kind of like potential energy. Electrical potential ΔV is

$$\Delta V = \frac{\Delta PE}{q} = \frac{-W}{q} = \frac{-\overrightarrow{F} \cdot \overrightarrow{d}}{q} = -\overrightarrow{E} \cdot \overrightarrow{d}$$

Positive charges accelerate as they move from higher to lower values of electric potential. Negative charges accelerate as they move from lower to higher values. Think of it like potential energy. Something with positive mass at a certain height has potential energy. When it's dropped, it accelerates downward and it loses potential energy. Negative charges are like balloons.

The unit for electrical potential is the Volt (V). $1\ V = 1\ J/C$, or $1\ V/m = 1\ N/C$.

Example

- Determine the number of particles (each with charge e) that pass between the terminals of a 12.0 V car battery when a 60.0 W headlight burns for an hour.

  We are given electrical potential ($V = 12\ Volts$) and we need to find charge. We can multiply power and time to get work, then divide that by electrical potential to get charge. Once we have charge, we can do a unit conversion and find the number of particles.

$$P = \frac{W}{t} \qquad W = Pt = (60.0\ W)(3600\ s) = 216\ kJ$$

$$V = \frac{W}{q} \qquad q = \frac{W}{V} = \frac{216\ kJ}{12.0\ V} = 18.0\ kC$$

$$\frac{18,000C}{1.6 \times 10^{-19}\ C} = 1.13 \times 10^{23}\ particles$$

4 The Electron Volt

The electron volt is a new unit of energy. Energy is acquired by an electron moving through a potential of 1V. Because a Volt is 1 J/C, the electron volt is

$$eV = (1.6 \times 10^{-19}\ C)\left(1\frac{J}{C}\right) = 1.6 \times 10^{-19}\ J$$