CSE 3380: Linear Algebra for CSE
University of Texas at Arlington
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Exam 1

1) (12 points) What is the minimum number of equations a linear system must have in $\mathbb{R}^n$ such that the equation $A\mathbf{x} = \mathbf{b}$ has the following? **Justify your answer, don't just write a value.**

   (a) A unique solution

   (b) Infinite solutions

   (c) No solutions

2) (10 points) Given a 4×5 coefficient matrix A for a system that is *consistent*, what is the minimum number of solutions it can have? Explain your answer.

3) (16 points) Given the following system:

$$\begin{aligned} -4x_1 \quad & -7x_3 + 7x_4 = -5 \\ x_1 + x_2 \quad & -4x_4 = 8 \\ -x_2 + 7x_3 & - 6x_4 = -1 \\ x_1 \quad & + 7x_3 + 2x_4 = -1 \end{aligned}$$

(a) Solve the system and write the solution set in parametric vector form.

(b) Describe the set geometrically.

4) (16 points) Given the following matrices

$$B = \begin{bmatrix} -4 & 4 & -3 \\ -7 & -4 & 6 \\ 7 & -5 & 6 \end{bmatrix} \text{ and } C = \begin{bmatrix} 7 & 9 & 4 \\ -2 & 1 & 0 \\ 3 & 8 & 3 \end{bmatrix}$$

(a) Calculate C^{-1}

(b) Let $B^2 = B(CAC^{-1})$. Solve for A then calculate A.

5) (10 points) Given the following block matrices

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \text{ and } B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \\ B_{31} & B_{32} \end{bmatrix}$$

Is the product defined? Why or why not?

6) (10 points) Calculate the determinant of A using cofactor expansion, where

$$A = \begin{bmatrix} 1 & -3 & 2 \\ -9 & 7 & 1 \\ 6 & -5 & -9 \end{bmatrix}$$

7) (10 points) Give a geometric description of $\text{Span}\{v_1, v_2, v_3\}$, where

$$v_1 = \begin{bmatrix} -1 \\ -7 \\ -5 \\ -2 \end{bmatrix}, v_2 = \begin{bmatrix} 2 \\ -5 \\ 2 \\ 4 \end{bmatrix}, \text{ and } v_3 = \begin{bmatrix} 4 \\ -10 \\ 4 \\ 8 \end{bmatrix}$$

8) (16 points) Given the following matrices

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \text{ and } B = \begin{bmatrix} a & b & c \\ g & h & i \\ 3d+a & 3e+b & 3f+c \end{bmatrix}$$

(a) Define the elementary matrices E_1, E_2, and E_3 such that $E_1E_2E_3A = B$.

(b) If det $A = 6$, what is det B?