PHYS 1444 - Capacitors
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1 Energy Storage

Capacitors are building up potential, so they are storing energy.

$$W = \int_0^Q V\ dq = \frac{1}{C}\int_0^Q q\ dq = \frac{Q^2}{2C}$$

A few different ways to find the same thing.

$$PE = \underbrace{\frac{1}{2}QV}_{\text{No C}} = \underbrace{\frac{1}{2}CV^2}_{\text{No Q}} = \underbrace{\frac{Q^2}{2C}}_{\text{No V}}$$

For a parallel plate capacitor (constant electric field across):

$$PE = \frac{1}{2}\varepsilon(Ad)E^2$$

$$\text{Energy Density} = \frac{\text{Energy}}{\text{Volume}} = \frac{1}{2}\varepsilon E^2$$

Example

- A 6 μF capacitor is charged to 12.0 V. (a) How much charge and energy are stared in the capacitor. (b) If the plates of the charged capacitor are then connected by a pair of thin conductors to the plates of a second, uncharged 12 μF capacitor, what is the shared electrical potential once equilibrium has been achieved and how much energy is stored by the pair.

  Part A is pretty easy, just remember that $Q = CV$.

  $$Q = (6.00\ \mu F)(12.0\ V) = 72\ \mu C$$

  For part B, we can use one of the equations from above

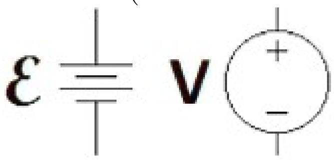
  $$PE = \frac{1}{2}CV^2 = \frac{1}{2}(6\ \mu F)(12\ V)^2 = 432\ \mu J$$

  The two plates have to have the same voltage across them. He finished the problem on the white board and I can't read it because the resolution on Echo360 is booty cheeks.

2 Circuits

A circuit is a closed path that allows electric current (electrons) to flow. We need a power source, which is a battery.

An electromotive force (emf or ε) maintains a constant electric potential between its terminals (also called a "voltage source"). They have a fixed voltage inside.



Elements are connected via conductors (wires), which are represented in the circuit as lines.

In conductors in static equilibrium, $E = 0$. This makes them equipotential. There is no voltage drop/rise along a conductor.

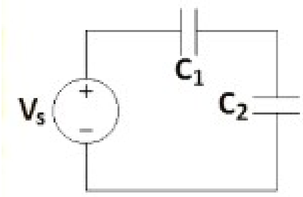
Series and Parallel

A **series connection** are elements that are in the same path with no junction between them. The order of elements in a series does not matter. They may be rearranged to simplify a circuit.

A **parallel connection** is a collection of elements that have the same potential. A direct path via conductors exists between the terminals of the two elements.

A component or gorup of components may be replaced by another (known as an **equivalent**) if it has no effect on outside components.

Capacitors in Series



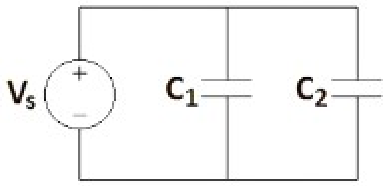
There is no potential change within the wires. Potential will rise and fall as current travels through the capacitor, I think. When capacitors are in series, they will have the same charge, and the voltage is summed.

$$Q_1 = Q_2 = Q \qquad V_s = V_1 + V_2$$

$$V_s = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right) Q = \left(\frac{1}{C_{EQ}}\right) Q$$

$$C_{EQ} = \frac{C_1 C_2}{C_1 + C_2}$$

Capacitors in Parallel



This is opposite series. Here, the voltages are equal and the charges are summed (in series, the charge is equal and the voltage is summed).

$$Q_1 + Q_2 = Q \qquad V_s = V_1 = V_2$$

$$Q = C_{EQ}V$$
$$C_{EQ} = C_1 + C_2$$

3 Current

An electric potential placed across a conductor creates an electric field, which in turn causes charges to move. The movement of charges is called **electric current** (not static equilibrium)

$$\underbrace{I}_{\text{Current (amp)}} = \frac{dq}{dt}$$

$$1\ A = 1\ C/s$$

- Direct current (DC) - Current always flows the same direction
- Alternating current (AC) - Direction of current flow changes periodically

Because electrons are negative, current is thought of as "conventional current", which is the fictitious flow of positive charges. In reality, it's electrons flowing the opposite direction.

4 Resistance

In a constant electric field, electrons have constant acceleration in free space. Through a material, however, electrons are slowed by interaction with atoms. Chance of collision is proportional to speed.

$$m\vec{a} = q\vec{E} - c\vec{v}$$

Electrons (on average) will accelerate until reaching "terminal velocity" ($v = \frac{qE}{c}$)

Ohm's Law

$$V = IR$$

V = voltage

I = current

R = resistance

Units of resistance: 1 Ohm (Ω) = 1 V/A

5 Conductors/Resistors

Resistance of conductors is very, very low. A device with significant resistance is called a resistor. It's represented by a zig-zag in a circuit

The resistance through a resistor is given by

$$R = \frac{V}{I}$$

When the area of the resistor grows, the resistance falls. When the voltage grows, the resistance grows with it.

$$R \propto \frac{1}{A} \qquad R \propto L$$

Given resistivity ρ

$$R = \rho \frac{L}{A}$$