PHYS 1444 - Lecture 2
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1 Fields

Instead of 2 charges acting on each other, one charge is going to create an electric field which spreads through space around it, which then acts on another object. Gravity is a field which we normally evaluate at a specific point. As you gain altitude, the force of gravity gets weaker because it's a field.

Electric fields are "vector fields", meaning they consist of a different vector (magnitude and direction) at every point in space.

We can work with fields through this equation

$$\overrightarrow{E} = \frac{\overrightarrow{F}}{q}$$

$\overrightarrow{F}$ is the force felt by a small positive "test charge"
$\overrightarrow{E}$ is the electric field
q is the small positive charge.

E and F are vectors. E points in the same direction as the force felt by a positive test charge. Because F is a force (Newton) and q is a charge (Coulombs), the units for E are Newtons per Coulomb (or volt per meter).

For a point charge Q:

$$E = \frac{F}{Q} = \frac{kQq/r^2}{q} = k\frac{Q}{r^2}$$

Note: When you see r^2, it's a vector and you don't put the signs of the charges in. If you see r, it's a scalar and you do.

Like forces, electric fields obey superposition (you can add them up).

Examples

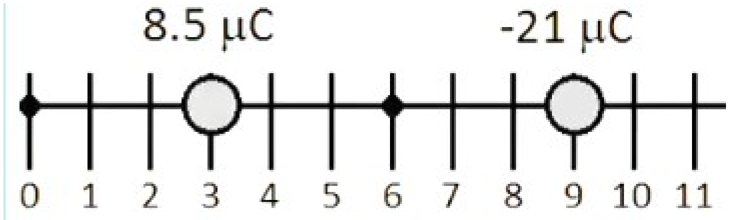
- An electric field of strength 260 kN/C points due west at a certain spot. What are the magnitude and direction of the force that acts on a -7.0 μC charge at this spot?

  We can use the equation from above $E = \frac{F}{Q}$ and solve for F. $F = QE$. We can multiply the charge and the electric field to get the force in Newtons. We're finding the magnitude, so we don't put the sign in.

  $$F = QE = (260,000\ N/C)(7 \cdot 10^{-6}) = 1.82\ N$$

  Because the electric field is pointing west, a positive charge would be pushed west. Therefore, a negative charge would be pointing east.

- Two charges are placed on the x-axis. One charge ($Q_1 = 8.5\ \mu C$) is at $x_1 = 3.0\ cm$, and the other ($Q_2 = -21\ \mu C$) is at $x_2 = 9.0\ cm$. Find the net electric field (magnitude and direction) at (a) $x = 0\ cm$ and (b) $x = 6.0\ cm$.



We need the equation

$$E = k\frac{Q}{r^2}$$

to find the magnitude of the field 4 times. Both charges will have an effect on both points. We can take the effect due to both charges on one point and add them together to get the net charge at that point. We'll call the net charge at each point $E_1(x = 0) + E_2(x = 0)$ and $E_1(x = 6) + E_2(x = 6)$ (the force due to each charge at each point).

After finding the magnitude, we can look at the signs to determine to direction of each field. This is a one dimensional problem. $k = 9 \cdot 10^9\ Nm^2/C^2$

$$E_1(x = 0) = k\frac{8.5 \cdot 10^{-6}\ C}{(0.03\ m)^2} = 8.5 \cdot 10^7$$
$$E_1(x = 6) = k\frac{8.5 \cdot 10^{-6}\ C}{(0.03\ m)^2} = 8.5 \cdot 10^7$$
$$E_2(x = 0) = k\frac{21 \cdot 10^{-6}\ C}{(0.09\ m)^2} = 2.3 \cdot 10^7$$
$$E_2(x = 6) = k\frac{21 \cdot 10^{-6}\ C}{(0.03\ m)^2} = 21 \cdot 10^7$$

Now we have all the magnitudes, we can put the correct sign on each force. Q_1 is positive and is to the right of $x = 0$, so it will push it in the negative direction. It will push the point $x = 6$ to the positive direction.

Q_2 is negative and to the right of both points, so it will pull both points to the positive direction.

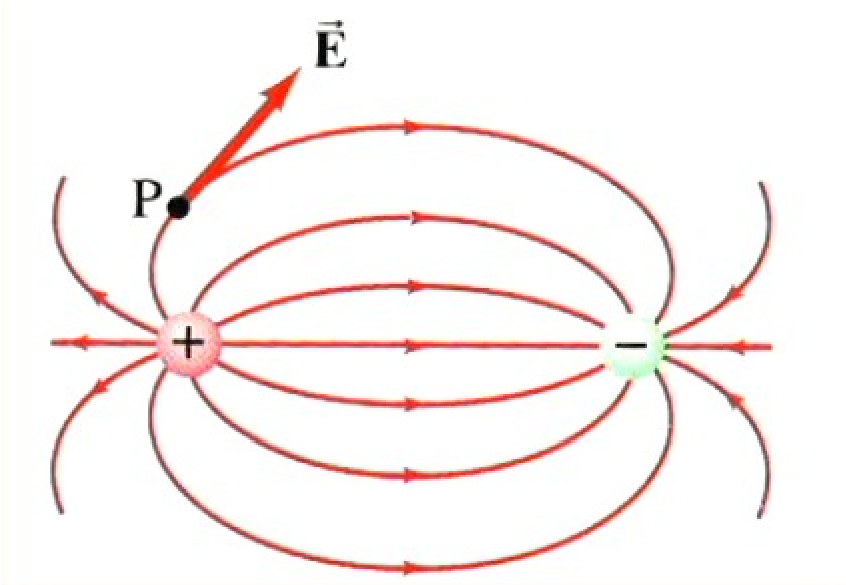
$$E_1(x = 0) = -8.5 \cdot 10^7 \qquad E_1(x = 6) = 8.5 \cdot 10^7$$
$$E_2(x = 0) = 2.3 \cdot 10^7 \qquad E_2(x = 6) = 21 \cdot 10^7$$

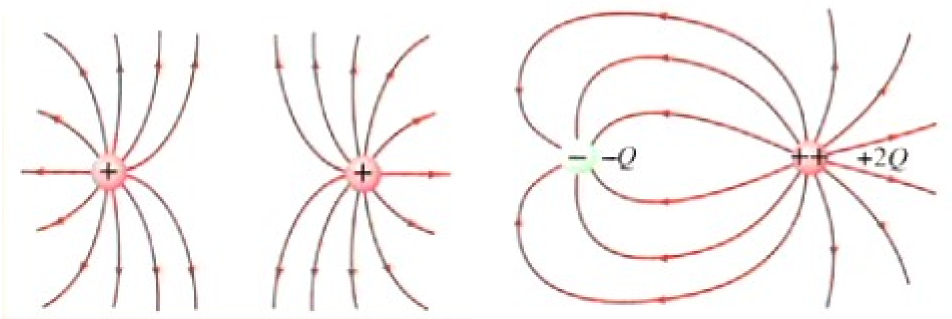
Now we can add all the magnitudes to find the force of the field. At $x = 0$, the force is

$$E_1(x = 0) + E_2(x = 0) = (-8.5 + 2.3) \cdot 10^7 = -6.2 \cdot 10^7\ N/C$$
$$E_1(x = 6) + E_2(x = 6) = (8.5 + 21) \cdot 10^7 = 3.0 \cdot 10^8\ N/C$$

2 Electric Field Lines

1. Lines indicate the direction the field vectors point (field vectors are tangent to the field lines).
2. The number of lines starting on a positive charge (or ending on a negative charge) is proportional to the charge.
3. The closer the lines, the stronger the field.

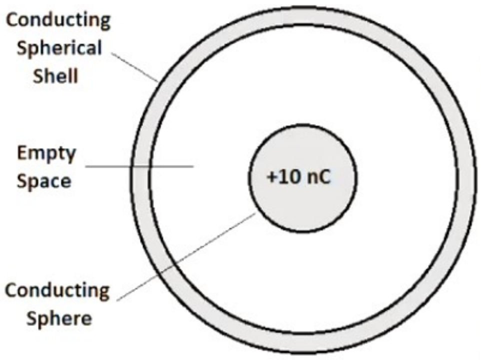


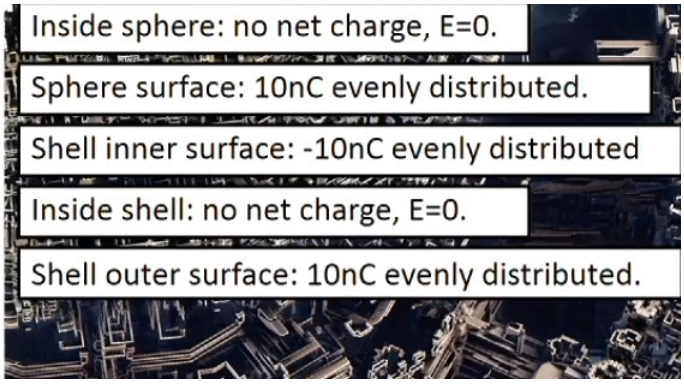


3 Electric Fields and Conductors

In **static equilibrium**, there is no acceleration in a system. Inside a conductor, there are free charges that can move. If there is an electric field inside a conductor, the charges will move and you can't have static equilibrium. So
In static equilibrium:

1. Inside a conductor, $E = 0$
2. Any net charge distributes itself evenly on the surface
3. At the surface of the conductor, the electric field is perpendicular to that surface





In the above image, the empty space has no net charge because it's empty. The inner sphere has a charge of 10 nC evenly distributed around it's surface area. The outer sphere is polarized, so the negative charge is attracted to the inner wall and the positive charge is pushed to the outer wall.

4 Motion of a Charged Particle in an Electric Field

$$\overrightarrow{a} = \frac{\overrightarrow{F}}{m} = \frac{q\overrightarrow{E}}{m}$$

If you are given the field, use that to find the acceleration and then solve the kinematics. If you are given the kinematic information, use that to find the acceleration, then use $a = qE/m$ to find q or E

Example

- An electron is accelerated in the uniform field ($E = 1.45 \times 10^4\ N/C$) between two parallel charged plates. The separation between the plates is 1.1 cm. The electron is accelerated from rest near the negative plate through a tiny hole in the positive plate. With what speed does it leave the hole?

  Remember that $a = qE/m$, and the mass of an electron $m_{electron} = 9.11 \times 10^{-31}\ kg$, and the charge of an electron $q = 1.6 \times 10^{-19}\ C$

  $$a = \frac{qE}{m} = \frac{(1.6 \times 10^{-19}\ C)(1.45 \times 10^4\ N/C)}{(9.11 \times 10^{-31}\ kg)} = 2.54665 \times 10^{15}\ m/s^2$$

  Now that we know the acceleration, we can write out what we know and use 1D kinematics to find the velocity.

  $$x_0 = 0 \quad x = 0.011\ m \quad v_0 = 0 \quad v = ? \quad a = 2.54665 \times 10^{15}\ m/s^2$$

  $$v^2 = v_0^2 + 2a(x - x_0) = 2ax$$

  $$v^2 = 2(2.54665 \times 10^{15}\ m/s^2)(0.011\ m) = 5.60263 \times 10^{13}\ m^2/s^2$$

  $$v = 7.49 \times 10^6\ m/s$$

5 Continuous Charge Distributions

Break the distribution into infinitesimally small point charges, determine the electric from each, and add them up. This is like integrating an infinite number of tiny point charges. Given a tiny point charge dQ which makes a little tiny bit of electric field dE

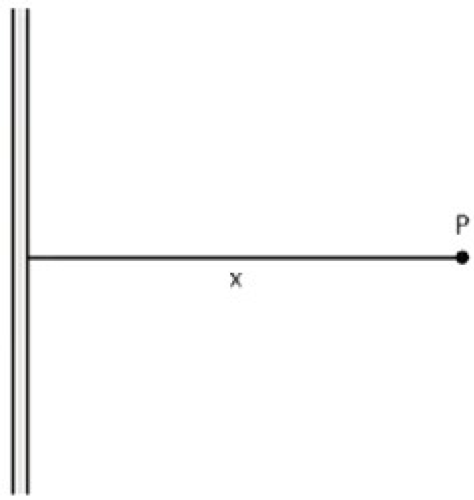
$$dE = k\frac{dQ}{r^2} \qquad \vec{E} = \int d\vec{E}$$

Important note: because you can only add the magnitude of a vector if they face the same direction, you'll need to break these up into their components.

$$E_x = \int dE_x \qquad E_y = \int dE_y$$
$$E = \sqrt{E_y^2 + E_x^2}$$

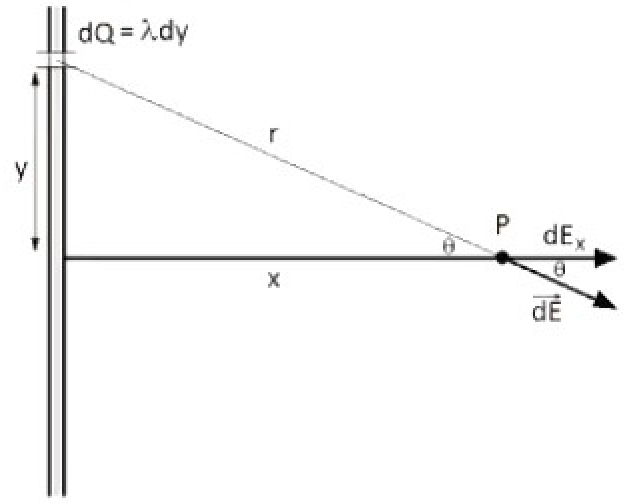
Example

- (This example is stupid complicated and I really don't understand it) Determine the magnitude of the electric field at a point P a distance X from the midpoint of a very long line of uniformly distributed positive charges. Assume X is much smaller that the length of the wire. Let λ be the charge per unit length (units C/m).



Select an infinitesimal part of the charge distribution (the vertical line of positive charges) and label is dQ, then write dQ in terms of your integration variable. $dQ = \lambda\ dy$. We can then substitute that into the above equation

$$dE = k\frac{dQ}{r^2} = k\frac{\lambda\ dy}{r^2}$$



Each unit of length on the y axis will have an effect on the point P, but because it is vertically symmetrical, all the y components of the force will be canceled out. We can only focus on the x components. To get the x components we need $E = \int dE_x = \int dE \cos(\theta)$.

$$\cos(\theta) = \left(\frac{x}{r}\right)$$

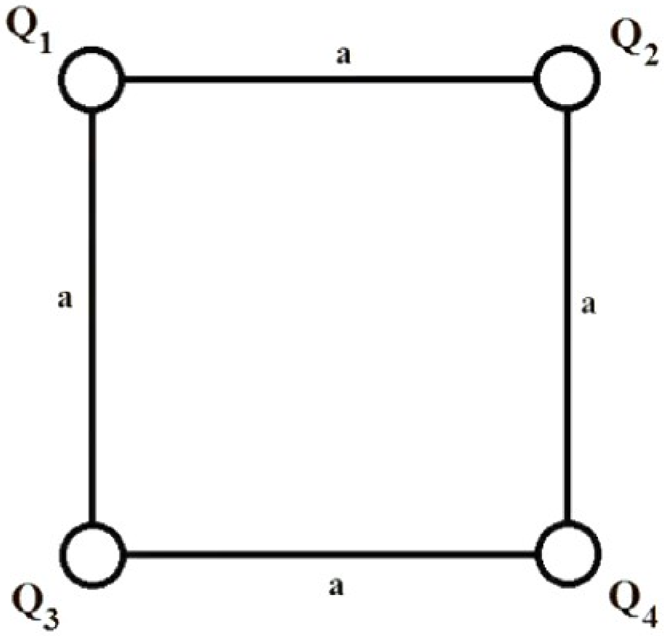
$$E = \int \left(k\frac{\lambda\, dy}{r^2}\right)\left(\frac{x}{r}\right)$$

$$E = k\lambda x \int \frac{dy}{r^3} = k\lambda x \int \frac{dy}{[x^2 + y^2]^{3/2}}$$

I lost him...

The Problem from Last Class

Four charges ($Q_1 = 2.00\ \mu C$, $Q_2 = 3.00\ \mu C$, $Q_3 = 5.00\ \mu C$, $Q_4 = 4.00\ \mu C$) sit at the corners of a square with sides of length $a = 3.00\ m$. Determine the electric field at the center of the square.



The opposite charges ($Q_1 \leftrightarrow Q_4$ and $Q_2 \leftrightarrow Q_3$) can be subtracted because they exactly counteract each other. That leaves only Q_3 and Q_4, which are both 2.00 μC. We can then find the electric field in the center. Because both are the same charge, the force will point 90° straight up with a certain magnitude.

$$r^2 + r^2 = a^2 \qquad r^2 = \frac{1}{2}a^2$$

$$Q = 2 \times 10^{-6}\ C$$

$$|E_3| = |E_4| = k\frac{Q}{\frac{1}{2}a^2} = 4000\frac{N}{C}$$

$$|E| = \sqrt{\left(4000\frac{N}{C}\right)^2 + \left(4000\frac{N}{C}\right)^2} = 5656\ \frac{N}{C}\ \angle 90^\circ$$